Assignment 8

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1 QUESTION No. 2.47

Solve 3x+2y ≤ 12,x ≥ 1,y ≥ 2

2 SOLUTION

The given system of inequality can be written in matrix form as

$$\begin{pmatrix} -3 & -2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -12 \\ 1 \\ 2 \end{pmatrix} \quad (2.0.1)$$

Let the surplus vector be

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq 0 \quad (2.0.2)$$

1)

$$\begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -12 \\ 1 \end{pmatrix} \quad (2.0.3)$$

$$\implies \begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -12 \\ 1 \end{pmatrix} + \mathbf{u} \quad (2.0.4)$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -12 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix}^{-1} \mathbf{u}$$
$$(2.0.5)$$

$$\implies \mathbf{x} = \begin{pmatrix} 1 \\ \frac{9}{2} \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ \frac{-1}{3} & \frac{-3}{2} \end{pmatrix} \mathbf{u} \quad (2.0.6)$$

2)

$$\begin{pmatrix} -3 & -2 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -12 \\ 2 \end{pmatrix} \quad (2.0.7)$$

$$\implies \begin{pmatrix} -3 & -2 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -12 \\ 2 \end{pmatrix} + \mathbf{u} \quad (2.0.8)$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -3 & -2 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -12 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 & -2 \\ 0 & 1 \end{pmatrix}^{-1} \mathbf{u}$$
$$(2.0.9)$$

$$\implies \mathbf{x} = \begin{pmatrix} \frac{8}{3} \\ 2 \end{pmatrix} + \begin{pmatrix} \frac{-1}{3} & \frac{-2}{3} \\ 0 & 1 \end{pmatrix} \mathbf{u} \quad (2.0.10)$$

Now,solution region which is common to regions of eq. (2.0.6) and eq. (2.0.10),is given by

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} \frac{-1}{3} & \frac{2}{3} \\ \frac{1}{9} & \frac{-7}{9} \end{pmatrix} \mathbf{u} \quad (2.0.11)$$
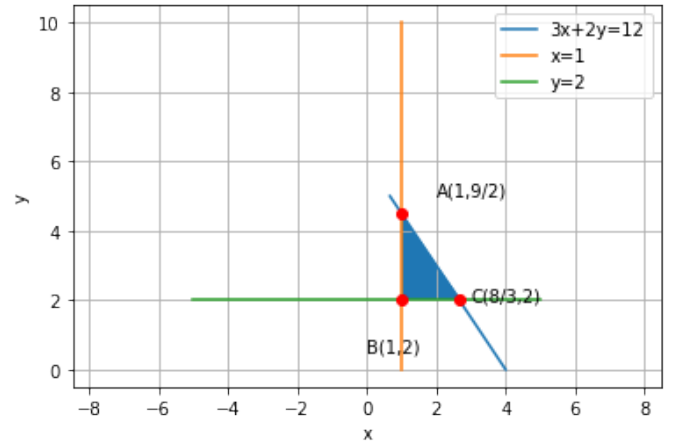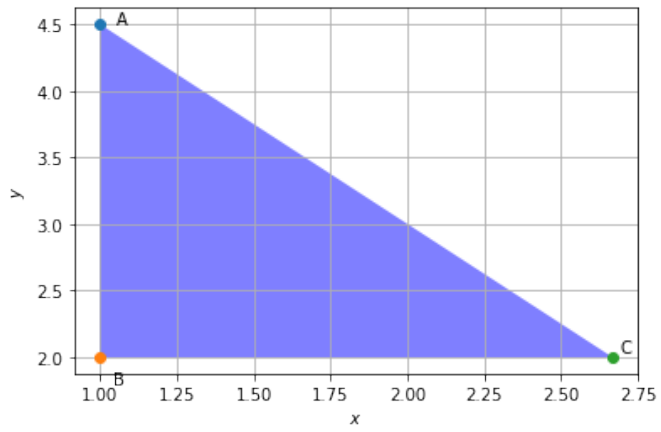

Fig. 2.1: Solution Region

Fig. 2.2: Magnified Solution Region