Reaction Diffusion and Pattern Formation

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We observe a lot of patterns in nature. The spots on a cheetah, stripes on a zebra, the arrangement of petals on a flower, the arrangement of thorns on a cactus plant, the way in which new leaf buds arise out of the growing shoot of a plant etc. We can observe patterns even in developmental phenomena as well. The following simulation tries to generate some animal coat patterns using reaction diffusion models.

In the simplest reaction diffusion model, there will be two species/substances/morphogens that diffuse in space as well as interact with each other. A guiding principle for reaction-diffusion systems that generate patterns are local self enhancement and long range inhibition of morphogens involved. i.e a network that combines short range positive feedback and long range negative feedback can lead to pattern formation [1], [2]. The reaction diffusion systems where first proposed by Turing and it goes without saying, the patterns generated by these systems are called Turing patterns.

All the models simulated below are models described in [1]. All the details required for simulating them, including the model formalism, parameter values, initial conditions etc are given in [1]. In all of the systems that follow, $D_\alpha, \rho_\alpha, \kappa_\alpha, \mu_\alpha, \sigma_\alpha$ are the diffusion constant, cross-reaction coefficient, saturation constant, removal rate, production rate respectively of the substance α involved in the reaction diffusion system. What each these mean is described in [1].

Activator-Inhibitor System.

This is one of the simplest system where two substances diffuse in space. The activator a is self enhancing and the inhibitor h inhibits a where it is present. Also h diffuses faster than a. These conditions ensures that a is enhanced locally and inhibited at long ranges.

$$\begin{aligned} \frac{\partial a}{\partial t} &= D_a \nabla^2 a + \rho_a \frac{a^2}{(1+\kappa_a a^2)h} - \mu_a a + \sigma_a \\ \frac{\partial h}{\partial t} &= D_h \nabla^2 h + \rho_h a^2 - \mu_h h + \sigma_h \end{aligned} \tag{1}$$

Spots using Activator inhibitor system

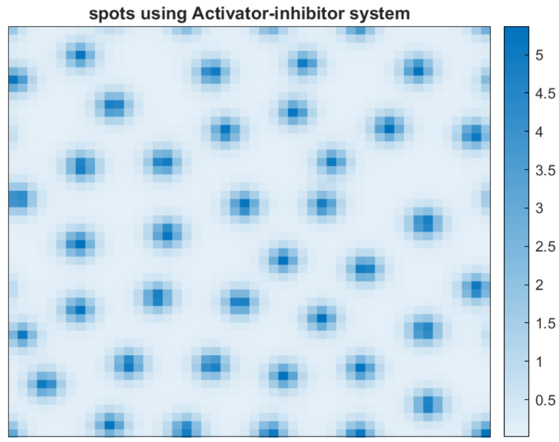

Figure 1: Spot pattern generated using activator-inhibitor model with appropriate parameter values

Stripes using Activator-Inhibitor system

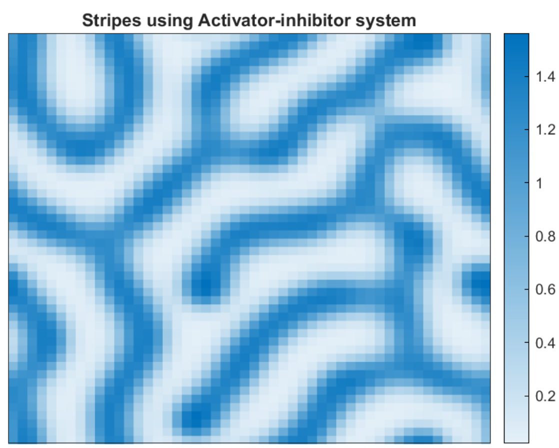

Figure 2: Stripe pattern generated using activator-inhibitor model with appropriate parameter values

Emergence of Periodic structures during growth

As a typical shoot grows, the formation of leaf takes place in a periodic manner. As a cactus plant grows, new thorns arise and there is a periodicity in the way these thorns are arranged. In the following simulation, we try to grow our domain by adding new rows and columns of cells after every 2000^{th} iteration along with simulation of the activator inhibitor system.

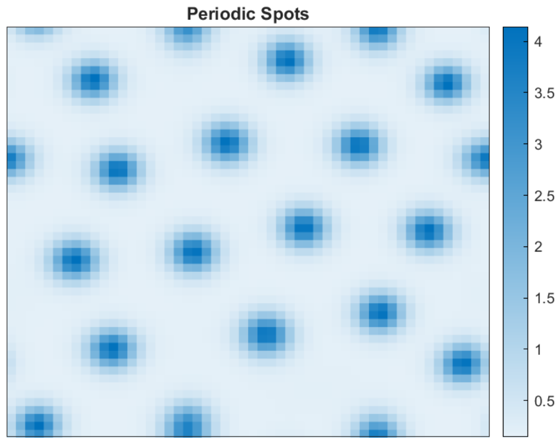

Figure 3: Periodic Spots in a growing field

To see a gif file of the above simulation,click here

Giraffe and Leopard

The model:

$$\begin{aligned}
\frac{\partial a}{\partial t} &= D_a \nabla^2 a + \rho_a \left[\frac{a^2 s}{1+\kappa_a a^2} - a \right] \\
\frac{\partial s}{\partial t} &= D_s \nabla^2 s + \frac{\sigma_s}{1+\kappa_s y} - \frac{\rho_s a^2 s}{1+\kappa_a a^2} - \mu_s s \\
\frac{\partial y}{\partial t} &= \rho_y \frac{y^2}{1+\kappa_y y^2} - \mu_y y + \sigma_y a
\end{aligned} \tag{2}$$

The skin color is given by y.

For appropriate choice of parameters and initial conditions we get the following patterns.

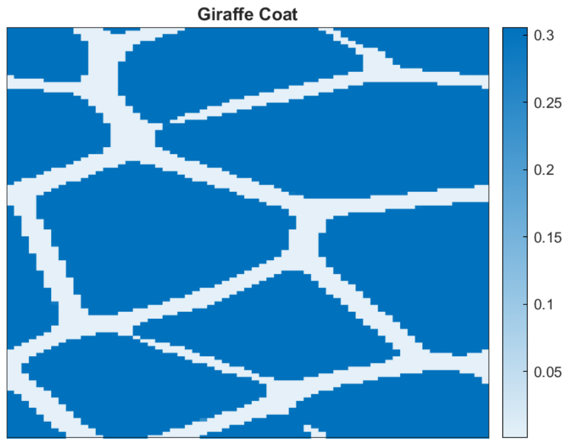

Figure 4: Giraffe

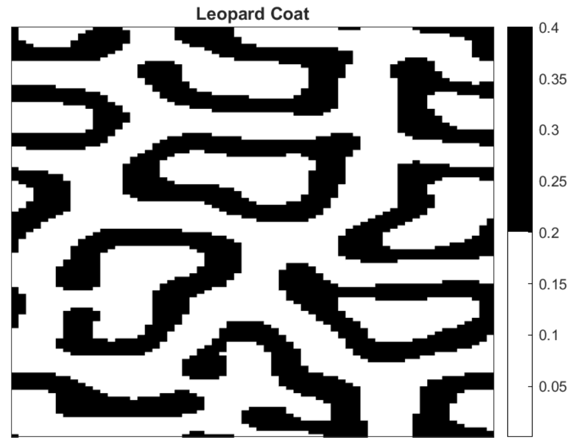

Figure 5: Leopard

References

[1] AJ Koch and Hans Meinhardt. “Biological pattern formation: from basic mechanisms to complex structures”. In: *Reviews of modern physics* 66.4 (1994), p. 1481.

[2] Shigeru Kondo and Takashi Miura. “Reaction-diffusion model as a framework for understanding biological pattern formation”. In: *science* 329.5999 (2010), pp. 1616–1620.