

Name - Raman Kumar Pandey

(Q.1): Find the rank of the matrix by reducing it to Row reduced echelon form.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Apply elementary row operations to get the matrix into row echelon form then reduce it to P.D.S. R.R.E.F.

⇒ First Column

$$\textcircled{1} \quad R_2 \rightarrow R_2 - 2R_1$$

$$\textcircled{2} \quad R_3 \rightarrow R_3 - 3R_1$$

$$\textcircled{3} \quad R_4 \rightarrow R_4 - 6R_1$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -5 & 5 \end{bmatrix}$$

Second Column

$$\textcircled{1} \quad R_3 \rightarrow R_3 - 4/3 R_2$$

$$\textcircled{2} \quad R_4 \rightarrow R_4 - 4/3 R_2$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & -4 & 0 & 7/3 \end{bmatrix}$$

Third Column

$$\textcircled{1} \quad R_4 \rightarrow R_4 - R_3$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & 0 & 0 & 13/3 \end{bmatrix}$$

Name - Raman Kumar Pandey

Roll No. ⇒ 2801010093



This is now row echelon form.

To convert it to RREF

$$1) R_3 = R_3 + \frac{4}{3}R_2$$

$$2) R_1 = R_1 - 2R_2$$

$$\left[\begin{array}{ccccc} 1 & 2 & 0 & -4 \\ 0 & 1 & 0 & 2 \\ 0 & (-4) & 0 & -1 \\ 0 & 0 & 0 & 13/3 \end{array} \right]$$

divide third row by -4

$$R_3 = -\frac{1}{4}R_3$$

$$\left[\begin{array}{ccccc} 1 & 2 & 0 & -4 \\ 0 & 0 & -3 & 2 \\ 0 & 1 & 0 & 1/4 \\ 0 & 0 & 0 & 13/3 \end{array} \right]$$

We have the matrix in RREF
so rank of matrix A is 3.

Q.2.

Let W be the vector space of all symmetric 2×2 matrices & let $T: W \rightarrow P_2$ be the linear transformation defined by $T \begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a+b) + (b-c)x + (c-a)x^2$

Find the rank & nullity of T.

⇒

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

By rank-nullity theorem

$$\text{rank}(T) + \text{nullity}(T) = \dim(W)$$

Find the images of the basis vectors of W under T.

Name - Raman Kumar Pandey

Date _____
Page _____

\Rightarrow ① for the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$:

$$T \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = (1-0) + (0-0)x + (0-1)x^2 \\ = 1 - x^2$$

2) for the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$:

$$T \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = (0-1) + (1-1)x + (1-0)x^2 \\ = -1 + x$$

3) for the matrix $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$:

$$T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = (0-0) + (0-0)x + (0-0)x^2 = 0$$

The images of the basis vectors $\{1-x^2, -1+x, 0\}$ are linearly independent.

They span the image of T . so the rank T is 3.

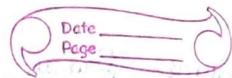
using the rank-nullity theorem

$$\text{rank}(T) + \text{nullity}(T) = \dim(W)$$

$$3 + \text{nullity}(T) = 3$$

$$\text{nullity}(T) = 0$$

So the rank of T is 3 & nullity of T is 0.



Name - Raman Kumar Pandey

Q: 3.

Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ find the eigenvalues & eigenvectors of A^{-1} and $A + 4I$.

Eigenvalues and Eigenvectors of matrices A^{-1} and $A + 4I$.

\Rightarrow ① Eigenvalues and Eigenvectors of A .

$$\det(A - \lambda I) = 0$$

$$\det(A - \lambda I) = \begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)^2 - (-1)(-1)$$

$$= \lambda^2 - 4\lambda + 4 - 1$$

$$= \lambda^2 - 4\lambda + 3$$

$$= (\lambda-1)(\lambda-3) = 0$$

So, $\lambda_1 = 1$ and $\lambda_2 = 3$

$$(A - \lambda I)y = 0$$

Solve y we get

for $\lambda_1 = 1$

$$(A - \lambda_1 I)v_1 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} v_1 = 0$$

$$\text{We get } v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

for $\lambda_2 = 3$

$$(A - \lambda_2 I)v_2 = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} v_2 = 0$$

$$\text{We get } v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Name - Raman Kumar Pandey

Date _____
Page _____

(2) Eigenvalues and Eigenvectors of A^{-1}

If λ is eigenvalue of A , then λ^{-1} is for A^{-1}
Same for the eigenvectors.

Eigenvalues of A^{-1} are λ_1^{-1} and λ_2^{-1}

$$\text{Eigenvalues of } A^{-1} : \frac{1}{\lambda_1}, \frac{1}{\lambda_2} = \frac{1}{1}, \frac{1}{3} = 1, \frac{1}{3}$$

The Eigenvectors remains the same.

(B) Eigenvalues & Eigenvectors of $A + 4I$:

If λ is eigenvalue of A , then $\lambda + 4$ is an eigenvalue of $A + 4I$, with same eigenvectors.

So, eigenvalues of A , then $\lambda + 4$ is an eigenvalue of $A + 4I$: $\lambda_1 + 4, \lambda_2 + 4$,

$$= 1+4, 3+4$$

The eigenvectors remains the same.

For A^{-1} :

$$\text{Eigenvalue } \lambda_1 = 1, \text{ Eigenvector } v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\text{Eigenvalue } \lambda_2 = 3, \text{ Eigenvector } v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

For $A + 4I$:

$$\text{Eigenvalue } \lambda_1 + 4 = 5, \text{ Eigenvector } v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\text{Eigenvalue } \lambda_2 + 4 = 7, \text{ Eigenvector } v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Q(4): Solve by Gauss Seidel Method (Take 3 iteration)

$$3x - 0.1y - 0.2z = 7.85$$

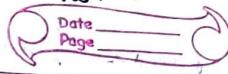
$$0.1x + 7y - 0.3z = -19.3$$

$$0.3x + 0.2y + 10z = 71.4$$

with initial values $x(0) = 0, y(0) = 0, z(0) = 0$

Name - Raman Kumar Pandey

Enrollment No. - 2301010093



→ Bgn to isolate x, y and z

$$1) x = (7.85 + 0.1y + 0.2z)/3$$

$$2) y = (-19.3 - 0.1x + 0.3z)/7$$

$$3) z = (71.4 - 0.3x - 0.2y)/10$$

Iteration 1

using initial values $x(0) = 0, y(0) = 0, z(0) = 0$

$$1) x(1) = \frac{17.85 + 0.1(0) + 0.2(0)}{3} = 2.61667$$

$$2) y(1) = \frac{-19.3 - 0.1(2.61667) + 0.3(0)}{7} = -2.77295$$

$$3) z(1) = \frac{71.4 - 0.3(2.61667) - 0.2(-2.77295)}{10} = 7.18943$$

Iteration 2

using $x(1) = 2.61667, y(1) = -2.77295, z(1) = 7.18943$

$$① x(2) = \frac{17.85 + 0.1(-2.77295) + 0.2(7.18943)}{3} = 3.00056$$

$$② y(2) = \frac{-19.3 - 0.1(3.00056) + 0.3(7.18943)}{7} = -2.99984$$

$$③ z(2) = \frac{71.4 - 0.3(3.00056) - 0.2(-2.99984)}{10} = 7.00004$$

Iteration 3

using $x(2) = 3.00056, y(2) = -2.99984, z(2) = 7.00004$

$$1) x(3) = \frac{17.85 + 0.1(-2.99984) + 0.2(7.00004)}{3} = 3.00002$$

$$2) y(3) = \frac{-19.3 - 0.1(3.00002) + 0.3(7.00004)}{7} = -3$$

$$3) z(3) = \frac{71.4 - 0.3(3.00002) - 0.2(-3)}{10} = 7$$

After 3 iteration

$$x = 3$$

$$y = -3$$

$$z = 7$$