Moran Process for competition between TFT and ALLD

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1 Brief Theory behind the simulation

Let PD be the standard prisoner's dilemma matrix

$$PD = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{1}$$

where,
a $\implies$ payoff to a cooperator when two cooperators interact.
b $\implies$ payoff to a cooperator when cooperator interacts with a defector.
c $\implies$ payoff to a defector when cooperator interacts with a defector.
d $\implies$ payoff to a defector when two defectors interact.

Now, ALLD is a defective strategy which defects in every round of the game where as TFT is a more cooperative strategy which starts with cooperating and chooses to cooperate or defect depending on whether it's opponent cooperates or defects in later rounds of the game.

Let the expected number of rounds the game is played be m.

Then the pay-off matrix for the interaction between TFT and ALLD is given by

$$M = \begin{bmatrix} ma & b + (m-1)d \\ c + (m-1)d & md \end{bmatrix} \tag{2}$$

Now for a population where,
Number of TFT players $= A$.
Number of ALLD players $= B$.
Population size $= A + B = N$

Expected pay-off to a TFT player,

$$F_{TFT} = \frac{M_{11}(A-1) + M_{12}B}{N-1}$$

Expected pay-off to an ALLD player,

$$F_{ALLD} = \frac{M_{21}A + M_{22}(B-1)}{N-1}$$

where Mij are elements of M.
Let w be the intensity of selection. It is a number between 0 and 1. It measures the strength with which the game contribute to fitness.

The fitness of TFT and ALLD is given by

$$f_{TFT} = 1 - w + wF_{TFT}$$

$$f_{ALLD} = 1 - w + wF_{ALLD}$$

The objective of this simulation is to see if the cooperative strategy TFT can invade a population of defective strategy $ALLD$. If the number of rounds played is just 1, (i.e $m = 1$), then cooperative strategy cannot invade $ALLD$. From the derivations in Martin Nowak's book,'Evolutionary Dynamics,Exploring the equations of life';in a finite population of size N, the selection would favour the fixation of TFT if:

$$M_{11}(N-2) + M_{12}(2N-1) > M_{21}(N+1) + M_{22}(2N-4) \tag{3}$$

which boils down to the condition on m:

$$m > \frac{c(N+1) + d(N-2) - b(2N-1)}{(a-d)(N-2)} \tag{4}$$

In the simulation the values of $a, b, c, d = 3, 0, 5, 1$. For these values the threshold on m that would allow for invasion of $ALLD$ by TFT is 10.5 For a very small population size of $N = 3$.

m is chosen to be 10 in the simulation. The fitness is calculated by the formulas derived above in each generation and the population is evolved through Moran process. Starting from a single TFT individual, the population is evolved till fixation. This simulation is carried out for 5000 trials and the number of times TFT gets fixed,n if calculated. Then the fixation probability from simulation can be estimated as $\rho_{TFT} = \frac{n}{5000}$. ρ_{TFT} is calculated in this way for different values of population size N. And the following plot was obtained.

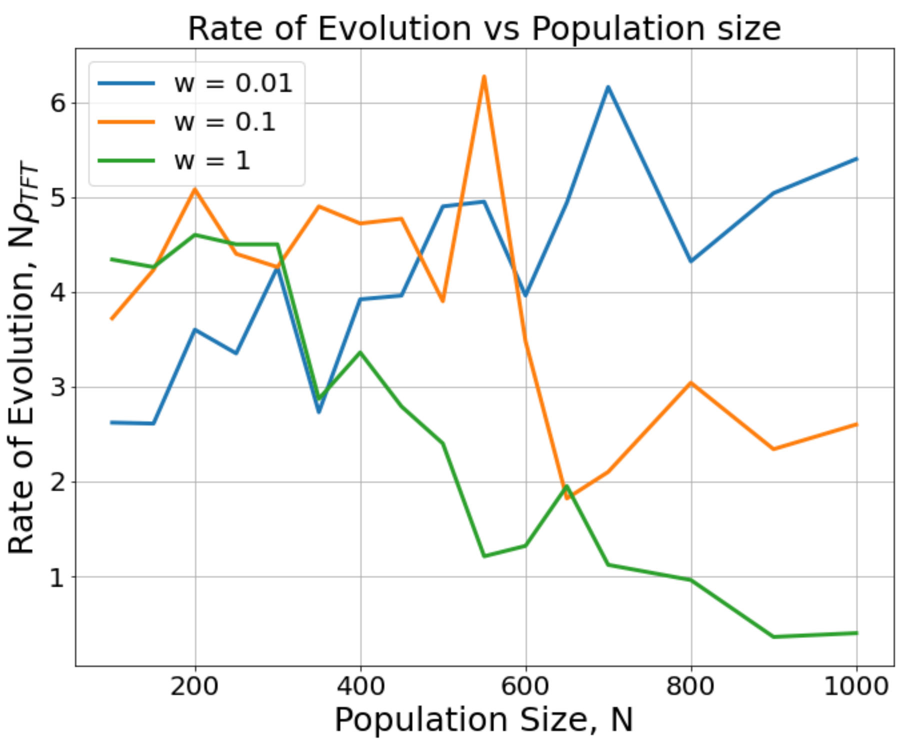

Figure 1

Again, from theory, for the fixation of TFT, $\rho_{TFT} > \frac{1}{N}$. i.e $N\rho_{TFT} > 1$. The simulations where carried out for N above 100.

- For $w = 1$, we see that $N\rho_{TFT}$ goes beneath 1 for large values of N. This is intuitive clear as for very large population size, it is difficult for a single mutant cooperator to multiply and finally invade/reach fixation in an sea of defectors.
- We have already seen that for small values of N, the number of rounds m has to be above a threshold for favouring the fixation of TFT.
- The inference from these two points is that, for selection to favour the invasion of $ALLD$ by TFT, i.e for the initiation of cooperation, the population size should neither be too small nor too large.

2 Reference

- Evolutionary Dynamics:Exploring the equations of life.
- Notes from the PH4209, Evolutionary Dynamics course offered at IISER Kolkata.