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Q.1: Find the rank of the matrix by reducing in Row reduced echelon form.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Apply elementary row operations to get the matrix into row echelon form then reduce it to ~~form~~. RREF.

$\Rightarrow$ First Coloumn

1. $R_2 \to R_2 - 2R_1$
2. $R_3 \to R_3 - 3R_1$
3. $R_4 \to R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -5 & 5 \end{bmatrix}$$

Second Coloumn

1. $R_3 = R_3 - 4/3 R_2$
2. $R_4 = R_4 - 4/3 R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & -4 & 0 & 7/3 \end{bmatrix}$$

Third Coloumn

1. $R_4 = R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & 0 & 0 & 13/3 \end{bmatrix}$$

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This is now row echelon form.

To convert it to RREF

1) $R_3 = R_3 + 4/3 R_2$

2) $R_1 = R_1 - 2R_2$

$$\begin{bmatrix} 1 & 2 & 0 & -4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & 0 & 0 & 13/3 \end{bmatrix}$$

divide third row by -4

$R_3 = -\frac{1}{4} R_3$

$$\begin{bmatrix} 1 & 2 & 0 & -4 \\ 0 & 0 & -3 & 2 \\ 0 & 1 & 0 & 1/4 \\ 0 & 0 & 0 & 13/3 \end{bmatrix}$$

We have the matrix in RREF

so rank of matrix A is 3.

Q.2.

Let W be the vector space of all symmetric 2×2 matrices & let $T: W \to P_2$ be the linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$

Find the rank & nullity of T.

⇒ $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

By rank-nullity theorem

rank (T) + nullity (T) = dim(W)

Find the images of the basis vectors of W under T.

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$\Rightarrow$ ① for the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$:

$$T\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = (1-0) + (0-0)x + (0-1)x^2$$

$$= 1 - x^2$$

2) for the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$:

$$T\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = (0-1) + (1-1)x + (1-0)x^2$$

$$= -1 + x$$

3) for the matrix $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$:

$$T\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = (0-0) + (0-0)x + (0-0)x^2 = 0$$

The images of the basis vectors $\{1-x^2, -1+x, 0\}$ are linearly independent.
They span the image of T. so the rank T is 3.

Using the rank - nullity theorem

$$\text{rank}(T) + \text{nullity}(T) = \dim(W)$$

$$3 + \text{nullity}(T) = 3$$

$$\text{nullity}(T) = 0$$

rank of T is 3 & nullity of T is 0.

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Q:3.

let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ find the eigen values & eigenvectors of A^{-1} and $A+4I$.

Eigen values and Eigenvectors of matrices A^{-1} and $A+4I$.

⇒ ① Eigenvalues and Eigenvectors of A.

$$\det(A - \lambda I) = 0$$

$$\det(A - \lambda I) = \begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)^2 - (-1)(-1)$$

$$= \lambda^2 - 4\lambda + 4 - 1$$

$$= \lambda^2 - 4\lambda + 3$$

$$= (\lambda - 1)(\lambda - 3) = 0$$

so, $\lambda_1 = 1$ and $\lambda_2 = 3$

$$(A - \lambda I)y = 0$$

Solve y we get

for $\lambda_1 = 1$

$$(A - \lambda_1 I)v_1 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} v_1 = 0$$

we get : $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

for $\lambda_2 = 3$

$$(A - \lambda_2 I)v_2 = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} v_2 = 0$$

We get $V_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

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(2) Eigenvalues and Eigenvectors of A^{-1}

If λ is eigenvalue of A, then λ^{-1} is for A^{-1}
Same for the eigenvectors

eigenvalues of A^{-1} & λ_1^{-1} and λ_2^{-1}

Eigenvalues of A^{-1} : $\frac{1}{\lambda_1}, \frac{1}{\lambda_2} = \frac{1}{1}, \frac{1}{3} = 1, \frac{1}{3}$

The Eigenvectors remains the same

(B) Eigenvalues & Eigenvectors of $A+4I$:

If μ is eigenvalues of A, than $\mu+4$ is an eigenvalue of $A+4I$, with same eigenvectors,

So, eigenvalues of A, then $\mu+4$ is an eigenvalue of $A+4I$: λ_1+4, λ_2+4,

$= 1+4, 3+4$

$= 5, 7$

The eigen vectors remains the same.

For A^{-1}

Eigenvalue $\lambda_1 = 1$, Eigenvector $V_1 = \begin{bmatrix}1\\1\end{bmatrix}$

Eigenvalue $\lambda_2 = 3$, Eigenvector $V_2 = \begin{bmatrix}1\\-1\end{bmatrix}$

For $A+4I$

Eigenvalue $\lambda_1 + 4 = 5$, Eigenvector $V_1 = \begin{bmatrix}1\\1\end{bmatrix}$

Eigenvalue $\lambda_2 + 4 = 7$, Eigenvector $V_2 = \begin{bmatrix}1\\-1\end{bmatrix}$

Q(4): Solve by Gauss Seidal Method (Take 3 iteration)

$3x - 0.1y - 0.2z = 7.85$

$0.1x + 7y - 0.3z = -19.3$

$0.3x + 0.2y + 10z = 71.4$

with initial values $x(0) = 0, y(0) = 0, z(0) = 0$

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⇒ Eqn to isolate x, y and z

1) $x = (7.85 + 0.1y + 0.2z)/3$

2) $y = (-19.3 - 0.1x + 0.3z)/7$

3) $z = (71.4 - 0.3x - 0.2y)/10$

Iteration 1

using initial values $x(0) = 0$, $y(0) = 0$, $z(0) = 0$

1) $x(1) = \dfrac{17.85 + 0.1(0) + 0.2(0)}{3} = 2.61667$

2) $y(1) = \dfrac{-19.3 - 0.1(2.61667) + 0.3(0)}{7} = -2.77295$

3) $z(1) = \dfrac{71.4 - 0.3(2.61667) - 0.2(-2.77295)}{10} = 7.18943$

Iteration 2

Using $x(1) = 2.61667$, $y(1) = -2.77295$, $z(1) = 7.18943$

1. $x(2) = \dfrac{17.85 + 0.1(-2.77295) + 0.2(7.189)}{3} = 3.00056$

2. $y(2) = \dfrac{-19.3 - 0.1(3.00056) + 0.3(7.18943)}{7} = -2.99984$

3. $z(2) = \dfrac{71.4 - 0.3(3.00056) - 0.2(-2.99984))}{10} = 7.00004$

Iteration 3

using $x(2) = 3.00056$, $y(2) = -2.9984$, $z(2) = 7.0004$

1) $x(3) = \dfrac{7.85 + 0.1(-2.99984) + 0.2(7.00004)}{3} = 3.00002$

2) $y(3) = \dfrac{(-19.3 - 0.1(3.00002) + 0.3(7.00004))}{7} = -3$

3) $z(3) = \dfrac{(71.4 - 0.3(3.00002) - 0.2(-3))}{10} = 7$

After 3 iteration

$x = 3$

$y = -3$

$z = 7$