AE312 AFM Assignment-1

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Usage of jet engine and propeller engine for an aircraft have been analysed using their characteristic envelopes and their performances have been compared.

I. INTRODUCTION

Jet engines and propeller engines are two main types of engines used in aircrafts to generate thrust. A jet engine is a type of reaction engine discharging a fast-moving jet that generates thrust. Propeller engines utilize a propeller to convert the shaft power to thrust and the shaft power is produced by various like reciprocating pistons or electric power. Both types of engines have their own advantages and disadvantages. In this exercise, we try to analyse their performances by plotting their envelopes (airspeed versus altitude plot) and excess thrust/power plots.

II. THEORY

II.1. Aircraft details given

The following details about the aircraft are given:

$$\begin{aligned}
m &= 750\ Kg \\
S &= 12\ m^2 \\
b &= 10\ m \\
C_{D_0} &= 0.036 \\
C_{L_{max}} &= 2.7 \\
e &= 0.87
\end{aligned}$$

The following aircraft details can be derived:

$$\begin{aligned}
W &= mg \\
AR &= \frac{b^2}{S} \\
K &= \frac{1}{\pi\, e\, AR} \\
v_{stall} &= \sqrt{\frac{2\,w}{S\,\rho\, C_{L_{max}}}}
\end{aligned}$$

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II.2. Engine models given

II.2.1. jet engine

Thrust model for the jet engine is given by:

$$T = T_{SL}\ \sigma^{1.5}$$

where $T_{SL} = 1140\, N$

II.2.2. Propeller engine

Power model for the propeller engine is given by:

$$T = T_{SL}\ \sigma^{0.5}$$

where $T_{SL} = 100\, hp\ = 74569.9872\, W$

II.3. Air density as a variation of altitude

Air density is approximated as a variation of altitude:

$$\sigma(h) = \begin{cases} \exp\left(-\frac{h}{9296}\right) & (0 < h \leq 11000) \\ 0.3063\exp\left(-\frac{h-11000}{6216}\right) & (h > 11000) \end{cases}$$

Where $\sigma = \dfrac{\rho}{\rho_{SL}}$.

II.4. equations of motion

The general equations of motion for an aircraft(approximated as a point mass) are:

$$\begin{aligned}
m\frac{dv}{dt} &= T\cos\alpha - D + W\sin\gamma \\
mv\frac{d\gamma}{dt} &= T\sin\alpha + L - W\cos\gamma \\
\frac{dx}{dt} &= v\cos\gamma \\
\frac{dh}{dt} &= v\sin\gamma
\end{aligned}$$

We assume steady level flight for analysing the engines. This makes $\dfrac{dv}{dt} = 0$ and $\gamma = 0$. Approximating $\alpha \approx 0$ for small angle of attacks, we get:

$$\begin{aligned} T &= D \\ L &= W \\ \frac{dx}{dt} &= v \\ \frac{dh}{dt} &= 0 \end{aligned}$$

We derive the further equations from these fundamental equations and C_L, C_D relations like $C_D = C_{D_0} + KC_L^2$.

II.5. Flight Envelopes

Plot of maximum and minimum velocity at a particular altitude versus altitude is called flight envelope. Flight envelopes gives an idea about the extent to which the aircraft can fly in air as the altitude and velocity varies.

II.5.1. jet engine

Since we assume steady level flight, thrust required is assumed to be equal to drag. Equating the thrust available and thrust required, expanding the drag expression and rearranging the equation gives us a bi-quadratic equation to solve for velocities at which thrust required is equal to thrust available. The equation is given by:

$$\frac{1}{4}\rho^2 S^2 C_{D_0}\, v^4 - \frac{1}{2}\rho S T_a\, v^2 + KW^2 = 0 \tag{1}$$

Given the biquadratic nature of thrust required (or drag) and the constant nature of thrust available at a certain altitude, we can see the real roots of eq. 1 will be the maximum and minimum velocities possible at a particular height (if the flight is steady level). Eq. 1 has a closed form solution given by:

$$v = \sqrt{\frac{W}{\rho S C_{D_0}} \left[\frac{T}{W} \pm \sqrt{\frac{T^2}{W^2} - 4C_{D_0}}\right]} \tag{2}$$

Ceiling is found by conditioning eq. 1 to have two real repeated roots and from eq. 2 we can see that it corresponds to:

$$\frac{T}{W} = 4KC_{D_0}$$

We T is a function of h, we back calculate h_{ceil} and using eq. 2 for the condition to get v_{ceil}.

Minimum thrust required is calculated by minimizing the drag expression:

$$T_{r,min} = 2W\sqrt{C_{D_0}K}$$

II.5.2. Propeller engine

Similar to the jet engine, thrust required is equal to drag. But since P_a is known as a function of altitude and it stays constant for varying velocity at a particular altitude, power available is equated with power required which is $P_r = T_r\, v = D\, v$. Expanding the drag expression and rearranging the equation gives us a bi-quadratic equation which is given by:

$$\frac{1}{4}\rho^2 S^2 C_{D_0}\, v^4 - \frac{1}{2}\rho S P_a\, v + KW^2 = 0 \tag{3}$$

Equation 3 has no closed form solution and has to be solved numerically to arrive at the solution.

Minimum required power is given by:

$$P_{r,min} = \sqrt{\frac{2W^3}{S}}\, \frac{1}{\rho^{0.5}}\, \frac{1}{\left(\frac{C_L^{1.5}}{C_D}\right)_{max}}$$

Observe that required power is minimum when $\frac{C_L^{1.5}}{C_D}$ is maximum. This happens when $C_L = \sqrt{\frac{3\,C_{D_0}}{K}}$ and $C_D = 4\,C_{D_0}$.

Ceiling occurs when power available is equal to minimum required power. We use that fact to calculate σ_{ceil} from that equation and hence calculate h_{ceil}. v_{ceil} can be calculated from the velocity equation:

$$v = \sqrt{\frac{2W}{S\rho C_L}}$$

III. RESULTS AND DISCUSSIONS

III.1. Jet engine

1. The envelope for jet engine shows very little aerodynamic limit region (Fig. 1) and hence the pilot has a lot of leeway to vary his velocity from minimum to maximum velocity at a particular altitude and worry less about stalling in cruise. As the altitude increases, the pilot has less range of velocity as which the aircraft can fly.
2. The ceiling is at $h = 16086.2\,m$ and the velocity at that ceiling is $v = 90.44\,m/s$. (Fig. 1)
3. The envelope with equivalent airspeed conveys the same information as the envelope but it enables the pilot to handle the velocity variations with more ease as equivalent velocity (airspeed) doesn't depend on the local air density variations.($v_{stall,eq} = 19.255\,m/s$). (Fig. 2)
4. At a constant altitude, we observe that the velocity range increases as the thrust required or drag increases. As altitude increases, the curve shifts rightward and flattens out. We observe the minimum thrust $T_{r,min} = 585.016\,N$ required is constant across different altitudes and velocities.(Fig. 3)
5. At a constant altitude, we observe that the velocity range decreases as the excess thrust increases. As altitude increases, the curve shifts rightward and flattens down (with decreasing peaks). (Fig. 4)

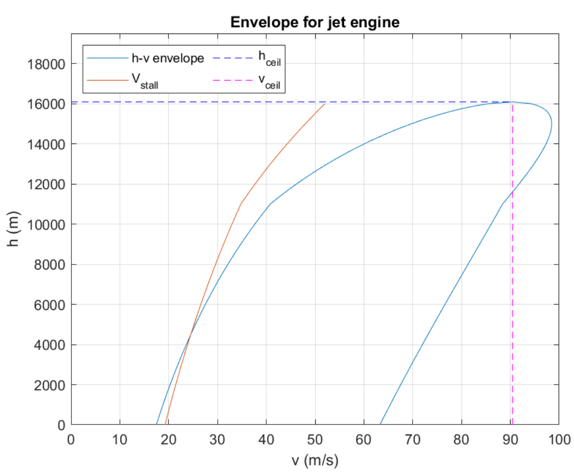

FIG. 1. Envelope for jet engine

III.2. Propeller engine

1. The envelope for propeller engine shows a significant aerodynamic limit region (Fig. 5) and hence

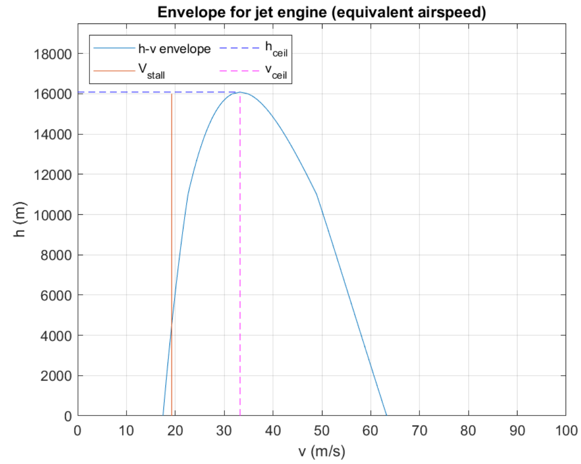

FIG. 2. Envelope for jet engine (equivalent airspeed)

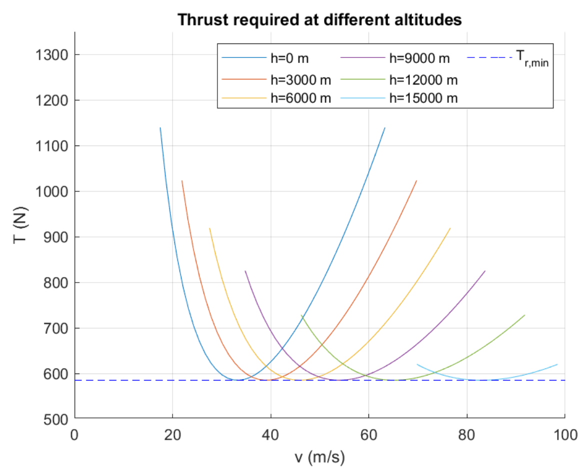

FIG. 3. Thrust required for jet engine

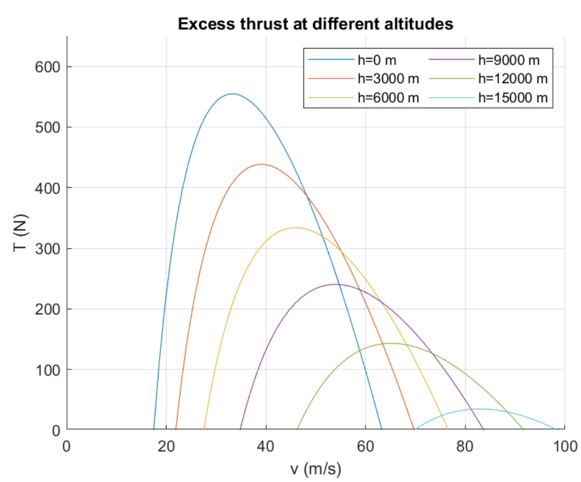

FIG. 4. Excess thrust for jet engine

the pilot has to be cautious not to decelerate below stall velocity at almost all altitudes the aircraft can fly. As the altitude increases, the pilot has less range of velocity as which the aircraft can fly.

2. The ceiling is at $h = 12811.7\,m$ and the velocity at that ceiling is $v = 52.81\,m/s$. (Fig. 5)

3. Analysis of the envelope with equivalent airspeed is similar to that of jet engine. ($v_{stall,eq} = 19.255\,m/s$). (Fig. 6)

4. Figure 7 shows power at different altitudes. Available power decreases with altitude and minimum required power increases with altitude and they meet at the ceiling altitude, Which is also shown by their difference or maximum excess thrust.

5. At a constant altitude, we observe that the velocity range decreases as the excess power increases. As altitude increases, the curve flattens down (with decreasing peaks).(Fig. 8)

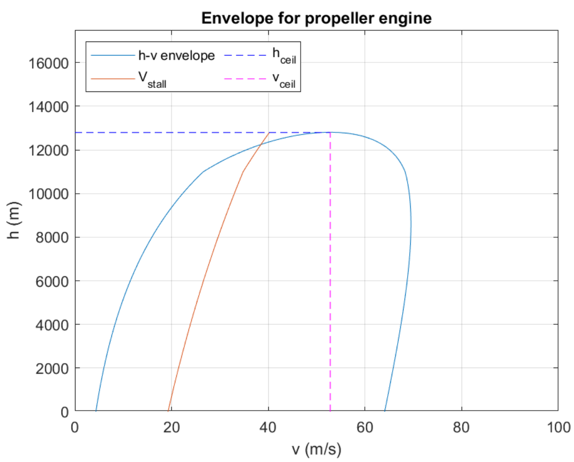

FIG. 5. Envelope for propeller engine

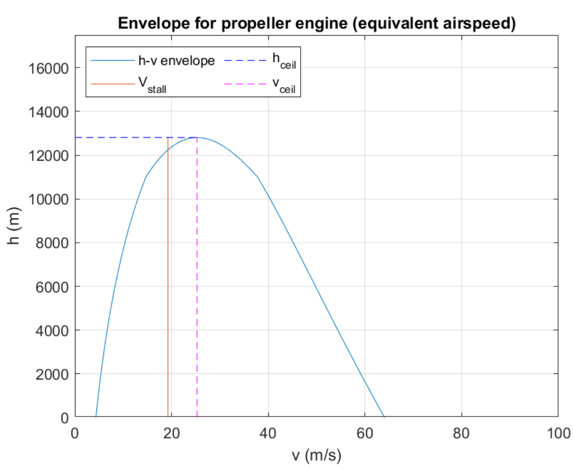

FIG. 6. Envelope for propeller engine (equivalent airspeed)

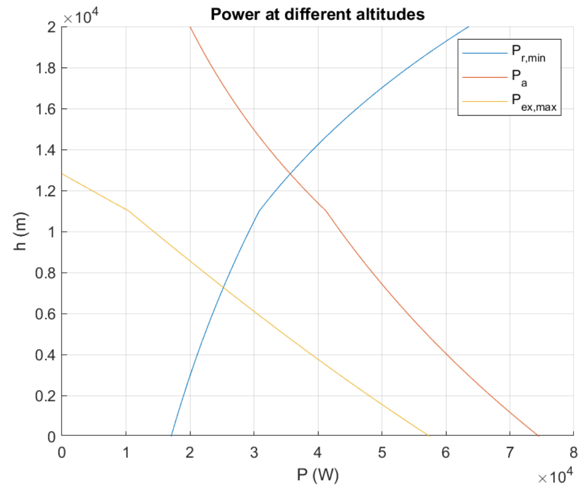

FIG. 7. Power at different altitudes for propeller engine

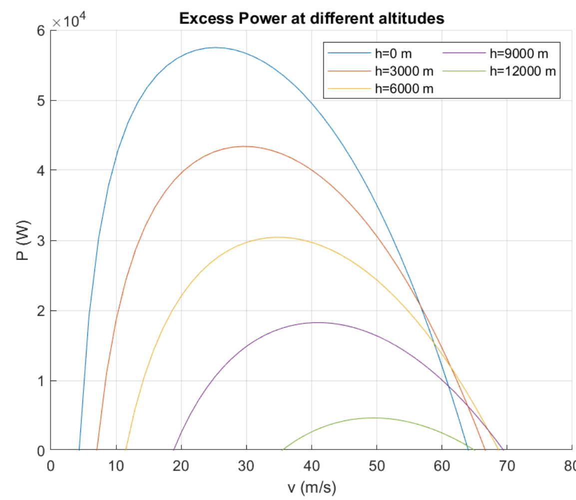

FIG. 8. Excess power for propeller engine

IV. CONCLUSION

On a general note, performance of jet engine is better as it has higher ceiling alttiude and it has less aerodynamic limit region in its envelope. Also, propeller engines produce a lot of noise during operation. But specific application use case and its requirements decide the suitability of a type of engine (jet or propeller) for that specific application.

Appendix A: MATLAB codes

1. Script for developing envelope and other plots for jet engine

```
% % jet engine

%given constant parameters
m=750;
S=12;
b=10;
C_Do=0.036;
C_lmax=2.7;
e=0.87;

% altitude and air density
h=(0:100:20000)';
sig=sigma(h);
rho=1.225.*sig;

%derived constant parameters
W=m*9.81;
AR=(b^2)/S;
K=(pi*e*AR)^-1;
v_stall=(2*W./(S.*rho.*C_lmax)).^0.5;
v_stall_eq=v_stall.*(sig.^0.5);

%Thrust model
T_sl=1140;
T=T_sl.*(sig.^(1/3));

% condition for real roots
check=(T/W).^2 >= 4*K*C_Do ;

% vector to store velocity
v_T=zeros(length(h),2);

figure;
hold on;
grid on;
%solving the biquadratic equation and plotting thrust plots
syms v;
for i=1:1:length(h)
    v_=vpasolve(0.25*(rho(i)^2)*S*C_Do*v^4-0.5*rho(i)*T(i)*v^2+(K*W^2)/S == 0,v,[0 Inf]);
    if isempty(v_)
        break;
    end
    v_T(i,:)=v_;

    if mod(h(i),3000)==0
        v_space=min(v_):0.025*(max(v_)-min(v_)):max(v_);
        D_=drag(rho(i),v_space,S,C_Do,K,W);
        T_ex=T(i)-D_;
        plot(v_space,T_ex);
%         plot(v_space,D_); %toggle comment for second graph

    end
end
```

```

% % toggle comment for second graph
% T_rmin=2*W*(C_Do*K)^0.5;
% plot([0 100],T_rmin*ones(2,1),'b--');
hold off;
xlim([0 100]);
ylim([0 650]);
% ylim([500 1350]);
xlabel("v (m/s)");
ylabel("T (N)");
legend({'h=0 m','h=3000 m','h=6000 m','h=9000 m','h=12000 m','h=15000 m'},'Location','northeast','NumColumns',2);
title('Excess thrust at different altitudes');
% % toggle comment to produce second graph
% legend({'h=0 m','h=3000 m','h=6000 m','h=9000 m','h=12000 m','h=15000 m','T_{r,min}'},'Location','northeast','NumColumns',3);
% title("Thrust required at different altitudes");

%equivalent airspeed
v_T_eq=v_T.*(sig).^0.5;

%ceiling calculations
T_ceil=W*(4*C_Do*K)^0.5;
sig_ceil=(T_ceil/T_sl)^3;
h_ceil=siginv(sig_ceil);
rho_ceil=1.225*sig_ceil;
v_ceil=(T_ceil/(rho_ceil*C_Do*S))^0.5;
v_ceil_eq=v_ceil*(sig_ceil)^0.5;

%plotting envelope
v_stall_plot=v_stall;
h1=h;
v_T(i:end,:)=[];
h1(i:end)=[];
v_stall_plot(i:end)=[];
h1_plot=cat(1,h1,h_ceil,flip(h1));
v_plot=cat(1,v_T(:,1),v_ceil,flip(v_T(:,2)));

figure;
plot(v_plot,h1_plot);
hold on;
plot(v_stall_plot,h1);
plot([0 v_ceil],h_ceil*ones(2,1),'b--');
plot(v_ceil*ones(2,1),[0 h_ceil],'m--');
hold off;
grid on;
xlim([0 100]);
ylim([0 19500]);
xlabel("v (m/s)");
ylabel("h (m)");
legend({'h-v envelope','V_{stall}','h_{ceil}','v_{ceil}'},'Location','northwest','NumColumns',2);
title('Envelope for jet engine');

%plotting envelope for equivalent airspeed
v_stall_eq_plot=v_stall_eq;
h1=h;
v_T_eq(i:end,:)=[];
h1(i:end)=[];
```

```
v_stall_eq_plot(i:end)=[];
h1_plot=cat(1,h1,h_ceil,flip(h1));
v_eq_plot=cat(1,v_T_eq(:,1),v_ceil_eq,flip(v_T_eq(:,2)));

figure;
plot(v_eq_plot,h1_plot);
hold on;
plot(v_stall_eq_plot,h1);
plot([0 v_ceil_eq],h_ceil*ones(2,1),'b--');
plot(v_ceil_eq*ones(2,1),[0 h_ceil],'m--');
hold off;
grid on;
xlim([0 100]);
ylim([0 19500]);
xlabel("v (m/s)");
ylabel("h (m)");
legend({'h-v envelope','V_{stall}','h_{ceil}','v_{ceil}'},'Location','northwest','NumColumns',2);
title('Envelope for jet engine (equivalent airspeed)');
```

2. Script for developing envelope and other plots for propeller engine

```
% % propeller engine

%given constant parameters
m=750;
S=12;
b=10;
C_Do=0.036;
C_lmax=2.7;
e=0.87;

% altitude and air density
h=(0:100:20000)';
sig=sigma(h);
rho=1.225.*sig;

%derived constant parameters
W=m*9.81;
AR=(b^2)/S;
K=(pi*e*AR)^-1;
v_stall=(2*W./(S.*rho.*C_lmax)).^0.5;
v_stall_eq=v_stall.*(sig.^0.5);


%Power model
P_sl=100*745.699872; %hp to W conversion
P=P_sl.*(sig.^0.5);

% vector to store velocity
v_P=zeros(length(h),2);

figure;
hold on;
grid on;
%solving the biquadratic equation and plotting thrust plots
syms v;
```

```
for i=1:1:length(h)
    v_=solve(0.25*(rho(i)^2)*S*C_Do*v^4-0.5*rho(i)*P(i)*v+(K*W^2)/S == 0,v,'Real',true);
    if isempty(v_)
        break;
    end
    v_P(i,:)=v_;

    if mod(h(i),3000)==0
        v_space=min(v_):0.025*(max(v_)-min(v_)):max(v_);
        D_=drag(rho(i),v_space,S,C_Do,K,W);
        P_=D_.*v_space;
        P_ex=P(i)-P_;
        plot(v_space,P_ex);

    end
end

hold off;
xlim([0 80]);
ylim([0 60000]);
xlabel("v (m/s)");
ylabel("P (W)");
legend({'h=0 m','h=3000 m','h=6000 m','h=9000 m','h=12000 m'},'Location','northeast','NumColumns'
    ,2);
title('Excess Power at different altitudes');

Ec_max=((3*C_Do/K)^0.75)/(4*C_Do);
P_rmin=(Ec_max^-1)*(2*W^3./(rho.*S)).^0.5;
figure;
grid on;
hold on;
plot(P_rmin,h);
plot(P,h);
plot(P-P_rmin,h);
hold off;
xlim([0 80000]);
ylim([0 20000]);

xlabel("P (W)");
ylabel("h (m)");
legend({'P_{r,min}','P_a','P_{ex,max}'},'Location','northeast');
title('Power at different altitudes');

%equivalent airspeed
v_P_eq=v_P.*(sig).^0.5;

%ceiling calculations
sig_ceil=((Ec_max*P_sl)^-1)*(2*W^3/(1.225*S))^0.5;
h_ceil=siginv(sig_ceil);
rho_ceil=1.225*sig_ceil;
P_ceil=P_sl*(sig_ceil^0.5);
v_ceil=(2*W/(S*rho_ceil*(3*C_Do/K)^0.5)).^0.5;
v_ceil_eq=v_ceil*(sig_ceil)^0.5;
v_ceil_2=vpasolve(0.25*(rho_ceil^2)*S*C_Do*v^4-0.5*rho_ceil*P_ceil*v+(K*W^2)/S == 0,v);

%plotting envelope
v_stall_plot=v_stall;
h1=h;
```

```
v_P(i:end,:)=[];
h1(i:end)=[];
v_stall_plot(i:end)=[];
h2_plot=cat(1,h1,h_ceil,flip(h1));
v_P_plot=cat(1,v_P(:,1),v_ceil,flip(v_P(:,2)));

figure;
plot(v_P_plot,h2_plot);
hold on;
plot(v_stall_plot,h1);
plot([0 v_ceil],max(h1)*ones(2,1),'b--');
plot(v_ceil*ones(2,1),[0 h_ceil],'m--');
hold off;
grid on;
xlim([0 100]);
ylim([0 17500]);
xlabel("v (m/s)");
ylabel("h (m)");
legend({'h-v envelope','V_{stall}','h_{ceil}','v_{ceil}'},'Location','northwest','NumColumns',2);
title('Envelope for propeller engine');

%plotting envelope for equivalent airspeed
v_stall_eq_plot=v_stall_eq;
h1=h;
v_P_eq(i:end,:)=[];
h1(i:end)=[];
v_stall_eq_plot(i:end)=[];
h1_plot=cat(1,h1,h_ceil,flip(h1));
v_eq_plot=cat(1,v_P_eq(:,1),v_ceil_eq,flip(v_P_eq(:,2)));

figure;
plot(v_eq_plot,h1_plot);
hold on;
plot(v_stall_eq_plot,h1);
plot([0 v_ceil_eq],h_ceil*ones(2,1),'b--');
plot(v_ceil_eq*ones(2,1),[0 h_ceil],'m--');
hold off;
grid on;
xlim([0 100]);
ylim([0 17500]);
xlabel("v (m/s)");
ylabel("h (m)");
legend({'h-v envelope','V_{stall}','h_{ceil}','v_{ceil}'},'Location','northwest','NumColumns',2);
title('Envelope for propeller engine (equivalent airspeed)');
```

3. Function for calculating relative air density

```
function sig=sigma(h_)
% function to calculate relative density as a variation of altitude.
    sig=ones(size(h_));
    for i=1:1:length(h_)
        h=h_(i);
        if h<11000
            e=1;
            ha=0;
            B=9296;
```

```
        else
            e=exp(-11000/9296);
            ha=11000;
            B=6216;
        end
        sig(i)=e.*exp(-(h-ha)/B);
    end
end
```

4. Function for calculating altitude given relative air density

```
function h=siginv(sig)
% function to calculate altitude as a variation of relative density.

    assert(sig<=1 & sig>=0);
    if sig>exp(-11000/9296)
        h=-9296*(log(sig));
    else
        h=11000-6216*(log(sig)-log(0.3063));
    end
end
```

5. Function for calculating drag force on an airfoil

```
function D=drag(rho,v,S,C_Do,K,W)
% function to calculate drag force.
    D=0.5*rho.*v.^2*S*C_Do+(K*W^2./(0.5*rho.*v.^2*S));
end
```