Assignment 8

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Download all python codes from

https://github.com/BatharajuRamana/Matrix-Theory/tree/main/Assignment11/Codes

and latex-tikz codes from

https://github.com/BatharajuRamana/Matrix-Theory/tree/main/Assignment11

1 Question No. 2.47

Solve 3x+2y $\leq$ 12,x $\geq$ 1,y $\geq$ 2

2 Solution

The given system of inequality can be written in matrix form as

$$\begin{pmatrix} -3 & -2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -12 \\ 1 \\ 2 \end{pmatrix} \tag{2.0.1}$$

Let the surplus vector be

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq 0 \tag{2.0.2}$$

1)

$$\begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -12 \\ 1 \end{pmatrix} \tag{2.0.3}$$

$$\implies \begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -12 \\ 1 \end{pmatrix} + \mathbf{u} \tag{2.0.4}$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -12 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix}^{-1} \mathbf{u} \tag{2.0.5}$$

$$\implies \mathbf{x} = \begin{pmatrix} 1 \\ \frac{9}{2} \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ \frac{-1}{3} & \frac{-3}{2} \end{pmatrix} \mathbf{u} \tag{2.0.6}$$

2)

$$\begin{pmatrix} -3 & -2 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -12 \\ 2 \end{pmatrix} \tag{2.0.7}$$

$$\implies \begin{pmatrix} -3 & -2 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -12 \\ 2 \end{pmatrix} + \mathbf{u} \tag{2.0.8}$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -3 & -2 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -12 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 & -2 \\ 0 & 1 \end{pmatrix}^{-1} \mathbf{u} \tag{2.0.9}$$

$$\implies \mathbf{x} = \begin{pmatrix} \frac{8}{3} \\ 2 \end{pmatrix} + \begin{pmatrix} \frac{-1}{3} & \frac{-2}{3} \\ 0 & 1 \end{pmatrix} \mathbf{u} \tag{2.0.10}$$

Now,solution region which is common to regions of eq. (2.0.6) and eq. (2.0.10),is given by

$$\boxed{\mathbf{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} \frac{-1}{3} & \frac{2}{3} \\ \frac{1}{9} & \frac{-7}{9} \end{pmatrix} \mathbf{u}} \tag{2.0.11}$$

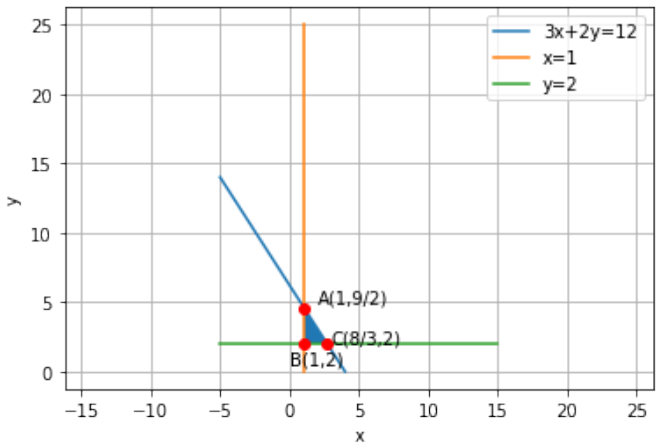

Fig. 2.1: Solution Region

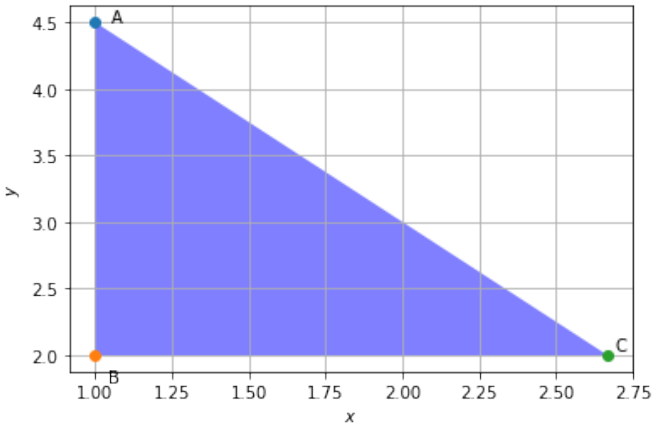

Fig. 2.2: Magnified Solution Region