Ramaneek Gill 1000005754

CSC373 Problem Set 6

How to solve partially conserved maximum flow using linear programming:
Want to maximize:

$$\sum_{(v,s)\in E} f(s,v) - \sum_{(s,v)\in E} f(v,s)$$

Subject to:
"normal network flow constraints":

$$f(u,v) \le c(u,v)\ for\ all\ edges\ (u,v)\ where\ (u,v) \in E$$
$$f(u,v) \ge 0$$

And given constraint:

$$for\ each\ vertex\ u \in V - \{s,t\}, \qquad \alpha_u f^{in}(u) \le f^{out}(u) \le \beta_u f^{in}(u)$$

The flow values will always have to satisfy each of these constraints, every valid flow produces a feasible solution in this linear program. Every flow is represented by a linear equality with a lower bound of ≥ 0 and an upper bound of $\le$ the capacity of the given connection, this is always fulfilled since $0 \le \alpha_u \le \beta_u$for each vertex $u \in V - \{s,t\}$. Therefore the maximum value of the objective function *cannot* be larger than the maximum flow therefore we get an assignment of flow values to every edge that maximizes $f^{out}(s)$.