Moran process with constant selection but without mutation

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1 Brief theory behind the simulation

Assuming two types of individuals 0 and 1 in the population. If the fitness of type 0 is r and type 1 is 1. Then for a population of size N with i individuals of type 0 and $N-i$ individuals of type 1,

Probability of type 0 to reproduce or equivalently, mean fitness of type 0 in the population is given by

$$\frac{ri}{ri+(N-i)}$$

Similarly the probability of type 1 to reproduce or the mean fitness of type 1 in the population is

$$\frac{N-i}{ri+(N-i)}$$

And the probabilities of type 0 and type 1 dying are $\frac{i}{N}$ and $\frac{N-i}{N}$ respectively.

Then the probability of type 0 reproducing and type 0 dying is

$$\frac{ri}{ri+(N-i)} \times \frac{i}{N}$$

Probability of type 1 reproducing and type 1 dying is

$$\frac{N-i}{ri+(N-i)} \times \frac{N-i}{N}$$

Probability of type 0 reproducing and type 1 dying is

$$\frac{ri}{ri+(N-i)} \times \frac{N-i}{N}$$

Probability of type 1 reproducing and type 0 dying is

$$\frac{N-i}{ri+(N-i)} \times \frac{i}{N}$$

The populations are evolved according to Moran Process with these probability weights for the

respective events.

The invasion probability of type 0 is given by

$$x_1 = \frac{1 - \frac{1}{r}}{1 - \frac{1}{r^N}} \tag{1}$$

The fixation probability of type 0 starting from an initial population with i individuals of it is given by

$$x_i = \frac{1 - \frac{1}{r^i}}{1 - \frac{1}{r^N}} \tag{2}$$

The derivations of 1 and 2 are given in Martin A Nowak's Book mentioned in reference.

Simulation 1

$r = 1.01$ and $N = 100$. The population starts from 1 type 0 individual and 99 type 1 individuals. The simulation is run till fixation for $Nt = 1000$ trials and the following results are obtained.

Figure 1: Evolution of frequencies over generations for one of the trials

Fraction of times type 0 was fixed = 0.015
Fraction of times type 1 was fixed = 0.985
Putting $r = 1.01$ in equation(1),we get the theoretical value of invasion probability of type 0 rounded to three decimal places as 0.016. It is observed that the theoretical value and the simulated value of the invasion probabilities match. If more trials are carried out say $Nt > 10000$ a more matching invasion probability will be obtained.

Simulation 2

$r = 0.99$ and $N = 100$. The population starts from 50 type 0 individuals and 50 type 1 individuals. The simulation is run till fixation for $Nt = 1000$ trials and the following results are obtained.

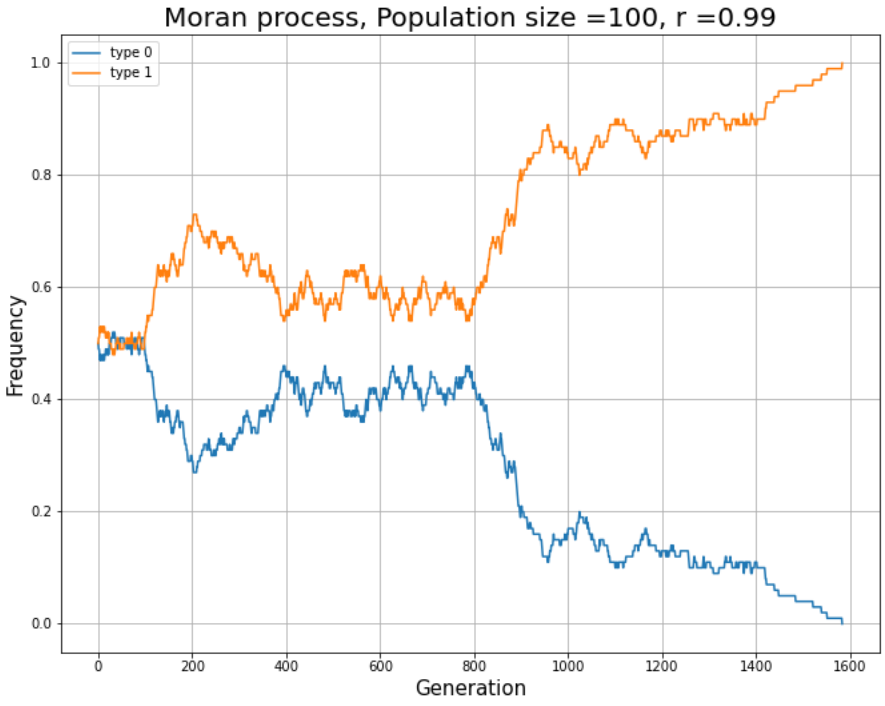

Figure 2: Evolution of frequencies over generations for one of the trials

Fraction of times type 0 was fixed = 0.373
Fraction of times type 1 was fixed = 0.627
Putting $r = 0.99$ in equation(2),we get the theoretical value of fixation probability of type 0 rounded to three decimal places as 0.377. It is observed that the theoretical value and the simulated value of the invasion probabilities match. Once again more trials are carried out say $Nt > 10000$ a more matching fixation probability will be obtained.

2 Reference

Evolutionary Dynamics, Exploring the equations of life. Martin A Nowak