# ASSIGNMENT 2

B.Ramana

Download all python codes from

https://github.com/BatharajuRamana/Assignment2/tree/main/Assignment2/codes

and latex-tikz codes from

https://github.com/BatharajuRamana/Assignment2/tree/main/Assignment2/main.tex

## 1 Question No 2.10

Find the intersection of the following lines

1)

$$\begin{pmatrix}\sqrt{2} & \sqrt{3}\end{pmatrix}\mathbf{x} = 0 \\ \begin{pmatrix}\sqrt{3} & \sqrt{8}\end{pmatrix}\mathbf{x} = 0 \tag{1.0.1}$$

2)

$$\begin{pmatrix}\frac{3}{2} & -\frac{5}{3}\end{pmatrix}\mathbf{x} = -2 \\ \begin{pmatrix}\frac{1}{3} & \frac{1}{2}\end{pmatrix}\mathbf{x} = \frac{13}{6} \tag{1.0.2}$$

## 2 Solution

1)

$$\begin{pmatrix}\sqrt{2} & \sqrt{3}\end{pmatrix}\mathbf{x} = 0 \\ \begin{pmatrix}\sqrt{3} & \sqrt{8}\end{pmatrix}\mathbf{x} = 0 \tag{2.0.1}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}\sqrt{2} & \sqrt{3} \\ \sqrt{3} & \sqrt{8}\end{pmatrix}\mathbf{x} = \begin{pmatrix}0 \\ 0\end{pmatrix} \tag{2.0.2}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}\sqrt{2} & \sqrt{3} & 0 \\ \sqrt{3} & \sqrt{8} & 0\end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - \frac{\sqrt{3}}{\sqrt{2}}R_1} \begin{pmatrix}\sqrt{2} & \sqrt{3} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0\end{pmatrix} \tag{2.0.3}$$

(2.0.4)

$\because$ row reduction of the $2 \times 3$ matrix

$$\begin{pmatrix}\sqrt{2} & \sqrt{3} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0\end{pmatrix} \tag{2.0.5}$$

results in a matrix with 2 nonzero row, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix}\sqrt{2} & \sqrt{3} \\ \sqrt{3} & \sqrt{8}\end{pmatrix} \tag{2.0.6}$$

is also 2.

$$\because Rank\begin{pmatrix}\sqrt{2} & \sqrt{3} \\ \sqrt{3} & \sqrt{8}\end{pmatrix} = \begin{pmatrix}\sqrt{2} & \sqrt{3} & 0 \\ \sqrt{3} & \sqrt{8} & 0\end{pmatrix} \tag{2.0.7}$$

$\therefore$ Given lines (1.0.1) have unique solution. $\therefore$ The given lines are intersection. PLOT OF GIVEN LINES -

Plot of (1.0.1) -

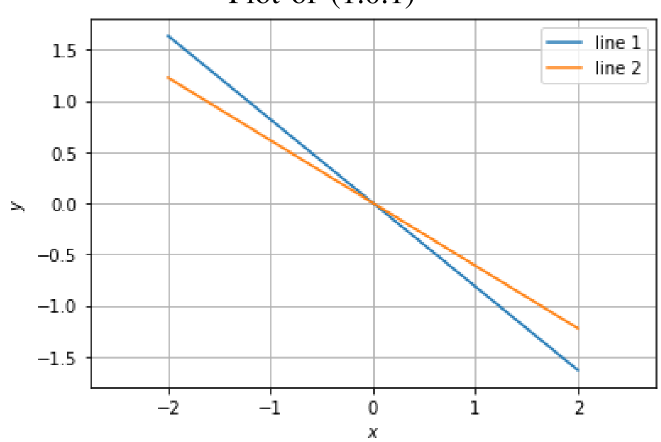


Fig. 2.1: intersecting lines

2)

$$\begin{pmatrix}\frac{3}{2} & -\frac{5}{3}\end{pmatrix}\mathbf{x} = -2 \\ \begin{pmatrix}\frac{1}{3} & \frac{1}{2}\end{pmatrix}\mathbf{x} = \frac{13}{6} \tag{2.0.8}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} \frac{3}{2} & -\frac{5}{3} \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} -2 \\ \frac{13}{6} \end{pmatrix} \tag{2.0.9}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} \frac{3}{2} & -\frac{5}{3} & -2 \\ \frac{1}{3} & \frac{1}{2} & \frac{13}{6} \end{pmatrix} \xleftrightarrow{R_1 \leftarrow 6R_1}, \xleftrightarrow{R_2 \leftarrow 6R_2} \begin{pmatrix} 9 & -10 & -12 \\ 2 & 3 & 13 \end{pmatrix} \tag{2.0.10}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 4R_2} \begin{pmatrix} 1 & -22 & -64 \\ 2 & 3 & 13 \end{pmatrix} \tag{2.0.11}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & -22 & -64 \\ 0 & 47 & 141 \end{pmatrix} \tag{2.0.12}$$

$$\xleftrightarrow{R_2 \leftarrow R_2/47} \begin{pmatrix} 1 & -22 & -64 \\ 0 & 1 & 3 \end{pmatrix} \tag{2.0.13}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + 22R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix} \tag{2.0.14}$$

$\because$ row reduction of the $2 \times 3$ matrix

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix} \tag{2.0.15}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} \frac{3}{2} & -\frac{5}{3} \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} \tag{2.0.16}$$

is also 2.

$$\because Rank \begin{pmatrix} \frac{3}{2} & -\frac{5}{3} \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix} \tag{2.0.17}$$

$\therefore$ Given lines (1.0.2) have unique solution. $\therefore$ The givens lines are intersection. PLOT OF GIVEN LINES -

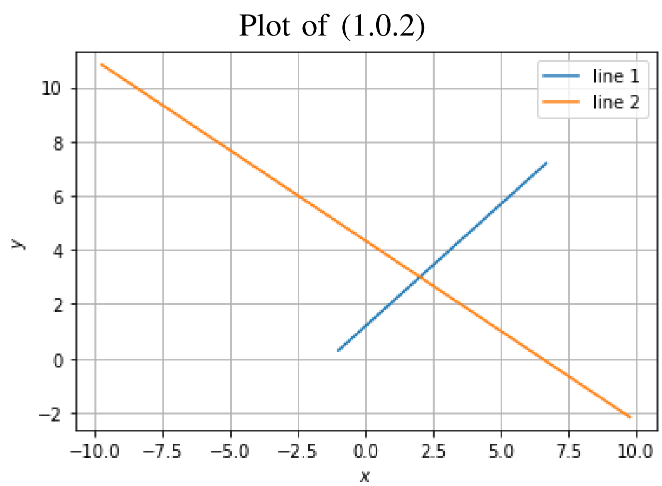


Fig. 2.2: intersecting lines