

a)

recursive structure:

$\text{return_value} = 3a + 10b + 25c$ where a, b, c are natural numbers. Then there are three cases:

$c > 0$:

$\text{return_value} = 3a + 10b + 25(c-1) + 25$

OR $b > 0$:

$\text{Return_value} = 3a + 10(b-1) + 10 + 25c$

OR $a > 0$:

$\text{Return_value} = 3(a-1) + 3 + 10b + 25c$

#this is just showing that each 'equation' of people can be calculated recursively in a multiple of 3, 10, or 25.

b)

each array value will store a boolean on whether or not x amount of nuggets can be purchased for up to N people:

for $x = -24, -23, -22, \dots, 0, 1, 2, \dots, N$; $A[x] = \text{true or false}$.

c)

Observations:

For any $x < 0$; $A[x] = \text{FALSE}$

$A[0] = \text{TRUE}$ #because 0 people implies 0 nuggets needed which is trivial case

#by simple boolean logic:

$A[x] = A[x-3] \text{ OR } A[x-10] \text{ OR } A[x-25]$ #try the mental math in your head to see it makes sense

This makes sense because if the number of people minus 3 OR 10 OR 25 can equally be bought exactly 1 nugget each then any multiple of another 3 OR 10 OR 25 people can equally get 1 nugget each.

d)

for x in $\text{range}(-24, 0)$: #python function 0 isn't included but -1 is

$A[x] = \text{FALSE}$ #initializing trivial values

$A[0] = \text{TRUE}$ #base case from c)

For x in $\text{range}(1, N+1)$:

$A[x] = A[x-3] \text{ OR } A[x-10] \text{ OR } A[x-25]$ #boolean ORs, also from c)

Return $A[N]$ #true if N people can be bought exactly 1 nugget each in multiples of 3 or 10 or 25

Correctness:

Recursive proof is trivial refer to a, b and c and follow the algorithm to see it is correct. Pitt said the same thing in his lectures.

Polynomial Time:

Yes it does run in polynomial time because each iteration calls a growing constant number of recursive calls each until they reach the base cases ($A[-24]$ all the way to and including $A[0]$), so $A[x-3]$ gets called recursively in *one current* iteration at most $O(3 \cdot (x-3 \text{ repeated recursively a constant amount of times until } x-3 \leq 0))$. The 3 times is in that big-O notation because it is a

very loose upper bound since $A[x-3]$ can get called in an $A[x-10]$ and $A[x-25]$ less times than in an $A[x-3]$ recursive call.