Probabilistic Language Models

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A Brief Introduction to probability
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Generative Language Model
Markov Assumption
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INTRODUCTION

How are _____? Can you guess the missing word?

INTRODUCTION

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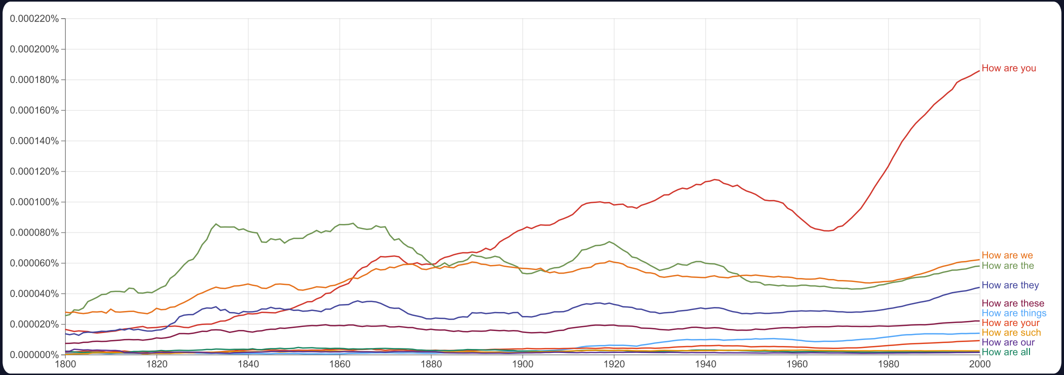

Source:Google NGram Viewer

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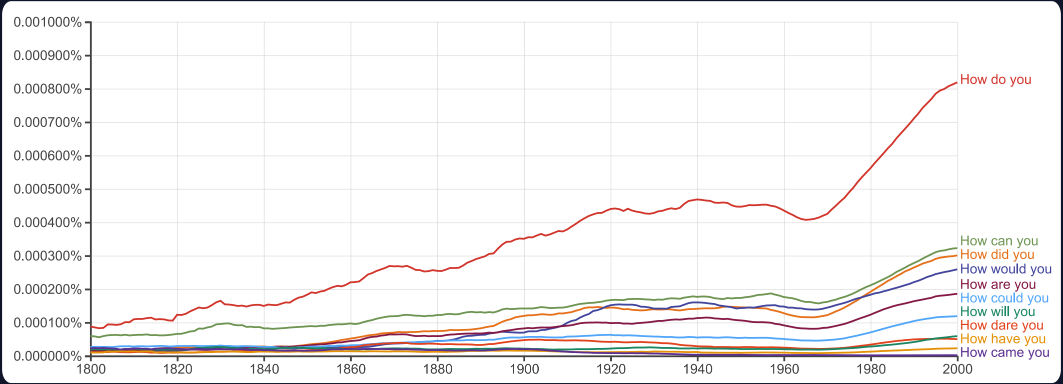

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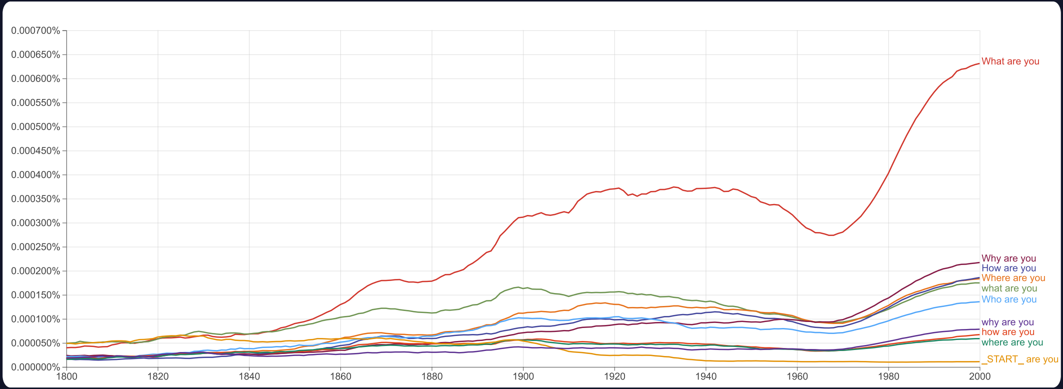
INTRODUCTION

_____ are you?

INTRODUCTION

_____ are you?



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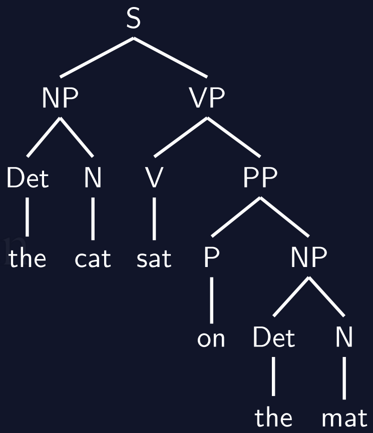
INTRODUCTION

How do humans predict the next word?

- Domain knowledge
- Syntactic knowledge
- Lexical knowledge
- Knowledge about the sentence structure
- Some words are hard to find. Why?
- Natural language is not deterministic in general
- Some sentences are familiar or had been heard/seen/used several times
- They are more likely to happen than others, hence we could guess

THE LANGUAGE MODEL

- Natural language sentences can be described by parse trees which use the morphology of words, syntax and semantics
- Probabilistic thinking - finding how likely a sentence occurs or formed, given the word sequence.
- In probabilistic world, the Language model is used to assign a probability $P(W)$ to every possible word sequence W.

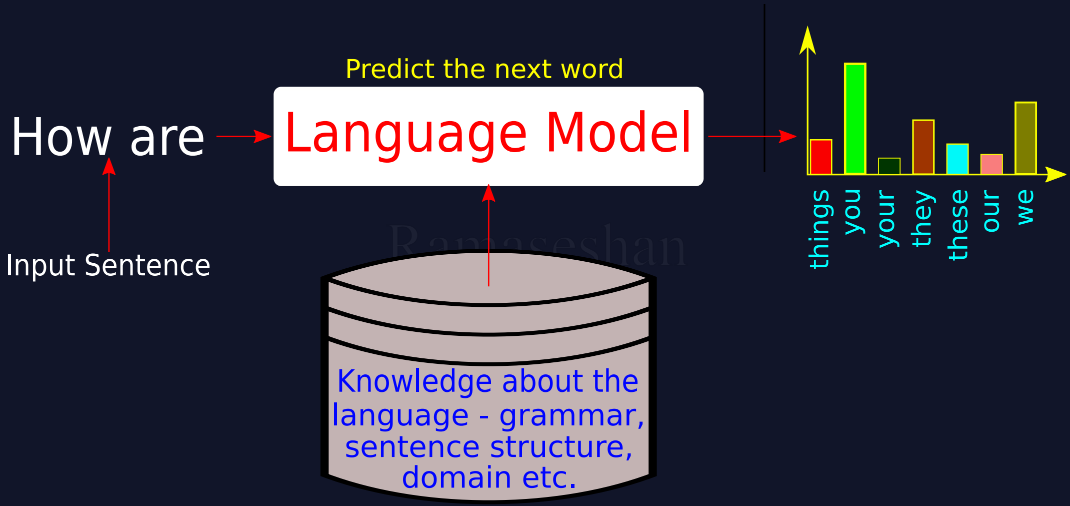


The current research in Language models focuses more on building the model from the huge corpus of text

APPLICATIONS

Application	Sample Sentences
Speech Recognition	Did you hear ***Recognize speech*** or Wreck a nice beach?
Context sensitive Spelling	One upon a ***tie***, ***Their*** lived aking
Machine translation	artwork is good $\longrightarrow$ l'oeuvre est bonne
Sentence Completion	Complete a sentence as the previous word is given - GMail

A SIMPLE LANGUAGE MODEL IMPLEMENTATION



WHY PROBABILISTIC MODEL

- Speech recognition systems cannot depend on the processed speech signals. It may require the help of a language model and context recognizer to convert a speech to correct text format.
- As there are multiple combinations for a word to be in the next slot in a sentence, it is important for language modeling to be probabilistic in nature - judgment about the fluency of a sequence of words returns the probability of the sequence
- The probability of the next word in a sequence is real number $[0, 1]$
- The combination of words with high-probability in a sentence are more likely to occur than low-probability ones
- A probabilistic model continuously estimates the rank of the words in a sequence or phrase or sentence in terms of frequency of occurrence

PROBABILISTIC ASSERTIONS

In the logical assertions, we strictly rule out possibilities - if-else-endif

- Probabilistic assertions are about possible worlds

  - how probable the various worlds are and the set of all possible worlds is called the sample space

FORMAL DEFINITION

A probability model is a mathematical framework that describes the likelihood of different outcomes occurring in a certain event or situation. A probability model associates a numerical probability value with each possible world.
Let V be the vocabulary, a finite set of symbols or words. Let us use $\triangleleft$ and $\triangleright$ as the start and stop symbols and let them be the part of V. Let $|V|$ denote the size of V.

Let W be infinite sequences of words from the collection of V. Every sequence in W starts with $\triangleleft$ and ends with $\triangleright$. Then a language model is a probability distribution of a random variable X which takes values from W.
Or p: $W \rightarrow \mathbb{R}$ such that

$$\forall x \in W,\ p(x) \geq 0 \text{ and } \sum_{x \in W} p(X = x) = 1 \tag{1}$$

PROBABILISTIC LANGUAGE MODEL

Goal: Compute the probability of a sequence of words

$$P(W) = P(w_1, w_2, w_3, \dots w_n) \tag{2}$$

Task: To predict the next word using probability. Given the context, find the next word using

$$P(w_n | w_1, w_2, w_3, \dots, w_{n-1}) \tag{3}$$

A model which computes the probability for (2) or predicting the next word (3) or complete the partial sentence is called as Probabilistic Language Model.

The goal is to learn the joint probability function of sequences of words in a language. The probability of $P(\text{The cat roars})$ is less likely to happen than $P(\text{The cat meows})$

CHAIN RULE

The chain rule states that the probability of a sequence of events occurring is the product of the probabilities of each event occurring. In the context of NLP, The sentence "I am a large language model" occurring is the product of the probabilities of the words "I", "am", "a", "large", "language", and "model" occurring in the corpus.

- Chain rule models joint probability of multiple events in NLP.
- For word sequence $(w_1, w_2, \ldots, w_n)$:

$$P(w_1, w_2, \ldots, w_n) = P(w_1) \cdot P(w_2|w_1) \cdot \ldots \cdot P(w_n|w_1, w_2, \ldots, w_{n-1})$$

- Breaks down into product of conditional probabilities.
- Essential for language models (n-grams, Markov models, RNNs, transformers).
- Enables language sequence modeling and likelihood estimation.

GENERATIVE MODEL

- A Generative model (GM) generates new sentences
- Produces sequences of words, sentences, or paragraphs that resemble any natural language
- Captures the underlying patterns and statistics of a corpus to create create new sentences
- Learns the underlying structure of language - includes grammar, syntax, semantics(?), and context

GENERATIVE LANGUAGE MODEL

- Ability to synthesize new sentences that are statistically indistinguishable from the training data (Ideal to pass the Turing Test)
- Describes the likelihood of any string (word or a sentence) using probability distribution
- Evaluates sentences - "The cat sat on the mat" is more likely than "The mat cat on the sat"
- Assists in predicting the next word or help in completing the sentence
- Provides alternate construct to a sentence
- Corrects spelling mistakes or grammar
- Provide translations or answer a question, if pairs of models are developed

Since natural languages are not like std-20 version of C++, the language models are not perfect :)

GENERATIVE LANGUAGE MODEL

- Rely on large amounts of training data to learn the patterns and statistics
- Depends on the diversity of the training data to have a good quality output

MATHEMATICAL DESCRIPTION OF A GENERATIVE MODEL

Here is the mathematical description of a generative model in NLP is the following:

$$P(w_1, w_2, ..., w_n) = P(w_1)P(w_2|w_1)P(w_3|w_1, w_2)...P(w_n|w_1, w_2, ..., w_{n-1}) \tag{4}$$

where $P(w_1, w_2, ..., w_n)$ is the probability of the sequence of words $w_1, w_2, \ldots, w_n$ and $P(w_i|w_1, w_2, \ldots, w_{i-1})$ is the probability of the word w_i given the previous words, or its context$w_1, w_2, \ldots, w_{i-1}$
What is the probability of the sentence ***The cat sat on the mat***?

$$\begin{aligned} P(The\ cat\ sat\ on\ the\ mat) =& P(The)P(cat|The) \times P(sat|The\ cat) \\ & \times P(on|The\ cat\ sat) \times P(the|The\ cat\ sat\ on) \\ & \times P(mat|The\ cat\ sat\ on\ the) \end{aligned} \tag{5}$$

The probability distribution of words in a language is used as the knowledge to generate new text that appears lexically similar to the text from the corpus.

MARKOV ASSUMPTION

Markov Assumption: The future behavior of a dynamic system depends on its recent history and not on the entire history

The product of the conditional probabilities can be written approximately for a bigram as

$$P(w_k|w_1^{k-1}) \approx P(w_k|w_{k-1}) \tag{6}$$

Equation (6) can be generalized for an *n-gram* as

$$P(w_k|w_1^{k-1}) \approx P(w_k|w_{k-K+1}^{k-1}) \tag{7}$$

Now, the joint probability of a sequence can be re-written as

$$\begin{aligned} P(W) &= P(w_1, w_2, w_3, \ldots, w_n) = P(w_1^n) && (8) \\ &= P(w_1)P(w2|w_1)P(w3|w_2, w_1)\ldots P(w_n|w_{n-1}, w_{n-2}, w_{n-3}, \ldots, w_1) && (9) \\ &= \prod_{k=1}^{n} P(w_k|w_1^{k-1}) && (10) \\ &\approx \prod_{k=1}^{n} P(w_k|w_{k-K+1}^{k-1}) && (11) \end{aligned}$$

TARGET AND CONTEXT WORDS

Next word in the sentence depends on its immediate past words, known as context words

$$P(w_{k+1}|\underbrace{w_{i-k}, w_{i-k+1}, \ldots, w_k}_{\text{Context words}})$$

n-grams

unigram	-	$P(w_{k+1})$
bigram	-	$P(w_{k+1}|w_k)$
trigram	-	$P(w_{k+1}|w_{k-1}, w_k)$
4-gram	-	$P(w_{k+1}|w_{k-2}, w_{k-1}, w_k)$

LANGUAGE MODELING USING UNIGRAMS

- All words are generated independent of its history $w_{,}w_2, w_3, \ldots..w_n$ and none of them depend on the other
- Not a good model for language generation
- It will have $|V|$ parameters
- $\theta_i = p(w_i) = \frac{c_{w_i}}{N}$, where c_{w_i} if the count of the word w_i and N is the total number of words in the vocabulary
- It may not be able to pick up regularities present tin the corpus
- It is more likely to generate ***the the the the*** as a sentence than a grammatically valid sentence

SMOOTHING

- One of the methods to find the unknown parameter(s) is the use of Maximum Likelihood Estimate
- Estimate the parameter value for which the observed data has the highest probability
- Training data may not have all the words in the vocabulary
- If a sentence with an unknown word is presented, then the MLE is zero.
- Add a smoothing parameter to the equation without affecting the overall probability requirements

$$P(\mathbf{W}) = \frac{C_{w_i} + \alpha}{C_W + \alpha|V|} \tag{12}$$

$$\text{If } \alpha = 1, \text{then it is called as Laplace smoothing} \tag{13}$$

$$P(\mathbf{W}) = \frac{C_{w_i} + 1}{C_W + |V|} \tag{14}$$

BIGRAM LANGUAGE MODEL

- This model generates a sequence one word at a time, starting with the first word and then generating each succeeding word conditioned on the previous one or its predecessor
- A bigram language model or the Markov model (first order)is defined as follows:

$$P(\mathbf{W}) = \prod_{i=1}^{n+1} P(w_i|w_{i-1}) \tag{15}$$

where $\mathbf{W} = w_1, w_2, w_3, \ldots, w_n$

BIGRAM LANGUAGE MODEL

- Estimate the parameter $P(w_i|w_{i-1})$ for all bigrams
- The parameter estimation does not depend on the location of the word
- If we consider the sentence as a sequence in time, then they are time-invariant
- MLE picks up the word that is $\frac{n_{w_c,w_i}}{n_{w_c}}$ where n_{w_c,w_i} is the number of times the words w_c, w_i occur together and n_{w_c} is the number of times the word w_c appears in the bigram sequence with any other word
- If V is the vocabulary of a corpus and V_c is the number of words in the vocabulary, what is the to number of parameters to be estimated?

COVID19 CORPUS

The bigram probabilities are estimated using a COVID19 corpus.[1] The corpus was created by reading the abstracts of all the papers found in the corpus.

$$\text{Total number of abstracts} = 224672$$
$$\text{The vocabulary count } |V| = 249737$$

[1] https://drive.google.com/file/d/1PVA5IR4bAn7RulAeuSgxRBNMtmRGXaUm/view?usp=share_link

BIGRAM LANGUAGE MODEL - EXAMPLE

Context - $<s>$	
$p(the\| <s>)$	0.13
$p(in\| <s>)$	0.05
$p(we\| <s>)$	0.05
$p(this\| <s>)$	0.03
$p(a\| <s>)$	0.02
$p(however\| <s>)$	0.02
$p(these\| <s>)$	0.02
$p(to\| <s>)$	0.01
$p(our\| <s>)$	0.01
$p(it\| <s>)$	0.01

Context - ***we***	
$p(have\|we)$	0.04
$p(also\|we)$	0.04
$p(found\|we)$	0.04
$p(show\|we)$	0.03
$p(present\|we)$	0.03
$p(propose\|we)$	0.03
$p(report\|we)$	0.03
$p(describe\|we)$	0.02
$p(used\|we)$	0.02
$p(identified\|we)$	0.02

Context - ***report***	
$p(the\|report)$	0.18
$p(a\|report)$	0.1
$p(of\|report)$	0.07
$p(on\|report)$	0.06
$p(that\|report)$	0.06
$p(we\|report)$	0.05
$p(here\|report)$	0.02
$p(.\|report)$	0.02
$p(describes\|report)$	0.02
$p(is\|report)$	0.02

The bigram probabilities are computed as follows:

$$P(w_n|w_{n-1}) = \frac{Count(w_{n-1}w_n)}{Counr(w_{n-1})} \tag{16}$$

$$P_L(w_n|w_{n-1}) = \frac{Count(w_{n-1}w_n)+1}{Count(w_{n-1})+|V|} \tag{17}$$

where P_L is the Laplacian smoothing and $|V|$ is the size of the vocabulary.

BUILDING A BIGRAM MODEL - CODE

```python
#compute the bigram model
def build_bigram_model():
    bigram_model = collections.defaultdict(
        lambda: collections.defaultdict(lambda: 0))
    for sentence in kinematics_corpus.sents():
        sentence = [word.lower() for word in sentence
                    if word.isalpha()]  # get alpha only
        #Collect all bigrams counts for (w1,w2)
        for w1, w2 in bigrams(sentence):
            bigram_model[w1][w2] += 1
        #compute the probability for the bigram containing w1
        for w1 in bigram_model:
            #total count of bigrams conaining w1
            total_count = float(sum(bigram_model[w1].values()))
            #distribute the probability mass for all bigrams starting with w1
            for w2 in bigram_model[w1]:
                bigram_model[w1][w2] /= total_count
    return bigram_model
```

BUILDING A BIGRAM MODEL - CODE

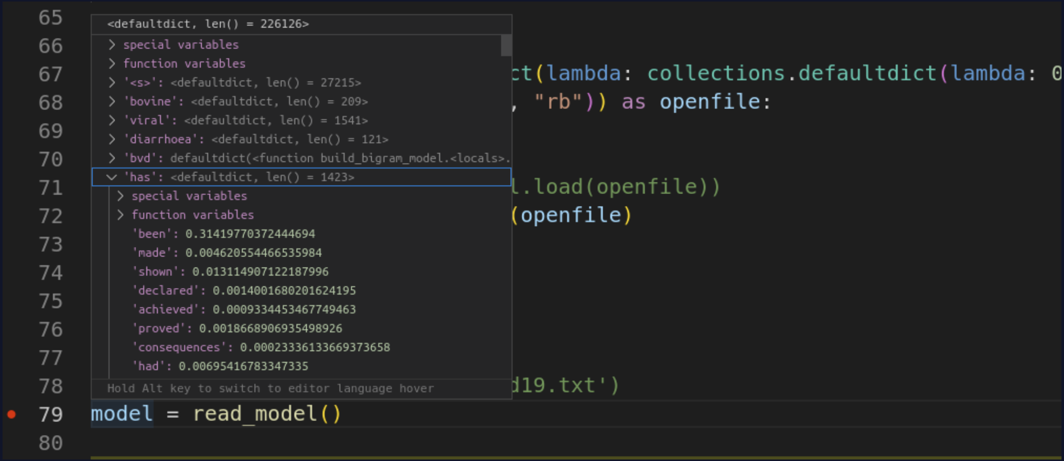
```python
def predict_next_word(first_word):
    #build the model
    model = build_bigram_model()
    #get the next for the bigram starting with 'word'
    second_word = model[first_word]
    #get the top 10 words whose first word is 'first_word'
    top10words = Counter(second_word).most_common(10)

    predicted_words = list(zip(*top10words))[0]
    probability_score = list(zip(*top10words))[1]
    x_pos = np.arange(len(predicted_words))

    plt.bar(x_pos, probability_score,align='center')
    plt.xticks(x_pos, predicted_words)
    plt.ylabel('Probability Score')
    plt.xlabel('Predicted Words')
    plt.title('Predicted words for ' + first_word)
    plt.show()

predict_next_word('how')
```

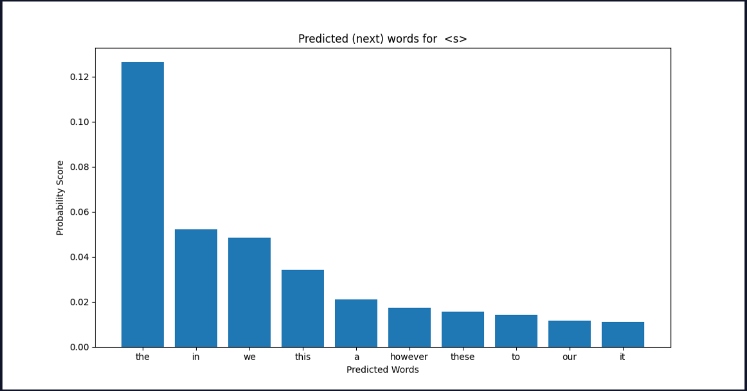
MODEL PARAMETERS - BIGRAM EXAMPLE



```
<defaultdict, len() = 226126>
> special variables
> function variables
> '<s>': <defaultdict, len() = 27215>
> 'bovine': <defaultdict, len() = 209>
> 'viral': <defaultdict, len() = 1541>
> 'diarrhoea': <defaultdict, len() = 121>
> 'bvd': defaultdict(<function build_bigram_model.<locals>.
v 'has': <defaultdict, len() = 1423>
  > special variables
  > function variables
    'been': 0.31419770372444694
    'made': 0.004620554466535984
    'shown': 0.013114907122187996
    'declared': 0.0014001680201624195
    'achieved': 0.0009334453467749463
    'proved': 0.0018668906935498926
    'consequences': 0.00023336133669373658
    'had': 0.00695416783347335
Hold Alt key to switch to editor language hover
```

```
ct(lambda: collections.defaultdict(lambda: 0
, "rb")) as openfile:
l.load(openfile))
(openfile)
d19.txt')
model = read_model()
```

BIGRAM MODEL - NEXT WORD PREDICTION



UNKNOWN WORDS

- In a closed vocabulary language model, there is no unknown words or ***out of vocabulary words (OOV)***
- In an open vocabulary system, you will find new words that are not present in the trained model
- Pick words below certain frequency and replace them as OOV.
- Treat every OOV as a regular word
- During testing, the new words would be treated as OOV and the corresponding frequency will be used for computation
- This eliminates zero probability for sentences containing OOV

Recap of

Probabilistic Language Model

CHAIN RULE

The chain rule allows us to compute probabilities using *N-gram* models. It states that the joint probability of a sequence can be expressed as the product of conditional probabilities. The chain rule is defined as:

$$P(w_1, w_2, \ldots, w_N) = P(w_1) \cdot P(w_2|w_1) \cdot P(w_3|w_1, w_2) \ldots P(w_N|w_1, w_2, \ldots, w_{N-1}) \tag{18}$$

Example

Consider a bigram model. To calculate the probability of a sentence, say "I love cats," we can use the chain rule: $P(\text{"I love cats"}) = P(\text{"I"}) \cdot P(\text{"love"}|\text{"I"}) \cdot P(\text{"cats"}|\text{"love"})$.

MLE

Given a training corpus of sentences, we can calculate the MLE probabilities.

Let w_i represent the i-th word in a sentence

w_1^{i-1} denote the context words before w_i

$$P(w_i|w_1^{i-1}) = \frac{\text{count}(w_1^{i-1}, w_i)}{\text{count}(w_1^{i-1})} \tag{19}$$

The MLE probability $P(w_i|w_1^{i-1})$ is estimated by counting the occurrences of w_i with the specific context and normalizing by the total count of that context: Here, $\text{count}(w_1^{i-1}, w_i)$ represents the number of times the context w_1^{i-1} is followed by the word w_i, and $\text{count}(w_1^{i-1})$ is the count of the context w_1^{i-1}.

BIGRAM LANGUAGE MODEL

Let's define the Bigram Language Model equation:

$$P(W) = P(w_1) \cdot P(w_2|w_1) \cdot P(w_3|w_2) \cdot \ldots \cdot P(w_n|w_{n-1}) \tag{20}$$

where

- $P(W)$: Probability of observing the entire sequence W.
- $P(w_i)$: Probability of the ith word in the sequence.
- $P(w_i|w_{i-1})$: Conditional probability of the ith word given the $(i-1)$th word.

BIGRAM LANGUAGE MODEL EXAMPLE

Now, let's consider a small corpus of sentences:

1. "I love cats."
2. "Cats love playing."
3. "Dogs chase cats."
4. "Many love cats."

We will build a bigram language model to calculate the probabilities of these sentences. The probabilities for each sentence can be calculated as follows:

$$P(\text{"I love cats."}) = P(\text{I}) \cdot P(\text{love} \mid \text{I}) \cdot P(\text{cats} \mid \text{love}) \cdot P(.\mid \text{cats}) \tag{21}$$

$$P(\text{"Cats love playing."}) = P(\text{cats}) \cdot P(\text{love}\mid \text{cats}) \cdot P(\text{playing}\mid \text{love}) \cdot P(.\mid \text{playing}) \tag{22}$$

$$P(\text{"Dogs chase cats."}) = P(\text{dogs}) \cdot P(\text{chase}\mid \text{dogs}) \cdot P(\text{cats}\mid \text{chase}) \cdot P(.\mid \text{cats}) \tag{23}$$

COMPUTING PROBABILITIES

To calculate the probabilities, we need to count the occurrences of each word and each bigram in the corpus. For example, for the sentence "I love cats.":

- Count(I) $=1$; Count(love$|$ I) $=1$;Count(cats$|$ love) $=2$; Count(.$|$ cats) $=1$

Using Maximum Likelihood Estimation (MLE), we can calculate the probabilities as follows:

$$P(\text{I}) = \frac{\text{Count(I)}}{\text{Total number of words in the corpus}} \tag{24}$$

$$P(\text{love}|\ \text{I}) = \frac{\text{Count(love}|\ \text{I)}}{\text{Count(I)}} \tag{25}$$

$$P(\text{cats}|\ \text{love}) = \frac{\text{Count(cats}|\ \text{love)}}{\text{Count(love)}} \tag{26}$$

$$P(.|\ \text{cats}) = \frac{\text{Count(.}|\ \text{cats)}}{\text{Count(cats)}} \tag{27}$$

Finally, we can plug these values into the bigram language model equation to estimate the probability of each sentence.

PERPLEXITY

- A measure to evaluate a language model
- Quantifies how well a language model predicts the next word in a sequence
- Lower the perplexity implies a better language model

The perplexity of the language model on the test set is a measure of how well the language model generalizes to unseen text.

PERPLEXITY OF A BIGRAM MODEL

The perplexity of a bigram model is calculated using the following equation:

$$\begin{aligned} perplexity &= \exp\left(-\frac{1}{N}\sum_{i=1}^{N}\log P(w_i|w_{i-1})\right) \\ &= \exp\left(-\frac{1}{N}\sum_{i=1}^{N}\log\left(\frac{count(w_i,w_{i-1})}{count(w_{i-1})}\right)\right) \end{aligned} \tag{28}$$

where

$perplexity$ is the measure of the language model
N is the number of words in the test set
w_i is the ith word in the test set
w_{i-1} is the $(i-1)$th word in the test set
$count(w_i,w_{i-1})$ is the number of times the bigram (w_i,w_{i-1}) appears in the training corpus
$count(w_{i-1})$ is the number of times the word w_{i-1} appears in the training corpus

The perplexity equation can be used to evaluate the performance of a bigram language model on a test set.

PERPLEXITY OF A TRIGRAM MODEL

The perplexity of a trigram model is calculated using the following equation:

$$perplexity = \exp\left(-\frac{1}{N}\sum_{i=1}^{N}\log\left(\frac{count(w_i, w_{i-1}, w_{i-2})}{count(w_{i-1}, w_{i-2})}\right)\right) \tag{29}$$

where:

- $perplexity$ is the perplexity of the language model
- N is the number of words in the test set
- w_i is the ith word in the test set, w_{i-1} is the $(i-1)$th word in the test set and w_{i-2} is the $(i-2)$th word in the test set
- $count(w_i, w_{i-1}, w_{i-2})$ is the number of times the trigram (w_i, w_{i-1}, w_{i-2}) appears in the training corpus
- $count(w_{i-1}, w_{i-2})$ is the number of times the bigram (w_{i-1}, w_{i-2}) appears in the training corpus

The only difference is that the trigram language model uses the probability of the trigram (w_i, w_{i-1}, w_{i-2}) to predict the next word, while the bigram language model uses the probability of the bigram (w_i, w_{i-1}) to predict the next word.

THE PERPLEXITY OF N-GRAM MODELS

For an N-gram model, perplexity is calculated using the following generalized equation:

$$perplexity = \exp\left(-\frac{1}{N}\sum_{i=1}^{N}\log\left(\frac{count(w_i, w_{i-1}, w_{i-2}, \ldots, w_{i-(n-1)})}{count(w_{i-1}, w_{i-2}, \ldots, w_{i-(n-1)})}\right)\right) \tag{30}$$

where

$perplexity$ is the perplexity of the language model
w_i is the ith word in the test set , w_{i-1} is the $(i-1)$th word in the test set and w_{i-2} is the $(i-2)$th word in the test set
$count(w_i, w_{i-1}, w_{i-2}, \ldots, w_{i-(n-1)})$ is the number of times the n-gram $(w_i, w_{i-1}, w_{i-2}, \ldots, w_{i-(n-1)})$ appears in the training corpus
$count(w_{i-1}, w_{i-2}, \ldots, w_{i-(n-1)})$ is the number of times the $(n-1)$-gram $(w_{i-1}, w_{i-2}, \ldots, w_{i-(n-1)})$ appears in the training corpus

The perplexity equation can be used to evaluate the performance of an n-gram language model on a test set. The perplexity equation for an n-gram language model is similar to the perplexity equations for bigram and trigram language models, but in a generalized form.

PERPLEXITY - EXAMPLE

Assuming we have estimated the probabilities of word sequences based on the MLE model, we can calculate the perplexity. The equation 30 can be rewritten as

$$\text{Perplexity} = \sqrt[N]{\frac{1}{P(w_1, w_2, \ldots, w_N)}} \tag{31}$$

where N is the number of words in the sentence and $P(w_1, w_2, \ldots, w_N)$ is the probability of the entire sentence.
Let's calculate the perplexity for both a seen sentence ("I like cats") and an unseen sentence ("I like rabbits") based on the estimated probabilities from the MLE language model.

PERPLEXITY - EXAMPLE

Assuming the estimated probabilities for the seen sentence "I like cats" are as follows:

$$P(\text{I}|\triangleleft\text{start}\triangleright) = 0.25;\ P(\text{like}|\text{I}) = 0.5$$
$$P(\text{cats}|\text{like}) = 0.5;\ P(\triangleleft\text{end}\triangleright|\text{cats}) = 0.05$$
$$P(\triangleleft\text{end}\triangleright|\text{dogs}) = 0.05$$

We can calculate the probability of the sentences as follows:

$$\begin{aligned} P(\text{I like cats}) &= P(\text{I}|\triangleleft\text{start}\triangleright)\cdot P(\text{like}|\text{I})\cdot P(\text{cats}|\text{like})\cdot P(\triangleleft\text{end}\triangleright|\text{cats}) \\ &= 0.25\cdot 0.5\cdot 0.5\cdot 0.05 = 0.003125 \\ \text{Perplexity}_{\text{seen}} &= \sqrt{\frac{1}{P(\text{I like cats})}} \\ &= \sqrt{\frac{1}{0.003125}} = 17.89 \end{aligned}$$

PERPLEXITY EXAMPLE

Now, let's calculate the perplexity for the unseen sentence "I like rabbits." Smoothing can be applied to "rabbits" as it is unseen in the training corpus. We can either use smoothing for all bigrams or use smoothing only for unseen word. In this case, we are using the smoothing only for the unseen tokens/words.
Assuming we use additive smoothing with $\alpha = 0.5$ and a vocabulary size of $V = 1000$, we can estimate:

$$\begin{aligned} P_{\text{smooth}}(\text{rabbits}|\text{like}) &= \frac{\text{count}(\text{like}, \text{rabbits}) + \alpha}{\text{count}(\text{like}) + \alpha \cdot V} \\ &= \frac{0 + 0.5}{2 + 0.5 \cdot 1000} \\ &= 0.001 \end{aligned}$$

The probability of the unseen sentence "I like rabbits" using the smoothing function can be calculated as:

$$P_{\text{smooth}}(\triangleleft\text{end}\triangleright|\text{rabbits}) = \frac{0 + 0.5}{2 + 0.5 \cdot 1000} = 0.001$$

$$
\begin{aligned}
P(\text{I like rabbits}) &= P(\text{I}|\triangleleft\text{start}\triangleright)\cdot P(\text{like}|\text{I})\cdot P_{\text{smooth}}(\text{rabbits}|\text{like})\cdot P(\triangleleft\text{end}\triangleright|\text{rabbits})\\
&= 0.25\cdot 0.5\cdot 0.001\cdot 0.001\\
&= 1.25e-07
\end{aligned}
$$

The perplexity of this unseen sentence is

$$
\begin{aligned}
\text{Perplexity}_{\text{unseen}} &= \sqrt{\frac{1}{P(\text{I like rabbits})}}\\
&= \sqrt{\frac{1}{1.25e-07}}\\
&= 2828.43
\end{aligned}
$$

In conclusion, the perplexity for the seen sentence "I like cats" is approximately 17.89. However, the perplexity for the unseen sentence "I like rabbits" is very high (2828.43) as it contains an unseen word, *rabbit*. Perplexity serves as a metric to evaluate language models, where lower values indicate better model performance in predicting given sentences.

CURSE OF DIMENSIONALITY

Vocabulary size $= V$. The parameters for

- Unigram model - V parameters
- Bigram model - V^2 parameters
- Trigram model - V^3 parameters
- n-gram model - V^n parameters

The curse of dimensionality refers to the exponential increase in the size of the parameter space as the dimensionality of the data increases. In the context of an n-gram language model, the curse of dimensionality becomes apparent due to the rapidly growing number of parameters as the value of n increases.

CHALLENGES

As the free parameter space becomes larger, several challenges arise:

- **Data Sparsity** - Many n-grams will have zero or very low counts, making it difficult to reliably estimate their probabilities.
- **Over fitting** - The model may become overly sensitive to specific patterns in the training data, which may not generalize well to unseen data
- **Computational Complexity** - Training and inference become computationally expensive as the number of parameters grows

TECHNIQUES

- Smoothing - Improves the estimation of probabilities for unseen n-grams
- Pruning - Low count n-grams can be removed to improve the estimation of probabilities and computational efficiency
- Back-off and interpolation - combines n-gram models of different orders to balance the trade-off between data sparsity and the complexity of the model

THANK YOU