1000005754
Ramaneek Gill

CSC336 Assignment 2

1.
It is possible for $||x||_1 > ||y||_1$ and $||x||_\infty < ||y||_\infty$
Let $x = [3, 2\ 2]^T$ and $y = [4\ 0\ 2]^T$

$||x||_1 = 3 + 2 + 2 = 7$ $\qquad$ $||y||_1 = 4 + 0 + 2 = 8$

$||x||_\infty = \max(3,2,2) = 3$ $\qquad$ $||y||_\infty = \max(4,0,2) = 4$

Therefore $||x||_1 > ||y||_1$ and $||x||_\infty < ||y||_\infty$

2. a)

$A = \begin{bmatrix} 5 & 2 \\ 5 & 1 \end{bmatrix}$

$||A||_\infty = \max(5 + 2, 5 + 1) = 7$

$||A||_1 = \max(5 + 5, 2 + 1) = 10$

Therefore $||A||_1 > ||A||_\infty$

b)

$B = \begin{bmatrix} 5 & 5 \\ 2 & 1 \end{bmatrix}$

$||B||_\infty = \max(5 + 5, 2 + 1) = 10$

$||B||_1 = \max(5 + 2, 5 + 1) = 7$

Therefore $||B||_1 < ||B||_\infty$

3.
Need to show: $\frac{1}{cond(A)} \frac{\|\Delta b\|}{\|b\|} \leq \frac{\|\Delta x\|}{\|x\|}$

Remember that $cond(A) = ||A||||A||^{-1}$, $\Delta x = x - \hat{x}$, $\Delta b = b - \hat{b}$

$$Ax = b$$
$$x = A^{-1}b$$
$$||x|| = ||A^{-1}b|| \leq ||A^{-1}||||b||$$
$$\frac{||b||}{||A||} \leq ||x|| \leq ||A^{-1}||||b||$$

This method also gets us
$\frac{||\Delta b||}{||A||} \leq ||\Delta x|| \leq ||A^{-1}||||\Delta b||$
By following very similar steps as the equation before
Now continuing on…
$$\frac{1}{||A^{-1}||||b||} \leq \frac{1}{||x||}$$
Done by just rearranging the variables to the other side

Now by combining the previous two equations we get:

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$$\frac{1}{||A||||A^{-1}||} \cdot \frac{||\Delta b||}{||b||} \leq \frac{||\Delta x||}{||x||}$$

This was done by multiplying the equations
Now we just apply the definitions of cond(A)

$$\frac{1}{cond(A)} \cdot \frac{||\Delta b||}{||b||} \leq \frac{||\Delta x||}{||x||}$$

Therefore under the conditions stated in the question the above equation holds true!

QUESTION 4
///
A)

A\B =

10.8579
-7.6777
10.0000
-25.0000
46.2132
-7.6777
0
-25.0000
3.5355
22.5000
20.0000
-31.8198
22.5000

B)

$$\frac{||f-\hat{f}||}{||f||} \leq cond(A) \cdot \frac{||r||}{||b||} \leq 404.2256$$

///
code: q4.m

```
A = zeros(13, 13);
B = zeros(13, 1);

B(2,1) = 10;
B(8,1) = 15;
B(10,1) = 20;

A(1,2) = 1;
A(1,6) = -1;
A(2,3) = 1;
A(3,1) = -sqrt(2)/2;
A(3,4) = 1;
```

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```
A(3,5) = sqrt(2)/2;
A(4,1) = sqrt(2)/2;
A(4,3) = 1;
A(4,4) = sqrt(2)/2;
A(5,4) = 1;
A(5,8) = -1;
A(6,7) = 1;
A(7,5) = sqrt(2)/2;
A(7,6) = 1;
A(7,9) = -sqrt(2)/2;
A(7,10) = -1;
A(8,5) = sqrt(2)/2;
A(8,7) = 1;
A(8,8) = sqrt(2)/2;
A(9,10) = 1;
A(9,13) = -1;
A(10,11) = 1;
A(11,8) = 1;
A(11,9) = sqrt(2)/2;
A(11,12) = -sqrt(2)/2;
A(12,9) = sqrt(2)/2;
A(12,11) = 1;
A(12,12) = sqrt(2)/2;
A(13,12) = sqrt(2)/2;
A(13,13) = 1;

A
B
A\B
cond = cond(A, 1)
r = norm(B - A*A\B, 1)
b = norm(B, 1)
disp(['||f-f.hat|| / ||f|| is less than or equal to ', num2str(cond*r/b)]);
```

output:

```
>> q4

A =

  Columns 1 through 7

         0    1.0000         0         0         0   -1.0000         0
         0         0    1.0000         0         0         0         0
   -0.7071         0         0    1.0000    0.7071         0         0
    0.7071         0    1.0000    0.7071         0         0         0
```

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```
       0         0         0    1.0000         0         0         0
       0         0         0         0         0         0    1.0000
       0         0         0         0    0.7071    1.0000         0
       0         0         0         0    0.7071         0    1.0000
       0         0         0         0         0         0         0
       0         0         0         0         0         0         0
       0         0         0         0         0         0         0
       0         0         0         0         0         0         0
       0         0         0         0         0         0         0

  Columns 8 through 13

       0         0         0         0         0         0
       0         0         0         0         0         0
       0         0         0         0         0         0
       0         0         0         0         0         0
 -1.0000         0         0         0         0         0
       0         0         0         0         0         0
       0   -0.7071   -1.0000         0         0         0
  0.7071         0         0         0         0         0
       0         0    1.0000         0         0   -1.0000
       0         0         0    1.0000         0         0
  1.0000    0.7071         0         0   -0.7071         0
       0    0.7071         0    1.0000    0.7071         0
       0         0         0         0    0.7071    1.0000


B =

     0
    10
     0
     0
     0
     0
     0
    15
     0
    20
     0
     0
     0


ans =

   10.8579
```

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```
   -7.6777
   10.0000
  -25.0000
   46.2132
   -7.6777
         0
  -25.0000
    3.5355
   22.5000
   20.0000
  -31.8198
   22.5000


cond =

   27.7279


r =

  656.0229


b =

    45
```

||f-f.hat|| / ||f|| is less than or equal to 404.2256

//

QUESTION 5
//
Refer to the program output below for the justification for these answers.

A) The Hilbert Matrix can go up to 13 dimensions before its relative error is greater than or equal to 1.

B) As n gets bigger so does the condition number for the hilbert matrix with a dimension of n. This shows that the bigger the hilbert matrix is the bigger the error will be in calculating x_hat. The condition number seems to grow exponentially based on n. This is observed by the log10 operation used condition number of the Hilbert Matrix.

C) As n grows by 1 each loop, the amount of correct digits seem to decrease

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by slightly more than 1. When n == 1 the correctness of x_hat is almost at the magnitude of a single precision floating point number. As n grows the correct digits continue to decrease to the point where no digit is in x_hat is correct when n = 13.

//
code: q5.m

```
n = 1;
H = hilb(n);
x = ones(n);
B = H*x;
xhat = H\B;
relerror = norm(x-xhat, inf)/norm(x,inf);

while relerror < 1
        H = hilb(n);
        con = cond(H, inf);
        x = ones(n);
        B = H*x;
        xhat = H\B;
        relerror = norm(x-xhat, inf)/norm(x,inf);
        digits = -log10(relerror);
        lcond = log10(con);

        disp(['N = ',num2str(n),' relative error: ',num2str(relerror),' condition number: ',num2str(con),'
log10(con): ',num2str(lcond),' # correct digits: ',num2str(digits)]);
        n = n+1;
end
```

output: //Please ignore the warnings from the program.

```
N = 1 relative error: 0 condition number: 1 log10(con): 0 # correct digits: Inf
N = 2 relative error: 7.7716e-16 condition number: 27 log10(con): 1.4314 # correct digits: 15.1095
N = 3 relative error: 4.885e-15 condition number: 748 log10(con): 2.8739 # correct digits: 14.3111
N = 4 relative error: 2.9587e-13 condition number: 28375 log10(con): 4.4529 # correct digits: 12.5289
N = 5 relative error: 1.9926e-12 condition number: 943656 log10(con): 5.9748 # correct digits: 11.7006
N = 6 relative error: 4.6634e-10 condition number: 29070279.0029 log10(con): 7.4634 # correct digits:
9.3313
N = 7 relative error: 2.0028e-08 condition number: 985194889.7198 log10(con): 8.9935 # correct digits:
7.6984
N = 8 relative error: 4.3834e-07 condition number: 33872790819.4947 log10(con): 10.5299 # correct
digits: 6.3582
N = 9 relative error: 1.9351e-05 condition number: 1099650991701.052 log10(con): 12.0413 # correct
digits: 4.7133
```

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N = 10 relative error: 0.00037952 condition number: 35353724553756.42 log10(con): 13.5484 # correct digits: 3.4208
N = 11 relative error: 0.0066196 condition number: 1230369938308720 log10(con): 15.09 # correct digits: 2.1792
Warning: Matrix is close to singular or badly scaled.
Results may be inaccurate. RCOND = 2.632766e-17.
> In cond at 48
In q5 at 11
Warning: Matrix is close to singular or badly scaled.
Results may be inaccurate. RCOND = 2.409320e-17.
> In q5 at 14
N = 12 relative error: 0.2395 condition number: 37983201226912104 log10(con): 16.5796 # correct digits: 0.62069
Warning: Matrix is close to singular or badly scaled.
Results may be inaccurate. RCOND = 2.339949e-18.
> In cond at 48
In q5 at 11
Warning: Matrix is close to singular or badly scaled.
Results may be inaccurate. RCOND = 2.339949e-18.
> In q5 at 14
N = 13 relative error: 5.7974 condition number: 427595335326831488 log10(con): 17.631 # correct digits: -0.76323