

AI1103-Assignment 2

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Download all python codes from

https://github.com/Ramanathan-Annamalai/AI1103-Probability_and_Random_Variables/tree/main/Assignment%202/Codes

and latex-tikz codes from

https://github.com/Ramanathan-Annamalai/AI1103-Probability_and_Random_Variables/blob/main/Assignment%202/Assignment_2.tex

QUESTION

Suppose we uniformly and randomly select a permutation from the $20!$ permutations of $1, 2, 3, \dots, 20$. What is the probability that 2 appears at an earlier position than any other even number in the selected permutation.

- (A) $\frac{1}{2}$
 (B) $\frac{1}{10}$
 (C) $\frac{9!}{20!}$
 (D) None of the above.

SOLUTION

Total number of permutations = T

$$T = 20! \quad (0.0.1)$$

No. of ways of choosing 10 places for the odd numbers

$$= {}^{20}C_{10} \quad (0.0.2)$$

No. of ways of arranging the odd numbers

$$= 10! \quad (0.0.3)$$

Since 2 should occur before other even numbers, the first blank place in the permutation has to be 2.

No. of ways of arranging the even numbers other than 2 in the remaining 9 places

$$= 9! \quad (0.0.4)$$

Number of favourable permutations

$$= {}^{20}C_{10} \times 10! \times 9! \quad (0.0.5)$$

o o o 2 e o e e o o e e o e o o e e e o

o → choose positions for odd numbers

2 → place 2 in the first empty space

e → place remaining even numbers

Now, Probability of 2 appears at an earlier position than any other even number in the selected permutation = p

$$\begin{aligned} p &= \frac{\text{No. of favourable permutations}}{\text{Total no. of permutations}} \\ &= \frac{{}^{20}C_{10} \times 10! \times 9!}{20!} && (0.0.6) \\ &= \frac{1}{10} && (0.0.7) \end{aligned}$$

Answer: **Option (B).**