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CSC263 Problem Set 8

1.
By the definition of a tree we know that it has n-1 edges where n is the amount of vertices in the graph/tree.
We also know that through these n-1 edges every single vertex in the graph/tree is connected to each other.

a)
Suppose there is an arbitrary vertex v and v', we know that there is exactly 1 path we can take through the edges of the
tree to travel from v to v'. If we were to add an edge to either v or v' to any other vertex v'' we would create another possible
path that could be traveled from v to v'' back to v through another path and then to v'. This by definition is the creation of a cycle in the tree.

b and 2)
Since there are only n-1 edges in the tree, if we were to remove one the tree would then have n-2 edges. Because there are n vertices in the
graph we need at least n-1 edges to connect all of them together to form some type of tree or graph. With only n-2 edges at least one vertex or a group of vertices will be disconnected from the other vertex or group of vertices. So by removing any edge e in the tree T it will create two subtrees S and U such that S + U = T when adding any edge from any vertex from S to U because all the vertices in S are not in U and vice versa. By adding any edge e between any vertex p in S and any vertex q in U it will create a tree since the graph will have n-1 edges and there will be no cycles since vertices in S and U are unique from one another and completely disconnected, so by definition this addition of the edge e creates a tree.