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Assignment 4

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Download all python codes from

https://github.com/BatharajuRamana/ ASSIGNMNT4/Assignment6.py

and latex-tikz codes from

https://github.com/BatharajuRamana/ ASSIGNMNT4/main.tex

1 Question No.2.70

In each of the following exercises, find the equation of the parabola that satisfies the following conditions:

f. vertex $\begin{pmatrix} 0 & 0 \end{pmatrix}$ passing through $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and symmetric with respect to the y-axis

2 solution

Given that axis is symmetric with respect to the y-axis.

So, vector equation of the parabola is,

$$\mathbf{x}^{T} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 & -2\mathbf{a} \end{pmatrix} \mathbf{x} + 0 = 0 \tag{2.0.1}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ -2\mathbf{a} \end{pmatrix}, f = 0 \tag{2.0.2}$$

|V| = 0 and $\lambda_1 = 0$ i.e. it is in standard form

$$\mathbf{P} = \mathbf{I} \implies \mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.3}$$

$$\eta = \mathbf{u}^T \mathbf{p_1} = -2\mathbf{a} \tag{2.0.4}$$

 \therefore (5 2) satisfies it.

$$\begin{pmatrix} 5 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 0 & -2\mathbf{a} \end{pmatrix} \begin{pmatrix} 5 \\ 2 \end{pmatrix} = 0 \qquad (2.0.5)$$

$$\implies (5 \quad 0) \begin{pmatrix} 5 \\ 2 \end{pmatrix} + (0 \quad -4\mathbf{a}) \begin{pmatrix} 5 \\ 2 \end{pmatrix} = 0 \qquad (2.0.6)$$

$$\implies 25 - 8\mathbf{a} = 0 \tag{2.0.7}$$

$$\implies$$
 25 = 8**a** (2.0.8)

$$\implies \mathbf{a} = \frac{25}{8} \tag{2.0.9}$$

... The equation is,

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 & -\frac{25}{4} \end{pmatrix} \mathbf{x} + 0 = 0 \qquad (2.0.10)$$

Plot of given parabola

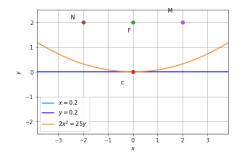


Fig. 0: Parabola $2x^2 = 25y$