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Evolution through mutation without selection

Assuming x and y to be the frequencies of A and B individuals in a population with mutation without selection, where,

$u_1 \rightarrow$ Rate at which A mutates to B

$u_2 \rightarrow$ Rate at which B mutates to A

The dynamical equation describing the rate of change of x and y,

$$\frac{dx}{dt} = -u_1 x + u_2 y$$

$$\frac{dy}{dt} = u_1 x - u_2 y$$

Additionally we have

$$x + y = 1$$

Setting the derivatives equal to 0,

We get the theoretical value of equilibrium frequencies as

$$x_{eq} = \frac{u_2}{u_1 + u_2}$$

$$y_{eq} = \frac{u_1}{u_1 + u_2}$$

a) Let **u1 = 0.003 and u2 = 0.001**

The theoretical prediction of equilibrium frequencies for **u1 = 0.003 and u2 = 0.001** is

x_{eq} = 0.25

y_{eq} = 0.75

Simulation results for different population sizes:

For N = 50,

x_{eq} = 0.26

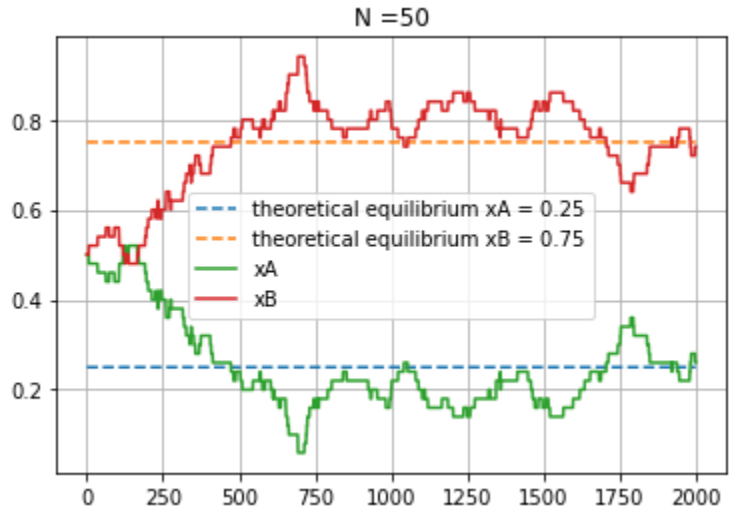
y_{eq} = 0.74

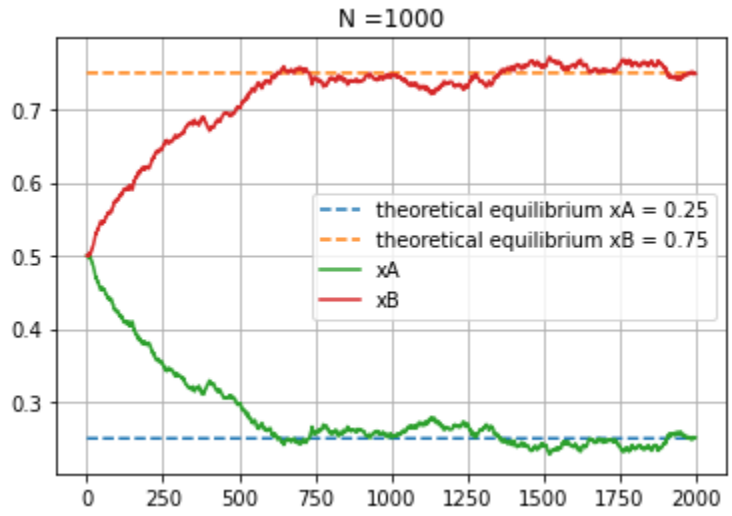
N = 1000
x_{eq} = 0.251
y_{eq} = 0.749

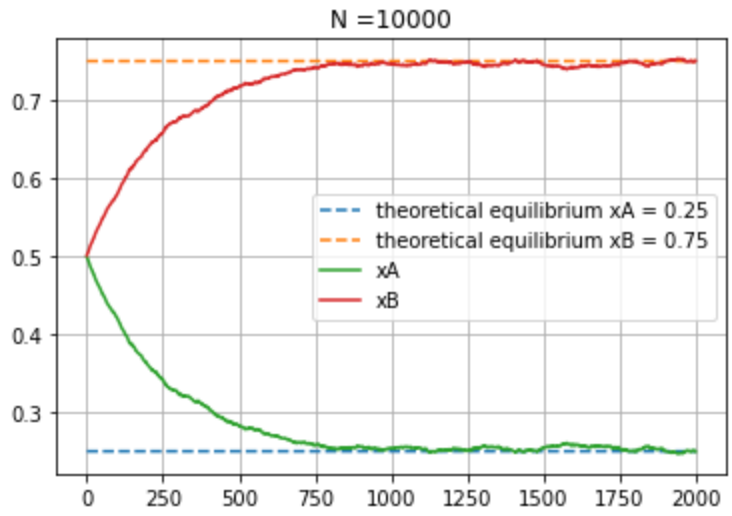
N = 10000
x_{eq} = 0.24960000000000004
y_{eq} = 0.7504

The equilibrium frequencies of the simulated population is practically close/equal to the theoretically predicted equilibrium frequencies. The simulation results become more and more closer to the theoretical prediction when the population size increase.

The plots have generation and frequency on the x and y axis respectively







b) Let **u1 = 0.07 and u2 = 0.001**

The theoretical prediction of equilibrium frequencies for **u1 = 0.07 and u2 = 0.001** is

x_{eq} ≈ 0.014

y_{eq} ≈ 0.986

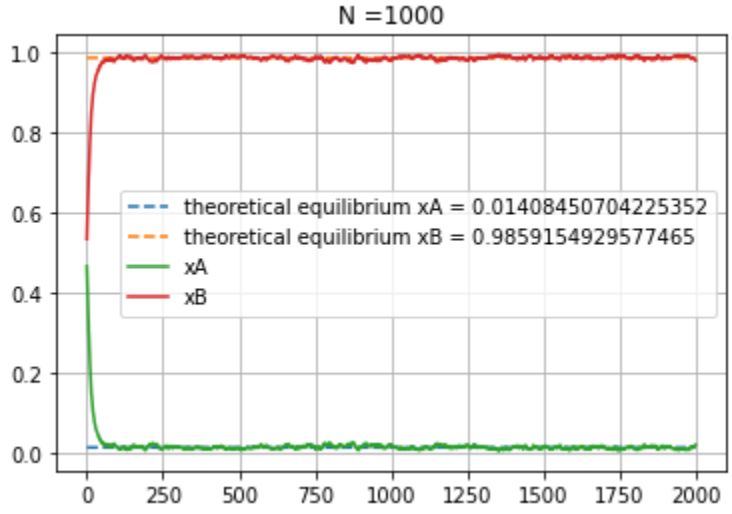
Simulation results:

For N = 1000

x_{eq} = 0.02100000000000002

y_{eq} = 0.979

The plot have generations and frequency on the x and y axis respectively



Again the simulated population equilibrium matches very much with the theoretical equilibrium. One can also note that on increasing the mutation rate u1 to 0.07, the population achieves equilibrium faster.

Reference:

Evolutionary dynamics, exploring the equations of life. Martin A Nowak