Assignment 02: Network Elements

Performance summary

✓ Assessed

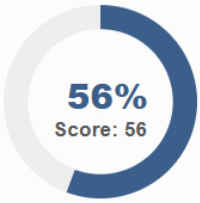
Success status	Score	Attempts
Undefined	56 of 100 points	1

Results

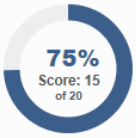
Course	**Network Theory and Dynamic Systems (SS 2025)** ID: 4669112833 / 109642102391946
Test	**Assignment 02** ID: 4548624789

This are your test results

Duration	5h 1m 55s 4/27/2025, 11:53 AM - 4/29/2025, 12:45 PM
Answered	13 of 13 questions (100%)
Your score	56 of 100 points (56%)

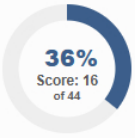


1. The Manhattan Road Map (20 points) 3



go to section

2. Social Networks (44 points) 5



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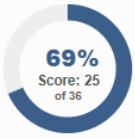
Adjacency matrix (36 points) 5



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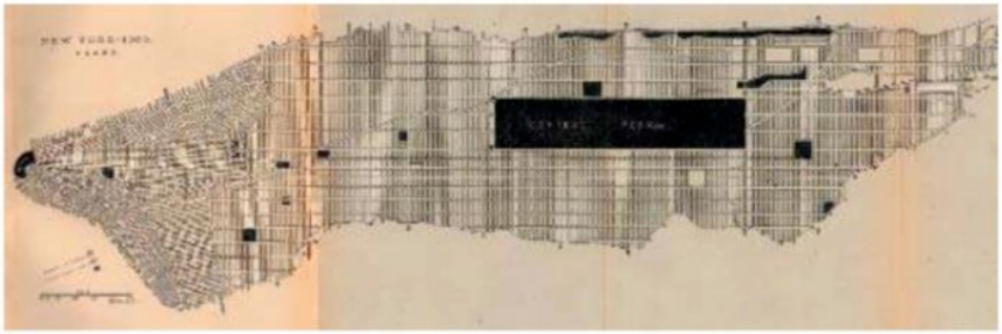
1. The Manhattan Road Map (20 points) 15 of 20 points (75%)



Map of New York in 1880. From Report on the Social Statistics of Cities, Compiled by George E. Waring, Jr., U.S. Census Office, 1886. Image courtesy of University of Texas Libraries.

Q1. Node Degree (5 points)

Status	Answered	
Your score	5 / 5	100%

Response

Consider the road map in the figure. The grid-like structure of this network means that most nodes have the same degree. What is the most common degree for nodes in this network? Write your answer without blank spaces before or after the number.

4.0

Solution

Q2. In- and Out-Degree (10 points)

Status	Answered	
Your score	10 / 10	100%

Response

Consider the road map in the above Figure. Manhattan has a lot of one-way streets. This implies that a good network model of traffic flow would probably have directed links. Consider a subgraph of this network with grid-like connectivity and all one-way streets (i.e, each node is a four-way intersection of two one-way streets).

What is the most common in-degree of nodes in this subgraph? Do not add blank spaces in your answer.

2.0

What is the most common out-degree?

2.0

Solution

··· Q3. Networks (5 points)

Status	Answered	
Your score	0 / 5	0%

Response

What **network quantity** can we use to represent the volume of traffic between each pair of adjacent intersections in the Manhattan road map? Do not add blank spaces in your answer.

edge weight

Solution

What **network quantity** can we use to represent the volume of traffic between each pair of adjacent intersections in the Manhattan road map? Do not add blank spaces in your answer.

link weight weight, Link weight, Link Weight, Weight

go back to overview

2. Social Networks (44 points) 16 of 44 points (36%)

··· Q4. Subnetworks (8 points)

Status	Answered	
Your score	8 / 8	100%

Response

Imagine that your social network has a subnetwork where you and 24 of your friends (25 people total) are all friends with each other.

What is such a subnetwork called?

Clique

And how many links are contained in the subnetwork?

300.0

Solution

Q5. Complete Networks (8 points)

Status	Answered	
Your score	0 / 8	0%

Response

Recall that in a complete network there exists a link between each pair of nodes. We know that a complete undirected network of N nodes has N(N - 1)/2 edges. Must any undirected network of N nodes and N(N - 1)/2 links be complete? Explain why or why not.

Yes, it is true that any undirected network of N nodes, having precisely N(N - 1)/2 connections, must necessarily be assumed to be a complete network.

Here's why: In an undirected network, there's a link to connect two nodes together in combination without indicating any particular direction. Importantly, there can be no more than one link connecting each distinct pair of nodes. To calculate the number of distinct pairs of nodes in the network, we can use the formula N(N - 1)/2. If a network contains precisely that many links, it means each potential pair of nodes in the network is indeed connected by a link – which, in turn, means there are quite simply no connections missing or absent between these nodes.

Had there been even one link missing from the network, the sum of all links available would fall below the formula N(N - 1)/2. Thus, having precisely N(N - 1)/2 links is a confirmation that the network is complete, in that each and every node is directly linked to every other node in the system.

In brief, to realize the maximum number of potential connections in an undirected network (N(N - 1)/2) is to have the network complete or fully connected.

182 words (min. 150)

▸ Solution

Q6. Bipartite Networks (8 points)

Status	Answered	
Your score	8 / 8	100%

Response

Given a complete network A with N nodes, and a bipartite network B also with N nodes, which of the following holds true for any N > 2?

- ○ Network A has fewer links than network B
- ⊘ Network A has more links than network B
- ○ None of these hold true for all such N > 2
- ○ Network A has the same number of links as network B

▸ Solution

Q7. Network Degree (10 ponints)

Status	Answered	
Your score	0 / 10	0%

Response

Imagine two different undirected networks, each with the same number of nodes and links. Must both networks have the same maximum and minimum degree? Explain why or why not. Must they have the same mean degree? Explain why or why not.

Let’s consider two different undirected networks that have the same number of nodes and the same number of links. You might wonder: do they also have to have the same maximum and minimum degrees (the highest and lowest number of connections a node has)?

The answer is no, they don’t have to. Even if two networks have the same number of nodes and links, the way the links are distributed can be very different. In one network, a few nodes may have a lot of connections (a very high degree), while others have very few connections (a low degree). In another network with the same number of nodes and links, the connections could be spread out more evenly among the nodes. So, while the total number of links is the same, the maximum and minimum degree values can vary based on how the connections are organized.

Now, what about the mean degree (the average number of connections per node)? Here, the answer is yes, they must have the same mean degree. That’s because the mean degree depends directly on the total number of links and the number of nodes. In an undirected network, each link is shared between two nodes, so the total degree across the network is simply twice the number of links. Dividing that by the number of nodes gives the mean degree. Since both networks have the same number of links and nodes, their mean degree will be identical, even if the individual node degrees are different.

In short: maximum and minimum degrees can differ, but the mean degree will always be the same if the number of nodes and links are the same.

272 words (min. 250)

▸ Solution

Q8. Facebook Network (10 points)

Status	Answered	
Your score	0 / 10	0%

Response

We have seen that Facebook’s network is incredibly sparse. Assume it has approximately 1 billion users, each with 1000 friends on average.

• Suppose Facebook releases its annual report and it shows that while the number of users in the network has stayed the same, the average number of friends per user has increased. Would this imply that the network density increased, decreased, or stayed the same?
• Suppose instead that both the number of users and the average number of friends per user doubled. Would this imply the network density increased, decreased, or stayed the same?

Scenario 1: Same Users, Increased Average Friends
If the number of users stays the same but the average number of friends per user rises, the network density increases. Network density measures how full the network is — it's the ratio between the number of actual connections and the maximum possible connections. When the number of users doesn't change, the maximum possible number of connections remains fixed. However, if every user has more friends, then the number of actual connections rises. Since the denominator (maximum possible connections) stays the same and the numerator (actual connections) gets bigger, the overall density increases.

For example, if users originally had 1,000 friends on average and now they have 2,000, the number of edges (friendship links) roughly doubles. Since the maximum number of possible edges is unchanged, the density also approximately doubles. Thus, when the average number of friends increases without a change in user count, the network becomes denser — more tightly connected relative to what is possible.

Scenario 2: Users and Average Friends Both Double
In case both the number of users and the average number of friends double, the circumstance is different.

Here's why:
-> The number of edges increases four times. (Edges are calculated as users X average friends/2). Doubling users and doubling friends results in four times as many total connections.)

-> The maximum possible edges also increase four times. (Maximum edges are users X (users – 1)/2, and if you double the users, the maximum possible connections increase roughly by a factor of 4.)

Since both the actual number of connections and the maximum possible number of connections grow at the same rate, network density remains constant.
In simple terms, even though everyone has more friends and there are more people, the network's "fullness" — how many of the possible connections are actually realized — stays about the same.

Why This Matters:
Understanding density is important for understanding how social networks behave as they grow. A denser network means that users are more tightly interconnected, which can impact how information spreads, how groups form, and how influence operates across the network.

In Facebook's case, even if the platform grows massively in users and friendships, it doesn't necessarily mean that the network becomes more interconnected in a relative sense. The overall network remains sparse unless friendships grow faster than the user base itself.
For example, if users increased by 50% but the number of friends doubled, then density would increase, because friendships are growing faster than the possible connections.

Final Summary
-> More friends per user (same users) → Higher density.
-> Doubling users and friends together → Density stays the same.

421 words (min. 400)

▸ Solution

go back to overview

Adjacency matrix (36 points) 25 of 36 points (69%)

Consider this adjacency matrix:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 1 & 3 & 1 & 1 & 0 \end{pmatrix} \end{array}.$$

Q9. Weighted Network (7 points)

Status	Answered	
Your score	7 / 7	100%

Response

An entry in the ith row and ith column indicates the weight of the link from node i to node j. For instance, the entry in the second row and third column is - meaning the weight of the link from node B to node C is 2. What kind of network does this matrix represent?

- ○ Undirected, weighted
- ○ Directed, unweighted
- ⊘ Directed, weighted
- ○ Undirected, unweighted

▸ Solution

▸ Solution

··· Q10. Network Representation (9 points)

Status	Answered	
Your score	9 / 9	100%

Response

Consider the network defined by the above adjacency matrix.

How many nodes are in this network?

6.0

How many links?

10.0

Are there any self-loops? (Yes or No)

No

▸ Solution

Q11. Nodes and Links (8 points)

Status	Answered	
Your score	0 / 8	0%

Response

Are there any nodes with outgoing links to every other node? If so, which nodes? Are there any nodes with in-links from every other node? If so, which nodes?

In this network, we are trying to figure out two things: first, if any node has outgoing links to every other node, and second, if any node has incoming links from every other node.

Let's start by looking at the outgoing links. Outgoing links mean that a node points to or connects to other nodes. When we check the matrix row by row, we notice that node A only connects to node B. Node B only connects to node C. Node C does not have any outgoing links at all. Node D has outgoing links to B and E, but not to everyone. Node E only connects to F. However, when we reach node F, something interesting happens. Node F has outgoing links to A, B, C, D, and E — basically to every other node except itself. This makes F the only node that has outgoing links to all the other nodes in the network.

Now, for incoming links, we check which nodes are receiving connections from others. For a node to have incoming links from every other node, it would need to appear in every other node's outgoing list. When we scan the matrix column by column, we find that no node meets this condition. For example, node A only receives a connection from F, not from everyone else. Similarly, node B receives links from A, D, and F, but not from C or E. Since no node gets links from all the others, there is no node that has incoming links from every other node.

In short, only node F has outgoing links to all other nodes, while no node has incoming links from all other nodes. This highlights that F is the most connected and central node in this network.

262 words (min. 250)

▸ Solution

⦿ Q12. Sink Node (6 points)

Status	Answered	
Your score	6 / 6	100%

Response

A sink is defined as a node with in-links but no out-links. Which nodes in the network, if any, have this property?

- ○ E
- ○ F
- ◉ C
- ○ A
- ○ None of them
- ○ D
- ○ B

▸ Solution

Q13. Strength (6 points)

Status	Answered	
Your score	3 / 6	50%

Response

What is the in-strength of node C?

6.0

What is its out-strength?

0.0

▾ Solution

What is the in-strength of node C?

5.0

What is its out-strength?

0.0

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Test execution

Information

Availability: Expired at 4/29/2025, 11:59 PM

Max. attempts: Unlimited

Results of this test are visible to administrators and tutors of this course.

Start test

▸ Change log