AE312 AFM Assignment-3

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Usage of jet engine and propeller engine for an aircraft have been analysed using their characteristics during steady climb and their performances have been compared.

I. INTRODUCTION

Jet engines and propeller engines are two main types of engines used in aircrafts to generate thrust. A jet engine is a type of reaction engine discharging a fast-moving jet that generates thrust. Propeller engines utilize a propeller to convert the shaft power to thrust and the shaft power is produced by various like reciprocating pistons or electric power. Both types of engines have their own advantages and disadvantages. In this exercise, we try to analyse their performances during steady climb.

II. THEORY

II.1. Aircraft details given

The following details about the aircraft are given:

$$m = 750\ Kg$$
$$S = 12\ m^2$$
$$b = 10\ m$$
$$C_{D_0} = 0.036$$
$$C_{L_{max}} = 2.7$$
$$e = 0.87$$

The following aircraft details can be derived:

$$W = mg$$
$$AR = \frac{b^2}{S}$$
$$K = \frac{1}{\pi\, e\, AR}$$
$$v_{stall} = \sqrt{\frac{2\, w}{S\, \rho\, C_{L_{max}}}}$$

II.2. Engine models given

II.2.1. jet engine

Thrust model for the jet engine is given by:

$$T = T_{SL}\ \sigma^{1.5}$$

where $T_{SL} = 1140\, N$

II.2.2. Propeller engine

Power model for the propeller engine is given by:

$$P = P_{SL}\ \sigma^{0.5}$$

where $P_{SL} = 100\, hp\ = 74569.9872\, W$

II.3. Air density as a variation of altitude

Air density is approximated as a variation of altitude:

$$\sigma(h) = \begin{cases} \exp\left(-\frac{h}{9296}\right) & (0 < h \leq 11000) \\ 0.3063 \exp\left(-\frac{h-11000}{6216}\right) & (h > 11000) \end{cases}$$

Where $\sigma = \dfrac{\rho}{\rho_{SL}}$.

II.4. Equations of motion

The general equations of motion for an aircraft(approximated as a point mass) are:

$$m\frac{dv}{dt} = T\cos\alpha - D + W\sin\gamma$$
$$mv\frac{d\gamma}{dt} = T\sin\alpha + L - W\cos\gamma$$
$$\frac{dx}{dt} = v\cos\gamma$$
$$\frac{dh}{dt} = v\sin\gamma$$

We assume steady climb flight for analysing the engines. This makes $\dfrac{dv}{dt} = 0$ and $\gamma = 0$. Approximating $\alpha \approx 0$ for small angle of attacks, we get:

$$0 = T - D - W\sin\gamma$$
$$0 = L - W\cos\gamma$$
$$\frac{dx}{dt} = v\cos\gamma$$
$$ROC = \frac{dh}{dt} = v\sin\gamma$$

We derive the further equations from these fundamental equations and C_L, C_D relations like $C_D = C_{D_0} + KC_L^2$.

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II.5. Steady climb

Optimized steady climb can either be the fastest climb or the steepest climb. Fastest climb is when the rate of climb (ROC) is maximized and steepest climb is when climb angle (γ) is maximised.

II.5.1. Fastest climb

For maximizing the rate of climb, rate of climb expression (specific excess power) is differentiated and equated to zero and velocity and climb angle required for fast climb are found out. Maximum rate of climb is given by:

$$ROC_{max} = V_{ROC_{max}} \sin\gamma_{ROC_{max}}$$

For jet engine:

$$V_{ROC_{max}} = \sqrt{\frac{T}{3\rho C_{D_o} S}\left[1 + \sqrt{1 + \frac{12KC_{D_o}W^2}{T^2}}\right]}$$

$$\gamma_{ROC_{max}} = \sin^{-1}\left[\frac{T}{W} - \frac{\rho V_{ROC_{max}}^2 SC_{D_o}}{2W} - \frac{2KW}{\rho V_{ROC_{max}}^2 S}\right]$$

For propeller engine:

$$V_{ROC_{max}} = \sqrt{\frac{2W}{\rho S\sqrt{\frac{3C_{D_o}}{K}}}}$$

$$\gamma_{ROC_{max}} = \sin^{-1}\left[\frac{P\eta_P}{V_{ROC_{max}}W} - \frac{\rho V_{ROC_{max}}^2 SC_{D_o}}{2W} - \frac{2KW}{\rho V_{ROC_{max}}^2 S}\right]$$

II.5.2. Steepest climb

For maximizing the climb angle, climb angle expression (arc-sin of specific excess thrust) is taken and excess thrust is maximized and velocity is found for that condition by differentiating $\sin\gamma$ expression and substituted back to find γ_{max}

For jet engine:

$$\gamma_{max} = \sin^{-1}\left[\frac{T_{max}}{W} - 2\sqrt{KC_{D_o}}\right]$$

For propeller engine:

$$V_{\gamma_{max}} = \frac{4W^2K}{P_{max}\eta_P\rho S}$$

$$\gamma_{max} = \sin^{-1}\left[\frac{P\eta_P}{V_{\gamma_{max}}W} - \frac{\rho V_{\gamma_{max}}^2 SC_{D_o}}{2W} - \frac{2KW}{\rho V_{\gamma_{max}}^2 S}\right]$$

II.6. Climb schedule

Climb schedule is the velocity vs altitude plot that gives the information about the velocities at which an aircraft will fly at a particular altitude at a constant rate of climb. We find it using the energy height expression or the rate of climb expression which is the specific excess power and substitute for drag and rearrange to get an bi-quadratic equation in v.

The equation is given by:

For jet engine:

$$\frac{1}{2}C_{D_o}\rho Sv^3 - Tv + \frac{2KW^2}{\rho vS} + P_S W = 0$$

For propeller engine:

$$\frac{1}{2}C_{D_o}\rho Sv^3 - P\eta_P + \frac{2KW^2}{\rho vS} + P_S W = 0$$

The value of specific excess power (P_S) or ROC is varied and the equations are plotted to get contours of the h-v plot.

III. RESULTS AND DISCUSSIONS

III.1. Jet engine

1. As altitude increases, maximum rate of climb decreases. The absolute ceiling is at $h = 16085\,m$ when $ROC_{max} = 0\,m/s$ and service ceiling is at $h = 14679\,m$ when $ROC_{max} = 0.5080\,m/s$. (Fig. 1)

2. As maximum rate of climb decreases, velocity at maximum rate of climb increases. (Fig. 2)

3. As maximum climb angle decreases, velocity at maximum climb angle increases at higher rate. (Fig. 3)

4. From the climb schedule contours, we see that difference between the minimum velocity and maximum velocity decreases as altitude increases. As specific excess power decreases, the ceiling altitude decreases. (Fig. 4)

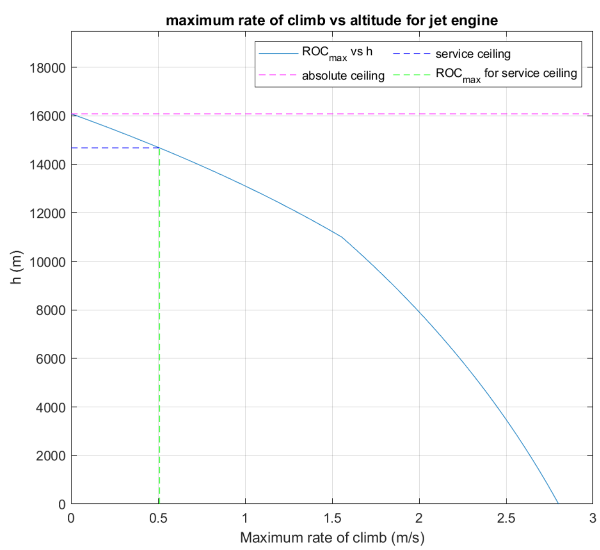

FIG. 1. Maximum rate of climb vs altitude for jet engine

III.2. Propeller engine

1. As altitude increases, maximum rate of climb decreases. The absolute ceiling is at $h = 12811\,m$ when $ROC_{max} = 0\,m/s$ and service ceiling is at $h = 12160\,m$ when $ROC_{max} = 0.5080\,m/s$. (Fig. 5)

2. As maximum rate of climb decreases, velocity at maximum rate of climb increases at higher rate. (Fig. 6)

3. As maximum climb angle decreases, velocity at

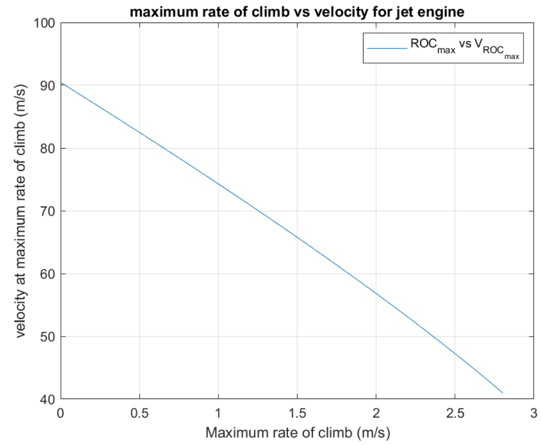

FIG. 2. Maximum rate of climb vs velocity for jet engine

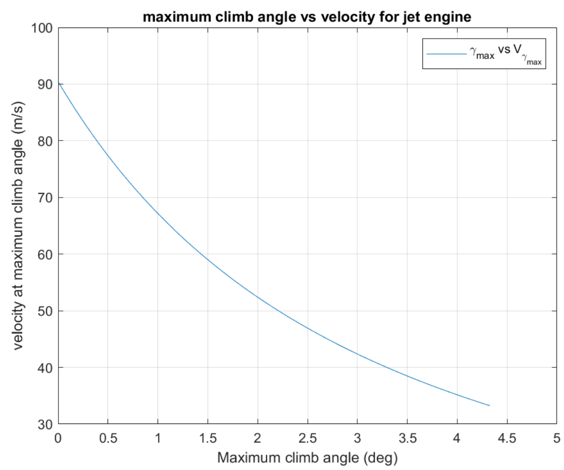

FIG. 3. maximum climb angle vs velocity for jet engine

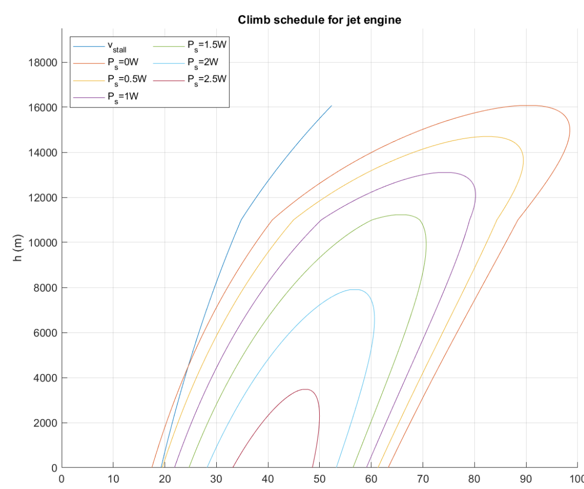

FIG. 4. Climb schedule for jet engine

maximum climb angle increases at exponential rate. (Fig. 7)

4. From the climb schedule contours , we see that difference between the minimum velocity and maximum velocity decreases as altitude increases. As specific excess power decreases, the ceiling altitude decreases. (Fig. 8)

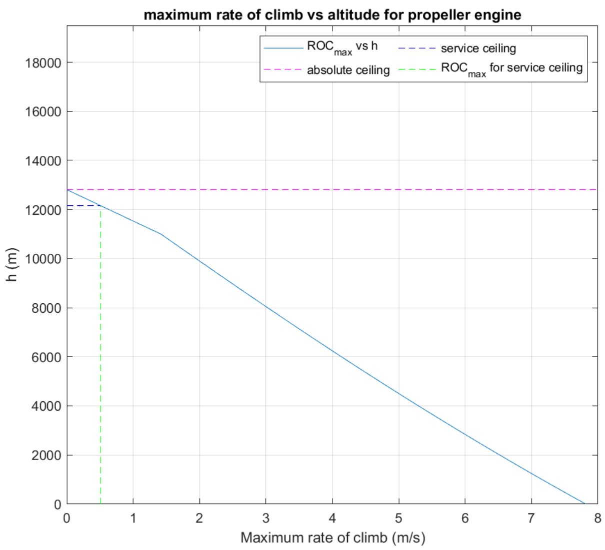

FIG. 5. Maximum rate of climb vs altitude for propeller engine

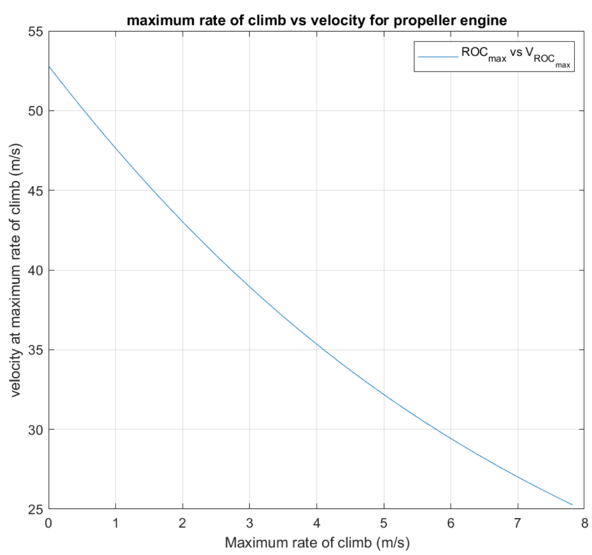

FIG. 6. Maximum rate of climb vs velocity for propeller engine

IV. CONCLUSION

On a general note, performance of jet engine is better as it has higher ceiling altitude and it has less aerodynamic limit region in its envelope. Also, propeller engines produce a lot of noise during operation. But specific application use case and its requirements decide the suitability of a type of engine (jet or propeller) for that specific application.

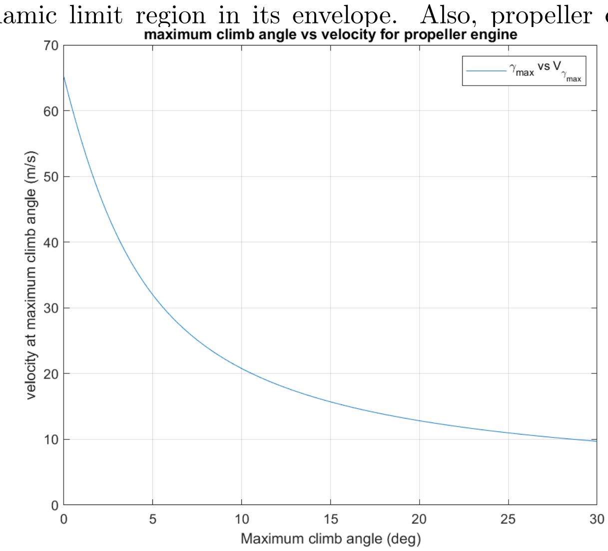

FIG. 7. maximum climb angle vs velocity for propeller engine

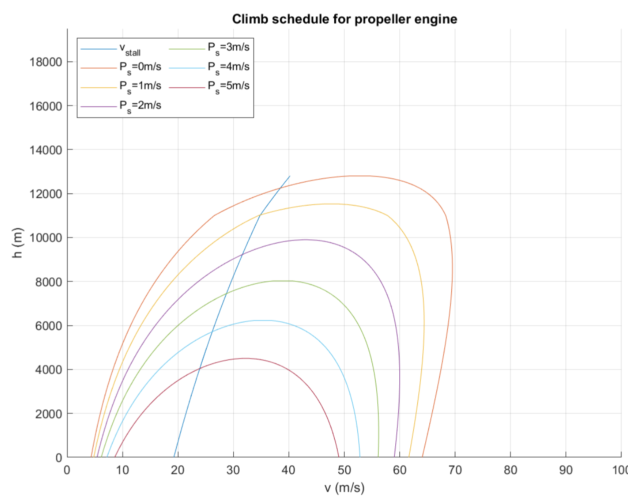

FIG. 8. Climb schedule for propeller engine

Appendix A: MATLAB codes

1. Script for analysing steady climb for jet engine

```
% % jet engine

%given constant parameters
m=750;
S=12;
b=10;
C_Do=0.036;
C_lmax=2.7;
e=0.87;

% altitude and air density
h=(0:25:20000)';
sig=sigma(h);
rho=1.225.*sig;

%derived constant parameters
W=m*9.81;
AR=(b^2)/S;
K=(pi*e*AR)^-1;
v_stall=(2*W./(S.*rho.*C_lmax)).^0.5;
v_stall_eq=v_stall.*(sig.^0.5);

%Thrust model
T_sl=1140;
T=T_sl.*(sig.^(1/3));

LD_max=0.5*(K*C_Do)^(-0.5);

% climb  calculations
v_roc_max=((T./(3*rho.*C_Do*S)).*(1+(1+3*(LD_max.*T./W).^(-2)).^0.5)).^0.5;
Y_roc_max=asind((T./W)-(0.5*rho.*v_roc_max.^2*S*C_Do/W)-((2*K*W)./(rho.*S.*v_roc_max.^2)));
roc_max=v_roc_max.*sind(Y_roc_max);
v_Y_max=(2*W./(rho.*S)).^0.5.*(K/C_Do)^0.25;
Y_max=asind((T./W)-(4*K*C_Do)^0.5);

% ceiling calculations
roc_max_f=@(hq) interp1(h,roc_max,hq);
roc_abs_ceil=0;
roc_ser_ceil=100*0.00508;

abs_ceil=fzero(@(hq) roc_max_f(hq)-roc_abs_ceil,[12000 20000]);
ser_ceil=fzero(@(hq) roc_max_f(hq)-roc_ser_ceil,[12000 20000]);

%plotting
figure;
plot(roc_max,h);
hold on;
plot([0 3],abs_ceil*ones(2,1),'m--');
plot([0 roc_ser_ceil],ser_ceil*ones(2,1),'b--');
plot(roc_ser_ceil*ones(2,1),[0 ser_ceil],'g--');
hold off;
grid on;
xlim([0 3]);
```

```
ylim([0 19500]);
xlabel("Maximum rate of climb (m/s)");
ylabel("h (m)");
legend({'ROC_{max} vs h','absolute ceiling','service ceiling','ROC_{max} for service ceiling'},'Location','northeast','NumColumns',2);
title('maximum rate of climb vs altitude for jet engine');

figure;
plot(roc_max,v_roc_max);
grid on;
xlim([0 3]);
xlabel("Maximum rate of climb (m/s)");
ylabel("velocity at maximum rate of climb (m/s)");
legend('ROC_{max} vs V_{ROC_{max}}','Location','northeast','NumColumns',2);
title('maximum rate of climb vs velocity for jet engine');

figure;
plot(Y_max,v_Y_max);
grid on;
xlim([0 5]);
xlabel("Maximum climb angle (deg)");
ylabel("velocity at maximum climb angle (m/s)");
legend('\gamma_{max} vs V_{\gamma_{max}}','Location','northeast','NumColumns',2);
title('maximum climb angle vs velocity for jet engine');

v_T=zeros(length(h),2);

% solving the biquadratic equation and plotting climb schedule
figure;
hold on;
grid on;
syms v;
Ps=0;
for j=0:1:6
    Ps=0.5*j;
    for i=1:1:length(h)
        v_=vpasolve(T(i)*v-Ps*W-0.5*C_Do*rho(i)*v^3*S-(2*K*W^2)/(rho(i)*v*S) == 0,v,[0 Inf]);
        if isempty(v_)
            break;
        end
        v_T(i,:)=v_;
    end

    h1=h;
    v_T1=v_T;
    v_T1(i:end,:)=[];
    h1(i:end)=[];

    h1_plot=cat(1,h1,flip(h1));
    v_plot=cat(1,v_T1(:,1),flip(v_T1(:,2)));

    if j==0
        v_stall_plot=v_stall;
        v_stall_plot(i:end)=[];
        plot(v_stall_plot,h1);
    end
    plot(v_plot,h1_plot);
end
```

```
% plotting climb schedule
hold off;
xlim([0 100]);
ylim([0 19500]);
xlabel("v (m/s)");
ylabel("h (m)");
legend({'v_{stall}','P_s='+string(0)+'W','P_s='+string(0.5)+'W','P_s='+string(1)+'W','P_s='+string(1.5)+'W','P_s='+string(2)+'W','P_s='+string(2.5)+'W'},...
'Location','northwest','NumColumns',2);
title('Climb schedule for jet engine');
```

2. Script for analysing steady climb for propeller engine

```
% % propeller engine

%given constant parameters
m=750;
S=12;
b=10;
C_Do=0.036;
C_lmax=2.7;
e=0.87;

% altitude and air density
h=(0:25:20000)';
sig=sigma(h);
rho=1.225.*sig;

%derived constant parameters
W=m*9.81;
AR=(b^2)/S;
K=(pi*e*AR)^-1;
v_stall=(2*W./(S.*rho.*C_lmax)).^0.5;
v_stall_eq=v_stall.*(sig.^0.5);


%Power model
P_sl=100*745.699872; %hp to W conversion
P=P_sl.*(sig.^0.5);
eta=1;

% climb  calculations
v_roc_max=(2*W./(rho.*S*(3*C_Do/K)^0.5)).^0.5;
Y_roc_max=asind((eta.*P./(v_roc_max.*W))-(0.5*rho.*v_roc_max.^2*S*C_Do/W)-((2*K*W)./(rho.*S.*v_roc_max.^2)));
roc_max=v_roc_max.*sind(Y_roc_max);
v_Y_max=(4*W^2*K)./(P.*rho.*eta*S);
Y_max=asind((eta.*P./(v_Y_max.*W))-(0.5*rho.*v_Y_max.^2*S*C_Do/W)-((2*K*W)./(rho.*S.*v_Y_max.^2)));

% ceiling calculations
roc_max_f=@(hq) interp1(h,roc_max,hq);
roc_abs_ceil=0;
roc_ser_ceil=100*0.00508;

abs_ceil=fzero(@(hq) roc_max_f(hq)-roc_abs_ceil,[10000 16000]);
```

```
ser_ceil=fzero(@(hq) roc_max_f(hq)-roc_ser_ceil,[8000 16000]);

% plotting
figure;
plot(roc_max,h);
hold on;
plot([0 8],abs_ceil*ones(2,1),'m--');
plot([0 roc_ser_ceil],ser_ceil*ones(2,1),'b--');
plot(roc_ser_ceil*ones(2,1),[0 ser_ceil],'g--');
hold off;
grid on;
xlim([0 8]);
ylim([0 19500]);
xlabel("Maximum rate of climb (m/s)");
ylabel("h (m)");
legend({'ROC_{max} vs h','absolute ceiling','service ceiling','ROC_{max} for service ceiling'},'Location','northeast','NumColumns',2);
title('maximum rate of climb vs altitude for propeller engine');

figure;
plot(roc_max,v_roc_max);
grid on;
xlim([0 8]);
xlabel("Maximum rate of climb (m/s)");
ylabel("velocity at maximum rate of climb (m/s)");
legend('ROC_{max} vs V_{ROC_{max}}','Location','northeast','NumColumns',2);
title('maximum rate of climb vs velocity for propeller engine');

figure;
plot(Y_max,v_Y_max);
grid on;
 xlim([0 30]);
xlabel("Maximum climb angle (deg)");
ylabel("velocity at maximum climb angle (m/s)");
legend('\gamma_{max} vs V_{\gamma_{max}}','Location','northeast','NumColumns',2);
title('maximum climb angle vs velocity for propeller engine');

v_P=zeros(length(h),2);

% solving the biquadratic equation and plotting climb schedule
figure;
hold on;
grid on;
syms v;
Ps=0;
for j=0:1:5
    Ps=j;
    for i=1:1:length(h)
        v_=vpasolve(P(i)*eta-Ps*W-0.5*C_Do*rho(i)*v^3*S-(2*K*W^2)/(rho(i)*v*S) == 0,v,[0 Inf]);
        if isempty(v_)
            break;
        end
        v_P(i,:)=v_;
    end

%plotting envelope

    h1=h;
```

```
    v_P1=v_P;
    v_P1(i:end,:)=[];
    h1(i:end)=[];

    h1_plot=cat(1,h1,flip(h1));
    v_plot=cat(1,v_P1(:,1),flip(v_P1(:,2)));

    if j==0
        v_stall_plot=v_stall;
        v_stall_plot(i:end)=[];
        plot(v_stall_plot,h1);
    end
    plot(v_plot,h1_plot);
end

% plotting climb schedule
hold off;
xlim([0 100]);
ylim([0 19500]);
xlabel("v (m/s)");
ylabel("h (m)");
legend({'v_{stall}','P_s='+string(0)+'m/s','P_s='+string(1)+'m/s','P_s='+string(2)+'m/s','P_s='+string(3)+'m/s','P_s='+string(4)+'m/s','P_s='+string(5)+'m/s'},...
'Location','northwest','NumColumns',2);
title('Climb schedule for propeller engine');
```

3. Function for calculating relative air density

```
function sig=sigma(h_)
% function to calculate relative density as a variation of altitude.
    sig=ones(size(h_));
    for i=1:1:length(h_)
        h=h_(i);
        if h<11000
            e=1;
            ha=0;
            B=9296;
        else
            e=exp(-11000/9296);
            ha=11000;
            B=6216;
        end
        sig(i)=e.*exp(-(h-ha)/B);
    end
end
```