ASSIGNMENT 2

B.Ramana

Download all python codes from

https://github.com/BatharajuRamana/Assignment2/tree/main/Assignment2/codes

and latex-tikz codes from

https://github.com/BatharajuRamana/Assignment2/tree/main/Assignment2/main.tex

1 Question No 2.10

Find the intersection of the following lines

1)
$$\begin{aligned}\begin{pmatrix}\sqrt{2} & \sqrt{3}\end{pmatrix}\mathbf{x} &= 0\\ \begin{pmatrix}\sqrt{3} & \sqrt{8}\end{pmatrix}\mathbf{x} &= 0\end{aligned} \tag{1.0.1}$$

2)
$$\begin{aligned}\begin{pmatrix}\frac{3}{2} & -\frac{5}{3}\end{pmatrix}\mathbf{x} &= -2\\ \begin{pmatrix}\frac{1}{3} & \frac{1}{2}\end{pmatrix}\mathbf{x} &= \frac{13}{6}\end{aligned} \tag{1.0.2}$$

2 SOLUTION

1)
$$\begin{aligned}\begin{pmatrix}\sqrt{2} & \sqrt{3}\end{pmatrix}\mathbf{x} &= 0\\ \begin{pmatrix}\sqrt{3} & \sqrt{8}\end{pmatrix}\mathbf{x} &= 0\end{aligned} \tag{2.0.1}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}\sqrt{2} & \sqrt{3}\\ \sqrt{3} & \sqrt{8}\end{pmatrix}\mathbf{x} = \begin{pmatrix}0\\0\end{pmatrix} \tag{2.0.2}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}\sqrt{2} & \sqrt{3} & 0\\ \sqrt{3} & \sqrt{8} & 0\end{pmatrix} \xleftrightarrow{R_2\leftarrow R_2-\frac{\sqrt{3}}{\sqrt{2}}R_1} \begin{pmatrix}\sqrt{2} & \sqrt{3} & 0\\ 0 & \frac{1}{\sqrt{2}} & 0\end{pmatrix} \tag{2.0.3}$$
(2.0.4)

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix}\sqrt{2} & \sqrt{3} & 0\\ 0 & \frac{1}{\sqrt{2}} & 0\end{pmatrix} \tag{2.0.5}$$

results in a matrix with 2 nonzero row, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix}\sqrt{2} & \sqrt{3}\\ \sqrt{3} & \sqrt{8}\end{pmatrix} \tag{2.0.6}$$

is also 2.

$$\begin{aligned}\because Rank\begin{pmatrix}\sqrt{2} & \sqrt{3}\\ \sqrt{3} & \sqrt{8}\end{pmatrix} &= Rank\begin{pmatrix}\sqrt{2} & \sqrt{3} & 0\\ \sqrt{3} & \sqrt{8} & 0\end{pmatrix}\\ &= dim\begin{pmatrix}\sqrt{2} & \sqrt{3}\\ \sqrt{3} & \sqrt{8}\end{pmatrix}\\ &= 2\end{aligned} \tag{2.0.7}$$

$$\because Rank\begin{pmatrix}\sqrt{2} & \sqrt{3}\\ \sqrt{3} & \sqrt{8}\end{pmatrix} = \begin{pmatrix}\sqrt{2} & \sqrt{3} & 0\\ \sqrt{3} & \sqrt{8} & 0\end{pmatrix} \tag{2.0.8}$$

$\therefore$ Given lines (1.0.1) have unique solution. $\therefore$ The given lines are intersection. PLOT OF GIVEN LINES -

Plot of (1.0.1) -

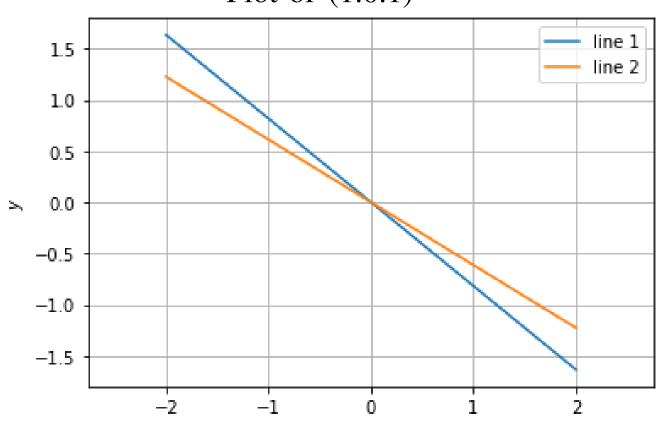

Fig. 2.1: intersecting lines

2)

$$\begin{pmatrix} \frac{3}{2} & -\frac{5}{3} \end{pmatrix} \mathbf{x} = -2$$
$$\begin{pmatrix} \frac{1}{3} & \frac{1}{2} \end{pmatrix} \mathbf{x} = \frac{13}{6} \tag{2.0.9}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} \frac{3}{2} & -\frac{5}{3} \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} -2 \\ \frac{13}{6} \end{pmatrix} \tag{2.0.10}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} \frac{3}{2} & -\frac{5}{3} & -2 \\ \frac{1}{3} & \frac{1}{2} & \frac{13}{6} \end{pmatrix} \xleftrightarrow{R_1 \leftarrow 6R_1}, \xleftrightarrow{R_2 \leftarrow 6R_2} \begin{pmatrix} 9 & -10 & -12 \\ 2 & 3 & 13 \end{pmatrix} \tag{2.0.11}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 4R_2} \begin{pmatrix} 1 & -22 & -64 \\ 2 & 3 & 13 \end{pmatrix} \tag{2.0.12}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & -22 & -64 \\ 0 & 47 & 141 \end{pmatrix} \tag{2.0.13}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 / 47} \begin{pmatrix} 1 & -22 & -64 \\ 0 & 1 & 3 \end{pmatrix} \tag{2.0.14}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + 22R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix} \tag{2.0.15}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix} \tag{2.0.16}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} \frac{3}{2} & -\frac{5}{3} \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} \tag{2.0.17}$$

is also 2.

$$\because Rank \begin{pmatrix} \frac{3}{2} & -\frac{5}{3} \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix} \tag{2.0.18}$$

$\therefore$ Given lines (1.0.2) have unique solution. $\therefore$ The givens lines are intersection. PLOT OF GIVEN LINES -

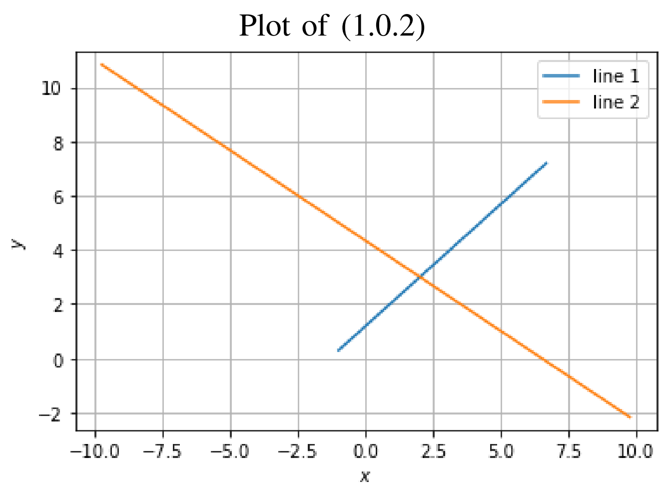

Fig. 2.2: intersecting lines