CSC336 Assignment 2

1.

It is possible for and

Let and

Therefore and

2. a)

Therefore

b)

Therefore

3.

Need to show:

Remember that

This method also gets us

By following very similar steps as the equation before

Now continuing on…

Done by just rearranging the variables to the other side

Now by combining the previous two equations we get:

This was done by multiplying the equations

Now we just apply the definitions of cond(A)

Therefore under the conditions stated in the question the  
 above equation holds true!

QUESTION 4

////////////////////////////////////////////////////////////////////////////////////////////////

A)

A\B =

10.8579

-7.6777

10.0000

-25.0000

46.2132

-7.6777

0

-25.0000

3.5355

22.5000

20.0000

-31.8198

22.5000

B)

////////////////////////////////////////////////////////////////////////////////////////////////

code: q4.m

A = zeros(13, 13);

B = zeros(13, 1);

B(2,1) = 10;

B(8,1) = 15;

B(10,1) = 20;

A(1,2) = 1;

A(1,6) = -1;

A(2,3) = 1;

A(3,1) = -sqrt(2)/2;

A(3,4) = 1;

A(3,5) = sqrt(2)/2;

A(4,1) = sqrt(2)/2;

A(4,3) = 1;

A(4,4) = sqrt(2)/2;

A(5,4) = 1;

A(5,8) = -1;

A(6,7) = 1;

A(7,5) = sqrt(2)/2;

A(7,6) = 1;

A(7,9) = -sqrt(2)/2;

A(7,10) = -1;

A(8,5) = sqrt(2)/2;

A(8,7) = 1;

A(8,8) = sqrt(2)/2;

A(9,10) = 1;

A(9,13) = -1;

A(10,11) = 1;

A(11,8) = 1;

A(11,9) = sqrt(2)/2;

A(11,12) = -sqrt(2)/2;

A(12,9) = sqrt(2)/2;

A(12,11) = 1;

A(12,12) = sqrt(2)/2;

A(13,12) = sqrt(2)/2;

A(13,13) = 1;

A

B

A\B

cond = cond(A, 1)

r = norm(B - A\*A\B, 1)

b = norm(B, 1)

disp(['||f-f.hat|| / ||f|| is less than or equal to ', num2str(cond\*r/b)]);

output:

>> q4

A =

Columns 1 through 7

0 1.0000 0 0 0 -1.0000 0

0 0 1.0000 0 0 0 0

-0.7071 0 0 1.0000 0.7071 0 0

0.7071 0 1.0000 0.7071 0 0 0

0 0 0 1.0000 0 0 0

0 0 0 0 0 0 1.0000

0 0 0 0 0.7071 1.0000 0

0 0 0 0 0.7071 0 1.0000

0 0 0 0 0 0 0

0 0 0 0 0 0 0

0 0 0 0 0 0 0

0 0 0 0 0 0 0

0 0 0 0 0 0 0

Columns 8 through 13

0 0 0 0 0 0

0 0 0 0 0 0

0 0 0 0 0 0

0 0 0 0 0 0

-1.0000 0 0 0 0 0

0 0 0 0 0 0

0 -0.7071 -1.0000 0 0 0

0.7071 0 0 0 0 0

0 0 1.0000 0 0 -1.0000

0 0 0 1.0000 0 0

1.0000 0.7071 0 0 -0.7071 0

0 0.7071 0 1.0000 0.7071 0

0 0 0 0 0.7071 1.0000

B =

0

10

0

0

0

0

0

15

0

20

0

0

0

ans =

10.8579

-7.6777

10.0000

-25.0000

46.2132

-7.6777

0

-25.0000

3.5355

22.5000

20.0000

-31.8198

22.5000

cond =

27.7279

r =

656.0229

b =

45

||f-f.hat|| / ||f|| is less than or equal to 404.2256

/////////////////////////////////////////////////////////////////////

QUESTION 5

/////////////////////////////////////////////////////////////////////

Refer to the program output below for the justification for these

answers.

A) The Hilbert Matrix can go up to 13 dimensions before its relative

error is greater than or equal to 1.

B) As n gets bigger so does the condition number for the hilbert matrix

with a dimension of n. This shows that the bigger the hilbert matrix is

the bigger the error will be in calculating x\_hat. The condition number

seems to grow exponentially based on n. This is observed by the log10

operation used condition number of the Hilbert Matrix.

C) As n grows by 1 each loop, the amount of correct digits seem to decrease

by slightly more than 1. When n == 1 the correctness of x\_hat is almost

at the magnitude of a single precision floating point number. As n

grows the correct digits continue to decrease to the point where no

digit is in x\_hat is correct when n = 13.

/////////////////////////////////////////////////////////////////////

code: q5.m

n = 1;

H = hilb(n);

x = ones(n);

B = H\*x;

xhat = H\B;

relerror = norm(x-xhat, inf)/norm(x,inf);

while relerror < 1

H = hilb(n);

con = cond(H, inf);

x = ones(n);

B = H\*x;

xhat = H\B;

relerror = norm(x-xhat, inf)/norm(x,inf);

digits = -log10(relerror);

lcond = log10(con);

disp(['N = ',num2str(n),' relative error: ',num2str(relerror),' condition number: ',num2str(con),' log10(con): ',num2str(lcond),' # correct digits: ',num2str(digits)]);

n = n+1;

end

output: //Please ignore the warnings from the program.

N = 1 relative error: 0 condition number: 1 log10(con): 0 # correct digits: Inf

N = 2 relative error: 7.7716e-16 condition number: 27 log10(con): 1.4314 # correct digits: 15.1095

N = 3 relative error: 4.885e-15 condition number: 748 log10(con): 2.8739 # correct digits: 14.3111

N = 4 relative error: 2.9587e-13 condition number: 28375 log10(con): 4.4529 # correct digits: 12.5289

N = 5 relative error: 1.9926e-12 condition number: 943656 log10(con): 5.9748 # correct digits: 11.7006

N = 6 relative error: 4.6634e-10 condition number: 29070279.0029 log10(con): 7.4634 # correct digits: 9.3313

N = 7 relative error: 2.0028e-08 condition number: 985194889.7198 log10(con): 8.9935 # correct digits: 7.6984

N = 8 relative error: 4.3834e-07 condition number: 33872790819.4947 log10(con): 10.5299 # correct digits: 6.3582

N = 9 relative error: 1.9351e-05 condition number: 1099650991701.052 log10(con): 12.0413 # correct digits: 4.7133

N = 10 relative error: 0.00037952 condition number: 35353724553756.42 log10(con): 13.5484 # correct digits: 3.4208

N = 11 relative error: 0.0066196 condition number: 1230369938308720 log10(con): 15.09 # correct digits: 2.1792

Warning: Matrix is close to singular or badly scaled.

Results may be inaccurate. RCOND = 2.632766e-17.

> In cond at 48

In q5 at 11

Warning: Matrix is close to singular or badly scaled.

Results may be inaccurate. RCOND = 2.409320e-17.

> In q5 at 14

N = 12 relative error: 0.2395 condition number: 37983201226912104 log10(con): 16.5796 # correct digits: 0.62069

Warning: Matrix is close to singular or badly scaled.

Results may be inaccurate. RCOND = 2.339949e-18.

> In cond at 48

In q5 at 11

Warning: Matrix is close to singular or badly scaled.

Results may be inaccurate. RCOND = 2.339949e-18.

> In q5 at 14

N = 13 relative error: 5.7974 condition number: 427595335326831488 log10(con): 17.631 # correct digits: -0.76323