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CSC373 Problem Set 2

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a)

Every input does not always have a solution. Counter Example: There are three computers and each one of them can only have one connection. This is an input that does not have a solution since at least one computer needs to have 2 connections to create an MST.

b)

I will use a modified Kruskal's algorithm for creating MSTs for the solution to this problem.

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Solution(G, w):
        V* = V-L
        E* = edges not containing an L
        W* = weights not containing an L
        T = MST-KRUSKAL(G: (V*, E* ), w*) #let the be an MST of vertices that aren't L

        E** = edges containing only 1 L #no L-L edges or V-V only L-V or V-L
        Sort E** in ascending order of weights
        For all edges u,v ∈ E**
                If endpoints of e_i are not connected in T:
                        T ← T ∪ (u, v)
        Return T
```

c)

First off we know that the 4th line of code returns an MST for V-L based on the proof in the textbook and in class. Also we shall assume that this input contains edges that make it possible for an MST to be created such that L will be a leaf in the optimum solution. Now with this assumption we can assume that an edge from any vertex v to a leaf L in this input there will be a minimum (and safe!) edge that can connect L to the MST created on the 4th line of code such that L will remain a leaf in the optimum final solution. Because of these assumptions we can follow the standard approach for any Kruskall MST algorithm to prove that this algorithm (which is just a modified Kruskall MST algorithm) will produce an MST, we know that any L will always be a leaf because we are inserting L into the tree after every normal vertex is already connected, consequently we know that every normal vertex can be connected without any L because of our trust in a 'good' input.

We don't have to worry about two leaves being connected to each other because E** doesn't contain those edges, the 4th line essentially creates an MST which is the body/middle and adding the L's after are just the leftover nodes on the outside.