Network Theory and Dynamic Systems

05. Strong and Weak Ties

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Recap from Previous Lecture

- Centrality Measures
- Centrality Distributions
- The Friendship Paradox
- Ultra-Small Worlds
- Robustness
- Core Decomposition

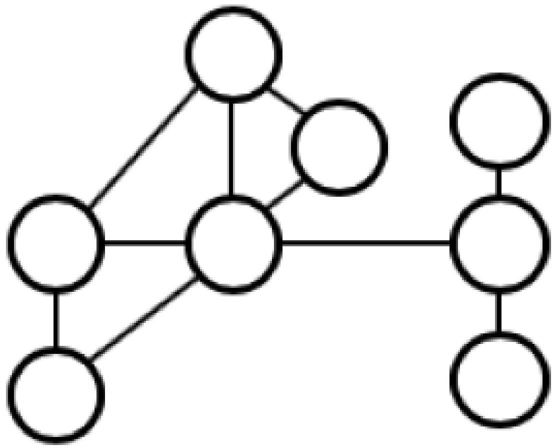
Objectives of this Lecture

- Triadic Closure
- The Strength of Weak Ties
- Tie Strength and Network Structure in Large-Scale Data Betweenness
- Tie Strength, Social Media, and Passive Engagement
- Closure, Structural Holes, and Social Capital
- Betweenness Measures and Graph Partitioning

1. “Strength of Weak Ties”

Edge Strength and Bridges in Social Networks

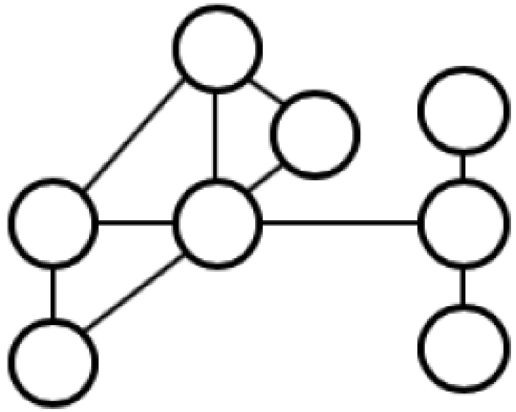
- Not all connections (edges) between people (nodes) in a social network are equally strong
  - *Example*: A close friend represents a strong connection, while someone you met once is a weak connection
- Some people are more connected than others—they link to many parts of the network
- These people often serve as bridges, forming connections between different groups or communities in the network
  - *Example*: A person who is part of both a sports team and a music club connects two otherwise separate groups
- These bridge connections are important because they help spread information across the network
  - *Example*: News shared with a bridge person can quickly reach distant parts of the network that wouldn't otherwise be connected



“Strength of Weak Ties” – A Hypothesis from Sociology

Based on interviews by **Mark Granovetter**:

- People who recently changed jobs were asked how they found those new opportunities
- Many reported that they got help **not from close friends**, but from **acquaintances** (weak ties)
- This was surprising, since it's often expected that **close friends** would be more willing to help
- Granovetter explained this using **network structure** and **social dynamics**:
  - Weak ties connect us to **different social circles**, giving access to **new information and opportunities**
- *Example:* A close friend likely knows the same people you do, but an acquaintance might work in a different company or industry



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2. Triadic Closure

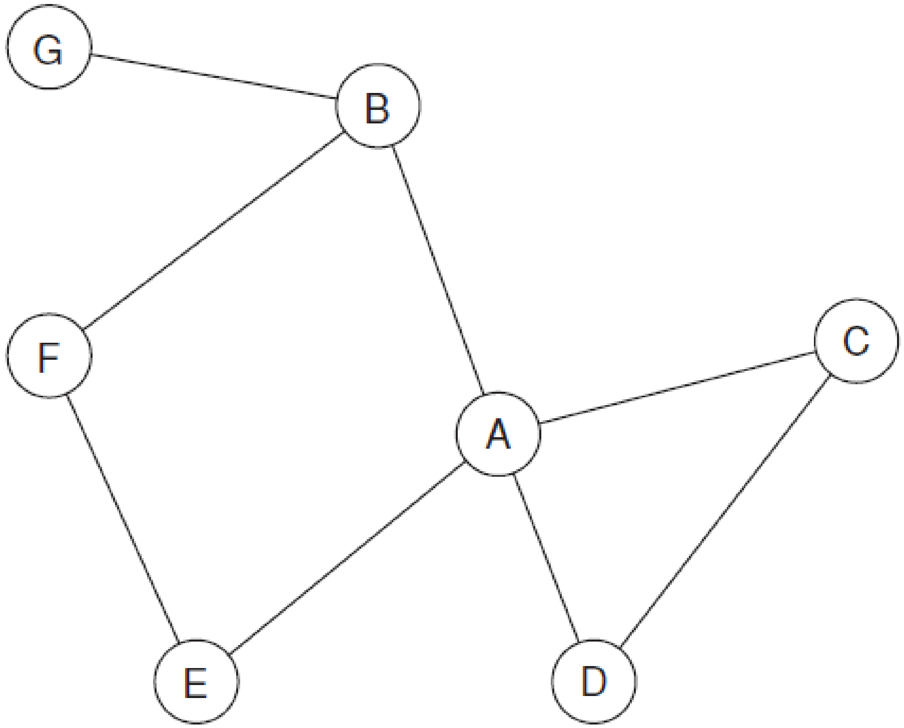
Triadic Closure: Why a Friend of a Friend Often Becomes Your Friend

Core Idea:

- Social networks are not static — they **change and grow** over time
- When two people have a **mutual friend**, there's a **higher likelihood** that they will also become friends
- This pattern is called **triadic closure**, and it's a common principle in how social networks *develop*

Triadic Closure: An Example (1/2)

- Imagine a small network with Alice (A), Bob (B), and Charlie (C)
- Alice is friends with both Bob and Charlie
- According to triadic closure, there's a higher chance that Bob and Charlie will become friends too — because they both know Alice



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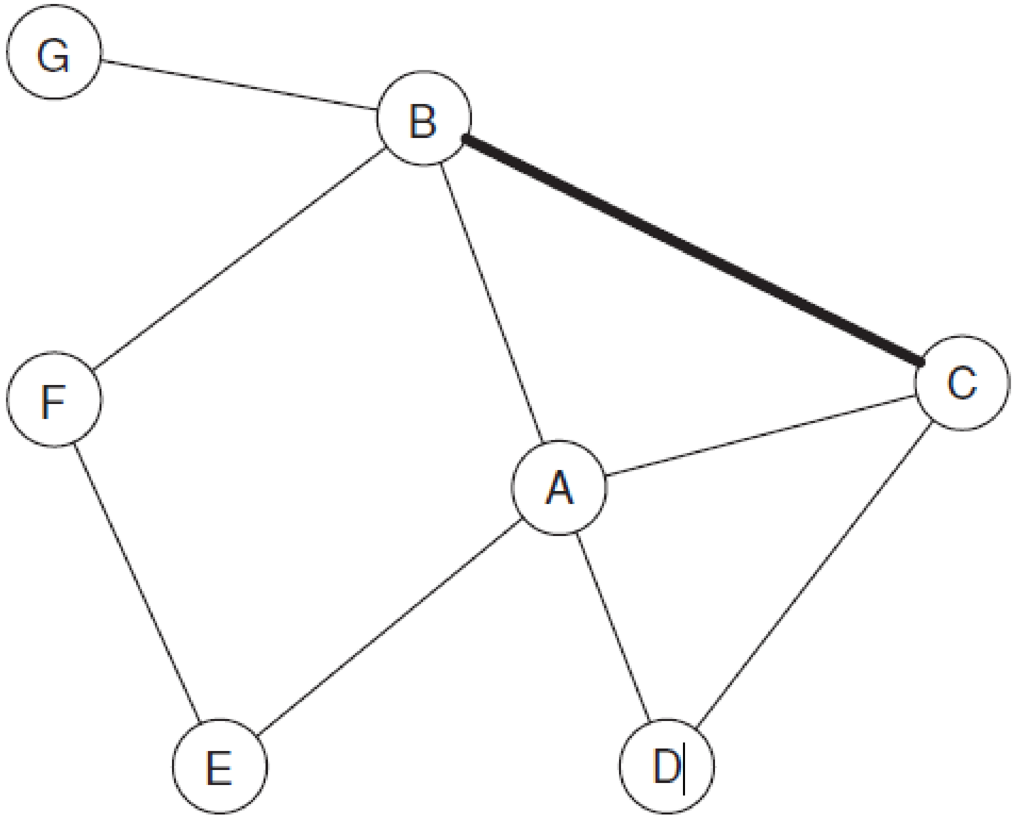
Triadic Closure: An Example (2/2)

- When Bob and Charlie form a connection, the triangle between Alice, Bob, and Charlie is complete
- This triangle is a visual sign of the triadic closure principle — when mutual connections increase the chance of new links forming
- *This triangle structure is a common pattern in real social networks*



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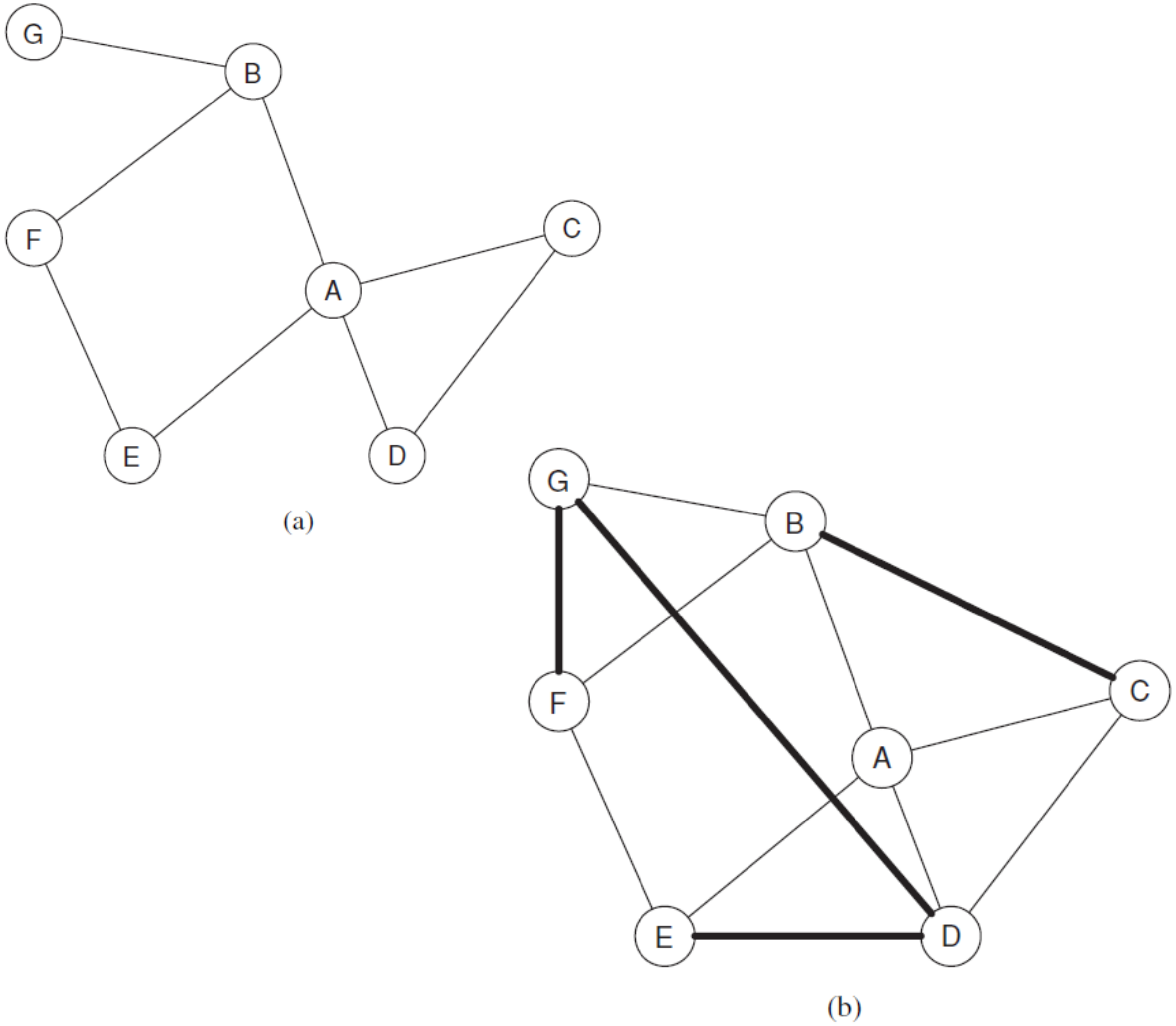
Triadic Closure: Network Evolution

- By observing a network over time, we can see new connections emerge due to triadic closure
  - Initially, Alice (A) connects with Bob (B), Charlie (C), David (D), and Emily (E). However, Bob and Charlie, or Charlie and David, might not be friends yet
  - Over time, triadic closure might lead to a new connection, for instance, between Charlie (C) and David (D)



Triadic Closure: Clustering Coefficient (1/2)

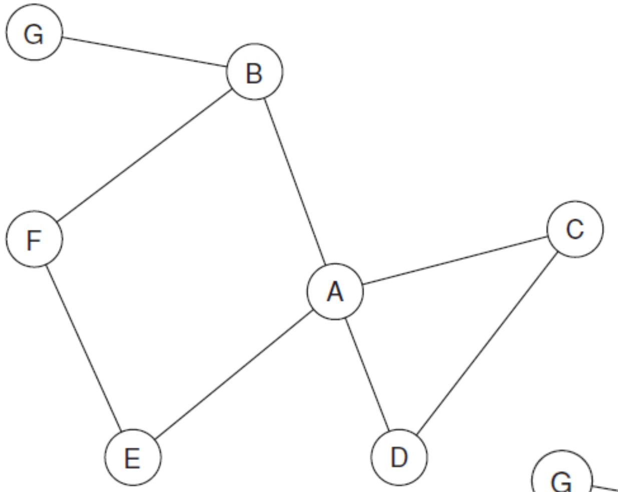
- Clustering Coefficient
  - A measure of triadic closure in a network
  - Indicates the likelihood that two friends of a node are also friends with each other
- How is it calculated?
  - The local clustering coefficient for a node v is calculated using (see also Lecture 03-Small Words)

$$C(v) = \frac{2 \times e(v)}{k(v) \times (k(v) - 1)}$$

    - $e(v)$ is the number of edges between the neighbors of v (number of triangles involving v)
    - $k(v)$ is the degree of node v (i.e., the number of neighbours node v has)
- Range: 0 (no friends connected) to 1 (all friends connected)

Triadic Closure: Clustering Coefficient (2/2)

- Example
  - Node A's clustering coefficient is 2/12 =1/6 (only the C-D edge connects its friends)
  - The coefficient increases to 6/12 = 1/2 (with B-C, C-D, and D-E edges)
- Interpretation
  - Higher coefficient suggests stronger triadic closure in a node’s neighborhood



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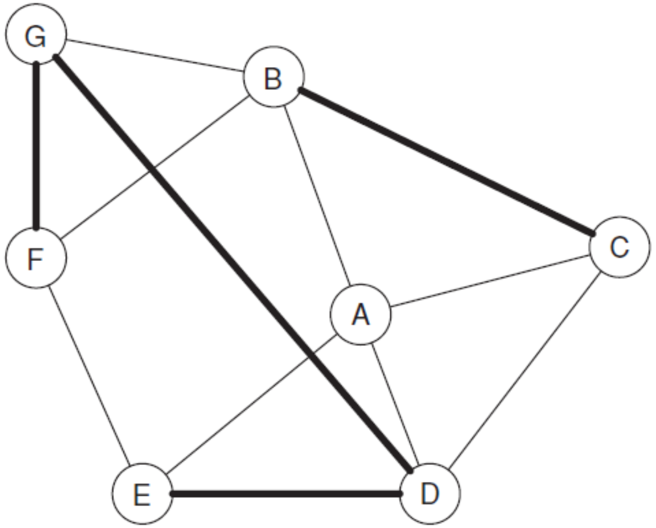

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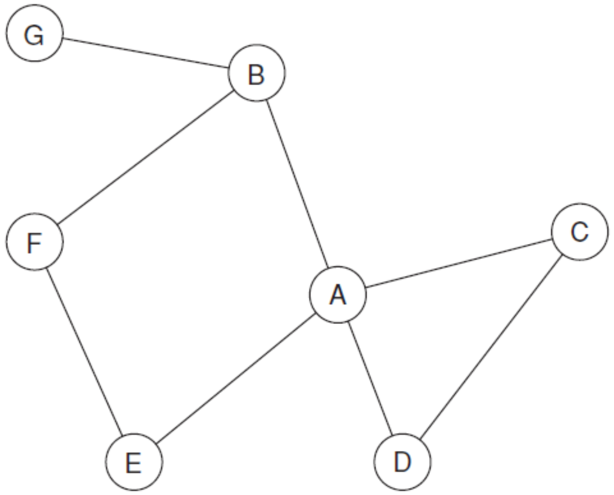
Why Triadic Closure Happens



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1. More Chances to Interact

- A shared friend (e.g., Alice) gives Bob and Charlie opportunities to meet
- Spending time together through this connection can lead to new friendships

2. Builds Trust

- If Alice trusts both Bob and Charlie, this trust can transfer between them
- Knowing there's a mutual connection makes people more open and trusting

3. Social Pressure

- Alice may feel responsible for introducing Bob and Charlie
- It can be uncomfortable if two of a person's friends don't get along, creating pressure to maintain harmony

3. The Strength of Weak Ties

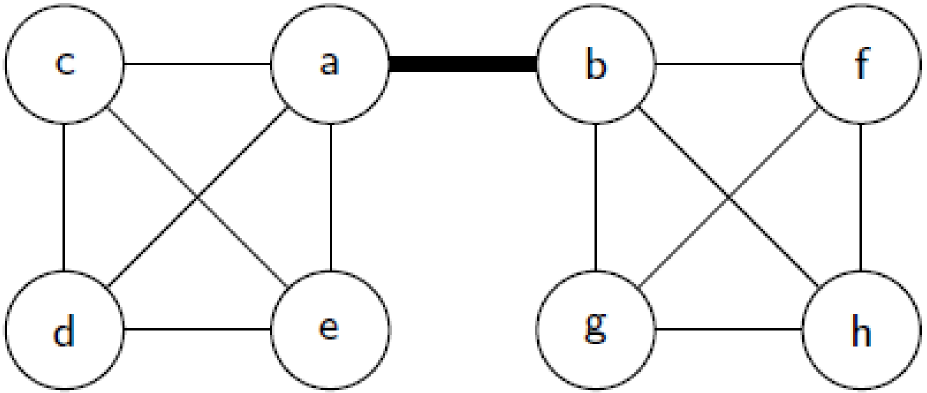
The Strength of Weak Ties

Based on key findings from Mark Granovetter's research:

- Weak ties refer to links with acquaintances, not close friends
- Granovetter found that people often find new job opportunities through these weak ties

Acquaintances connect us to different social circles and new information

Bridges



In this graph, the edge *{a, b}* is a bridge

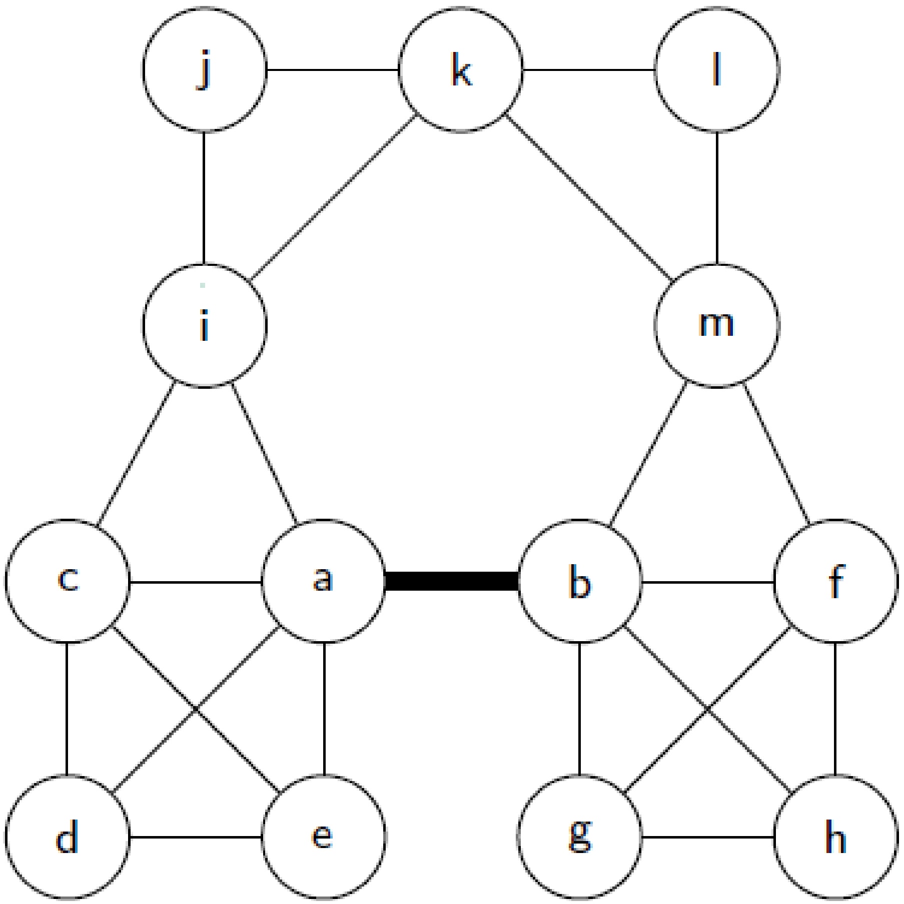
Bridge

- An edge in a graph is called a bridge if by dropping it, results the graph being split into two different components (disconnected parts)

Formally

- Let $G = (V, E)$ be an undirected grap
- An edge $\{a, b\} \in E$ is a *bridge* if a and b are in the same component in a G but different in $G' = (V, E \setminus \{\{a, b\}\}$

Local Bridges (1/2)



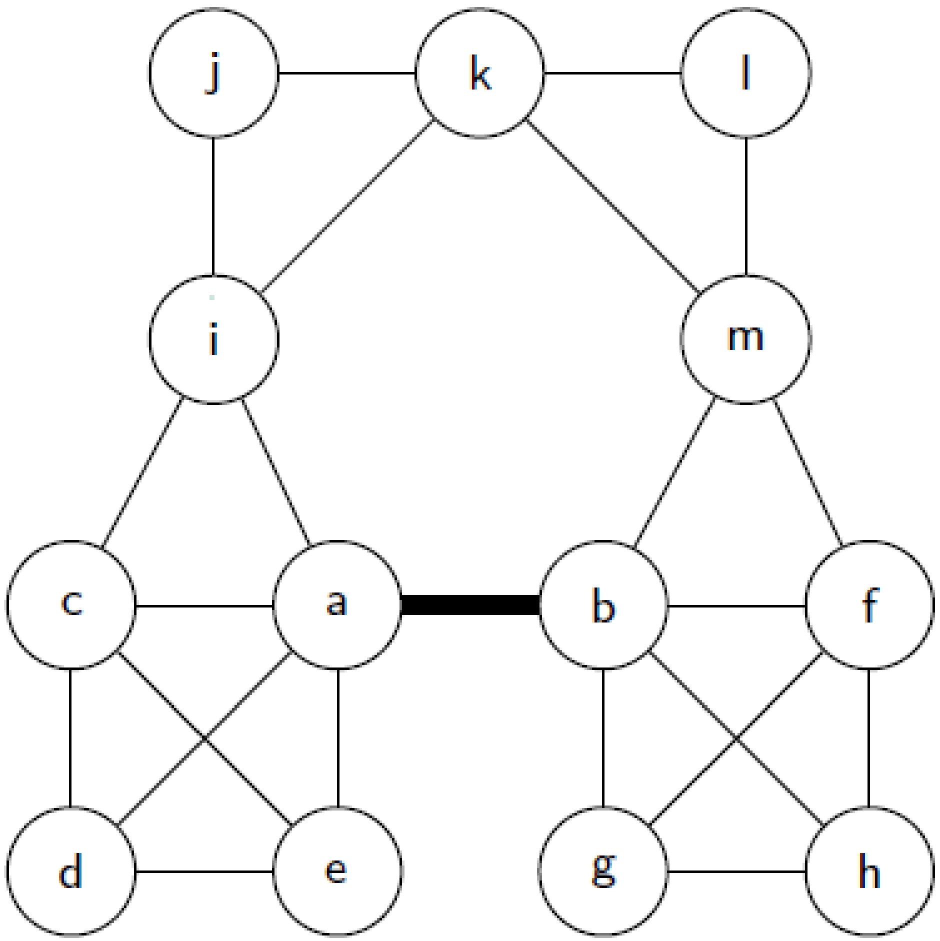
In this graph, the edge *{a, b}* is a local bridge

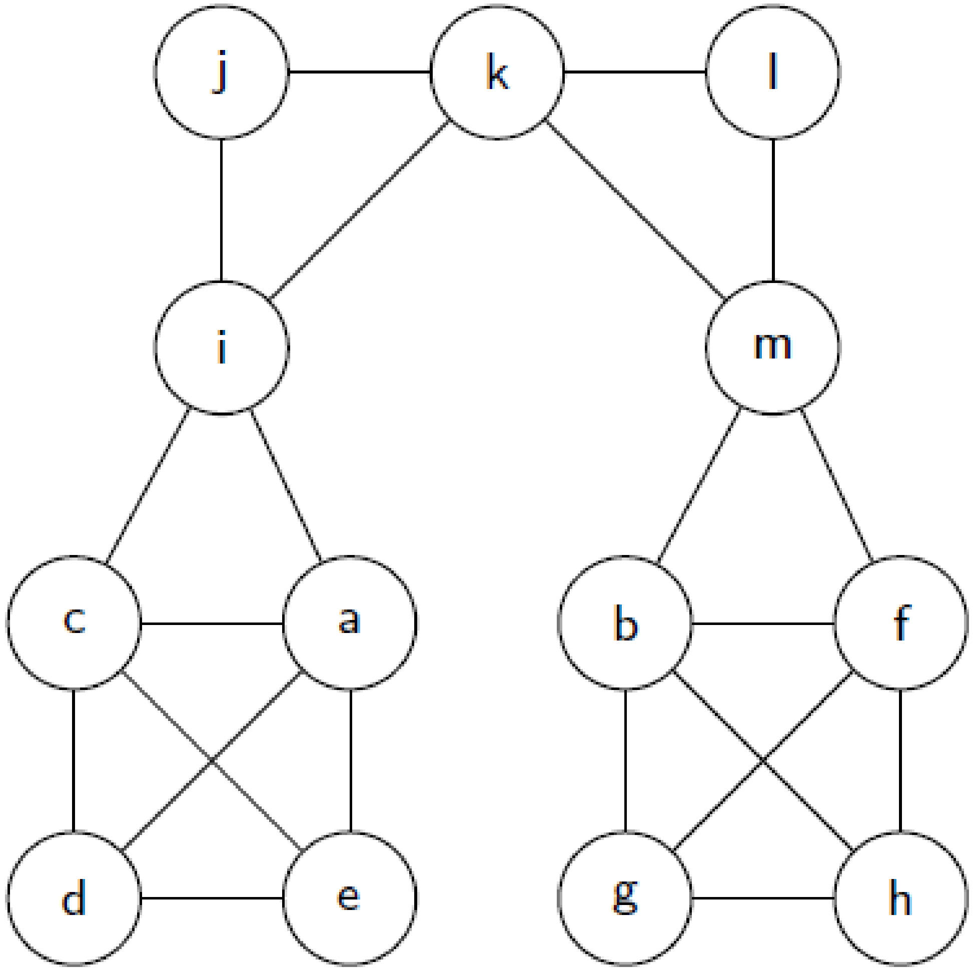
A local bridge is an edge that increases the distance between two nodes if removed but doesn't completely disconnect them

Formally

- Let $G = (V, E)$ be an undirected graph
- An edge $\{a, b\} \in E$ is a *local bridge* if there is no vertex $c \in V$ that is a neighbour of both a and b

Local Bridges (2/2)

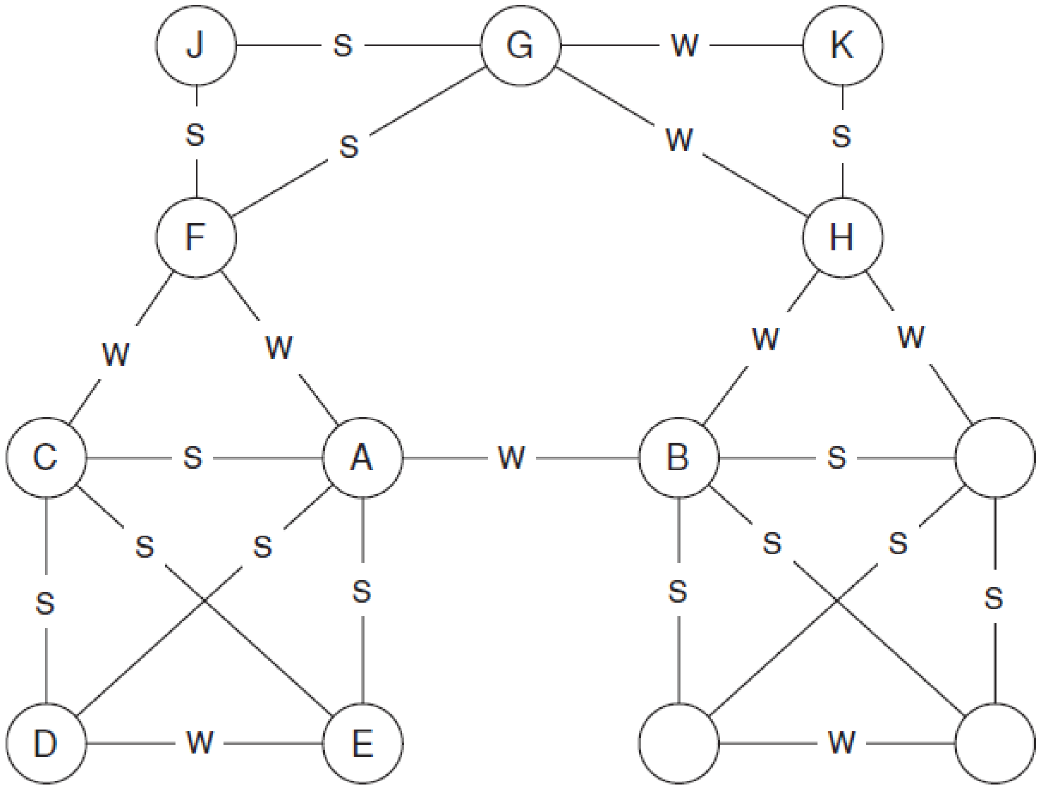




Here, *{a, b} is a local bridge with a span of 4*

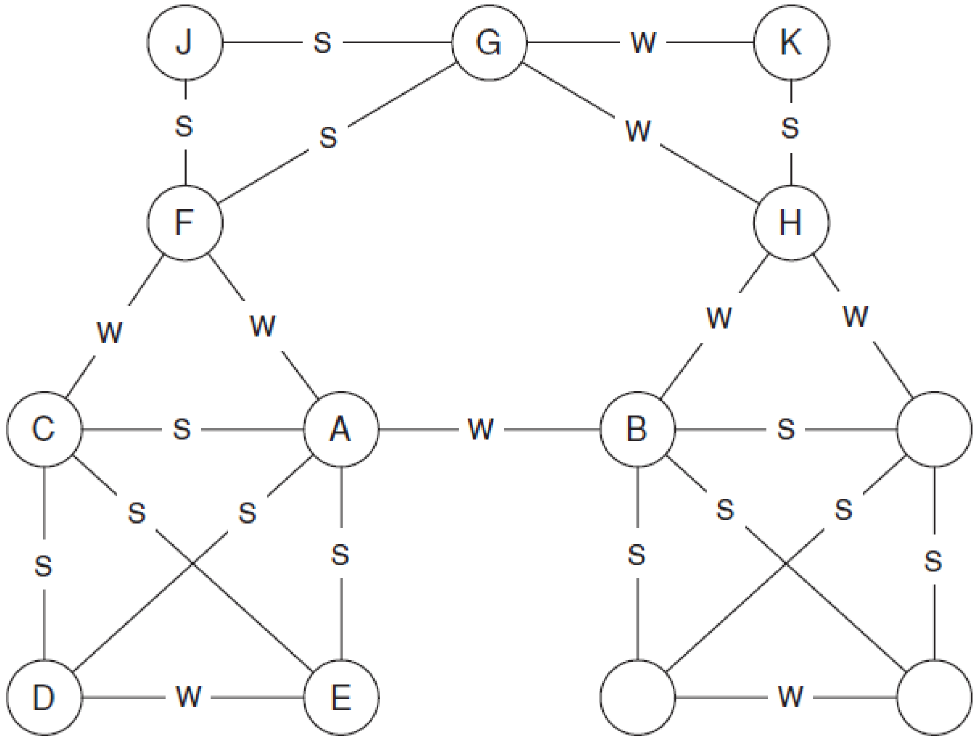
Strength in Ties

- In social networks, edges represent relationships between individuals
- These edges differ in strength:
    - Strong ties = close friendships
    - Weak ties = acquaintances
- Weak ties often connect different parts of the network, helping information spread widely



The Strong Triadic Closure Property (STCP)

- Triadic closure means that if two people have a common friend, they are likely to become friends too
- The Strong Triadic Closure Property adds precision: it looks at whether the connections are strong or weak ties
- A node violates this property if it has strong ties to two people, who are not connected to each other at all (by any tie)
- Example: If Alice is very close to both Bob and Charlie, we expect Bob and Charlie to know each other — otherwise, the closure is broken



4. Tie Strength and Network Structure in Large-Scale Data

Tie Strength and Network Structure

Theoretical Predictions

- The strength of social ties and the structure of the network help explain how social networks are organized
- Granovetter's early predictions about this were hard to test due to a lack of large-scale data

Role of Digital Communication

- With digital data, we now have access to "*who talks to whom*" at scale
- These patterns help us study both network structure and tie strength
- A common proxy for tie strength is how much time two people spend communicating

Case Study by Onnela et al.

Study Overview

- Data: Cell phone communication covering ~20% of a national population
- Nodes: Cell phone users
- Edges: Mutual phone calls over 18 weeks
- Network reflects personal communication within a significant fraction of a country's population

Findings

- The data set exhibits broad structural features of large social networks
- Contains a giant component with 84% of the individuals in the network

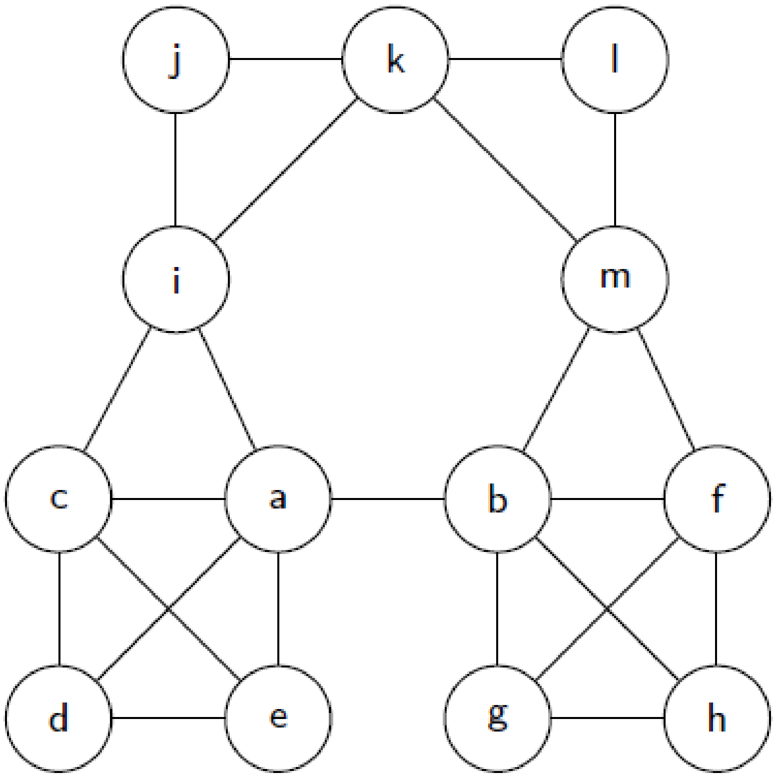
Measuring Tie Strength

Neigbourhood Overlap can be measured using the following formula

$$Overlap_G = \frac{\#\ of\ neighbouring\ nodes\ of\ both\ A\ and\ B}{\#\ of\ neighbouring\ nodes\ of\ at\ least\ one\ of\ A\ and\ B}$$

Computations for some of the edges

- $\mathrm{Overlap}_{\mathrm{G}}(\{a, b\}) = 0$
- $\mathrm{Overlap}_{\mathrm{G}}(\{l, m\}) = {}^{1}/_{3}$
- $\mathrm{Overlap}_{\mathrm{G}}(\{c, i\}) = 0.2$
- $\mathrm{Overlap}_{\mathrm{G}}(\{d, e\}) = 1$



Observations

- Local bridges: Neighborhood overlap of zero
- Edges with small neighborhood overlap are "almost" local bridges

Tie Strength and Neighborhood Overlap

Neighborhood overlap vs. Tie strength percentile

- Overlap increases with increasing tie strength
- Aligns with Granovetter's theoretical predictions

Local vs. Global Network Structure

- Local structure looks at individual connections
  - The neighborhood overlap of a specific edge (connection between two nodes)
  - Tells you how embedded that edge is in a tightly-knit group
- Global structure looks at the entire network
  - Focuses on how weak ties (i.e., edges with low tie strength and low overlap) play a crucial role
  - Weak ties are important for connecting separate communities — people who otherwise wouldn't be connected
  - Example: A casual acquaintance might link you to a completely new job opportunity in another social circle

Global Network Structure and Weak Ties

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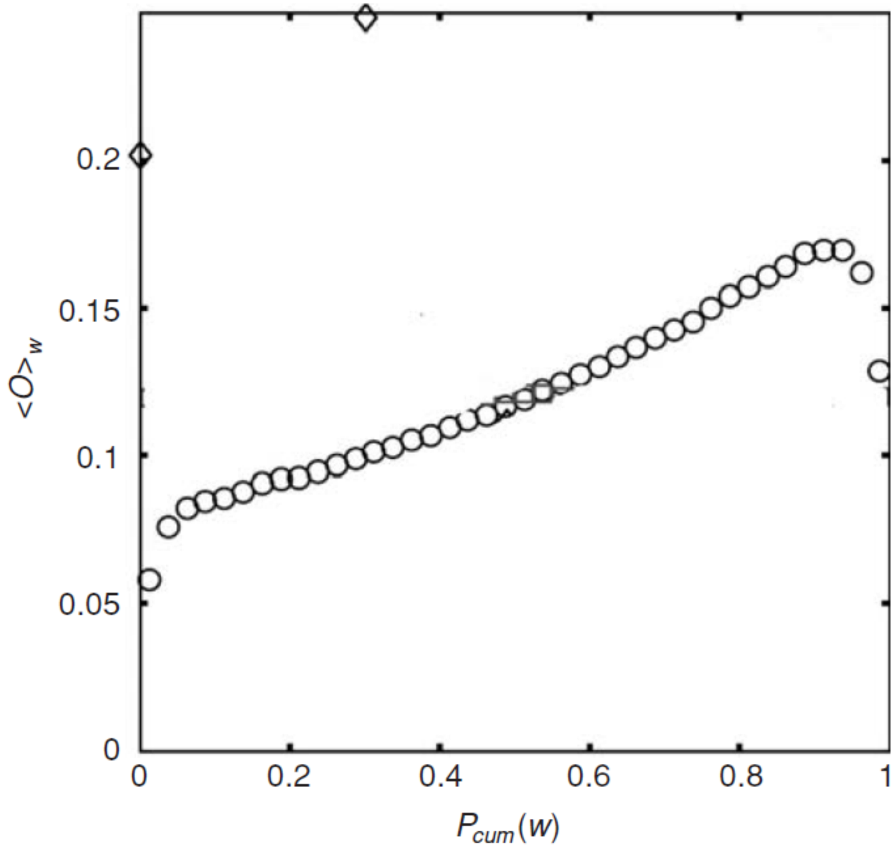
Experimental Analysis by Onnela et al.

- Method: Deleting edges by tie strength
- Strong ties first: Gradual shrinkage of the giant component
- Weak ties first: Rapid shrinkage and abrupt disintegration of the giant component

Conclusion: Weak ties are **crucial** for maintaining global network connectivity



5. Tie Strength, Social Media, and Passive Engagement

Tie Strength on Facebook (1/3)

Cameron Marlow and his team categorized Facebook edges based on user activity over a month

- Reciprocal Communication
  - User both sent and received messages from the friend (Strong Tie)
- One-Way Communication
  - User sent messages to the friend but may not have received any message in return (Medium Tie)
- Maintained Relationship
  - User followed the friend's information through News Feed or profile visits (without necessarily messaging) (Weak Tie)

Furthermore, the categories are not mutually exclusive

Tie Strength on Facebook (2/3)

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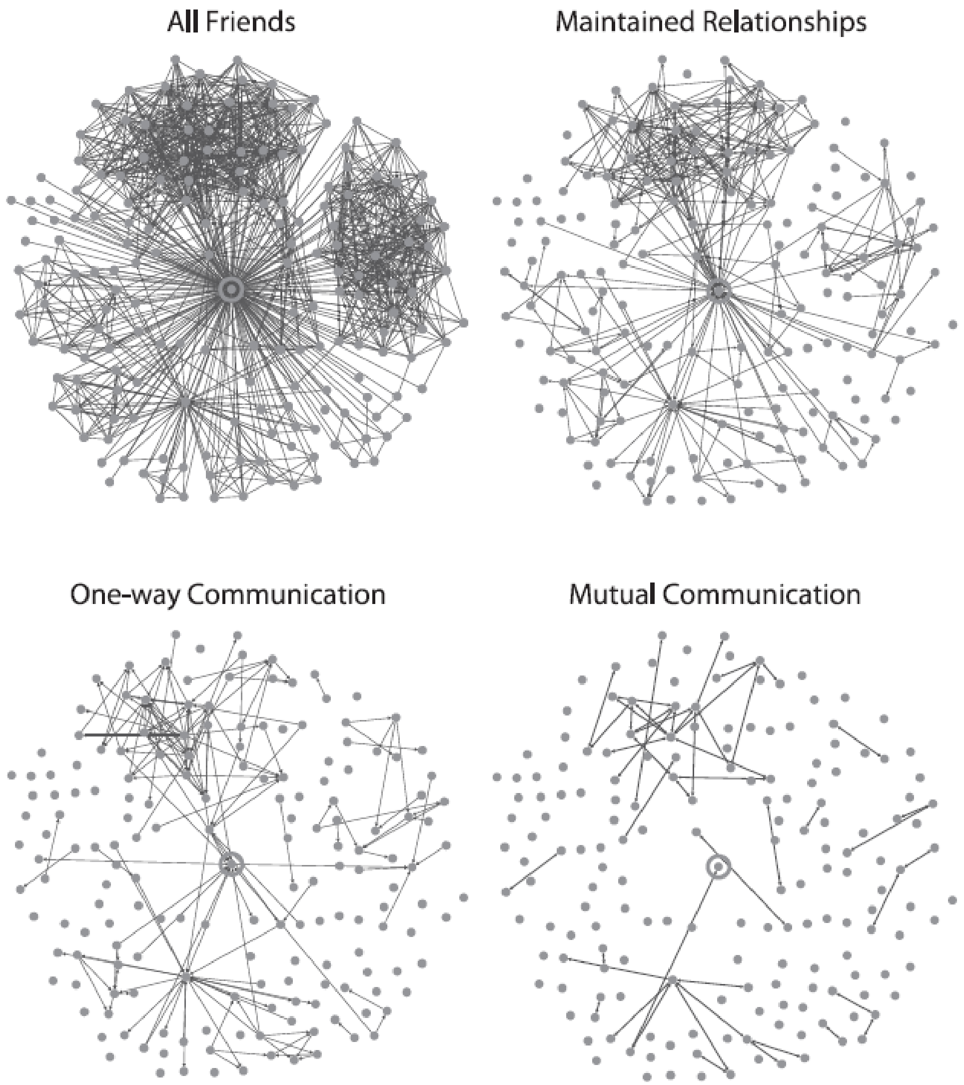
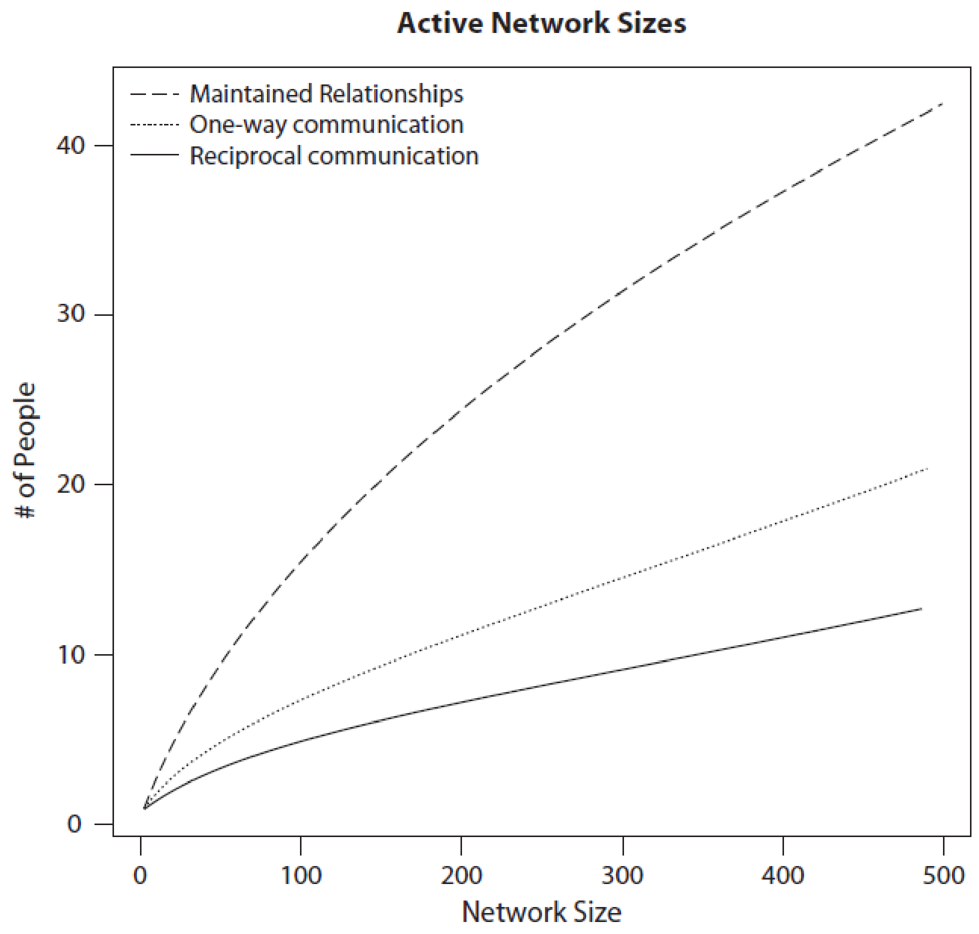
Observations

- Network sparsity increases with stronger tie definitions
- Different regions show varied levels of triadic closure



Tie Strength on Facebook (3/3)

- X-Axis: Total number of friends declared by a user
- Curves: Show numbers of other edge types as a function of total friends
- Users with 500 friends actively communicate with only 10-20
- Less than 50 friends are followed passively
- Passive Engagement
  - Enables users to keep up with friends without direct communication
  - Acts as a middle ground between strong and weak ties



Social Interactions on Twitter (1/2)

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Interaction Types

- Public Messages
  - Follow messages from various users without direct communication
  - Defines a social network based on passive, weak ties
- Directed Messages
  - Public messages marked for specific users
  - Indicates a stronger, more direct relationship

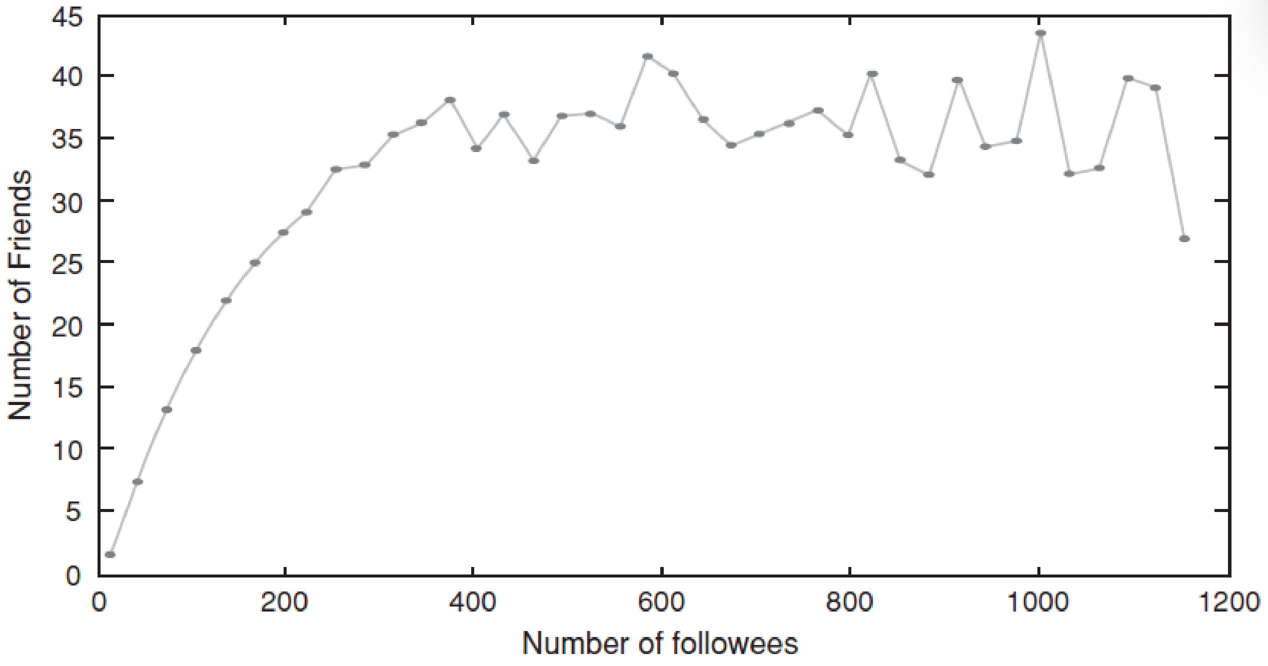
Edge Analysis

- Huberman, Romero, and Wu's Study
  - Analyzed the abundance of weak and strong ties on Twitter
  - Defined strong ties as users receiving at least two directed messages from another user

Social Interactions on Twitter (2/2)

Key Findings

- Even users with over 1,000 followees maintain fewer than 50 strong ties
- Pattern Consistency
  - Similar to Facebook, where a large number of weak ties contrast with a modest number of strong ties



Limits on Forming Social Ties

Strong Ties: High Effort, Limited Capacity

- Maintaining strong ties takes time and ongoing effort
- We're limited by the hours in a day
- As a result, there’s a natural cap on how many strong ties a person can sustain

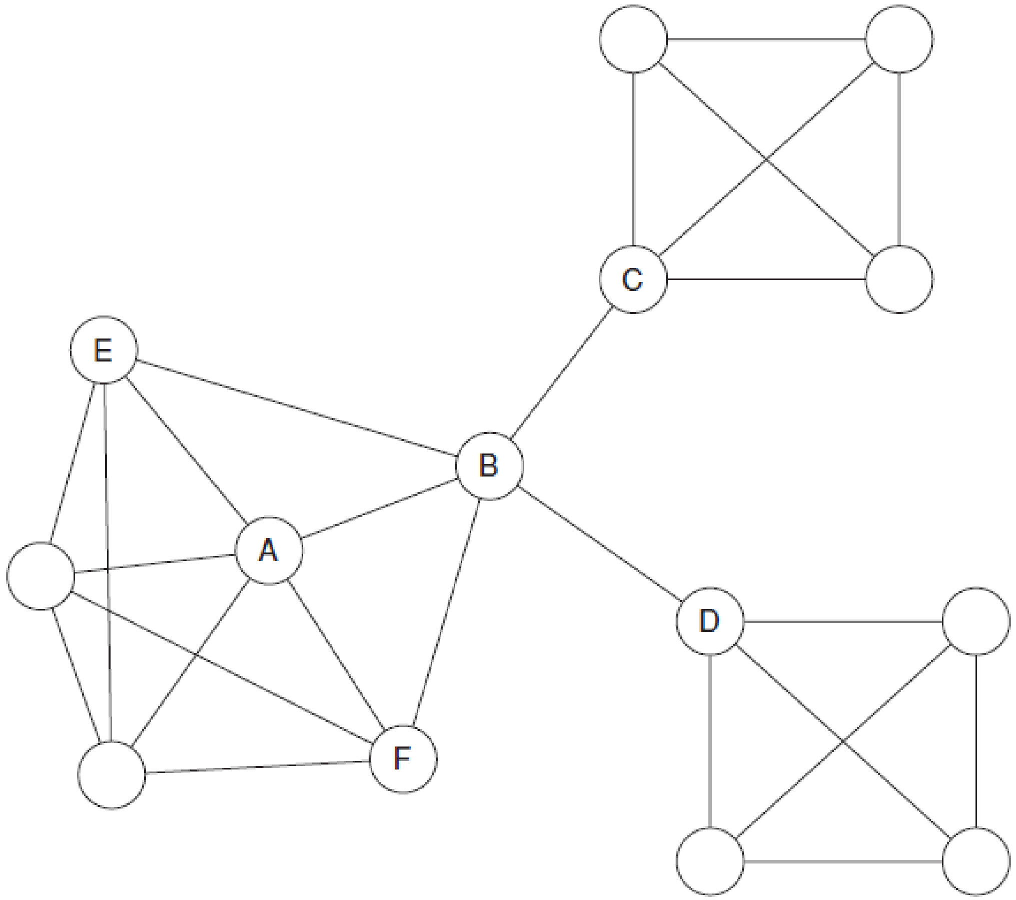
Weak Ties: Easy to Build, Easy to Keep

- Weak ties are easier to create—only initial contact is needed
- They require little or no ongoing effort
- This makes it possible to maintain many weak ties at the same time

6. Closure, Structural Holes, and Social Capital

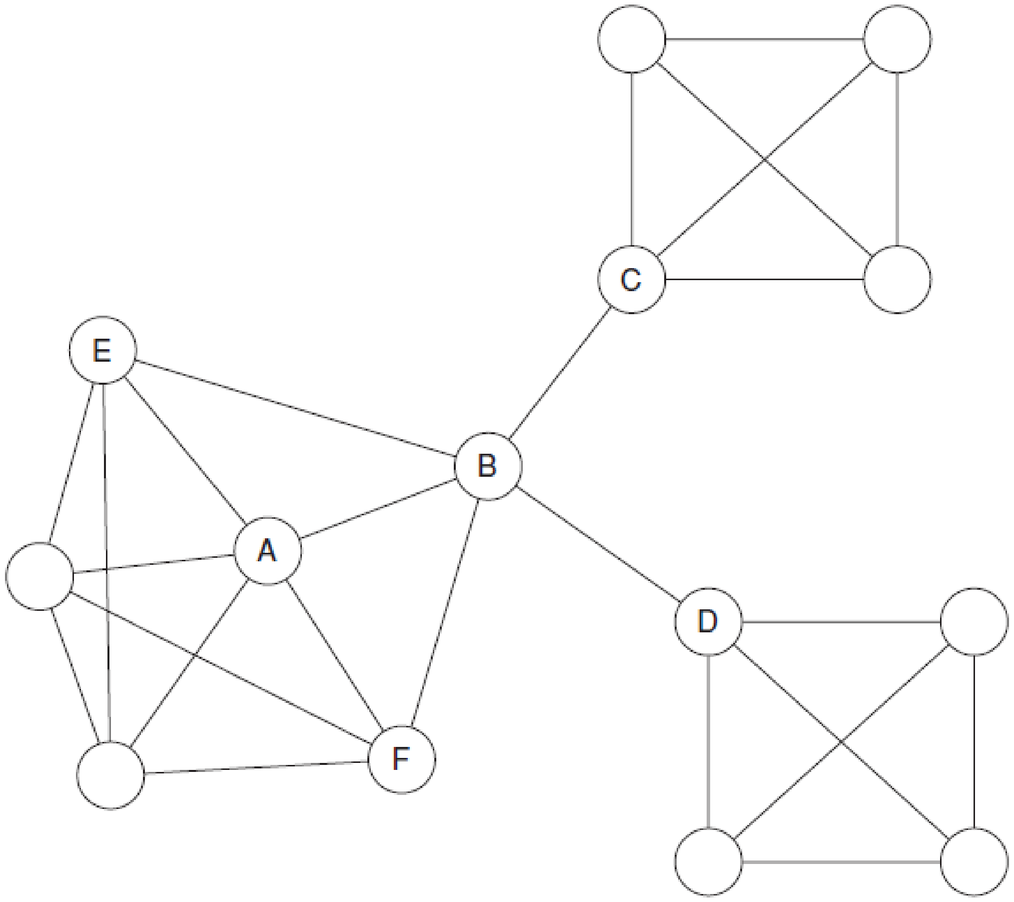
Embeddedness and Clustering Coefficient (1/2)

- Clustering Coefficient
  - The clustering coefficient of a node is the fraction of pairs of its neighbors that are themselves neighbors
  - High Clustering Coefficient: Indicates considerable triadic closure
- Embeddedness
  - The embeddedness of an edge is the number of common neighbors shared by the two endpoints of the edge



Embeddedness and Clustering Coefficient (2/2)

- Relationship to Neighborhood Overlap
  - Numerator: The embeddedness of an edge equals the numerator in the ratio defining neighborhood overlap
  - $Overlap_G = \dfrac{\#\, of\ neighbouring\ nodes\ of\ both\ A\ and\ B}{\#\, of\ neighbouring\ nodes\ of\ at\ least\ one\ of\ A\ and\ B}$



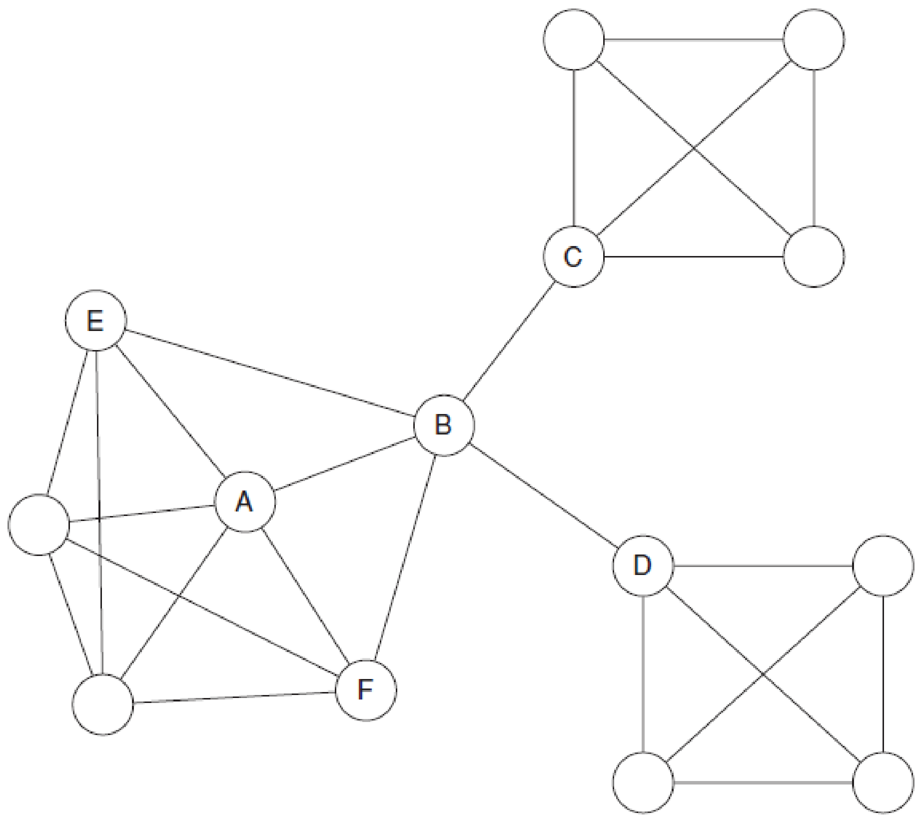
Embeddness in Social Networks

- Local Bridges
  - Edges with zero embeddedness, meaning their endpoints have no neighbors in common
  - Local bridges connect distinct parts of the network, facilitating the flow of information across diverse groups
- Embedded Edges and Trust
  - High Embeddedness: Indicates stronger and more trustworthy connections
    - ✓ Integrity of Transactions: High embeddedness supports confidence in social, economic, and other transactions
  - Research shows: People connected by highly embedded edges are more likely to trust each other

Structural Holes in Networks

Structural holes represent the "empty space" in a network between two sets of nodes that do not interact closely

- Node B's Position: At the end of multiple (local) bridges, spans a structural hole in the organization
- Node B has early access informational advantage from multiple noninteracting parts of the network
- Node B’s position can foster innovation through the synthesis of diverse ideas from noninteracting groups
- Innovations often arise from combining well-known ideas from distinct and unrelated bodies of expertise



Social Capital in Social Networks (1/2)

Social capital is the benefit people gain from being part of social groups or networks

- As Alejandro Portes puts it:

Social capital refers to the value people gain by being part of social networks or social structures

Forms of Capital

- Physical Capital: Tools and technologies that support work (e.g., machines, equipment)
- Human Capital: Skills and knowledge from education and training
- Economic Capital: Financial and physical assets like money, investments, and property
- Cultural Capital: Knowledge and cultural understanding gained through education and social institutions

Social Capital in Social Networks (2/2)

Variations in Social Capital

- Group vs. Individual
  - Group Property: Social capital is seen as a feature of a whole group
    - Example: A well-connected neighborhood or community that supports its members
  - Individual Property: Social capital depends on a person's position in the network
  - Example: Someone in a professional network with many influential contacts

- Intrinsic vs. Extrinsic
  - Intrinsic: Comes from interactions within the group
    - Example: Trust and collaboration within a work team
  - Extrinsic: Comes from connections to outside groups
    - Example: Networking with people in other departments or industries

7. Betweenness Measures and Graph Partitioning

Graph Partitioning

- **Goal:** Divide a network into tightly connected groups (clusters) with few connections between the groups
- Graph partitioning means breaking a graph into subgraphs where the nodes are split into non-overlapping groups
- Applications:
- Social networks: Detecting communities or interest groups
- Infrastructure: Optimizing traffic or road networks by analyzing structure

Example: Coauthorship Network

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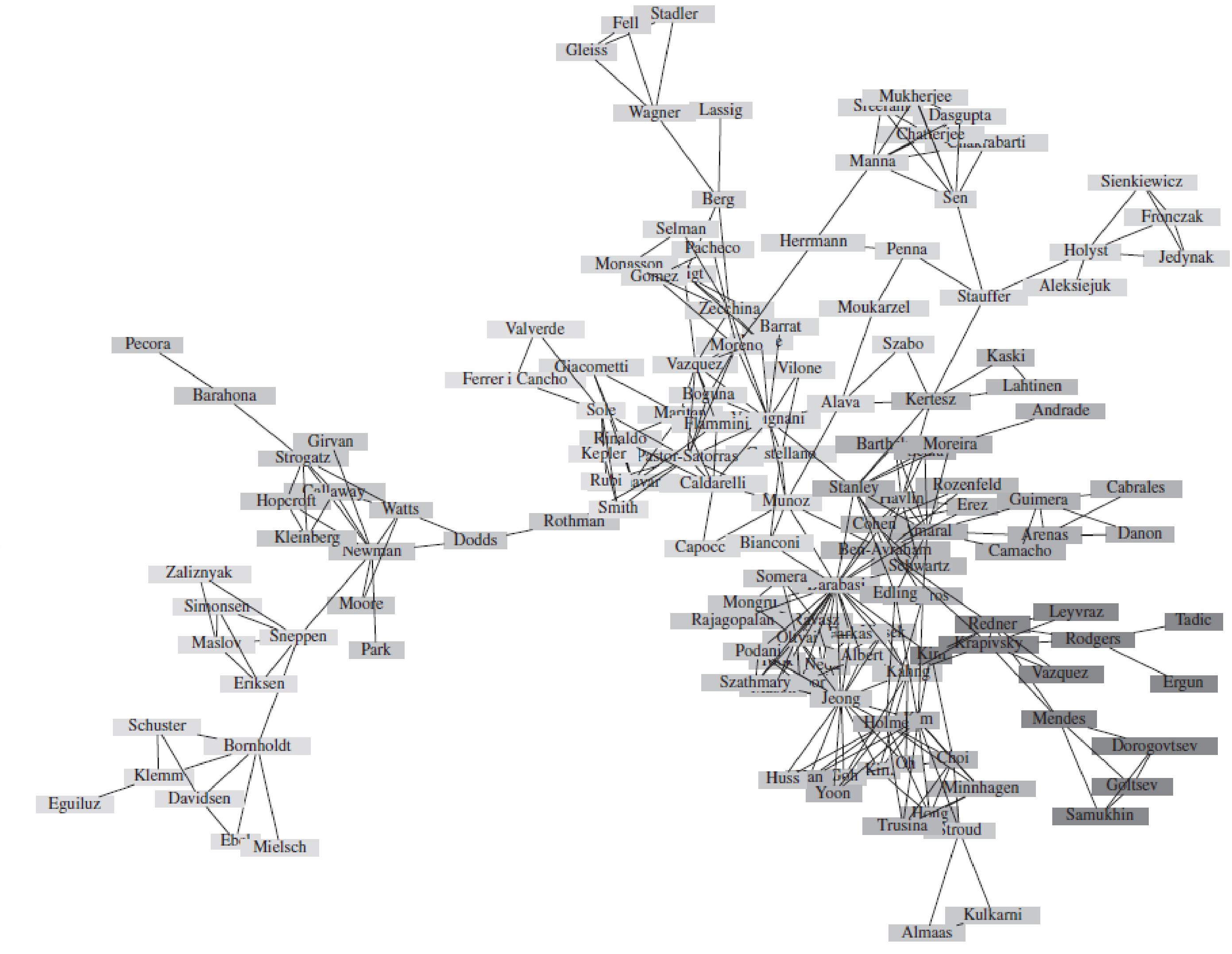
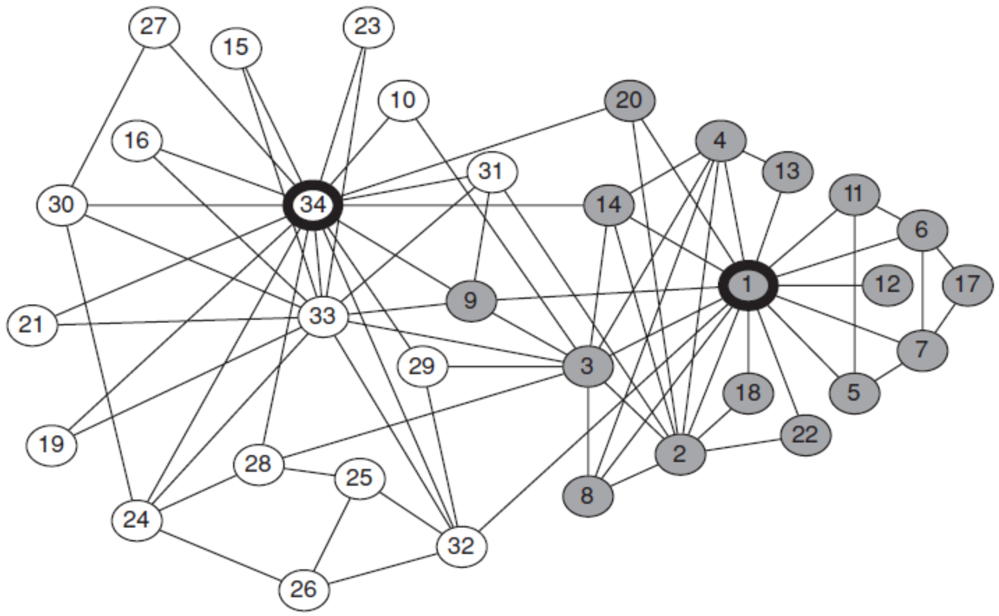


- Coauthorships among physicists and applied mathematicians
- Tightly-knit groups and boundary individuals evident from the structure
- Question: *Can we formally identify these groups beyond visual intuition?*

A coauthorship network of physicists and applied mathematicians working on networks. Within this professional community, more tightly-knit subgroups are evident from the network structure. (Image from the American Physical Society)

Example: Karate Club Network

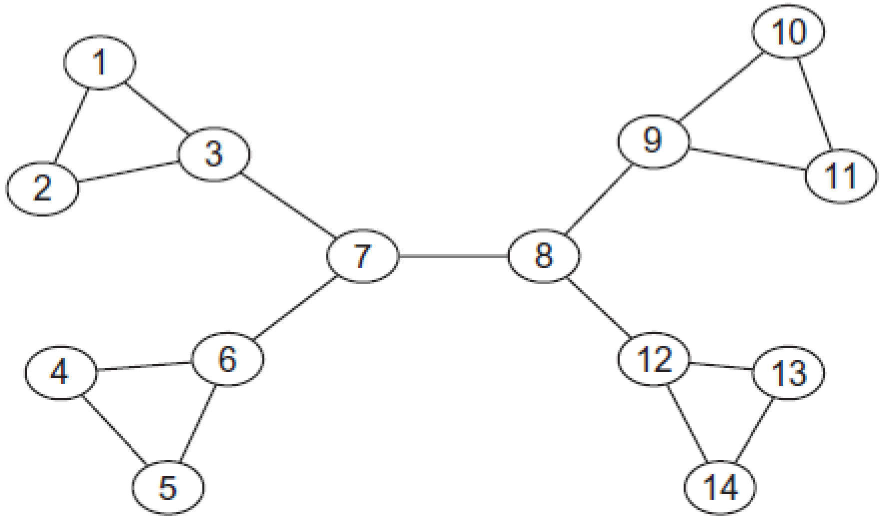


A karate club studied by Wayne Zachary. A dispute during the study caused it to split into two clubs, indicated by the shading of the nodes. Could the boundaries of the two clubs have been predicted from the network structure? (Image from the Journal of Anthropological Research)

- Social network of a karate club studied by Wayne Zachary
- Split into two rival clubs due to a dispute
- Question
  - Does the structure predict the split?
  - Are there subtle signals indicating lower edge density between conflicting groups?

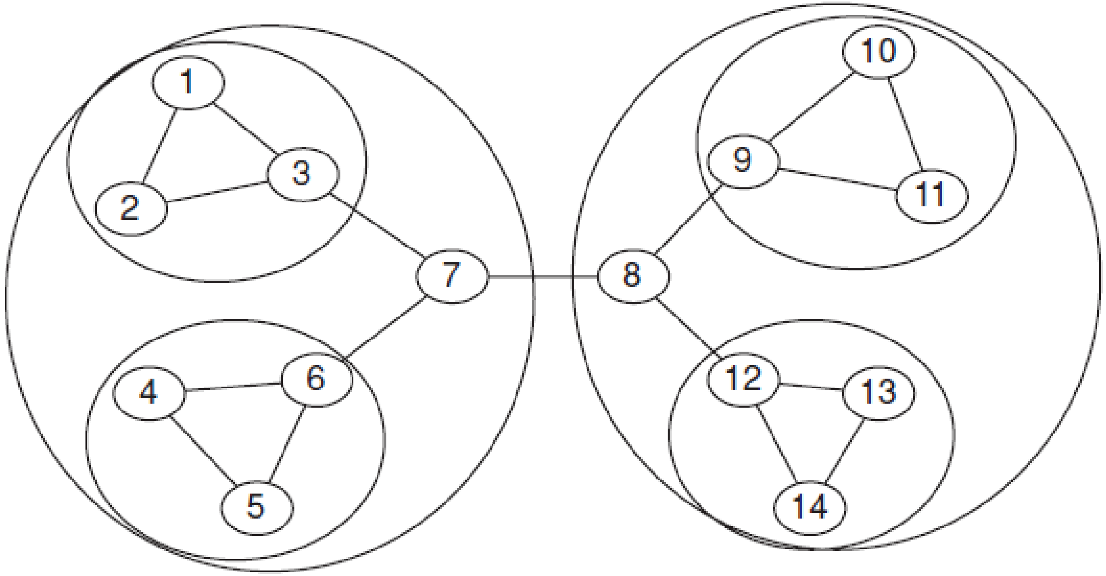
Graph Partitioning: Divisive Methods

- Process starts by identifying and removing spanning edges that are bridges between densely connected regions to progressively divide the network into subnetworks
- The process is iterative and removing weaker edges within each piece, systematically breaks down the network into smaller sections
- Girvan-Newman method is an example of divisive methods that makes use of the measure betweenness to identify and remove key edges



Consider this network, which is only held by a few edges. Removing the edge *{7, 8}* (that is also a weak edge) splits the network into two major parts. Iterating yields smaller networks.

Graph Partitioning: Agglomerative Methods



Tightly-knit regions and their nested structure. In many networks, there are tightly-knit regions that are intuitively apparent, and they can even display a nested structure, with smaller regions nesting inside larger ones.

- Process starts with merging the most densely connected nodes into small clusters, forming the seeds of larger regions
- These small clusters are then combined based on the connectivity to build larger interconnected regions

The Notion of Edge Betweenness

- Betweenness (already defined in Lecture 04: Hubs) is a measure used to identify the strength of an edge in a network
  - Quantifies the number of times an edge acts as a bridge between other nodes
- Formally, edge betweenness of an edge *e* is defined as:

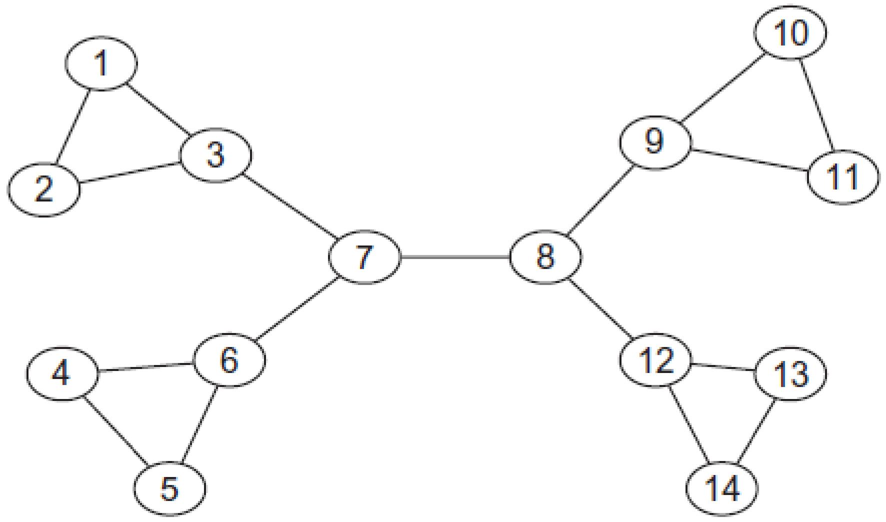
$$EB(e) = \sum_{v,\, w \in V} \frac{\sigma_{v,\, w}(e)}{\sigma_{v,\, w}}$$

where

  - $\sigma_{v,w}$ denote the number of shortest paths between *v* and *w*
  - $\sigma_{v,w}(e)$ denote the number of shortest paths between *v* and *w* that pass through the edge *e*

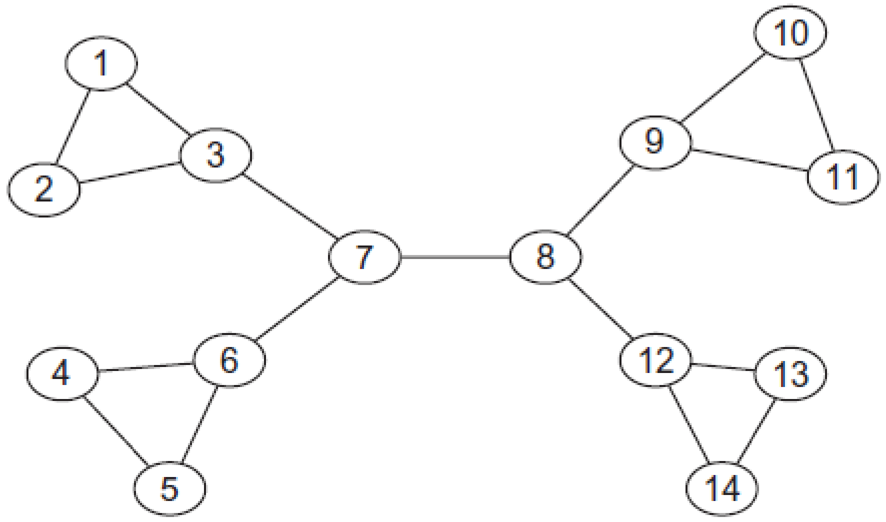
Example: Edge Betweenness (1/2)

- **Edge 7-8**
  - All nodes in the left half (nodes 1-7) connect to all nodes in the right half (nodes 8-14) via this edge
  - Betweenness: 7×7=49
- **Edge 3-7**
  - Carries flow from nodes 1, 2, and 3 to nodes 4-14
  - Betweenness: 3×11=33
  - Same betweenness for edges 6-7, 8-9, and 8-12

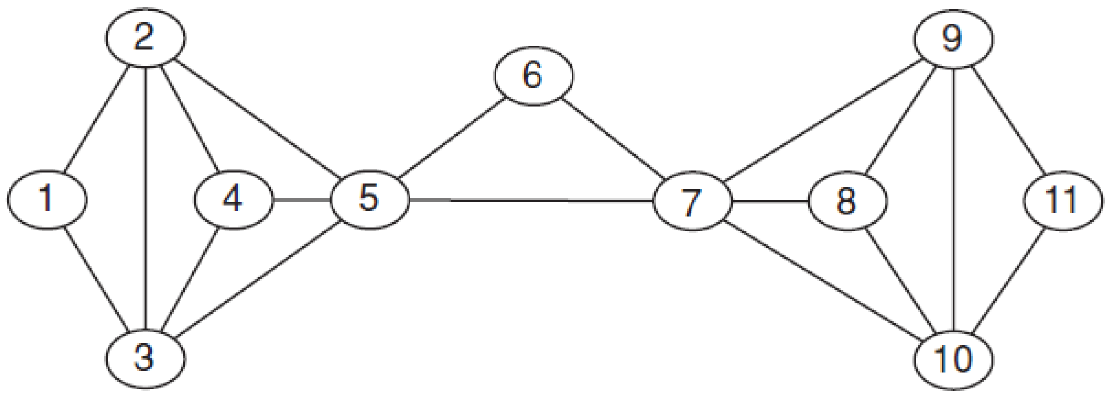


Example: Edge Betweenness (2/2)

- **Edge 1-3**
  - Carries flow from node 1 to every node except node 2
  - Betweenness: 12
  - Similar betweenness for edges linked from nodes 3, 6, 9, and 12 into their respective triangles
- **Edge 1-2**
  - Only carries flow between its endpoints (nodes 1 and 2)
  - Betweenness: 1
  - Same betweenness for edges 4-5, 10-11, and 13-14



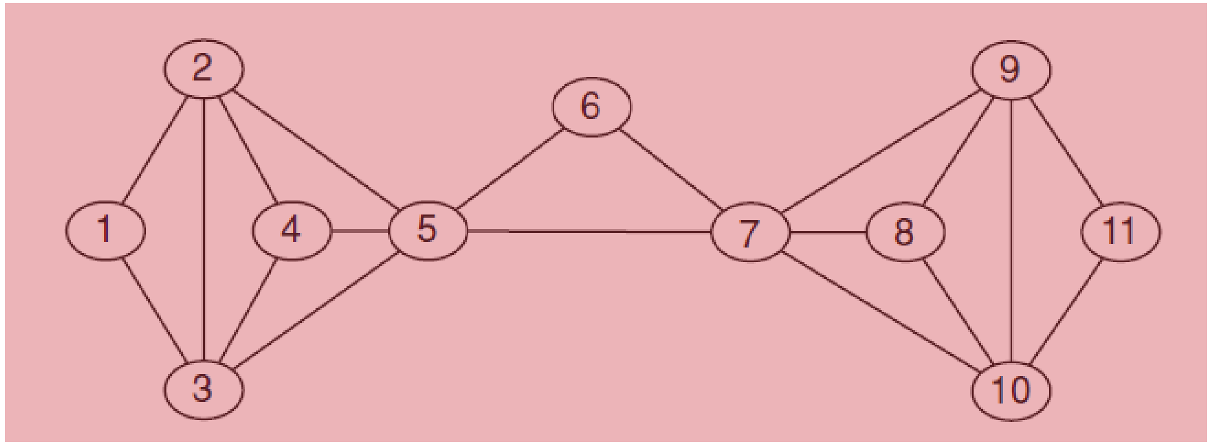
Girvan–Newman Method (1/3)

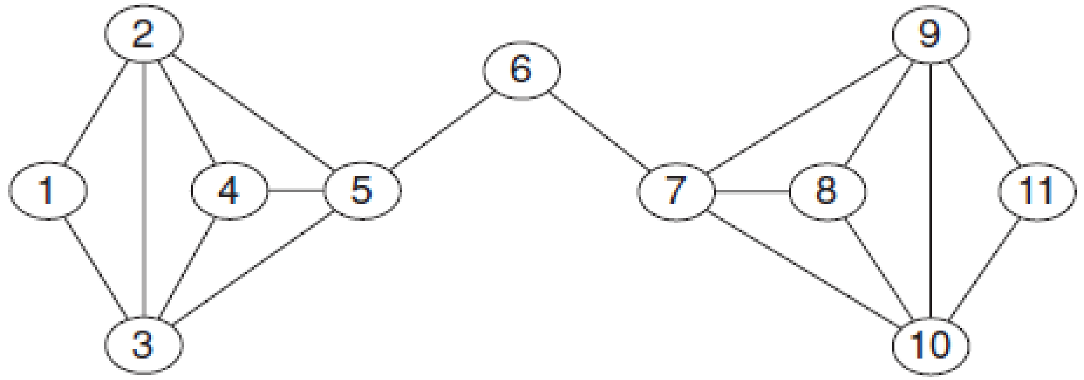


A network can display tightly-knit regions even when there are no bridges or local bridges along which to separate it.

- The Girvan–Newman Method is a pivotal algorithm in network analysis, particularly for uncovering community structures within complex networks
- The method involves iteratively removing edges of high betweenness centrality
- High-betweenness edges are those that facilitate the most traffic flow along shortest paths between nodes

Girvan–Newman Method (2/3)





A network can display tightly-knit regions even when there are no bridges or local bridges along which to separate it.

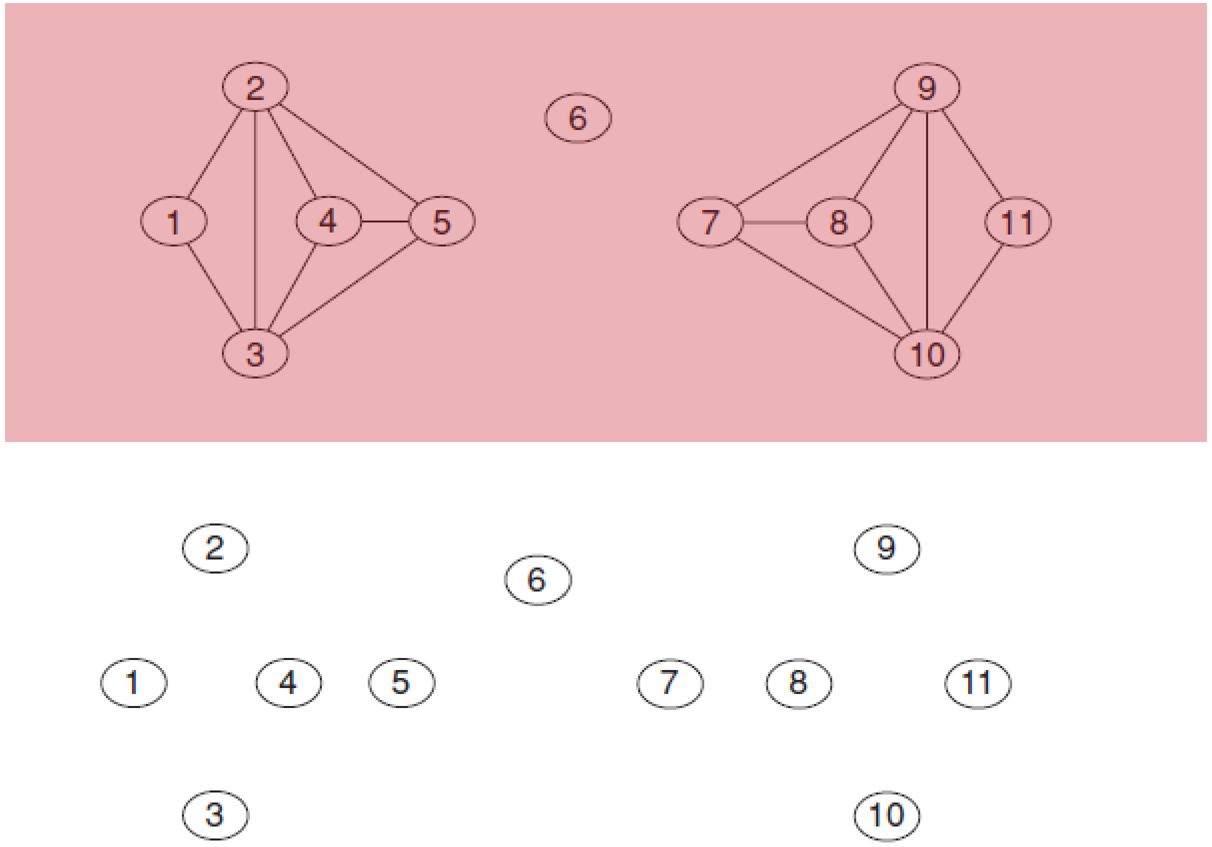
Purpose: Detect community structure by removing important edges.

How It Works:

1. Find edges with the highest betweenness centrality
   - These edges lie on many shortest paths between nodes
2. Remove these edges
   - This may split the network into separate parts (communities)
3. Recalculate betweenness centrality for the new graph
4. Repeat the process
   - Continue removing high-centrality edges until the graph breaks into meaningful clusters

Note: This method works even when there are no clear bridges, by identifying structural weak points using centrality

Girvan–Newman Method (3/3)



A network can display tightly-knit regions even when there are no bridges or local bridges along which to separate it.

- The algorithm gradually reveals the structure of the network by repeatedly removing central edges
- This process helps uncover tightly-knit regions (communities) that were not obvious before
- In the end, the network is split into smaller, well-connected groups, showing the underlying community structure

8. Summary

Summary (1/2)

- Network Edges
  - The strength of social ties varies: some connections are close friendships, others are acquaintances
- Bridges
  - Some individuals act as bridges, linking different parts of the network
  - These bridges are key for spreading information and enabling network evolution
- Granovetter's Hypothesis
  - People often find jobs through weak ties, not close friends
  - This challenges the assumption that close friends are always more helpful

Summary (2/2)

- Triadic Closure
  - If two people share a mutual friend, they are more likely to become friends
  - The Clustering Coefficient measures how likely it is that a person’s friends are also friends with each other
  - It is the ratio of actual connections among a node’s neighbors to the total possible connections
- Tie Strength
  - Weak Ties: Acquaintances - useful for spreading information across distant parts of the network
  - Strong Ties: Close friendships - require ongoing effort but are limited in number
  - Granovetter’s Insight: Weak ties often act as bridges, linking different social groups
  - Strong Triadic Closure Property (STCP): If a person has strong ties to two others who are not connected, the structure is considered unstable
  - Weak ties can help maintain overall network structure when STCP is not fulfilled

References

[1] Easley & Kleinberg, J. (2010). *Networks, Crowds and Markets*. Cambridge: Cambridge University Press.

- Chapter 3: Strong & Weak Ties

[2] OLAT course page: https://olat.vcrp.de/url/RepositoryEntry/4669112833