Evolution via mutation and selection

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1 Brief theory

The sequence length L = 1
The mutation rate from type 1 to type 0 = 0
The mutation rate from type 0 to type 1 = $u_{01} = u$
The fitness of type 0 = f_0
The fitness of type 1 = 1

Let $q = 1 - u01$ and the frequency of type 0 be x.

Then the replicator equation:

$$\frac{dx}{dt} = x_0 f_0 q + x_1 f_1 u_{10} - \phi x$$

Since $u_{10} = 0$

$$\frac{dx}{dt} = x_0 f_0 q - (f_0 x + f_1 y)\, x$$

Since $x + y = 1$ and $f_1 = 1$

Finally, we have,

$$\frac{dx}{dt} = x\,(f_0 q - 1) - x^2\,(f_0 - 1) \tag{1}$$

Case 1: If $f_0 q < 1$:

$$\implies \frac{dx}{dt} < 0$$

In this case type 0 will go extinct from the population and only type 1 will prevail. Coexistence is not possible.

Case 2: If $f_0 q > 1$:
Coexistence of the two types is possible. The equilibrium frequency is given by:

$$x_* = \frac{f_0 q - 1}{f_0 - 1} \tag{2}$$

Simulation 1

$u = 0.01$,$f_0 = 1.001$,$N = 10000$

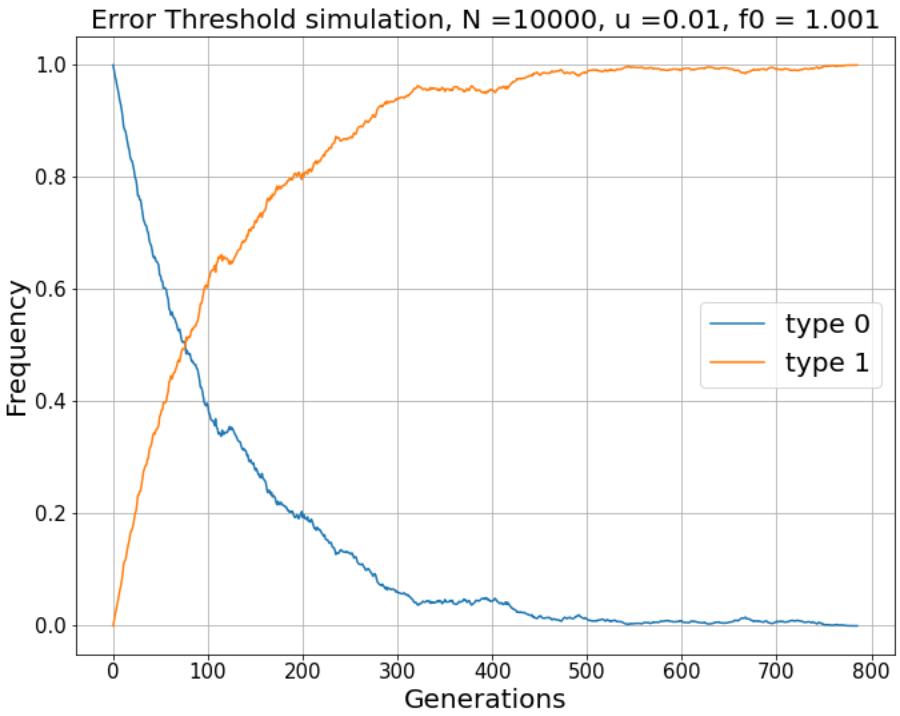

Figure 1: Simulation result for $u = 0.01, f_0 = 1.001, N = 10000$

$f_0 q = 1.001 \times (1 - 0.01) = 0.9909 < 1$. Hence, coexistence not possible and type zero will get extinct from the population in time. The simulation results agree with theoretical predictions.

Simulation 2

$u = 0.01, f_0 = 1.1, N = 10000$

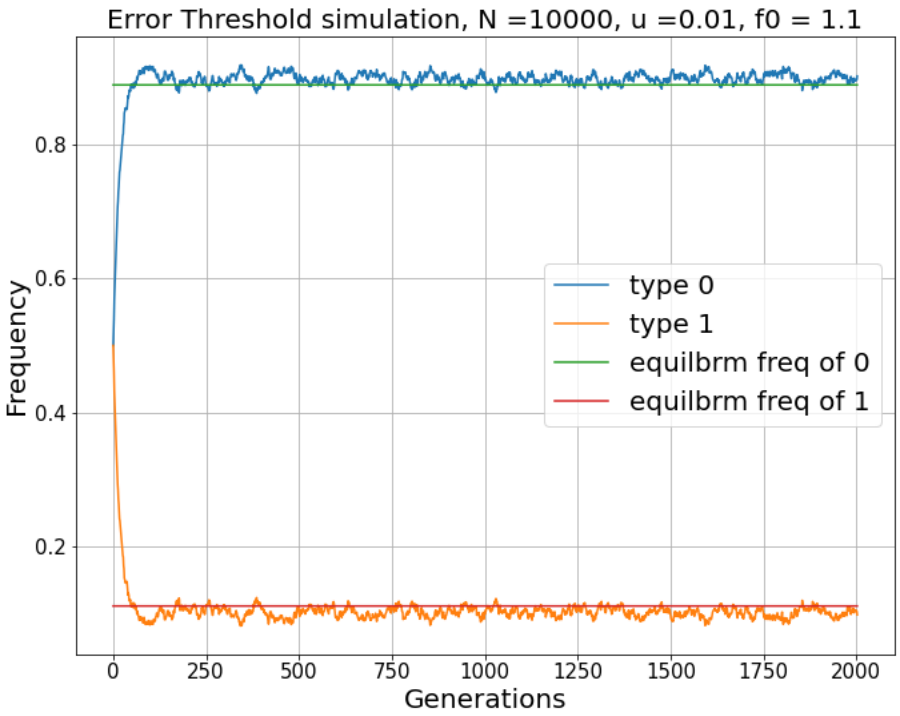

Figure 2: Simulation result for $u = 0.01, f_0 = 1.1, N = 10000$

$f_0 q = 1.1 \times (1 - 0.01) = 1.089 > 1$. Here, coexistence is possible. The theoretical equilibrium frequency of type 0(rounded to 4 decimal places) upon substituting the values in equation 2 above, we obtain

$$x_* = 0.8900$$

for type 1:

$$y_* = 0.1100$$

The simulation results agree with theoretical predictions. The theoretical equilibrium is plotted in the above plot.

1.1 Simulation 3

$u = 0.01, f_0 = 1.1, N = 100$

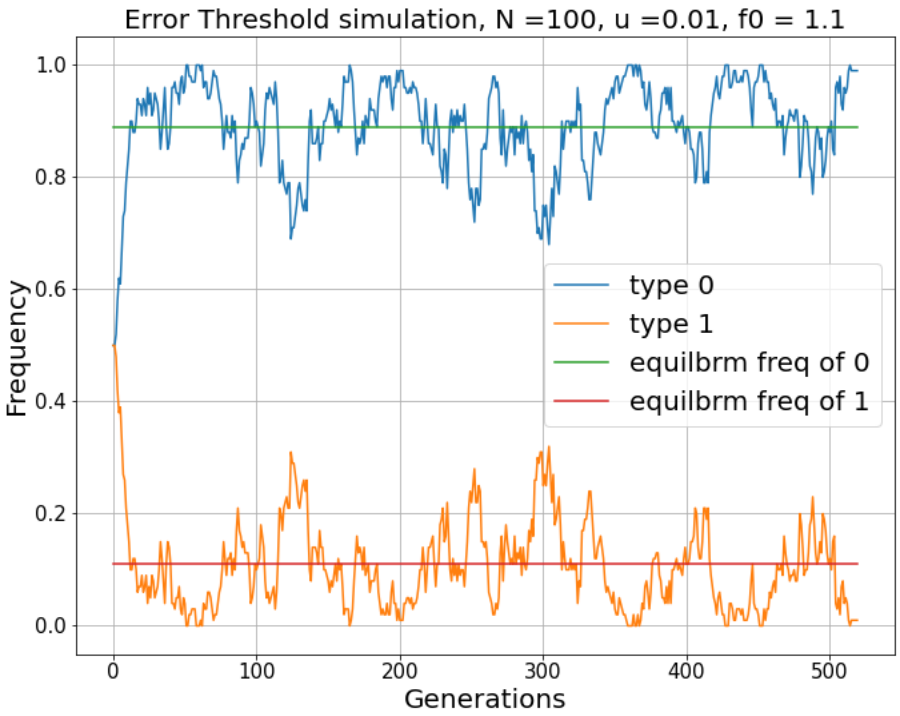

Figure 3: Simulation result for $u = 0.01$,$f_0 = 1.1$,$N = 100$

We have the same parameters as simulation 2 but the population size N is 100. As in simulation 2, we have same values for u and f_0.Thus we should be expecting coexistence, and the theoretical equilibrium frequency to be the same as in simulation 2. But the simulation results show extinction of type 1 after a while. The reason is small population size. We know that the equilibrium fluctuations are inversely proportional to the population size. Since our population size is small we are observing large fluctuations around the theoretically expected equilibrium frequency.