Artificial Intelligence: Solutions to Exercise 6

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1. Unify (if possible) the following terms and give the moste general unification (MGU) and the resulting terms. Here P, Q are predicate symbols, f, g are function symbols and X, Y, Z, U are variables.

   (a) $P(X, f(Y))$, $P(f(Z), U)$

   (b) $P(X, f(X))$, $P(Y, Y)$

   (c) $Q\left(f(X, Y, Z),\, f\left(g(W, W), g(X, X), g(Y, Y)\right)\right),\, Q(U, U)$

   Solution:

   (a) $P(X, f(Y))$, $P(f(Z), U)$

   - The predicates have the same name and same arity, so we can proceed.
   - We substitute the variables (never constants or function symbols)

     $$\sigma : X/f(Z) \quad \text{and} \quad U/f(Y)$$

     and obtain

     $$P(X, f(Y))_\sigma \equiv P(f(Z), f(Y)) \equiv P(f(Z), U)_\sigma.$$

   (b) $P(X, f(X))$, $P(Y, Y)$

   - The predicates have the same name and same arity, so we can proceed.
   - We can not unify. If we would substitute X/Y to unify the first argument, we would obtain $P(Y, f(Y))$ and $P(Y, Y)$. In the last literal, the argumenst are always the same, whereas this is not necessarily the case in $P(Y, f(Y))$. Thus, the two literals are not unifiable.

   (c) $Q\left(f(X, Y, Z),\, f\left(g(W, W), g(X, X), g(Y, Y)\right)\right),\, Q(U, U)$

   - For the unification to be successful, the arguments of $Q(\cdot, \cdot)$ must be equal.
   - We start with the unifications of the arguments of the function f. We substitute $X/g(W, W)$ and then $Y/g(g(W, W), g(W, W))$. With this substitution σ we find

     $$\begin{aligned} &f\left(g(W, W), g(X, X), g(Y, Y)\right)_\sigma \\ &\quad \equiv f\left(g(W, W)),\, g(g(W, W), g(W, W)),\, g(g(g(W, W), g(W, W)), g(g(W, W), g(W, W))))\right) \end{aligned}$$

- This implies the substitution $Z/g(g(g(W,W),g(W,W)),g(g(W,W),g(W,W)))$
- To further unify we substitute

  $$U/f\,(g(W,W)),\ g(g(W,W),g(W,W)),\ g(g(g(W,W),g(W,W)),g(g(W,W),g(W,W))))$$

Overall, we have the MGU

$$\begin{aligned}\sigma: &X/g(W,W), \quad Y/g(g(W,W),g(W,W)),\\ &Z/g(g(g(W,W),g(W,W)),g(g(W,W),g(W,W))),\\ &U/f\,(g(W,W)),\ g(g(W,W),g(W,W)),\ g(g(g(W,W),g(W,W)),g(g(W,W),g(W,W))))\end{aligned}$$

and the unified term reads

$$\begin{aligned}Q(&f\,(g(W,W)),\ g(g(W,W),g(W,W)),\ g(g(g(W,W),g(W,W)),g(g(W,W),g(W,W))))\,,\\ &f\,(g(W,W)),\ g(g(W,W),g(W,W)),\ g(g(g(W,W),g(W,W)),g(g(W,W),g(W,W)))))\end{aligned}$$

2. Russell's paradox reads "There is a barber who shaves everyone who does not shave himself." We can write this as

$$\forall X\, Shaves(barber, X) \Leftrightarrow \neg Shaves(X, X)$$

(a) Transform this into the conjunctive normal form

$$\begin{aligned}&(\neg Shaves(barber, X) \vee \neg Shaves(X, X))\\ &\quad \wedge (Shaves(barber, X) \vee Shaves(X, X))\end{aligned}$$

(b) Express the statement without the universal quantifier and use an existential quantifier instead.

Solution:

(a) The formula contains the universal quantifier "$\forall$". It is already in a prenex normal form, because all quantifiers are at the left. To replace the the implication we use $(P \Rightarrow Q) \equiv Q \vee \neg P$ or, more specifically, $(P \Leftrightarrow Q) \equiv (Q \vee \neg P) \wedge (P \vee \neg Q)$. Thus, with $P \equiv (Shaves(barber, X)$ and $Q \equiv \neg Shaves(X, X)$

$$\begin{aligned}&Shaves(barber, X) \Leftrightarrow \neg Shaves(X, X)\\ &\quad \equiv (\neg(Shaves(X, X) \vee \neg Shaves(barber, X))\\ &\quad\quad \wedge (Shaves(barber, X) \vee Shaves(X, X))\end{aligned}$$

The forall quantifier can be put in front.

(b) We use the de Morgan's law in the special form $\forall X P \equiv \neg\exists X \neg P$. Now we apply this

$$\begin{aligned}&\forall X\,(\neg(Shaves(X, X) \vee \neg Shaves(barber, X))\\ &\quad \wedge (Shaves(barber, X) \vee Shaves(X, X))\\ &\equiv \neg\exists X \neg((\neg(Shaves(X, X) \vee \neg Shaves(barber, X))\\ &\quad \wedge (Shaves(barber, X) \vee Shaves(X, X)))\end{aligned}$$

The inner formula can be further manipulated.

3. Monitoring safety belt usage.
In a car, the usage of the safety belt is monitored. The person might even be called by its name to remind her or him of using the safety belt. The monitoring is based on a weight sensor in the seat and a system which checks, whether the safety belt is fastened (i.e. the belt is locked). The triggering of the alarm is described by the following first order predicate logic (PL1) expressions

$$Occupant(Name, Weight, belted) \Rightarrow Alarm(Name, off) \quad (1)$$

$$Occupant(Name, less10kg, belted) \Rightarrow Alarm(Name, off) \quad (2)$$

$$\neg ok(Sensors) \Rightarrow Alarm(Name, off) \quad (3)$$

$$Occupant(Name, more10kg, unbelted) \wedge ok(Sensors) \Rightarrow Alarm(Name, on) \quad (4)$$

The implication (1) means that no alarm is triggered, if the occupant is belted. If the weight is below the minimal weight of $10kg$ (2), or the sensors are not correctly working (3), there will be no alarm. An alarm is triggered if the weight of the occupant is higher than $10kg$ and the occupant is unbelted and the sensors are ok (4).

The following facts are known

- There are two occupants in the car. The known facts about the occupants can be seen in the tabular.
- The sensors in the car are working properly.

Occupant	Name	Weight	Safety belt
1	Karl	more than 10 kg	unbelted
2	Hans	less than 10 kg	belted

(a) Summarize the facts stated above as clauses in PL1 using the predicated defined in Eqs. (1)-(4).

(b) Summarize the facts and the definitions in Eqs. (1)-(4) to set of axioms. The axioms should be clauses.

(c) Use resolutional calculus to proof that the alarm was triggerd by occupant Karl.

Solution:

(a) The known facts can be expresed in PL1 as

$$\begin{aligned}&Occupant(karl, more10kg, unbelted)\\&Occupant(hans, less10kg, belted)\\&ok(Sensors)\end{aligned}$$

(b) First, we need to transform the axioms into clauses

$$\begin{aligned}&Occupant(Name, Weight, belted) \Rightarrow Alarm(Name, off)\\&\equiv Alarm(Name, off) \vee \neg Occupant(Name, Weight, belted)\\&Occupant(Name, less10kg, belted) \Rightarrow Alarm(Name, off)\\&\equiv Alarm(Name, off) \vee \neg Occupant(Name, less10kg, belted)\\&\neg ok(Sensors) \Rightarrow Alarm(Name, off)\\&\equiv Alarm(Name, off) \vee ok(Sensors)\\&Occupant(Name, more10kg, unbelted) \wedge ok(Sensors) \Rightarrow Alarm(Name, on)\\&\equiv Alarm(Name, on) \vee \neg\left(Occupant(Name, more10kg, unbelted) \wedge ok(Sensors)\right)\\&\equiv Alarm(Name, on) \vee \neg Occupant(Name, more10kg, unbelted) \vee \neg ok(Sensors)\end{aligned}$$

Together with the known facts we obtain the following clauses as axioms

$$Occupant(karl, more10kg, unbelted) \tag{5}$$
$$Occupant(hans, less10kg, belted) \tag{6}$$
$$ok(Sensors) \tag{7}$$
$$Alarm(Name, off) \vee \neg Occupant(Name, Weight, belted) \tag{8}$$
$$Alarm(Name, off) \vee \neg Occupant(Name, less10kg, belted) \tag{9}$$
$$Alarm(Name, off) \vee ok(Sensors) \tag{10}$$
$$Alarm(Name, on) \vee \neg Occupant(Name, more10kg, unbelted) \vee \neg ok(Sensors)\,. \tag{11}$$

(c) To proofe the assertion $Alarm(karl, on)$ we add the negated assertion

$$\neg Alarm(karl, on)$$

to the axioms (5). The substitution $\sigma_1 : Name/karl$ is used in the first resolution step

$$\frac{\begin{array}{l}\neg Alarm(karl, on)\\ Alarm(Name, on) \vee \neg Occupant(Name, more10kg, unbelted) \vee \neg ok(Sensors)\end{array}}{\left(\neg Occupant(Name, more10kg, unbelted) \vee \neg ok(Sensors)\right)_{\sigma_1}} \tag{12}$$

and we have the resolvent

$$\neg Occupant(karl, more10kg, unbelted) \vee \neg ok(Sensors)$$

The next two resolution steps

$$\begin{array}{l} \neg Occupant(karl, more10kg, unbelted) \vee \neg ok(Sensors) \\ Occupant(karl, more10kg, unbelted) \\ \hline \neg ok(Sensors) \end{array}$$

and

$$\begin{array}{l} \neg ok(Sensors) \\ ok(Sensors) \\ \hline \emptyset \end{array}$$

generate a contradiction and we proof the (negated) assertion.