This assignment is due at the **start** of your lecture on Friday, 31 October 2014.

For the questions that require you to write a MatLab program, hand-in the program and its output as well as any written answers requested in the question. Your program and its output, as well as your written answers, will be marked. Your program should conform to the usual CS standards for comments, good programming style, etc. When first learning to program in MatLab, students often produce long, messy output. Try to format the output from your program so that it is easy for your TA to read and to understand your results. To this end, you might find it helpful to read "A short description of fprintf" on the course webpage. Marks will be awarded for well-formatted, easy-to-read output.

1. [5 marks]

   Assume x and y are vectors in $\mathbb{R}^2$. Is it possible to have $\|x\|_1 > \|y\|_1$ and $\|x\|_\infty < \|y\|_\infty$?

   If so, give an example.

   If not, explain why not.

2. [4 marks: 2 marks for each part]

   I mentioned in class that $\|x\|_\infty \le \|x\|_2 \le \|x\|_1$ for all vectors $x \in \mathbb{R}^n$ and all integers $n \ge 1$.

   A similar result does <u>not</u> hold for matrices. To illustrate this,

   (a) find a 2×2 matrix A such $\|A\|_\infty < \|A\|_1$;

   (b) find another 2×2 matrix B such that $\|B\|_\infty > \|B\|_1$.

3. [5 marks]

   We showed in class that, if A is nonsingular, $Ax = b$, $A\hat{x} = \hat{b}$ and $b \ne \underset{\sim}{0}$, where $\underset{\sim}{0}$ is the vector with all elements equal to 0, then

$$\frac{\|\Delta x\|}{\|x\|} \le \text{cond}(A) \frac{\|\Delta b\|}{\|b\|} \tag{1}$$

   where $\Delta x = \hat{x} - x$, $\Delta b = \hat{b} - b$, $\|\cdot\|$ is any vector norm, $\text{cond}(A) = \|A\|\,\|A^{-1}\|$ and the matrix norm used for cond(A) is the matrix norm *subordinate to* the vector norm used in (1).

I mentioned in class that you can also show that, if A is nonsingular, $Ax = b$, $A\hat{x} = \hat{b}$ and $b \neq \underset{\sim}{0}$, where $\underset{\sim}{0}$ is the vector with all elements equal to 0, then

$$\frac{1}{\text{cond}(A)} \frac{\|\Delta b\|}{\|b\|} \leq \frac{\|\Delta x\|}{\|x\|} \tag{2}$$

where $\Delta x = \hat{x} - x$, $\Delta b = \hat{b} - b$, $\|\cdot\|$ is any vector norm, $\text{cond}(A) = \|A\| \, \|A^{-1}\|$ and the matrix norm used for $\text{cond}(A)$ is the matrix norm *subordinate to* the vector norm used in (1).

Show that, under the conditions stated above, (2) is true.

4. [10 marks: 5 marks for each part]

(a) Do problem 2.3 on page 100 of your textbook. (It starts on page 100 and continues onto page 101 of your textbook.)

Your textbook says to "use a library routine to solve the system of linear equations for the vector f of member forces". Use the MatLab backslash operator \ instead to solve the linear system of equations $Af = b$ that arises in this problem for the vector f of member forces.

Hint: read "help mldivide" in MatLab.

Suggestion: you might find it easiest to hard-code the matrix, A, and right side vector, b, into your MatLab program. If you choose this option, you may want to start by using the MatLab function "zeros" (read "help zeros" in MatLab) to construct a matrix and a vector with all elements equal to zero and then change the appropriate elements to their nonzero values.

(b) Let $\hat{f}$ be the computed solution to the linear system $Af = b$ from part (a). Use the error bound (1) in Question 3 above to bound the relative error, $\|\hat{f} - f\|/\|f\|$, associated with the computed solution $\hat{f}$ in terms of the condition number of the matrix A associated with the system $Af = b$ and the relative residual, $\|r\|/\|b\|$, where $r = b - A\hat{f}$ is the residual associated with the computed solution $\hat{f}$.

Hint: read "help cond" in MatLab and make sure that the norm associated with the condition number that you use agrees with the vector norm that you use for the relative error and the relative residual.

Hand in your program and its output.

5. [10 marks]

Do problem 2.6 on page 101 of your textbook. (It starts on page 101 and continues onto page 102 of your textbook.)

Your textbook says to use a library routine for Gaussian elimination to solve the problem. Instead you should use MatLab to solve this problem. In particular, use the MatLab backslash operator \ to solve the linear system of equations $Hx = b$, where H is the Hilbert matrix.

Read "help mldivide" and "help hilb" in MatLab.

Use the MatLab function "cond" for the condition number estimator.

Read "help cond" in MatLab.

Since you are asked to use the ∞-norm for the vectors in this question, make sure that you use the condition number estimator associated with the ∞-norm also.

Your textbook asks, "How large can you take n before the error is 100 percent?" The n here is the dimension of the Hilbert matrix, H (i.e., $H = \text{hilb}(n)$). This question is not very well-defined. So, instead, determine how large can you take n before the relative error in the solution, $\|x - \hat{x}\|/\|x\|$, is greater than or equal to 1? To support your answer, print out a table of values of n and $\|x - \hat{x}\|/\|x\|$ for that n.

Your textbook also asks you to "try to characterize the condition number as a function of n". To do this, you might find it helpful to plot n versus the log of the condition number of hilb(n), for $n = 2, 3, \ldots, 13$. What does this tell you about the relationship between n and the condition number of hilb(n)?

Finally, your textbook asks you "as n varies, how does the number of correct digits in the components of the computed solution relate to the condition number of the matrix" (i.e., hilb(n))? Print a table of appropriate values to support your answer.

Hand in your program, its output and your answers to all the questions above and in problem 2.6.