Artificial Intelligence: Solutions to Exercise 5

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1. Let the three-place predicate "Child" and the one-place predicate "Female" from the family tree example in the lecture be given.
   Define:
   (a) A one-place predicate "Male".
   (b) A two-place predicate "Father" and "Mother".
   (c) A two-place predicate "Siblings".
   (d) A predicate "Parents(X, Y, Z)", which is true if and only if X is the father and Y is the mother of Z.
   (e) A predicate "Uncle(X, Y)", which is true if and only if X is the uncle of Y (use the predicates that have already been defined).
   (f) A two-place predicate "Ancestor" with the meaning: ancestors are parents, grandparents, etc. of arbitrarily many generations.

   Solution:

   (a) A one-place predicate "Male".
   We can first define the one-place relation

$$male = \{FranzA., OscarA., OscarB., HenryA., ClydeB.\}$$

   and then the predicate

$$\mathcal{I}(\text{Male}(X)) = T \equiv (\mathcal{I}(X)) \in male.$$

   In this family we have only persons who identify as man or women and who are in line with their biological sex[1]. Therefore, it is also possible to define the predicate in an alternative way via

$$\forall X \left(\text{Male}(X) \Leftrightarrow \neg\text{Female}(X)\right),$$

   where the predicate Female was defined in the lecture. This formula is only correct in our domain and it will be incorrect in a family where there are people identifying as with a non-binary gender

[1] With the term gender we don't mean biological sex, but the gender people identify with. Biological sex is in most cases a well defined concept from the genetics point of view.

(b) The two-place predicates “Mother” and “Father”.

$$\forall X \forall Y \, \text{Mother}(X,Y) \Leftrightarrow \exists Z \, \text{Child}(Y,X,Z) \wedge \text{Female}(X)$$

This can be rewritten as

$$\forall X \forall Y \exists Z \, \text{Mother}(X,Y) \Leftrightarrow \text{Child}(Y,X,Z) \wedge \text{Female}(X)$$

Analogously

$$\forall X \forall Y \exists Z \, \text{Father}(X,Y) \Leftrightarrow \text{Child}(Y,X,Z) \wedge \text{Male}(X).$$

(c) A two-place predicate “Siblings”.

$$\forall X \forall Y \, \text{Siblings}(X,Y) \Leftrightarrow$$
$$(\exists U \, \text{Father}(U,X) \wedge \text{Father}(U,Y)) \wedge (\exists V \, \text{Mother}(V,Y) \wedge \text{Mother}(V,Y)))$$

(d) A predicate “Parents(X,Y,Z)”, which is true if and only if X is the father and Y is the mother of Z.

$$\forall X \forall Y \forall Z \, \text{Parents}(X,Y,Z) \Leftrightarrow \text{Father}(X,Z) \wedge \text{Mother}(Y,Z)$$

(e) A predicate “Uncle(X,Y)”, which is true if and only if X is the uncle of Y (use the predicates that have already been defined).

$$\forall X \forall Y \, \text{Uncle}(X,Y) \Leftrightarrow \exists U \exists V \, \text{Child}(Y,U,V) \wedge \text{Male}(X) \wedge \text{Siblings}(U,X)$$

(f) A two-place predicate “Ancestor” with the meaning: ancestors are parents, grandparents, etc. of arbitrarily many generations.

$$\forall X \forall Y \, \text{Ancestor}(X,Y) \Leftrightarrow$$
$$\exists Z \, \text{Child}(Y,X,Z) \vee \exists U \exists V \, \text{Child}(Y,U,V) \wedge \text{Ancestor}(X,V)$$

2. Adapt Exercise 1 (b) and replace the predicate “Mother” by a one-place function symbol. How can the function be defined using the predicates Female(X) and Child(X,Y,Z)?
Solution:
$\forall X \forall Y \exists Z \; X = \mathit{mother}(Y) \Leftrightarrow \text{Female}(X) \wedge \text{Child}(Y,X,Z)$

3. Formalize the following statements in predicate logic:

   (a) Every person has a father and a mother.
   (b) Some people have children.
   (c) All birds fly.
   (d) There is an animal that eats (some) plant-eating animals.
   (e) Every animal eats plants or plant-eating animals which are much smaller than itself.

Solution:

(a) Every person has a father and a mother.
We introcduce the predecate Person(X) to indicate that an object X is a person.

$$\forall X \, \text{Person}(X) \Rightarrow \exists U \text{Mother}(U, X) \wedge \exists V \text{Father}(V, X)$$

(b) Some people have children.

$$\exists X \exists Y \exists Z \, (\text{Person}(X) \wedge \text{Child}(Y, X, Z))$$

(c) All birds fly.

$$\forall X \, \text{Bird}(X) \Rightarrow \text{Fly}(X)$$

(d) There is an animal that eats (some) plant-eating animals. We introduce the one-place predicates Animal and Plant, and the two-place predicate Eats(X, Y), which indicates that X eats Y.

$$\exists X \exists Y \exists Z \, \text{Animal}(X) \wedge \text{Animal}(Y) \wedge \text{Eats}(X, Y) \wedge \text{Eats}(Y, Z) \wedge \text{Plant}(Z)$$

(e) Every animal eats plants or plant-eating animals which are much smaller than itself.

$$\begin{aligned} \forall X \, \text{Animal}(X) \Rightarrow & \; (\exists Y \text{Eats}(X, Y) \wedge \text{Plant}(Y)) \\ & \vee (\exists Z \exists U \text{Eats}(X, Z) \wedge (\text{Eats}(Z, U) \wedge \text{Plant}(U) \wedge \text{Smaller}(Z, X))) \end{aligned}$$

4. Give predicate logic axioms for the two-place relation "<" as a total order. For a total order we must have (1) Any two elements are comparable. (2) It is asymmetric. (3) It is transitive.
Solution:
We define the predicate > in the following way

(a) Any two elements are comparable:

$$\forall X \forall Y \, (X < Y) \vee (X < Y) \vee X = Y$$

(b) Asymmetry;

$$\forall X \forall Y \, X < Y \Rightarrow \neg(Y < X)$$

(c) Transitivity:

$$\forall X \forall Y \forall Z \, (X < Y) \wedge (Y < Z) \Rightarrow (X < Z)$$