AI1103-Assignment 2

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Download all python codes from

https://github.com/Ramanathan−Annamalai/AI1103−Probability_and_Random_Variables/tree/main/Assignment%202/Codes

and latex-tikz codes from

https://github.com/Ramanathan−Annamalai/AI1103−Probability_and_Random_Variables/blob/main/Assignment%202/Assignment_2.tex

Question

Suppose we uniformly and randomly select a permutation from the 20! permutations of 1,2,3,...,20. What is the probability that 2 appears at an earlier position than any other even number in the selected permutation.

(A) $\frac{1}{2}$

(B) $\frac{1}{10}$

(C) $\frac{9!}{20!}$

(D) None of the above.

Solution

Total number of permutations = T

$$T = 20! \tag{0.0.1}$$

No. of ways of choosing 10 places for the odd numbers

$$= {}^{20}C_{10} \tag{0.0.2}$$

No. of ways of arranging the odd numbers

$$= 10! \tag{0.0.3}$$

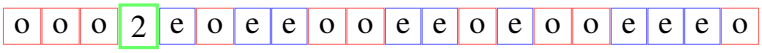
Since 2 should occur before other even numbers, the first blank place in the permutation has to be 2.

No. of ways of arranging the even numbers other than 2 in the remaining 9 places

$$= 9! \tag{0.0.4}$$

Number of favourable permutations

$$= {}^{20}C_{10} \times 10! \times 9! \tag{0.0.5}$$



o ⟶ choose random positions for odd numbers

2 ⟶ place 2 in the first empty space

e ⟶ place remaining even numbers

Now, Probability of 2 appears at an earlier position than any other even number in the selected permutation = p

$$p = \frac{\text{No. of favourable permutations}}{\text{Total no. of permutations}}$$

$$= \frac{{}^{20}C_{10} \times 10! \times 9!}{20!} \tag{0.0.6}$$

$$= \frac{1}{10} \tag{0.0.7}$$

Answer: **Option (B)**.