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CSC373 Problem Set 1

A)
To minimize the average completion time naturally we want to count the largest numbers as little as possible.

ALGORITHM:
Sort process so $t_{\pi[1]} \leq t_{\pi[2]} \leq \dots \leq t_{\pi[n]}$ //sort processes by their time in ascending order
$L = \emptyset$ //list of processes
For i=1,2,…,n:
    $L = L.append\{t_{\pi[i]}\}$
return L

B)
Notation: let L_0, L_1,…,L_n be values of L at the end of each loop iteration
- The partial solutions constructed by the algorithm
- These are the parts of the solution being built up with L_n being the maximum solution

L_0 is the empty list
L_1 = $\{t_{\pi[1]}\}$ = {2}
L_2 = $\{t_{\pi[1]}, t_{\pi[4]}\}$ = {2,2}
L_3 = $\{t_{\pi[1]}, t_{\pi[4]}, t_{\pi[3]}\}$ = {2, 2, 3}
L_4 = $\{t_{\pi[1]}, t_{\pi[4]}, t_{\pi[3]}, t_{\pi[2]}\}$ = {2, 2, 3, 5}

C)
A partial solution is promising if it is a subset of OPT (the optimum solution)
L_i-->optimum solution for this subproblem (the permutation of t created so far)

For any optimum solution (OPT):
OPT extends L_i if and only if:
- L_i is a subset of OPT
- OPT is a subset of $L_i \cup \{t_{\pi[i+1]}, \dots, t_{\pi[n]}\}$

We say "L_i Is promising" if and only if there exists an OPT that extends L_i

D, E, F, G)

PROOF:

Want to prove: L_i is promising, for all i by induction on i (number of iterations)
A partial solution is a subset of OPT. So after every iteration i, L_i is a subset of the OPT for the problem. In this case L_i is a permutated list of processes such that $\frac{C_1+\dots+C_n}{n}$ is as small as possible. This implies that the permutated list is a list of process in ascending processing time so that larger number are computed as little as possible in the average.

Using Induction:

Base Case:

$L_0 = \emptyset$ every optimum solution OPT extends L_0: $\emptyset \subset OPT \subset \{t_{\pi[1]}, \dots, t_{\pi[n]}\}$

Induction Hypothesis:

Suppose $i \geq 0$ and $\exists OPT$ extends L_i

Induction Step:

Consider L_{i+1}

$L_{i+1} = L_i.\text{append}\{t_{\pi[i+1]}\}$ because $\{t_{\pi[i+1]}\}$ is the next smallest processing time. Therefore $L_{i+1}$is a subset of OPT since $L_i \cup \{t_{\pi[i+2]}, \dots, t_{\pi[n]}\} \subset OPT$.

No cases can occur and subsequently no subcases can occur. This is because the solution was already created in the first step of the algorithm when the list of processes were sorted in ascending processing time order.

Conclusion:

Every L_i is "promising". In praticular, L_n is promising:

$$\exists OPT: L_n \subset OPT \subset L_n \cup \emptyset$$
$$\Rightarrow L_n = OPT$$

NOTE: They are no subcases or cases because the solution is essentially generated by the first line of the algorithm (the sorting function) and the ‘greedy’ algorithm is just copying the elements from the sorted list over to L.