

MMC-GRADE XI

2081 (2024)

Mathematics

Time: 3 Hours

FM: 75

Group-A (11*1=11)

- A compound statement which is always true is called
a) Contradiction ☒ b) Tautology c) Negation d) inverse
- The inequality $-1 \leq x \leq 7$ is in absolute value sign as
a) $|x - 2| \leq 4$ ☒ b) $|x - 3| \leq 4$ c) $|x - 1| \leq 2$ d) $|x - 5| \leq 2$
- Let a function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 7$, $x \in \mathbb{R}$ then f^{-1} is
a) $f^{-1}(x) = \sqrt{x - 7}$ b) $f^{-1}(x) = \frac{x-7}{2}$ c) $f^{-1}(x) = \frac{\sqrt{x-7}}{2}$ ☒ d) $f^{-1}(x) = \sqrt[3]{x - 7}$
- The value of x for $\log_4 X = 1/2$, is
a) -1 b) 0 c) 1 d) 2
- The vertex of the quadratic function $f(x) = x^2 + 4x + 3$ is
a) (-2, -1) b) (-1, -2) c) (1, 2) d) (2, 1)
- The sum of the infinite series $16 + 8 + 4 + \dots$ is
a) 4 ☒ b) 8 c) 16 ☒ d) 32
- If determinant $\begin{vmatrix} 5 & -6 \\ x-2 & 3 \end{vmatrix} = 0$, then the value of x is
a) 2 ☒ b) -1/2 c) 1/2 d) 1/4
- If a line makes 45° with positive x -axis and 60° with positive y -axis then the angle made by the line with z -axis is
a) 60° b) 120° c) both a and b d) 180°
- If mean = 50, mode = 56 and S.D. = 15, then the Karl Pearson's coefficient of skewness is
a) -0.1 b) -0.2 c) -0.3 ☒ d) -0.4
- Two coins are tossed simultaneously, the probability that both are heads is
a) 0 b) 1/2 ☒ c) 1/4 d) 1
- The value of $\int \sec^2(ax + b) dx$ is
a) $\tan(ax+b) + c$ b) $a \tan(ax+b) + c$ c) $\frac{\tan(ax+b)}{b} + c$ ☒ d) $\frac{\tan(ax+b)}{a} + c$

Group-B (5*8 = 40)

- a) prove that $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$ [2]
a) solve $x^2 - 2x - 3 \geq 0$ [3]
- a) prove that the $f: \mathbb{R} - \{3\} \rightarrow \mathbb{R} - \{1\}$ defined by $f(x) = \frac{x}{x-3}$ is bijective function [2]
b) sketch the graph of $y = -x^2 + 4x - 3$. [3]

OR,

prove that the equation of the lines joining the origin and the point of intersection of the line $y = mx + c$ and the circle $x^2 + y^2 = a^2$ will be at right angles if $2c^2 = a^2(1 + m^2)$.

- Prove that: $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$ [5]

15. a) If $A = \begin{bmatrix} 4 & -5 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ -1 & -2 \end{bmatrix}$, prove that $(AB)^T = B^T A^T$ [2]

b) find the sum to infinity of the series $1+2x+3x^2+4x^3+\dots$ [3]

16. a) find the square roots of $-7+24i$. [2]

b) if α and β be the root roots of the equation $px^2+qx+q=0$, prove that:

$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$$
 [3]

17. a) find the general solution of $\cos^2 x = 1/2$ [2]

b) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, prove that : $x + y + z = xyz$. [3]

18. a) find the ratio in which the xy-plane divides the join of the points $(-2,4,7)$ and $(3,-5,-8)$.

b) Define direction cosines of a line . Find the direction cosine of a line which is equally inclined with the axes of coordinates. [2+3]

19. calculate the coefficient of skewness : [5]

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
F	5	12	15	20	10	4	2

Group- C (8*3 = 24)

20. a) Evaluate: $\lim_{x \rightarrow \theta} \frac{x \tan \theta - \theta \tan x}{x - \theta}$ [3+2+3]

b) Find the derivative of $y = \tan(\sin(ax+b))$

c) Prove that the function defined as $f(x) = \begin{cases} 2x+1, & \text{for } x < 1 \\ 3, & \text{for } x = 1 \\ 3x, & \text{for } x > 1 \end{cases}$ is continuous

at $x=1$

21. a) Find the maxima or minima of the function $f(x) = 4x^3 - 15x^2 + 12x + 7$. [3+2+3]

b) Evaluate $\int \frac{1}{9+x^2} dx$

c) Evaluate $\int x \sin x dx$.

22. a) If $f(x) = x^2 + 2$ and $x_0 = 2$, find x_1 & x_2 using Newton-Rapson method. [2+3+3]

b) Prove that the vectors $\vec{i} - 2\vec{j} + \vec{k}$, $2\vec{i} + \vec{j} - \vec{k}$ and $7\vec{i} - 4\vec{j} + \vec{k}$ are linearly dependent.

c) If p and q are the lengths of perpendiculars from the origin upon the straight lines $x \sec \theta + y \csc \theta = a$ and $x \cos \theta - y \sin \theta = a \cos 2\theta$, prove that $4p^2 + q^2 = a^2$.

OR,

a) state sine law and use it to prove Lami's theorem. [5+3]

b) A train moving with a velocity of 360 km/hr has the uniform acceleration 40 m/s^2 . Obtain the distance covered by the train in $\frac{1}{2}$ minute.

$$\frac{d \cos \alpha}{d \alpha} = -\sin \alpha$$

$$\sec^2 \alpha = \tan^2 \alpha + 1$$

$$\frac{d \sin \alpha}{d \alpha} = \cos \alpha$$

$$a^2 - x^2 = a^2 \sin^2 \theta$$

$$a^2 + x^2 = a^2 \tan^2 \theta$$

$$x^2 - a^2 = a^2 \sec^2 \theta$$