Network Theory and Dynamic Systems

09. Dynamics I

SOSE 2025

Dr. -Ing. Stefania Zourlidou

Institute for Web Science and Technologies
Universität Koblenz

universität koblenz
weiter:denken

WeST
Web Science and Technologies

Recap from Previous Lecture

universität koblenz
weiter:denken

- The Configuration Model
- Preferential Attachment
- Other Preferential Models

Outline

- Ideas, Information, Influence
- Epidemic Spreading
  - Diseases
  - Rumor
- Opinion
- Search
  - Local Search
  - Searchability

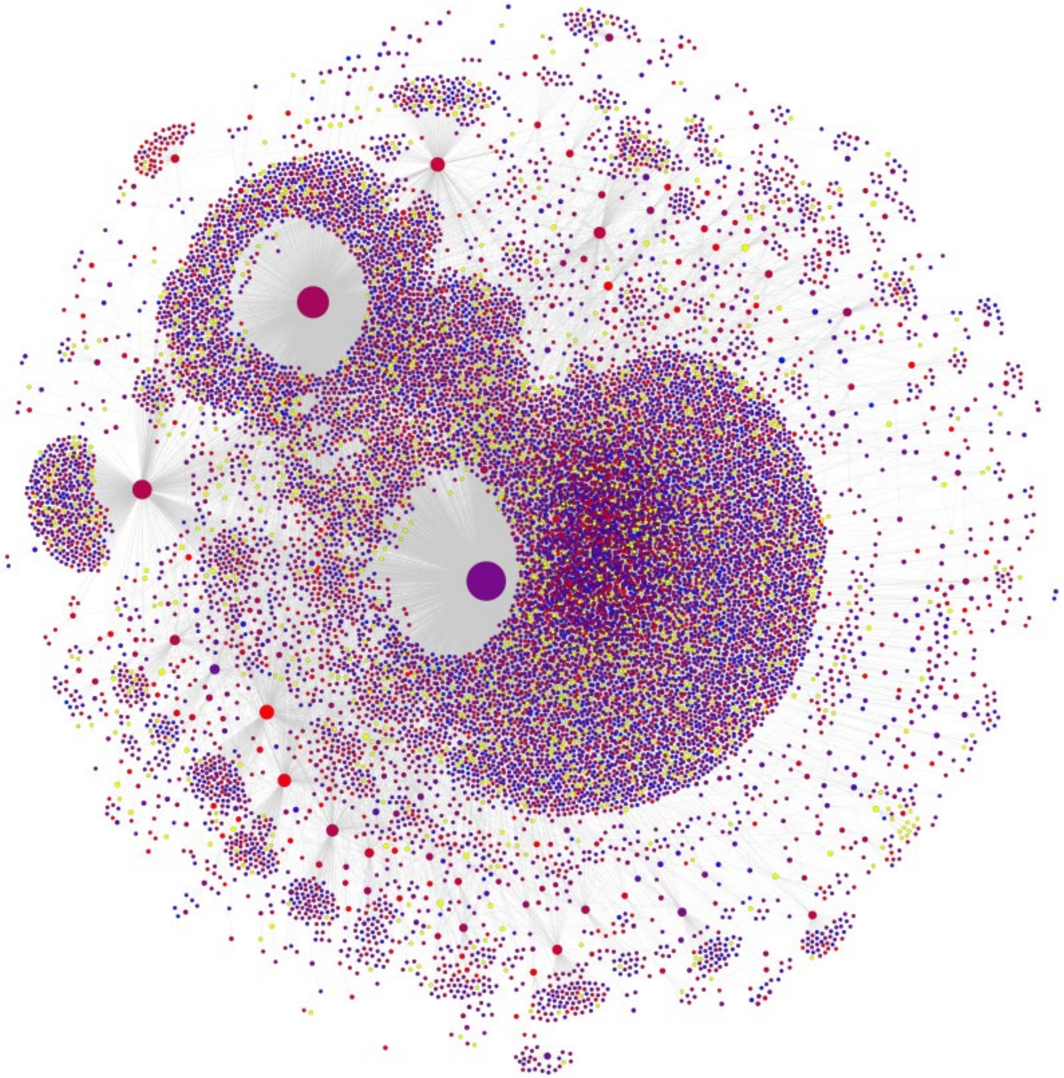
Dynamics I (this Lecture!)

Dynamics II (next Lecture!)

1. Ideas, Information and Influence as Networks

Example: Fake News Spreading

- During the 2016 US election, misinformation and fake news articles were widely shared across Twitter
- Nodes (Dots): Each dot represents a Twitter account that either posted or shared (retweeted) the fake news article
- Edges (Lines): These represent retweet or mention relationships—i.e., one account spreading content from another
- Red Nodes: These are accounts likely to be bots—automated accounts programmed to amplify specific content



Dynamics on Networks (1/2)

- **Misinformation diffusion** is a key example of how dynamic processes unfold on networks
  - Nodes (e.g., users or devices) interact with neighbors and **can adopt, modify, or pass on information**
- **What Changes Over Time?**
  - Node features: beliefs, knowledge, infection status, query targets, etc.
  - Influences often follow the structure of the network — **local neighbors impact a node's state**

Dynamics on Networks (2/2)

- **Common Examples:**
  - **Information Diffusion**: Like fake news or viral content spreading across social media
  - **Epidemic Spreading**: Diseases (e.g., COVID-19) propagating via human contact networks
  - **Opinion Dynamics**: How people's beliefs or preferences evolve (e.g., polarization, consensus)
  - **Network Search**: How information or targets are located through decentralized queries (e.g., peer-to-peer networks)

Information Diffusion (1/3)

universität koblenz
weiter:denken

- Networks play a central role **in how information spreads across society**
  - Individuals are influenced by their connections—family, friends, followers
- **Everyday Examples:**
  - **Consumer behavior:** We might buy a new phone because our friend did
  - **News exposure:** We often learn about breaking news through social media reposts or forwarded messages
- **Social Contagion:**
  - The process by which opinions, ideas, or behaviors spread from person to person
  - Just like viruses, information "infects" nodes via contact with neighbors in a social network

Information Diffusion (2/3)

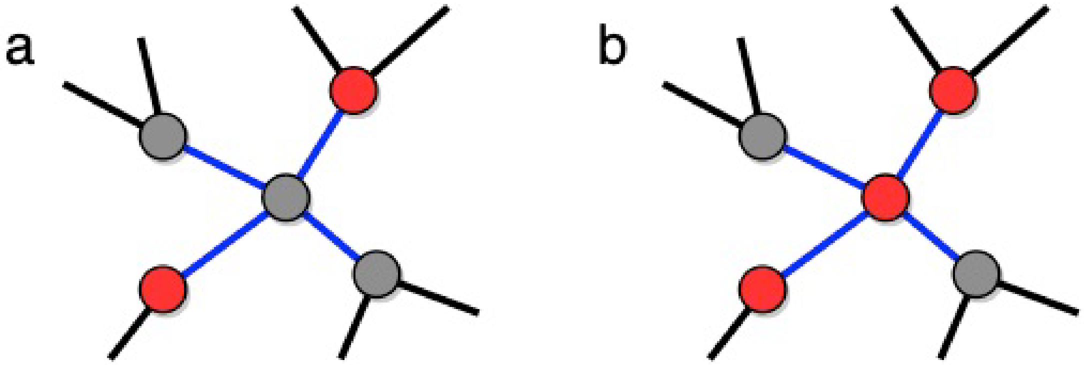
universität koblenz
weiter:denken

- We use **models** to simulate how information, behaviors, or innovations spread in a network
- **Basic Setup:**
  - **Seed nodes (influencers)**: A small set of individuals is initially activated (e.g., early adopters of an idea or technology)
  - **Activation rule**: Other nodes become active based on how many of their neighbors are active, and possibly other factors (e.g., thresholds, probabilities)

Information Diffusion (3/3)

universität koblenz
weiter:denken

- **Outcome**: The process results in **influence cascades** — a chain reaction where nodes activate one after another, triggered by their neighbors
- **What Do Cascades Look Like?**
  - **Small cascades**: Only a **few nodes** are influenced before the spread dies out
  - **Global cascades:** A **large part** of the network becomes activated — possibly reaching everyone
- **Example**
  - **a:** Initial state with red (active) influencers and inactive gray nodes
  - **b:** After applying the activation rule, more nodes turn red → showing how activation spreads



2. Threshold Models

Threshold Models (1/4)

universität koblenz
weiter:denken

- Principle: A node becomes active **only if the total influence** from its active neighbors **exceeds a predefined threshold**
  - This models resistance or hesitation to adopt new behaviors, ideas, or technologies
- Linear threshold model: the influence on a node *i* is defined as a sum over its active neighbors, in which the contribution of each neighbor is given by the weight of the link joining it to the node

$$I(i) = \sum_{j:\,active} w_{ji}$$

  - w_{ji} = weight of the link from *j* to *i*

Threshold Models (2/4)

- Activation condition $I(i) \geq \theta_i$

  where θ_i is the threshold of node *i*, indicating its tendency to be influenced
- On unweighted networks $I(i) = n_i^{on} \geq \theta_i$

  where n_i^{on} is the number of active neighbors of node *i*
- If all nodes have the same threshold θ

$$I(i) = n_i^{on} \geq \theta$$

Threshold Models (3/4)

universität koblenz
weiter:denken

- **Model dynamics**
  - Start with some initially activated nodes — chosen randomly or based on a proportion.
  - Activation is permanent: once a node becomes active, it stays active.
  - Each inactive node checks whether the influence from active neighbors meets or exceeds its threshold

Threshold Models (4/4)

- **Two Update Strategies:**
  - **Asynchronous Update**
    - Nodes update **one at a time**, using the **current (possibly already updated)** states of their neighbors
    - Example: A node updates based on its neighbor who just became active moments earlier
  - **Synchronous Update**
    - All nodes evaluate their states **simultaneously**, using the neighbor states **from the previous step**
    - Example: The system evolves in rounds, where all nodes update together after each round

3. Fractional Threshold Models

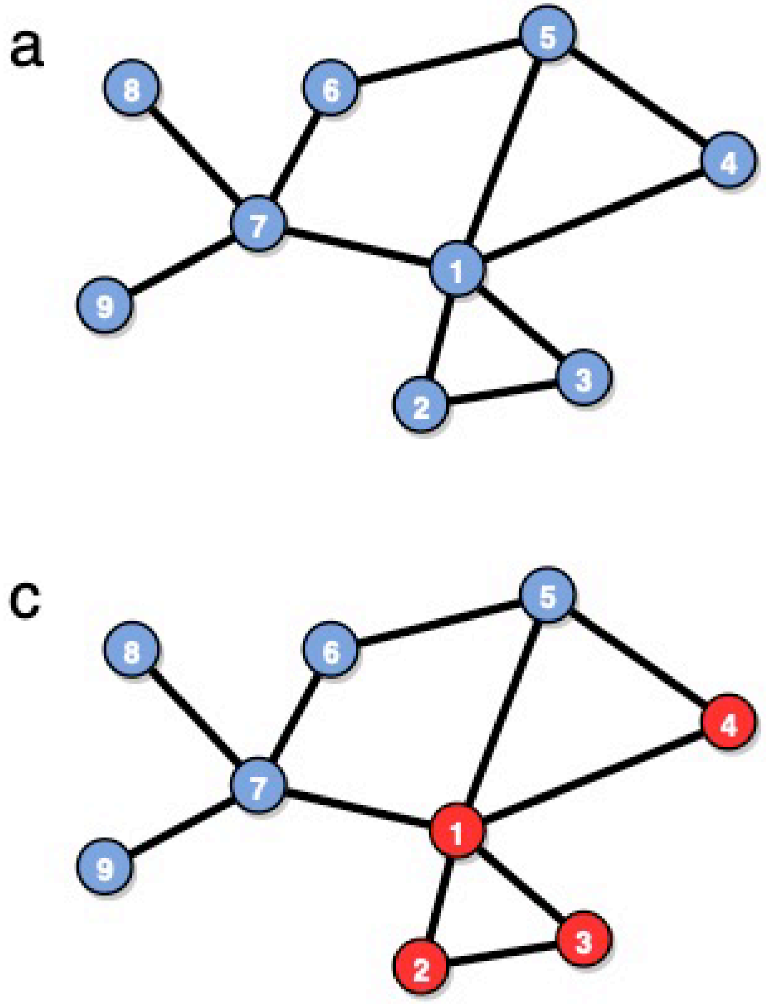
Fractional Threshold Model (1/7)

- **Principle:** Instead of counting how many neighbors are active, we focus on the fraction of active neighbors
- A node activates if this fraction exceeds its threshold θ_i
- **Example**
  - if θ=1/2, then at least 50% of the neighbors must be active for node i to activate
- **Activation condition**

$$\boxed{\frac{n_i^{on}}{k_i} \geq \theta_i}$$

- n_i^{on}: number of active neighbors of node i
- k_i: total number of neighbors (degree of node i)
- θ_i: activation threshold (fractional)

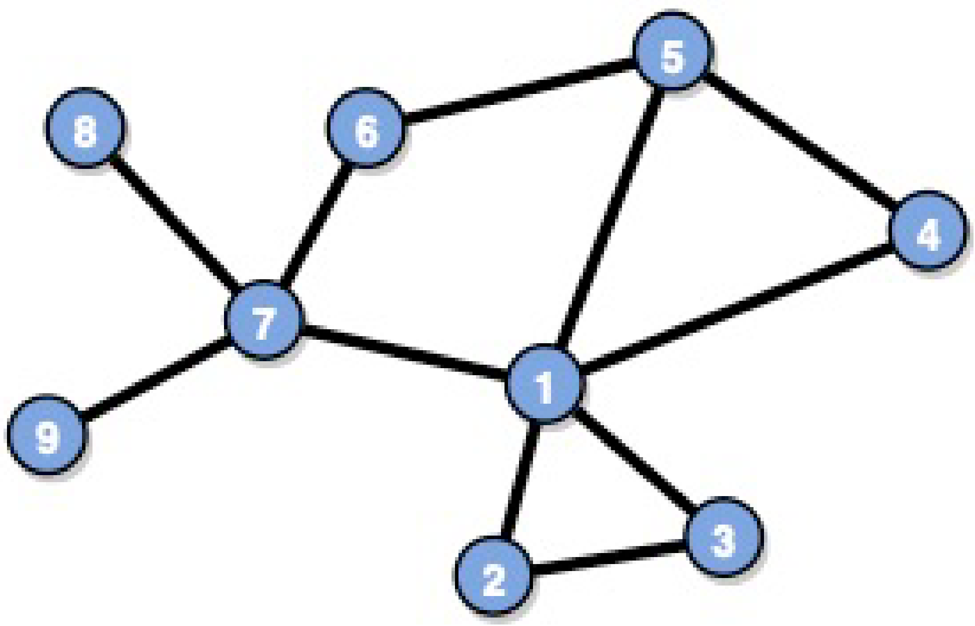
Fractional Threshold Model (2/7)



$\theta = 1/2$

Fractional Threshold Model (3/7)

What if the initial influencer is node 7?



Fractional Threshold Model (4/7)

- In **sparse networks**, whether a cascade spreads or not depends heavily on the **network structure**—not just thresholds
- Key driver: **Vulnerable Nodes**
  - These are nodes that can be activated by just one active neighbor
  - They are crucial to the onset and continuation of cascades
- **Condition for a node to be vulnerable**

$$k_i \leq \frac{1}{\theta_i}$$

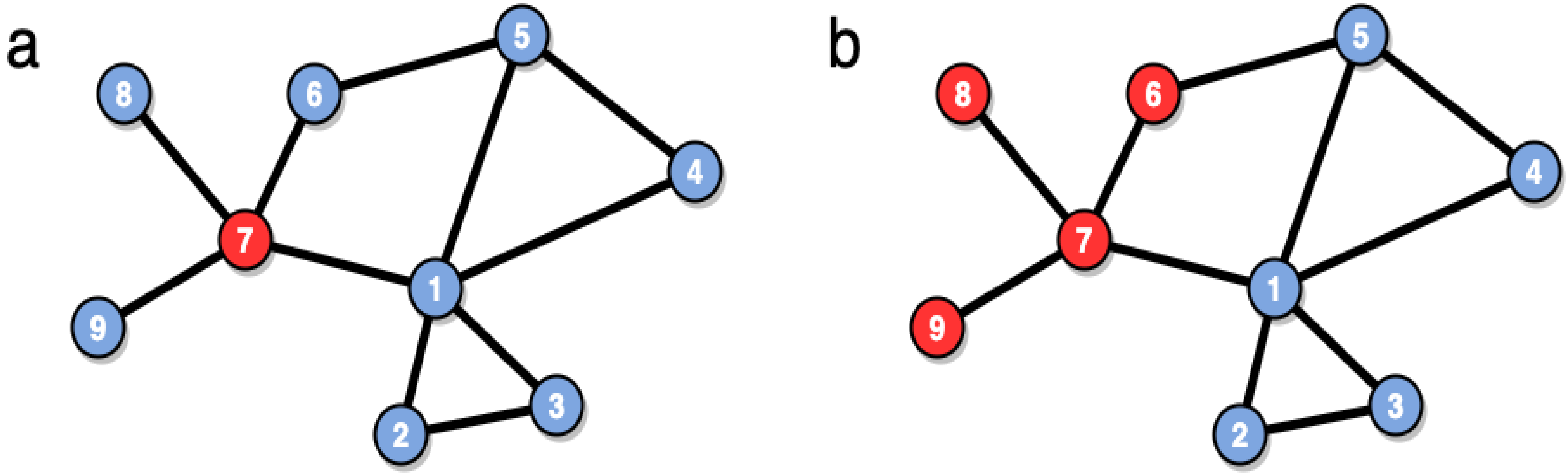
- **Global Cascades**:
  - To trigger large-scale (global) cascades, the network must contain **enough vulnerable nodes**
  - Their presence creates "weak spots" where activation can propagate quickly

Fractional Threshold Model (5/7)

- **Hubs and Influence:**
  - Hubs (high-degree nodes) are usually strong influencers
  - **But**: Being a hub doesn't guarantee effective influence—position and context matter
- Importance of Node Position:
  - Nodes in the core of the network are more likely to trigger global cascades
  - Peripheral nodes (on the edges) are less impactful in spreading activation widely
- Importance of Community Structure:
  - Spreading is easier within dense communities (many intra-connections)
  - Cross-community influence is harder:
    - Nodes must activate across sparser links
    - Few inter-community connections mean activation thresholds are harder to meet

Fractional Threshold Model (6/7)

$\theta=1/2$



Fractional Threshold Model (7/7)

universität koblenz
weiter:denken

- **Cascade Control**
  - Understanding the network structure enables us to influence or guide cascades
  - Even small initial activations can lead to large cascades—if the right nodes are chosen
- **Identifying Key Influencers**
  - Pinpointing the most impactful nodes is crucial for spreading ideas, behaviors, or innovations.
  - Especially important in marketing, public health, and political campaigns
- **Viral Marketing**
  - Social networks are used to trigger large-scale diffusion with minimal effort
  - Strategy: target influential nodes to initiate word-of-mouth effects

4. Independent Cascade Models

Independent Cascade Models (1/4)

- Principle of threshold models
  - Based on **peer pressure**: the **more neighbors** try to influence you, the **higher the chance** you'll adopt their behavior

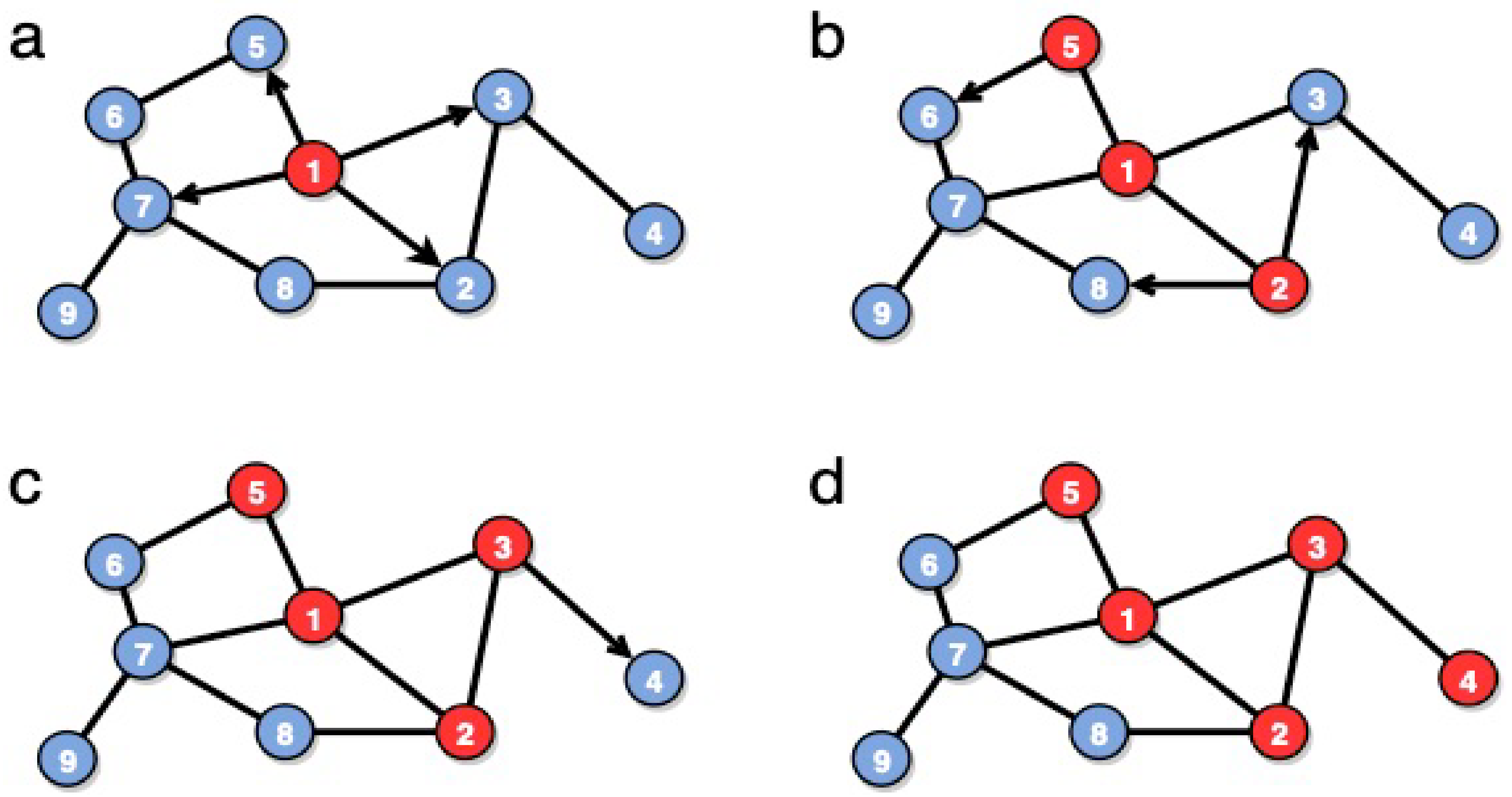
  But

  - In reality, we may be persuaded by just **one** passionate friend
  - Influence happens in individual interactions, not just group pressure
- **Alternative principle**
  - Each contact has a separate, independent chance to influence you.
  - Influence spreads via pairwise (node-to-node) interactions
- **Independent cascade models** are based on node-node interactions!

Independent Cascade Models (2/4)

- **Model dynamics**
  - An active node *i* has a probability p_{ij} to convince its inactive neighbor *j* ($p_{ij} \neq p_{ji}$, in general)
  - All active nodes are considered in sequence
    - The inactive neighbor *j* of the active node *i* is activated with probability p_{ij}
    - All inactive neighbors of *i* have one chance to be persuaded by *i*
  - If a node *j* is activated, it has only one chance to activate its inactive neighbors

Independent Cascade Models (3/4)



Independent Cascade Models (4/4)

universität koblenz
weiter:denken

- **Remark:** The more active neighbors a node has, the higher the chance it will eventually be activated — but not guaranteed
- **Independent Cascade vs. Threshold Models**
  - Threshold models: check inactive nodes and whether their thresholds are met
  - Independent cascade: focus on active nodes and how they try to activate others
  - Threshold models are typically deterministic: activation happens if a rule is satisfied
  - Independent cascade models are probabilistic: activation is uncertain, based on fixed activation probabilities per edge
    - Probabilistic nature makes independent cascades harder to predict and control

Information Diffusion

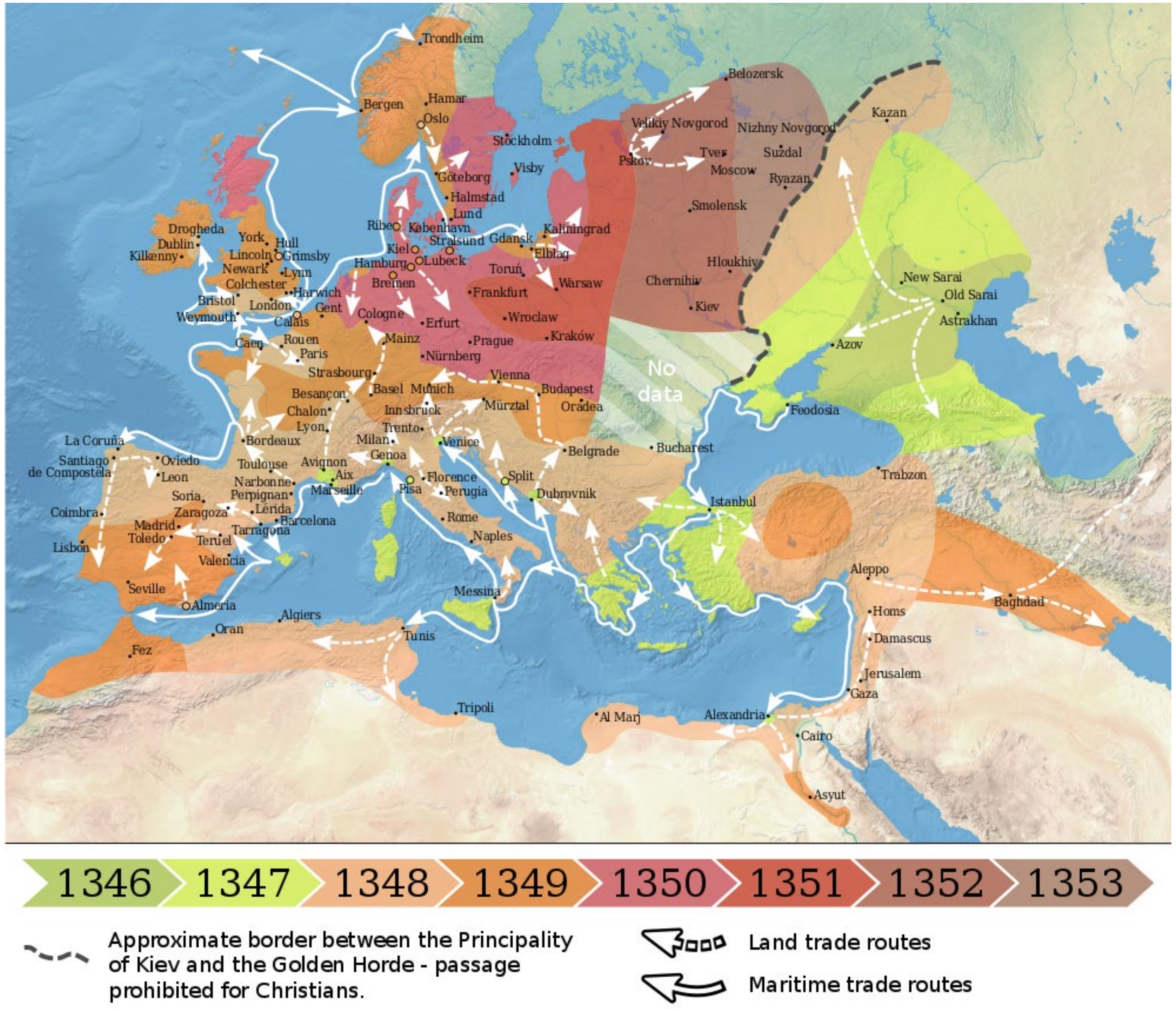
- **Problem:** Basic models are often **too simplistic** to capture real-world dynamics
- **Solution:** Use **more sophisticated variants** that better reflect how information spreads in practice
- **Example**
  - **Probabilistic version of threshold model**: activation probability increases with the number of active neighbors, rather than using a strict yes/no rule
  - **Modified Cascade Model**: Similar to independent cascade model, except that the active neighbors **do not exert influence independently of each other!**
  - **Complex contagion:** Influence **strengthens with repetition**—the more people expose us to an idea, the more likely we adopt it
    (e.g., seeing a product multiple times from different friends boosts adoption)

5. Epidemic Spreading: Diseases and Rumors

5.1 Network Models for Disease Spreading

Epidemic Spreading: the Black Death

- Probably originated in Central Asia, it spread throughout all of Europe between 1346 and 1353
- The Black Death is estimated to have killed 30-60% of Europe's population



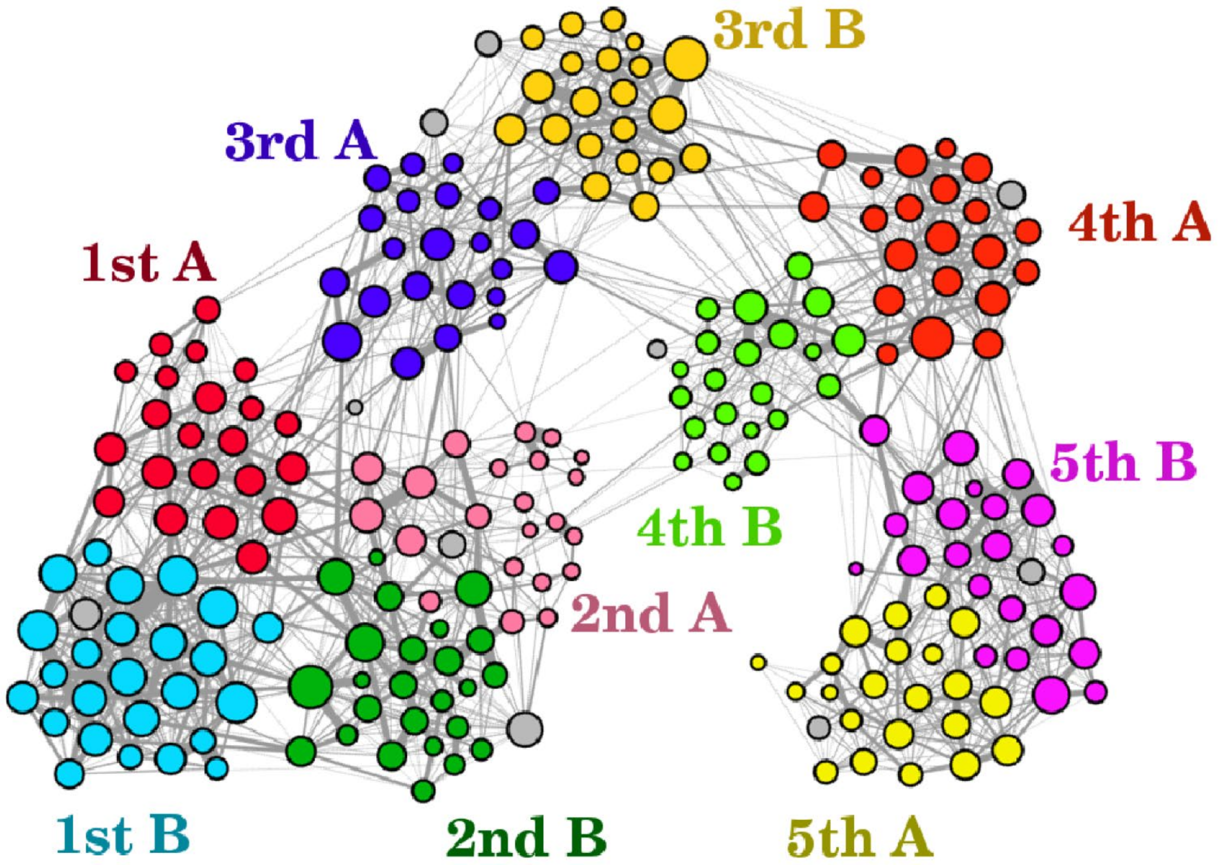
Epidemic Spreading

universität koblenz
weiter:denken

- **Problems**
  - Faster Global Transmission
    - Due to modern transportation, diseases now spread across continents in hours
    - Example: A person contracting Ebola in Africa could reach Europe or Asia before symptoms appear, unknowingly spreading the virus
  - New Forms of Epidemics via Technology
    - Epidemics aren't just biological anymore — digital and informational outbreaks are now common
      - Computer viruses & malware spread rapidly over the Internet, e.g., via email, downloads, or vulnerabilities
      - Mobile phone viruses spread via Bluetooth or MMS
      - Misinformation spreads virally across social media platforms, mimicking biological contagion

Contact Networks

- Epidemics propagate through contact networks
- These networks can model:
  - Physical contact (e.g., shaking hands)
  - Transportation systems (e.g., flights, buses)
  - Digital channels (e.g., email, social media)
  - Mobile communication (e.g., Bluetooth, messaging)



Epidemic Models

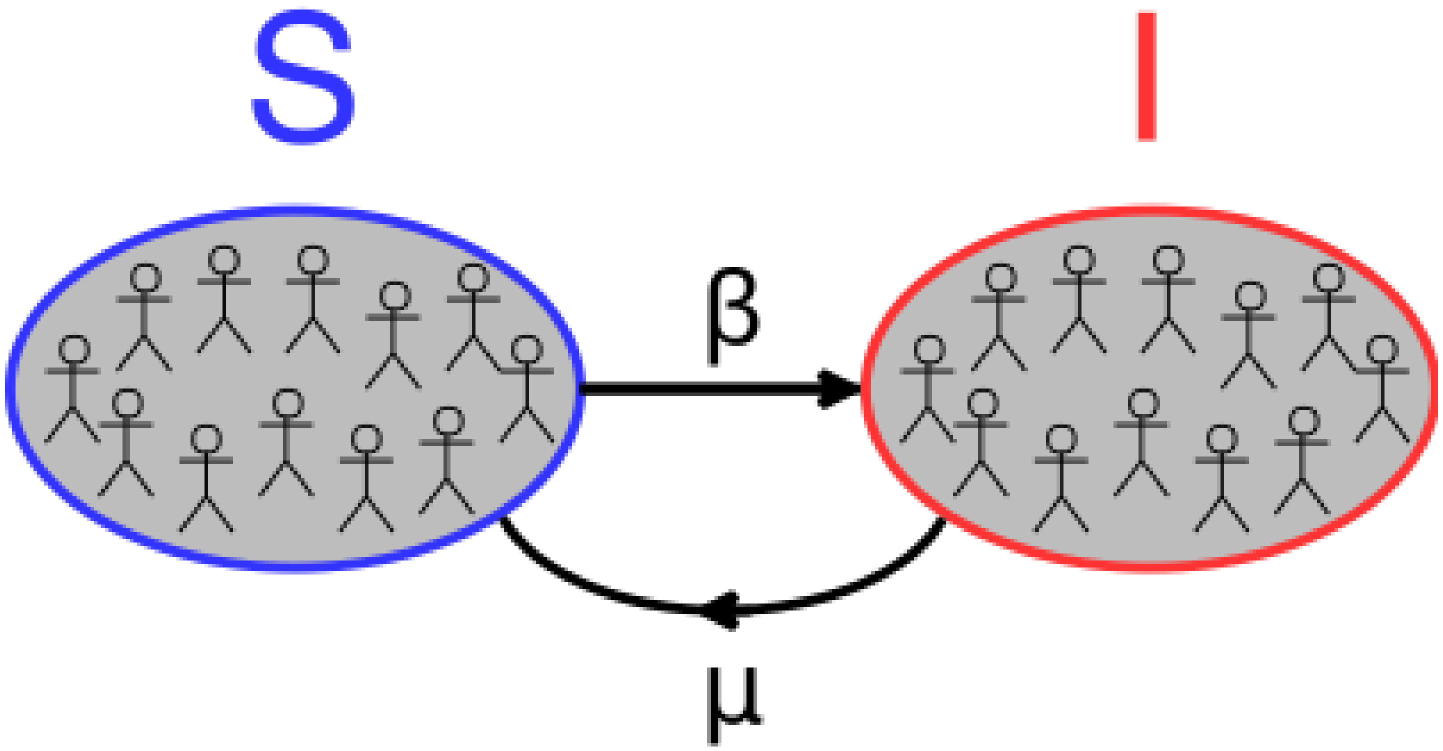
universität koblenz
weiter:denken

- Classic epidemic models divide the population into **compartments**, corresponding to different stages of the disease
- Key compartments
  - Susceptible (S): individuals who can contract the disease
  - Infected (I): individuals who have contracted the disease and can transmit it to susceptible individuals
  - Recovered (R): individuals who recovered from the disease and cannot be infected anymore

The SIS Model (1/3)

- Just **two compartments**: Susceptible (S) and Infected (I)
- **Dynamics**
  - A susceptible individual gets infected with a probability β (**infection rate**)
  - An infected individual recovers and becomes susceptible again with a probability μ (**recovery rate**)
  - The model applies to diseases that do not confer long-lasting immunity (e.g., common cold)

The SIS Model (2/3)

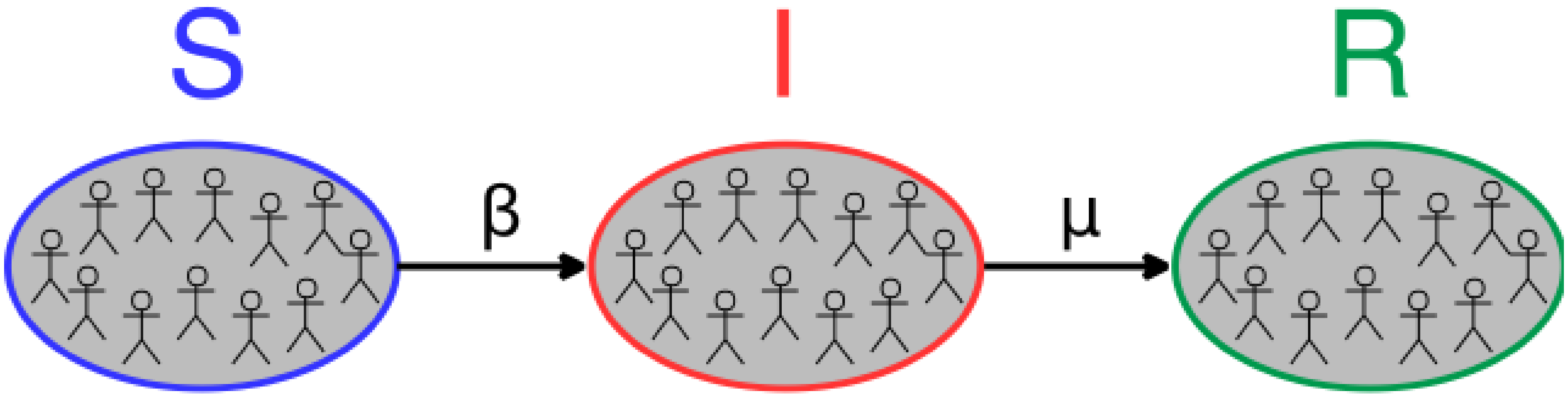


The SIS Model (3/3)

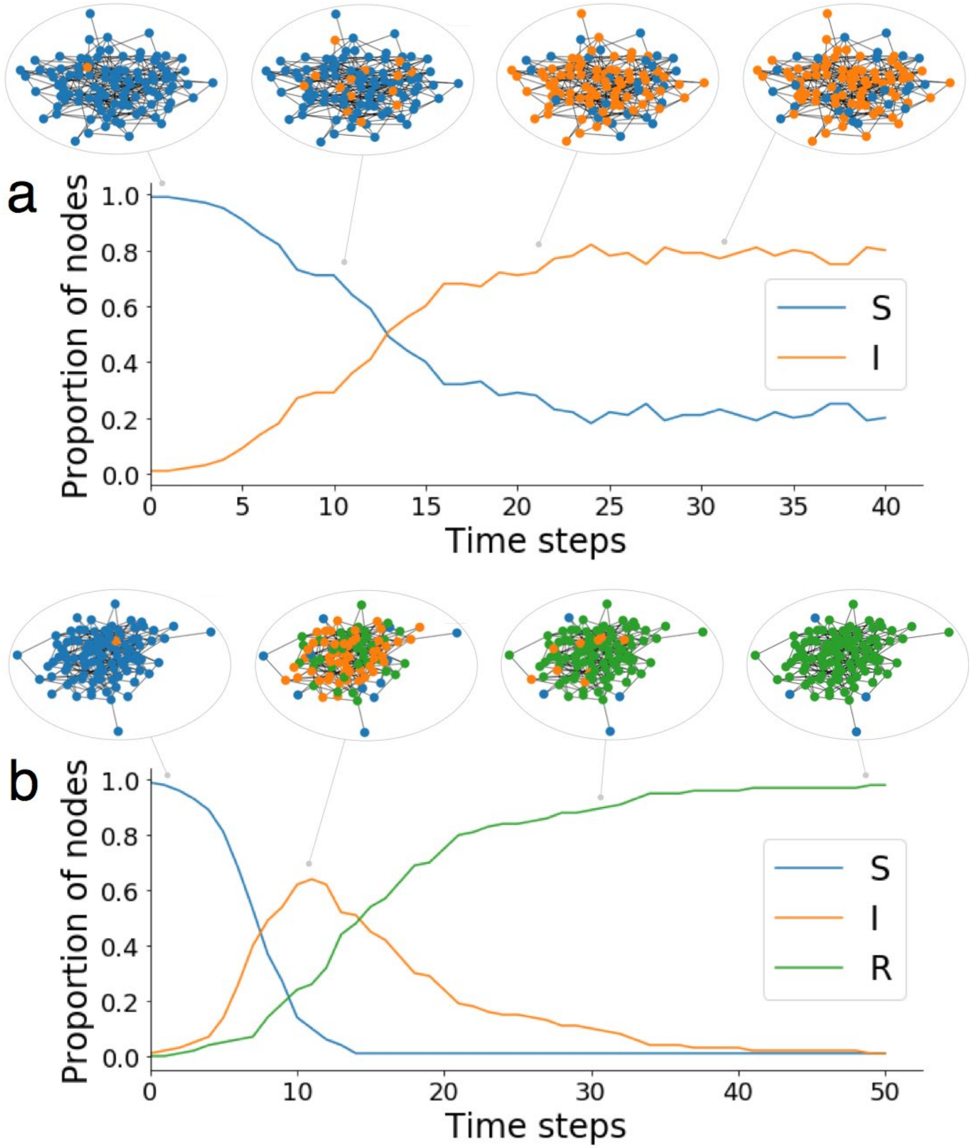
- **Simulation of SIS dynamics on networks**
  - Take a network (e.g., a random network or a real contact network)
  - A number (fraction) of the nodes are infected (e.g., at random), all others are susceptible
  - All nodes are visited in sequence
  - For each node i:
    - If i is susceptible, loop over its neighbors: for each infected neighbor, i becomes infected with probability β
    - If i is infected, it becomes susceptible with probability μ

The SIR Model

- **Difference from SIS model:** when infected individuals recover, they do not become susceptible again, but they are moved to the compartment R and play no further role in the dynamics
- The model applies to diseases that confer long-lasting immunity (e.g., measles, mumps, rubella, etc.)



Epidemic Spreading



- Three characteristic stages of the dynamics
  - **Initial stage:** just a few people are infected, and the diffusion of the epidemic is irregular and slow
  - **Ramp-up phase of exponential growth**, that can quickly affect a large number of people
  - **Stationary state**, in which the disease is either **endemic**, i.e. it affects a stable fraction of the population over time, or **eradicated**

Homogeneous Mixing

- **Hypothesis:** every individual is in contact with every other
- **Consequence:** all individuals in the same compartment have identical behavior and only the relative proportions of people in the various compartments matter for the model dynamics
- Justified for a small population, e.g., the inhabitants of a little village where all people are in touch with each other.
- In real large-scale epidemics, individuals can only be infected by the people they come in contact with
  - In this case it is **necessary to reconstruct the actual network of contacts**

SIS & SIR Models on Networks (1/3)

- **Start:** homogeneous contact network, with all nodes having degree approximately equal to $<k>$
- **Early stage:** few people are infected, so we can assume that every infected individual is in contact with mostly susceptible individuals
- Each infected individual can transmit the disease to about $<k>$ people at each iteration —> the expected number of people infected by a single person after one iteration is $\beta<k>$
- If there are I infected individuals, we expect to have $I_{sec} = \beta<k>I$ new infected people after one iteration and $I_{rec} = \mu I$ recovered people

SIS & SIR Models on Networks (2/3)

- Threshold condition for epidemic spreading: $I_{sec} > I_{rec}$

$$\beta\langle k\rangle I > \mu I \implies R_0 = \frac{\beta}{\mu}\langle k\rangle > 1$$

- $R_0 = \beta<k>/\mu$ is the **basic reproduction number**
- If $R_0 < 1$, the **initial outbreak dies out in a short time**, affecting only a few individuals
- If $R_0 > 1$, the **epidemic keeps spreading**

SIS & SIR Models on Networks (3/3)

- Problem: real contact networks are not homogeneous
- Hubs drastically change the scenario. **On contact networks with hubs there is effectively no epidemic threshold —>** even diseases with low infection rate and/or high recovery rate may end up affecting a sizable fraction of the population!
- Reason: even if the infection rate is low, the process is likely to eventually infect a hub, via one of its many contacts; the hub can in turn infect a large number of susceptible individuals, including possibly other hubs, and so on
- Effective disease containment strategies should aim at **isolating/vaccinating individuals with many contacts**. The latter can be identified by picking the endpoints of randomly selected links, as this increases the chance to bump into hubs. So, don't vaccinate a random sample of the population: **vaccinate their friends**!

5.2 Network Models for Rumor Spreading

Rumor Spreading (1/5)

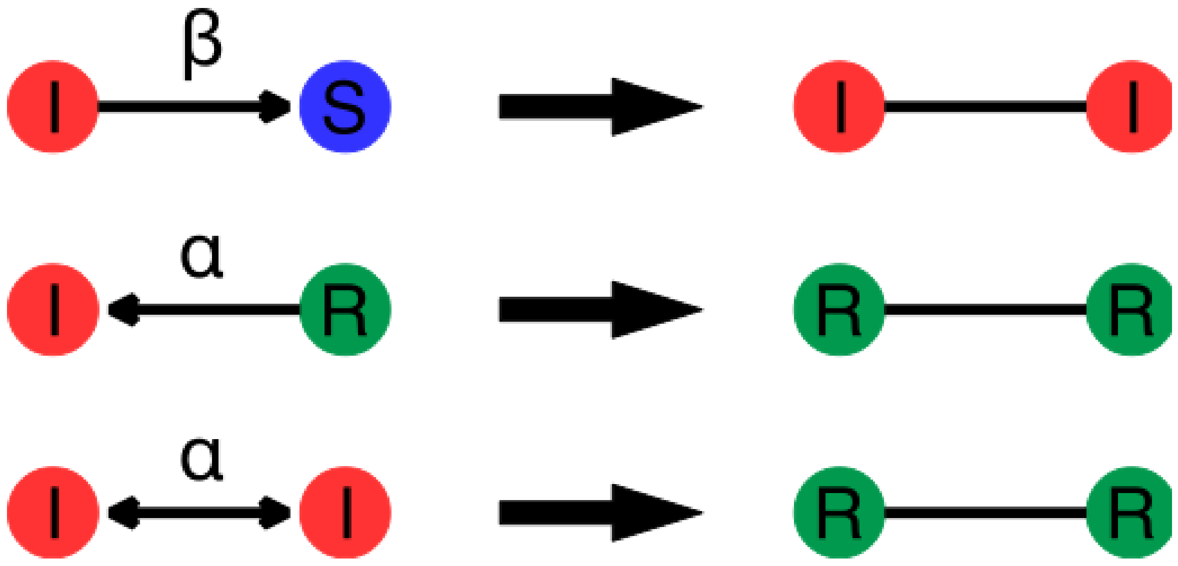
- Rumor spreading can be described as an epidemic spreading process
- **Simple model**: variant of SIR
- **Three compartments:** ignorant (S), spreaders (I) and stiflers (R). Stiflers are people who know the rumor but do not spread it
- **Basic idea:** people are engaged in the diffusion of the rumor as long as they find people who are unaware of it, otherwise they lose interest and stop spreading the rumor

Rumor Spreading (2/5)

- Model
  - When a spreader approaches an ignorant, the rumor is told and the ignorant becomes a spreader with a **transmission probability**
  - When a spreader meets a stifler, the spreader becomes a stifler with a **stop probability**
  - When two spreaders meet, they both turn to stiflers with the same stop probability

Rumor Spreading (3/5)

- Two parameters
  - **Transmission probability β**
  - **Stop probability α**
- Setup: network, real or computer-generated, all nodes are in state S (unaware of the rumor), except a few of them, which are in state I (aware of the rumor and willing to spread it)



ignorant (S), spreaders (I), stiflers (R)

Rumor Spreading (4/5)

- Dynamics
  - At each iteration all nodes are visited *synchronously* or *asynchronously* in random order. For each *i*:
    - If *i* is ignorant, loop over its neighbors: for each spreader neighbor, *i* becomes a spreader with probability β
    - If *i* is a spreader, loop over its neighbors
      - For each stifler neighbor, i becomes a stifler with probability α
      - For each spreader neighbor, i and the neighbor both become stiflers with probability α

Rumor Spreading (5/5)

- **Important difference from SIR model:** the transition from I to R does not occur spontaneously (in that a sick person recovers from the disease), but depends on the interaction between individuals
- As in the SIR model, starting from a few spreaders, eventually all individuals will be either ignorant or stiflers, as in this final state the dynamics cannot produce any change
- **No threshold effect:** the rumor can reach a large number of people even if the transmission probability is low, both on homogeneous and heterogeneous networks

6. Summary

Summary (1/3)

- Threshold Models of Influence Diffusion
  - Nodes/individuals are influenced by the combined effect of all neighboring influencers
  - Influence effect must exceed a threshold for the node to be affected
- Fractional Threshold Models
  - A node is influenced when the proportion of its neighbors that are influencers exceeds a certain fractional threshold
- Independent Cascade Models
  - Nodes/individuals are convinced by each neighbor influencer with a certain probability
  - Most effective influencers have a large degree and central network position

Summary (2/3)

- Susceptible–Infected–Susceptible (SIS) Model of Epidemic Spreading
  - Infected individuals recover and become susceptible again
  - Individuals can contract the disease multiple times
- Susceptible–Infected-Recovered (SIR) Model of Epidemic Spreading
  - Infected individuals recover and cannot be infected again
  - Recovered individuals play no further role in the dynamics

Summary (3/3)

- Impact of Contact Networks with Hubs
  - Diseases spreading according to both SIR and SIS dynamics can affect a large population fraction, even with low infection probability
  - Hubs can easily be infected and become dangerous spreaders

- Rumor-Spreading Model
  - Similar to SIR, but recovery (decision to stop spreading the rumor) results from encounters between individuals who know the rumor
  - Rumor can reach a significant portion of the network even with low transmission probability

References

[1] Menczer, F., Fortunato, S., & Davis, C. A. (2020). A First Course in Network Science Cambridge: Cambridge University Press.

- Chapter 7.1, 7.2

[2] OLAT course page: https://olat.vcrp.de/url/RepositoryEntry/4669112833