This assignment is due at the **start** of the class on **Monday, 1 December 2014**.
(Note that the assignment is due on a Monday, not the usual Friday.)

For the questions that require you to write a MatLab program, hand-in the program and its output as well as any written answers requested in the question. Your program and its output, as well as your written answers, will be marked. Your program should conform to the usual CS standards for comments, good programming style, etc. When first learning to program in MatLab, students often produce long, messy output, but you should be an experienced MatLab programmer now. So, try to format the output from your programs so that it is easy for your TA to read and to understand your results.

1. [10 marks: 5 marks each for parts (a) and (b)]

   Consider the equation
   $$f(x) = x^2 - 2$$
   The roots of $f(x)$ are obviously $\pm\sqrt{2}$.

   (a) Write a MatLab program that starts with the initial guess $x_0 = 1$ and uses Newton's method to compute the positive root of $f(x)$. Your program should print a table of values with the header

n	x_n	$x_n - \sqrt{2}$

   You may not be able to do subscripts properly in MatLab; if so, you can use $x(n)$ instead of x_n. Similarly for $\sqrt{2}$. Also, add additional spaces between the n, x_n and $x_n - \sqrt{2}$ as necessary to make the table "look nice".

   Your program should then print six lines, one line for each of $n = 0, 1, 2, \ldots, 5$. Each line should contain the values for n, x_n and $x_n - \sqrt{2}$ for the n associated with that line. Print x_n and $x_n - \sqrt{2}$ to 16 significant digits.

   (b) Write a MatLab program that starts with the initial guesses $x_0 = 1$ and $x_1 = 2$ and uses the secant method to compute the positive root of $f(x)$. Your program should print a table of values with the header

n	x_n	$x_n - \sqrt{2}$

   You may not be able to do subscripts properly in MatLab; if so, you can use $x(n)$ instead of x_n. Similarly for $\sqrt{2}$. Also, add additional spaces between the n, x_n and $x_n - \sqrt{2}$ as necessary to make the table "look nice".

   Your program should then print eight lines, one line for each of $n = 0, 1, 2, \ldots, 7$. Each line should contain the values for n, x_n and $x_n - \sqrt{2}$ for the n associated with that line. Print x_n and $x_n - \sqrt{2}$ to 16 significant digits.

2. [10 marks: 5 marks each for parts (a) and (b)]

   Do question 5.2 on page 250 of your textbook.

3. [10 marks]

   Do question 5.14 on page 251 of your textbook.

   The "zero finder" that you should use for this question is the MatLab function fzero. In particular, use the version of fzero described under "Root Starting From an Interval" on the webpage `http://www.mathworks.com/help/matlab/ref/fzero.html`.

4. [12 marks: 5 marks each for parts (a) and (b); 2 marks for part (c)]

   This question is taken from the December 2011 CSC 336 exam.

   The Cray 1 supercomputer did not have a divide unit. Instead, to compute a/b for $b \neq 0$, it first computed the reciprocal $r = 1/b$ and then computed the product $a \cdot r$.

   To compute the reciprocal $r = 1/b$, for $b \neq 0$, it used the fact that r is the solution of the equation

   $$f(x) = 1/x - b = 0 \tag{1}$$

   It first found an initial approximation r_0 to r that was accurate to about half the digits in a floating-point number. Then, using r_0 as an initial guess, it did one iteration of Newton's method applied to (1) to compute a final approximation r_1 to r.

   (a) Show that, if you apply Newton's method to (1), it is possible to re-arrange the terms in the resulting formula so that no divisions are required.

   Write the formula in as computationally effective form as possible.

   (b) Show that the error satisfies

   $$\frac{r - r_1}{r} = \left(\frac{r - r_0}{r} \right)^2 \tag{2}$$

   where $r = 1/b$.

   (c) Why does (2) imply that r_1 has roughly twice as many correct digits as r_0 has?