Collective Dynamics using Vicsek Model

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The following study tries to simulate collective dynamics using Vicsek Model. Implementation of Vicsek Model demonstrates collective behaviour in-silico. Various properties of the collective dynamics described in Vicsek et al [1] is replicated in the simulations carried out in this study. The various physical properties demonstrated by collective dynamics of agents described by Vicsek Model is studied.

I. INTRODUCTION

Active matter systems are those systems which consists of self-driven particles which convert stored energy into directed motion. Such systems are perpetually driven out of equilibrium. Interactions among such self-propelled particles give rise to novel collective behaviour (flocking, swarming etc.). Some examples of active matter systems include bacterial suspensions, motor protein-cytoskeletal filament assays, terrestrial, aerial and aquatic ocks, vibrated granular rods etc.

Vicsek model is a simple agent-based model that describes flocking in self driven particles. It is very popular due to it's simplicity and computational tractability. The model demonstrates an onset of kinetic order as certain parameters relevant to the model are varied. This study tries to simulate the Vicsek model and replicate some results obtained in [1]. This study also aims to understand the various behaviours demonstrated by active matter systems which are described by Vicsek model.

II. MODEL FORMALISM

The simulation is carried out in a field of size L $\times$ L. Let there be N agents in the field. The density ρ is $\frac{N}{L^2}$. All the agents have a velocity associated with them. The magnitude of velocity of all the agents across all time points is set to a constant, v. Thus the velocities differ only in their direction. r is the interaction radius.The position of the agents are updated in the following way:

i Let $\theta(t)$ be the angle which the velocity of i^{th} agent makes with respect to the x-axis at time t.

ii We now check for all those agents whose position fall within the interaction radius of agent i . The average angle made by the velocity vector of all these agents is calculated. Let this be $\langle\theta(t)\rangle_r$. Note that the angle of the focal agent i is also included for calculating $\langle\theta(t)\rangle_r$.

iii The angle of the velocity vector at $t+1$ is given by:

$$\theta(t+1) = \langle\theta(t)\rangle_r + \Delta\theta \tag{1}$$

iv The average direction,$\langle\theta(t)\rangle_r$ is calculated as $\tan^{-1}\frac{\langle\sin\theta(t)\rangle}{\langle\cos\theta(t)\rangle}$. $\Delta\theta$ is represents a noise term. It is a random number sampled from $U\left(-\frac{\eta}{2},\frac{\eta}{2}\right)$, the uniform distribution.

v Once we have $\theta(t+1)$, we can easily calculate the components of velocity at $t+1$ as

$$v_x(t+1) = v\cos\theta(t+1)$$
$$v_y(t+1) = v\sin\theta(t+1)$$

vi $v_x(t+1)$ and $v_y(t+1)$ is calculated simultaneously for all agents. Once we have velocity components of the agents at $t+1$ their position can be easily calculated.

vii The position of the agents at $t+1$ is given by

$$x(t+1) = x(t) + v_x(t)\Delta t$$
$$y(t+1) = y(t) + v_y(t)\Delta t \tag{2}$$

III. RESULTS

A. Simulation Results

This section includes the results of the simulations that I have carried out by implementing the Vicsek model as well as the results from

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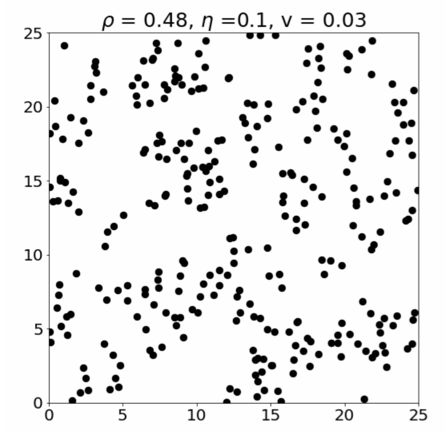

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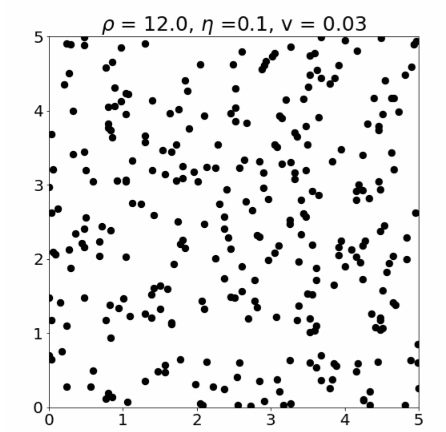


(b)

FIG. 1. Snapshots of a single frame from the simulation of Vicsek Model.a) demonstrates agents forming groups and moving coherently b) no group formation rather a well ordered motion where all agents move in same direction is observed

the original Viscek model paper [1]. In all of the simulations that were carried out, the initial condition was N agents that were randomly distributed across the field with their velocity vectors aligned in random directions.

FIG.1 shows snapshots of a frame from simulation of Viscek Model. The input parameters of these simulations were taken to be the same as those used to obtain FIG.1 b and d in the original Vicsek model paper[1]. In [1], it is claimed that for small densities and noise, the agents would form groups that move together. FIG 1a is simulated using $N = 300$, $L = 25$ and $\eta = 0.1$, the same parameters used to simulate FIG.1.b in [1]. It is observed from the simulations carried out by the author of this term paper that this is indeed true. The animations of the simulation showed that the agents tend to form groups and the groups as a whole try to move coherently. This behaviour is demonstrated in the .gif file of FIG. 1.a which can be viewed here.

In FIG.1.d of Vicsek et al [1] , it is mentioned that at high densities and small noise the motion of the particles become ordered. The simulation in FIG.1.b is in agreement with this. This simulation is carried out using $N = 300$, $L = 5$ and $\eta = 0.1$, the same parameters used to simulate FIG.1.d in [1]. The animations of the simulations showed that agents moved in a very ordered fashion. This is demonstrated in the .gif file of FIG.1.b which can be viewed here

B. Kinetic Order Parameter

Vicsek et al defines a parameter v_a where

$$v_a = \frac{1}{Nv}\left|\sum_{i=1}^{N} \vec{v_i}\right| \tag{3}$$

This is nothing but the absolute value of the average velocity over all agents, normalized by dividing by Nv which is the maximum possible value of the magnitude of average velocity.

i If the velocity of the agents are aligned in random directions, then v_a would be zero.

ii If the velocity of all agents are aligned in the same direction, then the system attains maximum order in terms of the direction of the velocities. v_a would also give us 1 in this case.

iii If there is any amount of order by virtue of collective motion of particle then v_a would be non-zero. Greater the order greater is the value of v_a.

C. Phase transition

As already observed in FIG.1 , the order in the system changes when the parameter ρ is varied. It would also change when η is varied. In order to study the effect of change in parameter η to the order, the following analysis is carried out. A range of η between $0 - 5$ is chosen. For each value of η in this range, the simulation is run for 5 times with different initial conditions. The v_a for the final frame(300^{th} frame as per the simulations of the author of this term paper) of the simulation is stored and the average v_a

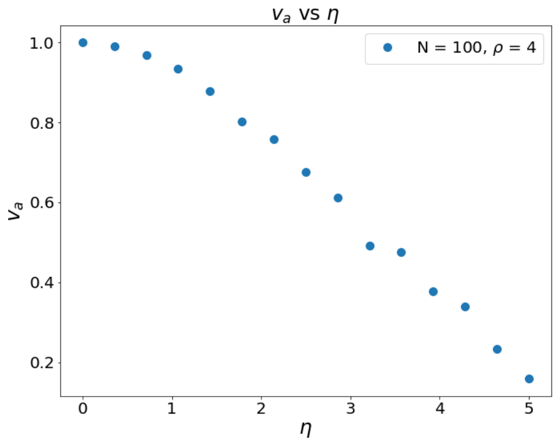
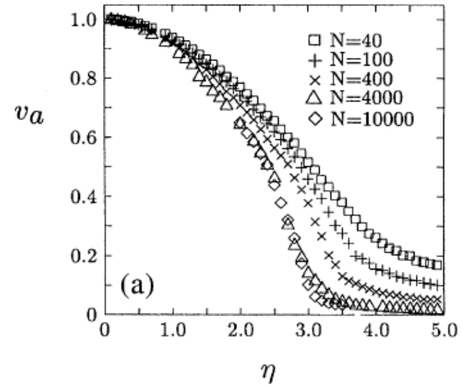


FIG. 2. Plot of v_a vs η. Field size was 5×5, with magnitude of velocity, $v = 0.03$. The figure demonstrates increasing order with decreasing noise strength η



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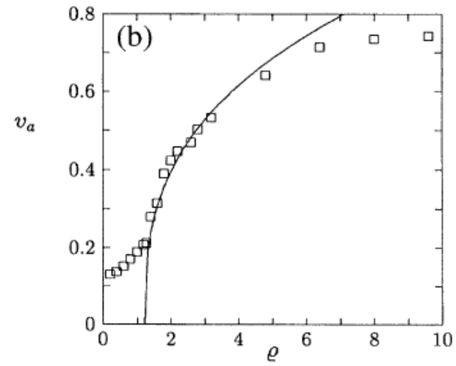


(b)

FIG. 3. Figures from Vicsek et al [1]

across the 5 simulations is computed and plotted against the particular value of η. The idea is to calculate and ensemble average of v_a. Larger the number of trials taken the better. Here only 5 trials was taken because of the computational costs of the simulation

FIG.2 shows the plot obtained after carrying out the analysis. It is observed that as the value of η or equivalently the amount of noise is decreased, keeping ρ constant, the kinetic order parameter increases. With decreasing noise, we observe an increase in order in the system.

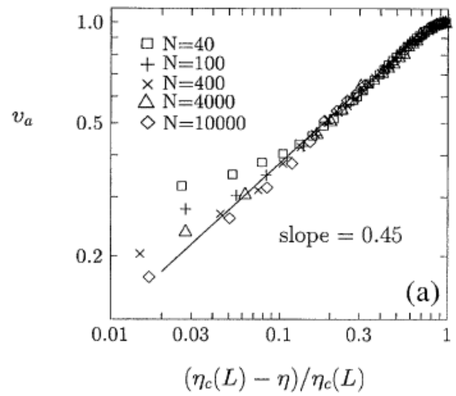
FIG.3. shows the plots from Vicsek et al [1]. It is clear that FIG.2 captures the qualitative aspects of FIG.3.a . Any difference in the two figures has to be attributed to the number of η values at which the simulation is carried out and the number of trials for which the simulation is done for a particular value of η. It is to be noted that these simulations are computationally expensive; hence there is a limitation to the number of runs of the simulation for obtaining the plot in FIG.2 .

Refering to FIG.3.a . All of the plots in the figure are obtained by setting a constant value of ρ and varying the parameter η. Vicsek et al observes that the behaviour of the kinetic order parameter v_a is similar to that of order parameters of equilibrium systems close to the critical point. One can make the observation that Vicsek model is similar to the ferromagnet Ising model [2] considering that the Hamiltonian which tends to align the Ising spins in same direction is replaced by the tendency of the individual agents to align their velocities in the same direction described by 1. The amplitude η of the random perturbation $\Delta\theta$ plays the role of temperature.

FIG.4 a and b are obtained by plotting $\ln v_a$ vs $\ln \frac{\eta_c(L)-\eta}{\eta_c(L)}$ at a fixed ρ and $\ln \frac{\rho-\rho_c(L)}{\rho_c(L)}$ at a fixed η respectively. For finite size $\eta_c(L)$ and $\rho_c(L)$ would depend on L. Thus the authors of [1] obtained these critical values by employing an indirect method which involves the analysing the 'straightness' of the data sets, i.e those values of quantities for which the plots in FIG.4 were straightest in relevant regions of noise or density values were used.

A key observation to be made here is that the plot in FIG.4.a obeyed the scaling relation for over a larger region as the system size L increase. This strongly suggests a kinetic phase transition in the limit $L \to \infty$, that is continuous, and follows the scaling relation analogous to phase transition in equilibruim systems:

$$\begin{aligned} v_a &\sim [\eta_c(\rho) - \eta]^{\beta} \\ v_a &\sim [\rho - \rho_c(\eta)]^{\delta} \end{aligned} \tag{4}$$



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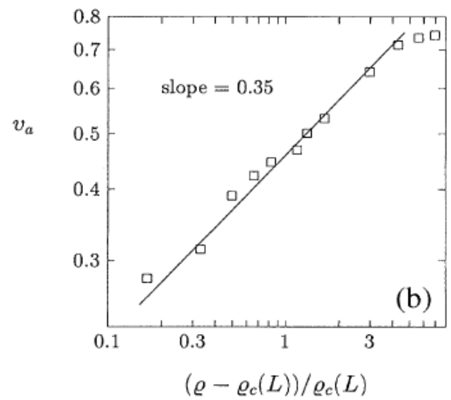

(b)

FIG. 4. Figures from Vicsek et al [1].a)Plot of $\ln v_a$ vs $\ln \frac{\eta_c(L)-\eta}{\eta_c(L)}$ at $\rho = 0.4$ with a linear fit.b)Plot of $\ln v_a vs \ln \frac{\rho-\rho_c(L)}{\rho_c(L)}$ at $L = 20$ and $\eta = 2$ along with a linear fit

From FIG.4.a and b, the critical exponents β and δ are estimated to be $0.45 \pm 0/07$ and 0.35 ± 0.06 respectively. These are conservative estimates owing to the sensitivity of their calculation on $\eta_c(L)$ and $\rho_c(L)$ combined with the fact that both $\eta_c(L)$ and $\rho_c(L)$ are calculated using indirect methods mentioned previously.Using a finite size scaling analysis of $\eta_c(L)$ Vicsek et al obtained the value of $\eta_c(\infty)$ as 2.9 ± 0.05 for $\rho = 0.4$

IV. DISCUSSION

In the following study, I tried to simulate collective dynamics using Vicsek Model. A few short-comings of my simulations needs to be pointed out. In the Viscek et al paper the figures corresponding to the simulation of the model demonstrated the particles with the direction of their velocity using an arrow along with the trajectory of the last 20 steps using a continuous curve. Presenting the simulations in this manner would allow for easier and better comprehension of finer aspects of the system dynamics at various parameter values. In my simulations,the agents are depicted as dots without any arrows. This form of depiction makes it difficult to comprehend the instantaneous direction of velocities of individual agents and other finer details concerning the dynamics. Nevertheless the simulations did capture those relevant features of the dynamics, which Vicsek et al claimed to have observed in parameter values mentioned in relation with FIG.1. b and d of the Vicsek et al paper.The simulations also gave FIG.2 which retained the qualitative aspects of FIG.2.a of Vicsek et al.

Further study of Vicsek et al revealed that the sytem described by Vicsek model demonstrates a kinetic phase transition analogous to the continuous phase transition observed in equilibrium systems, following the scaling relations given by 4. The authors of [1] also obtained the values of these critical exponents. Using a finite size scaling analysis of $\eta_c(L)$ they obtained the value of $\eta_c(\infty)$ as 2.9 ± 0.05 for $\rho = 0.4$.

The Vicsek Model is a very simple model that was able to demonstrate self-propelled collective behavior. Further studies of the model can be done by tweaking other parameters of the model and making changes in the update rules of the model formalism. In doing so one can simulate a wide variety of active matter systems.

V. ACKNOWLEDGEMENTS

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VI. REFERENCE

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