Assignment1

January 31, 2024

questions

1 vector

1. X And Y are the two points with position vectors $3\overrightarrow{a} + \overrightarrow{b}$ and $3\overrightarrow{a} - \overrightarrow{b}$ respectively. write the position vector of the point Z which divides the line segment XY in the ratio $2 : 1$ externally.

2. let $\overrightarrow{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\overrightarrow{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ be two vectors. show that the vectors $\left(\overrightarrow{a} + \overrightarrow{b}\right)$ and $\left(\overrightarrow{a} - \overrightarrow{b}\right)$ are perpendicular to each other.

3. Find the vector equation of a line passing through the point $(2, 3, 2)$ and parallel to the line $\overrightarrow{r} = \left(-2\hat{i} + 3\hat{j}\right) + \lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$. Also find the distance between the lines

4. find the vector equation of the plane determined by the points $A(3, -1, 2)$, $B(5, 2, 4)$, $C(-1, -1, 6)$. Hence find the distance of the plane, thus obtained, from the origin

2 Linear Forms

5. Find the length of the intercept, cut off by the plane $2x + y - z = 5$ on the x-axis.

6. find the coordinates of the foot of the perpendicular Q drawn from $P(3, 2, 1)$ to the plane $2x - y + z + 1 = 0$. Also, find the distance PQ and the image of the point P treating this plane as a mirror .

7. find the value of x, for which the four points $A(x, 1, -1)$, $B(4, 5, 1)$, $C(3, 9, 4)$ and $D(-4, 4, 4)$ are coplannar.

8. A ladder 13 m long is leaning against a vertical wall. the bottom of the ladder away from the wall along the ground at the rate of 2 cm/sec.how fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

3 matrices

9. if A is a square matrix of order 3, with $|A| = 9$, then write the value of $|2 \cdot \text{ Adj } A|$.

10. if A and B are symmetric matrices, such that AB and BA are both defined, then prove that $AB - BA$ is a skew symmetric matrix.

11. using properties of determinants, find the value of x for which

$$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

12. using elementary row transformations , find the inverse o the matrix

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

13. using matrices, solve the following system of linear equations

$$\begin{aligned} x + 2y - 3z &= -4 \\ 2x + 3y + 2z &= 2 \\ 3x - 3y - 4z &= 11 \end{aligned}$$

4 differentiation

14. From the differential equation representing the family of curves $y = \frac{A}{x} + 5$, by eliminating the Orbitary constant A.

15. if $y = \log(\cos e^x)$, then find $\dfrac{dy}{dx}$

16. solve the fallowing differential equation: $\dfrac{dy}{dx} + y = \cos x - \sin x$

17. solve the differential equation

$$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2,$$

given that $y = 1$ when $x = 0$

18. Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{xy}{x^2 + y^2},$$

given that $y = 1$ when $x = 0$

19. if $X = \sin t$, $Y = \sin pt$, prove that $(1 - x^2) \cdot \dfrac{d^2y}{dx^2} - x \cdot \dfrac{dy}{dx} + p^2y = 0.$

20. differentiate $\tan^{-1}\left[\frac{\sqrt{(1+x^2)} - \sqrt{(1-x^2)}}{\sqrt{(1+x^2)} + \sqrt{(1-x^2)}}\right]$ with respect to $\cos^{-1} x^2$.

21. if

$$y = (x)^{\cos x} + (\cos x)^x, find \frac{dy}{dx}$$

5 integration

22. find

$$\int_{-\frac{\pi}{4}}^{0} \frac{1 + \tan x}{1 - tanx}\, dx$$

23. Find:

$$\int x \cdot \tan^{-1} x \, dx$$

24. Find :

$$l \int \frac{dx}{\sqrt{5 - 4x - 2x^2}}$$

25. integrate the function $\dfrac{\cos(x + a)}{\sin(x + b)}$ w · r ·t ·x.

26. prove that

$$\int_0^a f(x)dx = \int_0^a f(a - x)dx, \text{and hence evaluate} \int_0^1 (x^2)(1 - x)^n dx$$

27. using integration, find the area of the greatest rectangle that can be incribed in an elipse $\dfrac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

28. using integration, find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$.

29. using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1$=0, $2x + 3y - 21 = 0$,and $x - 5y + 9 = 0$

6 function

30. let $*$ be a operator defined as :$\mathbf{R} \times \mathbf{R} \to \mathbf{R}$ such that $a * b = 2a + b$, a,$b \in \mathbf{R}$ .Check if $*$ is a binary operation. If yes, find if it is association too.

31. Let A=R−{2} and $B = R$−{1}. if $f : A$→B is a function defined by $f(x)$=$\frac{x-1}{x-2}$, show that f is one-one and onto. Hence, find f^{-1}

32. show that the relation S in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b|\}$ is divisible by 3 in an equivalence relation.

33. prove that:$\cos^{-1}(\frac{12}{13}) + \sin^{-1}(\frac{3}{5})$=$\sin^{-1}(\frac{56}{65})$

34. A dietician wishes to mix two types of food in such a way the vitamin contents of the mixtures contain at least ₹8 units of vitamin A and ₹10 units of vitamin C. it costs ₹50 per Kg to produce food I. Food II contain 1 unit/kg of vitamin C .It costs ₹70 per kg to produce food II. Formulate this problem as a LPP to minimise the cost of the mixture that will produce the required diet. Also find the minimum cost.

7 probability

35. 12 cards numbered 1 to 12 (one number on one end), are placed in a box and mixed up thoroughly. then a card is drawn at random from the box.If it is known that the number on the card is greater than 5, find the probability that the card bears an odd number.

36. out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for quiz competition.find the probability that 2 boys and 2 girls are selected.

37. in a multiple choice examination with three possible answers with for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

38. an insurance company insured 3000 cyclists,6000 scooter drivers and 9000 car drivers.the probability of an accident involving a cyclist, a scooter driver are 0.3 , 0.5 and 0.02 respectively. One of the insured person meets with an accident. what is the probability that he is a cyclist ?