Bits of probability

Preliminary knowledge

Set Theory

A ***set*** is a collection of objects under consideration, sometime it is also called the ***space***. If an object x belongs to a set $\mathcal{C}$, it is said to be an ***element*** of the set, denoted by $x \in \mathcal{C}$.

- Definition 1.2.1 (subset)

  If for every $x \in C_1$ it is also true that $x \in C_2$, then the set C_1 is called a **subset** of the set C_2, denoted by $C_1 \subset C_2$. If $C_1 \subset C_2$ and $C_2 \subset C_1$, then $C_1 = C_2$.

- Definition 1.2.2 (null set)

  If a set C has no elements, C is called the **null set** and denoted by $C = \phi$.

- Definition 1.2.3 (union)

  The **union** of C_1 and C_2, written $C_1 \cup C_2$, is the set of all elements belong to either C_1 or C_2 or both.

- Definition 1.2.4 (intersection)

  The **intersection** of C_1 and C_2, written $C_1 \cap C_2$, is the set of all elements belong to both C_1 and C_2.

  Note: The union and intersection of multiple sets are defined in a similar manner

- Definition 1.2.6 (Complement)

  If $C \subset \mathcal{C}$, then the **complement** of C consists of all elements of $\mathcal{C}$ that are not elements of C, denoted by C^c. In particular, $\mathcal{C}^c = \phi$. complement of A= all element that do not belong to the A set

- Mutually exclusive sets

  Two sets A and B are said to be mutually exclusive if they have no elements in common - that is $A \cap B = \phi$.

- DeMorgan's Laws

  $$(C_1 \cap C_2)^c = {C_1}^c \cup {C_2}^c$$

  $$(C_1 \cup C_2)^c = C_1^c \cap C_2^c$$

- Manipulating sets

  Commutative law: $A \cup B = B \cup A$, $A \cap B = B \cap A$

  Associative law:

  $(A \cup B) \cup C = (A \cup C) \cup (B \cup C)$, $(A \cap B) \cap C = (A \cap C) \cap (B \cap C)$

Why?

We need a concept of probability to make judgements about our hypotheses in the scientific method. Is the data consistent with our hypotheses?

Probability theory is a mathematical model for random phenomena

A phenomenon is *probabilistic* if individual outcomes of the experiment are uncertain but there is nonetheless a regular distribution of outcomes in a large number of repetitions.

Three key words for describing probabilistic phenomenon:

Experiment: any procedure that

1. **can be repeated, theoretically, an infinite number of times;**
2. **has a well-defined set of possible outcomes.**

Sample space: set of all possible outcomes of an experiment.

Event: any subset of the sample

Examples

Flip a coin one time.

Is it a experiment?
(1) coin may be repeatedly tossed under the same conditions and (2) only two possible outcomes

Sample space?
$C=\{T,H\}$

Event?
$C_1 = \{T\}, C_2 = \{H\}$

Examples

Flip a coin three times.

Sample space $C=\{TTT,HTT,THT,TTH,HHT,HTH,THH,HHH\}$

Event: majority of coin show heads $C_1=\{HHT,HTH,THH,HHH\}$

Flip a coin until the first tail appears.

Sample space $C=\{T,HT,HHT,HHHT,HHHHT,....\}$

Event: The first tail appears at the third toss or after

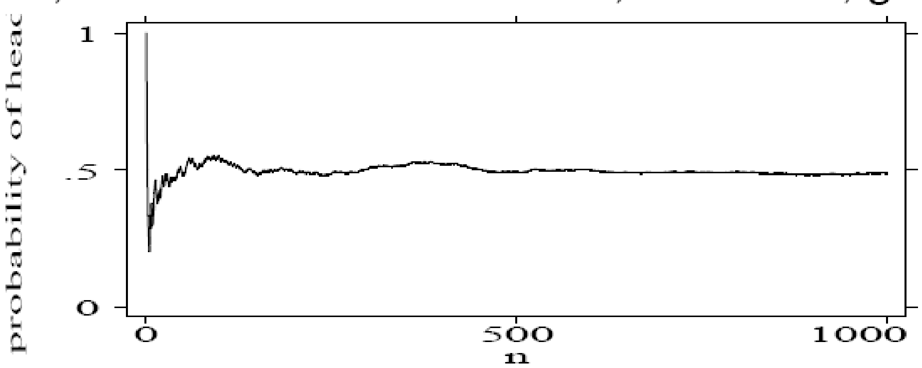
What is probability?

- **relative frequency**:

If an experiment is repeated a large number of times, *n*, and if the event, E, occurs *m* times, then the relative frequency, *m/n*, of *E* will be approximately equal to the probability of *E*.

$P(E) \approx m / n$

» Around 1900, the English statistician Karl Pearson heroically tossed a co 24,000 times and recorded 12,012 heads, giving a proportion of 0.5005.



Stata: graph twoway line prob n, yscale(r(0.0,1.0)) title("Probability of heads") ylabel(0(0.5)1)

- **personal probability**

What is the probability of life on Mars?

Probability Properties

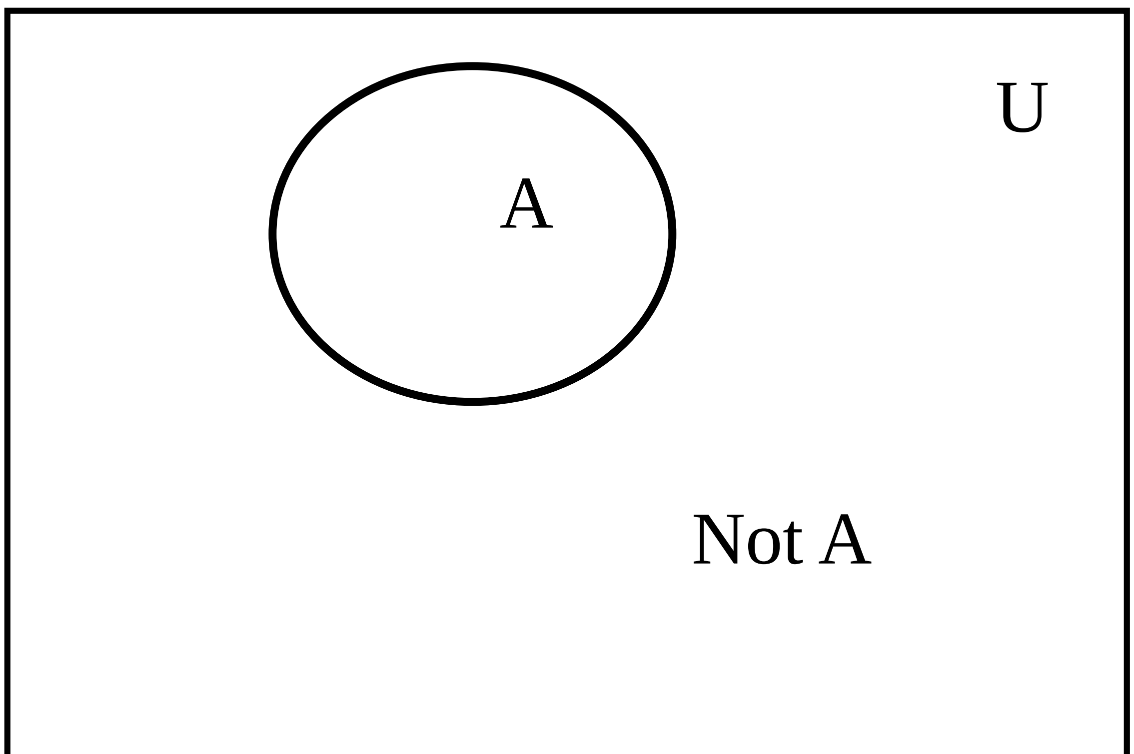
1. $0 \leq P(E_i) \leq 1$ The probability of E_i is always a number between 0 e 1

2. $P(\cup E_i) = 1$ The sum of all the outcomes $E_i \in C$ (the sample space) is = 1

3. Additivity : $P(E_1 \cup E_2) = ?$

Pictures

P(A) = (Area of A)/(Area of U) = implicitly P(A|U)



Sample space

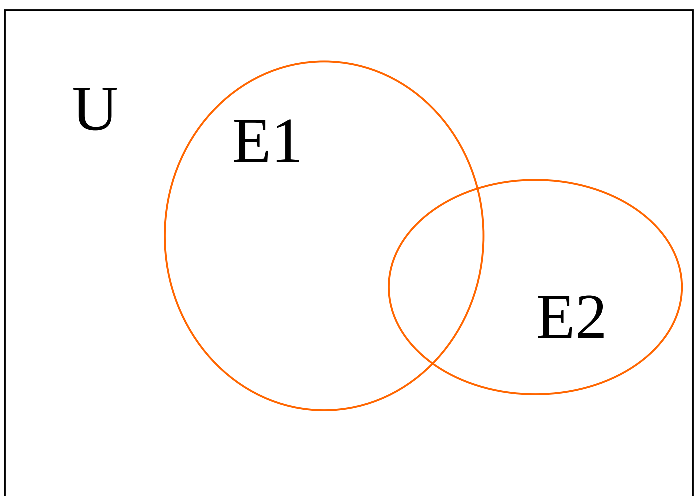
Operation on event sets

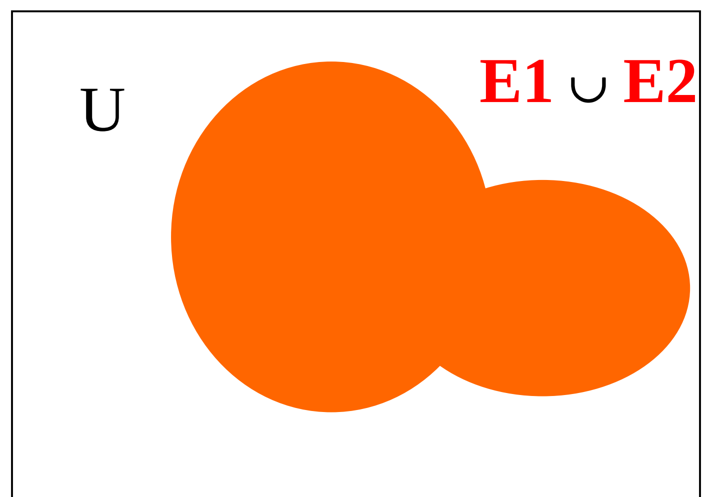
.OR.

Union of 2 events = probability(union)

$$= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$$

Ptot=Pa+Pb-intersection.
so you can sum only
if the intersection is null



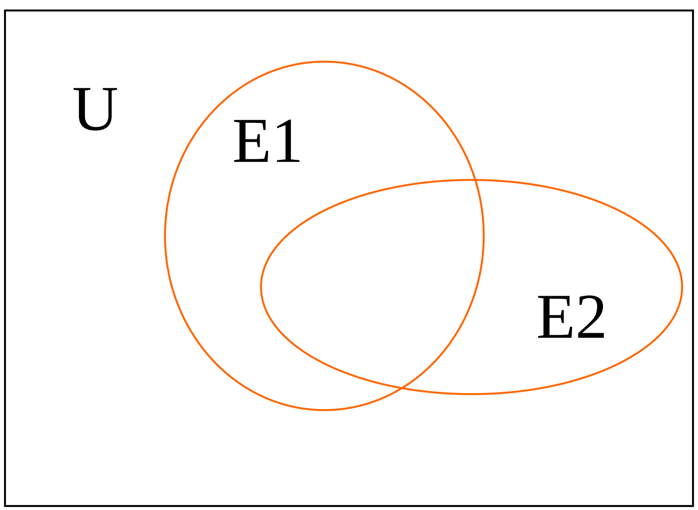


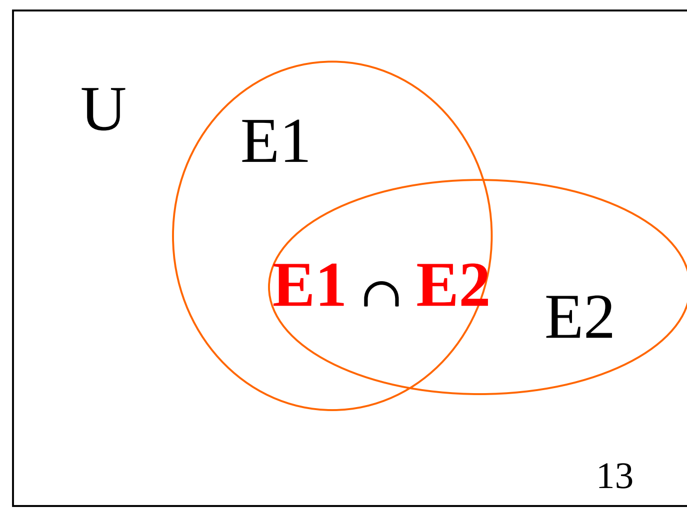
Operation on event sets

Intersection of 2 events = **probability(intersection)**

$$= P(E_1 \text{ and } E_2) = P(E_1 \cap E_2)$$

.AND.





Probability Additivity

if $\mathbf{E_1}$ and $\mathbf{E_2}$ are *mutually exclusive* then

$$\mathbf{P(E_1 \cup E_2) = P(E_1) + P(E_2)}$$

For instance, roll two dice

$$P(\text{sum} = 2 \cup 3) = \frac{1}{36} + \frac{2}{36} = \frac{3}{36}.$$

Basic exercises

1. A fair die is rolled once. Write down the probability that the score seen is: a) and odd number; b) greater than 4; c) and odd or a prime number.
2. A bag contains six red, three white and one blue disc. A single disc is removed at random. What is the probability that it is: a) not white; b) blue; c) white or blue; d) not red or blue?
3. A school has 5 houses, A, B, C, D, E. A class in the school contains 23 pupils: 4 from A, 8 from B, 5 from C, 2 from D and the rest from E. A student is selected at random: What is the probability that is: a) from B, b) from D or E, c) from A or C?
4. A bag contains 12 balls of which x are red, 2x are white and 3x are blue (x is an integer). A single ball is selected at random. What is the probability that it is: a) red, b) not white, c) white or blue?

Basic exercises

1. A ball is drawn at random from an urn containing colored balls. The balls can be either red or blue (no other colors are possible). The probability of drawing a blue ball is 0.30 What is the probability of drawing a red ball?
2. Consider a sample space comprising three possible (mutually exclusive) outcomes: A, B, C. Suppose the probabilities assigned to the three possible outcomes are: $P(A)=P(B)=1/4$, $P(C)=1/2$. Can you find an event whose probability is 3/4?
3. Consider a sample space comprising four possible (mutually exclusive) outcomes {a,b,c,d}. Consider the three events E= {a}, F= {a,b}, and G= {a,b,c}, with $P(E)=1/10$, $P(F)=5/10$ and $P(G)= 7/10$. What is the P(H) (H= {c,d})?

Probability Properties

1. $0 \leq P(E_i) \leq 1$ The probability of E_i is always a number between 0 e 1

2. $P(UE_i) = 1$ The sum of all the outcomes $E_i \in C$ (the sample space) is = 1

3. Additivity : $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

1° experiment = toss 2 dice || results = sum of the outcomes

P(sum is even **or** ≥ 7) = P(sum even) + P(sum ≥7) - P(sum = 8,10,12)

x= sum of the 2 dies

x	Possible outcomes						p(x)		
2	1,1						1/36		
3	1,2	2,1					2/36		
4	2,2	3,1	1,3				3/36		
5	2,3	3,2	4,1	1,4			4/36		
6	3,3	5,1	1,5	4,2	2,4		5/36		
7	3,4	4,3	5,2	2,5	1,6	6,1	6/36		
8	4,4	3,5	5,3	2,6	6,2		5/36		
9	6,3	3,6	5,4	4,5			4/36		
10	5,5	6,4	4,6				3/36		
11	5,6	6,5					2/36		
12	6,6						1/36		

1° experiment = toss 2 dice || results = sum of the outcomes

$$P(\text{sum is even } \mathbf{or} \geq 7) = P(\text{sum even}) + P(\text{sum} \geq 7) - P(\text{sum} = 8,10,12)$$
$$= 18/36 + 21/36 - 9/36 = 30/36$$

x= sum of the 2 dies

x	Possible outcomes						p(x)		
2	1,1						1/36		
3	1,2	2,1					2/36		
4	2,2	3,1	1,3				3/36		
5	2,3	3,2	4,1	1,4			4/36		
6	3,3	5,1	1,5	4,2	2,4		5/36		
7	3,4	4,3	5,2	2,5	1,6	6,1	6/36		
8	4,4	3,5	5,3	2,6	6,2		5/36		
9	6,3	3,6	5,4	4,5			4/36		
10	5,5	6,4	4,6				3/36		
11	5,6	6,5					2/36		
12	6,6						1/36		

2° experiment = joint probability of parents-children

Event = a pair of values: one for each variable

		Parent title		
		primary	High school	degree
Children title	primary	0,04	0,01	0,00
	High school	0,06	0,24	0,05
	degree	0,05	0,30	0,25

Marginal probability :

$P(P_d)$ = P(parent title = degree) = ?

$P(C_d)$ = P(child title = degree) = ?

2° experiment = joint probability of parents-children

Event = a pair of values: one for each variable

		Parent title			
		primary	High school	degree	total
Children title	primary	0,04	0,01	0,00	0,05
	High school	0,06	0,24	0,05	0,35
	degree	0,05	0,30	0,25	0,60
	total	0,15	0,55	0,30	1,00

Marginal probability :

$P(P_d)$ = P(parent title = degree) = 0,30

$P(C_d)$ = P(child title = degree) = 0,60

MARGINALIZE:sum a row\column to see the total probability of an event

2° experiment = joint probability of parents-children

Event = a pair of values: one for each variable

		Parent title			
		primary	High school	degree	total
Children title	primary	0,04	0,01	0,00	0,05
	High school	0,06	0,24	0,05	0,35
	degree	0,05	0,30	0,25	0,60
	total	0,15	0,55	0,30	1,00

Marginal probability :

$P(P_d) = P(\text{parent title} = \text{degree}) = 0{,}30$

$P(C_d) = P(\text{child title} = \text{degree}) = 0{,}60$

Union probabilities

$P(P_d \cup C_d) = P[(\text{parent} = \text{degree}) \text{ or } (\text{child}=\text{degree})] = ?$

$P(P_p \cup C_p) = P[(\text{parent}=\text{primary}) \text{ or } (\text{child}=\text{primary})] = ?$

$P(P_d \cup C_p) = P[(\text{parent}=\text{degree}) \text{ or } (\text{child}=\text{primary})] = ?$

2° experiment = joint probability of parents-children

Event = a pair of values: one for each variable

		Parent title			
		primary	High school	degree	total
Children title	primary	0,04	0,01	0,00	0,05
	High school	0,06	0,24	0,05	0,35
	degree	0,05	0,30	0,25	0,60
	total	0,15	0,55	0,30	1,00

Marginal probability :

$P(P_d)$ = P(parent title = degree) = 0,30

$P(C_d)$ = P(child title = degree) = 0,60

Union probabilities

subtract the intersection

$P(P_d \cup C_d)$ =P[(parent = degree) or (child=degree)] = 0,30+0,60-0,25 = 0,65

$P(P_p \cup C_p)$ = P[(parent=primary) or (child=primary)] =

$P(P_d \cup C_p)$ = P[(parent=degree) or (child=primary) =

2° experiment = joint probability of parents-children

Event = a pair of values: one for each variable

		Parent title			
		primary	High school	degree	total
Children title	primary	0,04	0,01	0,00	0,05
	High school	0,06	0,24	0,05	0,35
	degree	0,05	0,30	0,25	0,60
	total	0,15	0,55	0,30	1,00

Marginal probability :

$P(P_d) = P(\text{parent title = degree}) \quad = 0{,}30$

$P(C_d) = P(\text{child title = degree}) \quad = 0{,}60$

Union probabilities

$P(P_d \cup C_d) = P[(\text{parent = degree}) \text{ or } (\text{child=degree})] \quad = 0{,}30+0{,}60-0{,}25 = 0{,}65$

$P(P_p \cup C_p) = P[(\text{parent=primary}) \text{ or } (\text{child=primary})] \quad = 0{,}15+0{,}05-0{,}04= 0{,}16$

$P(P_d \cup C_p) = P[(\text{parent=degree}) \text{ or } (\text{child=primary})] \quad =$

2° experiment = joint probability of parents-children

Event = a pair of values: one for each variable

		Parent title			
		primary	High school	degree	total
Children title	primary	0,04	0,01	0,00	0,05
	High school	0,06	0,24	0,05	0,35
	degree	0,05	0,30	0,25	0,60
	total	0,15	0,55	0,30	1,00

Marginal probability :

$P(P_d)$ = P(parent title = degree) = 0,30

$P(C_d)$ = P(child title = degree) = 0,60

Union probabilities

$P(P_d \cup C_d)$ =P[(parent = degree) or (child=degree)] = 0,30+0,60-0,25 = 0,65

$P(P_p \cup C_p)$ = P[(parent=primary) or (child=primary)] = 0,15+0,05-0,04= 0,16

$P(P_d \cup C_p)$ = P[(parent=degree) or (child=primary) = 0,30+0,05-0,00= 0,35

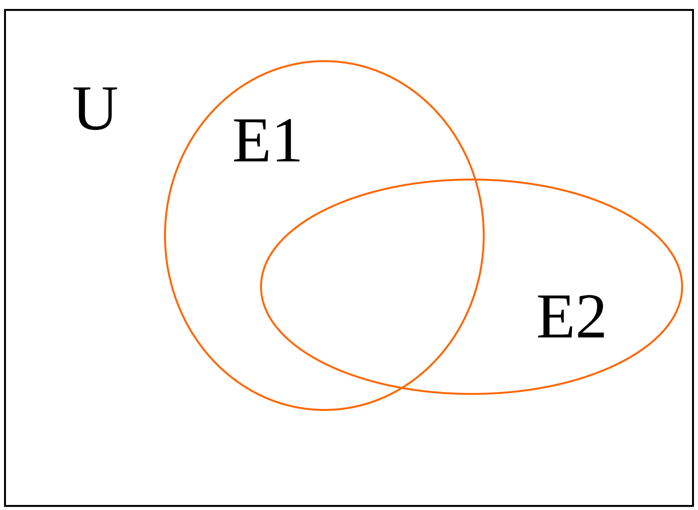
there is no interseption

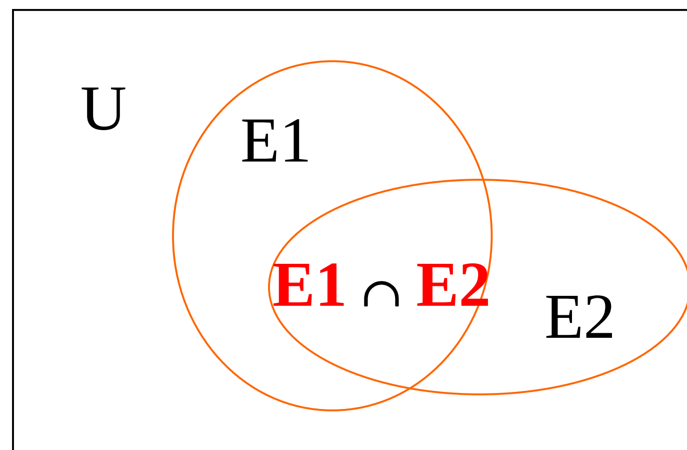
Operation on event sets

Intersection of 2 events = **probability(intersection)**

$$= P(E_1 \text{ and } E_2) = P(E_1 \cap E_2)$$

.AND.





$$= = P(E_1)\, P(E_2) \;\; ??$$

Conditional probability

$$P(E_1|E_2)=\frac{P(E_1 \cap E_2)}{P(E_2)}$$

$$P(E_1 \cap E_2)=P(E_1|E_2)\cdot P(E_2)$$

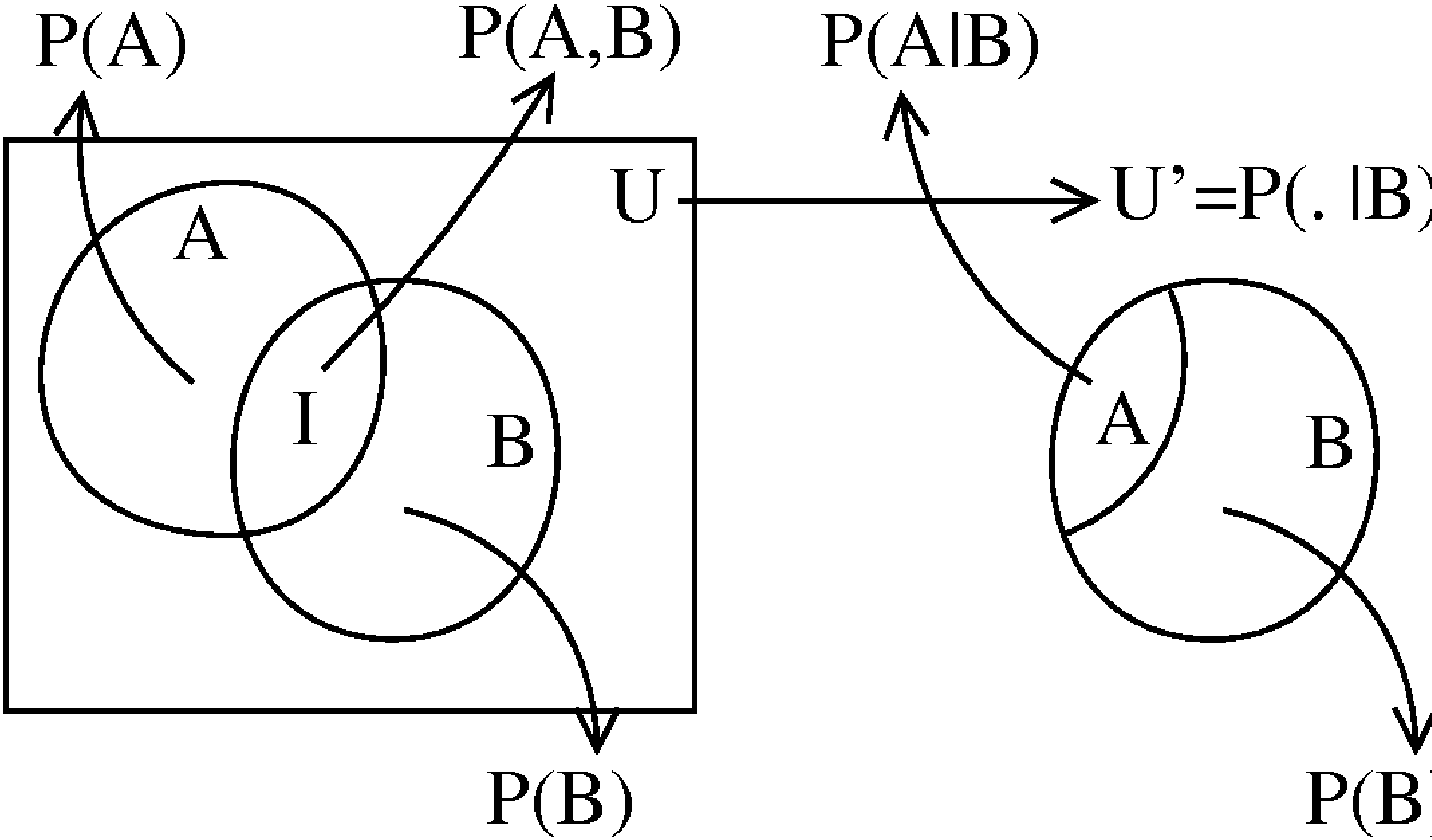
"Conditioning on an event" implies that the new total event space is reduced to that event. This is why we divide by its probability

Independent event

Ptot=Pa*Pb

2 outcomes E_1 and E_2 are independent when

$P(E_1 |E_2) = P(E_1)$ and $P(E_2|E_1) = P(E_2)$ both holds



"Conditioning on an event" implies that the new total event space is reduced to that event.

Example

1. Roll two fair dice, what is the probability of the sum is 3?
2. Roll one die first and you obtain as outcome 1. What is the probability that after rolling the second die the sum is 3?

Conditional probability

		Parent level of study			
		primary	High school	degree	total
Children level	primary	0,04	0,01	0,00	0,05
	High school	0,06	0,24	0,05	0,35
	degree	0,05	0,30	0,25	0,60
	total	0,15	0,55	0,30	1,00

$$P(E_1|E_2)=\frac{P(E_1 \cap E_2)}{P(E_2)}$$

$$P(E_1 \cap E_2)=P(E_1|E_2)\cdot P(E_2)$$

$P(C_d \mid P_p)$= P[(child=degree) given (parent=primary)] = ?

$P(C_d \mid P_{hs})$= P[(child=degree) given (parent=high school)] = ?

$P(C_d \mid P_d)$= P[(child=degree) given (parent=degree)] = ?

Conditional probability

		Parent level of study			
		primary	High school	degree	total
Children level	primary	0,04	0,01	0,00	0,05
	High school	0,06	0,24	0,05	0,35
	degree	0,05	0,30	0,25	0,60
	total	0,15	0,55	0,30	1,00

$$P(E_1|E_2)=\frac{P(E_1 \cap E_2)}{P(E_2)}$$

$$P(E_1 \cap E_2)=P(E_1|E_2)\cdot P(E_2)$$

$P(C_d \mid P_p)$= P[(child=degree) given (parent=primary)] = 0,05/0,15 = 0,33
$P(C_d \mid P_{hs})$= P[(child=degree) given (parent=high school)] =
$P(C_d \mid P_d)$= P[(child=degree) given (parent=degree)] =

Conditional probability

		Parent level of study			
		primary	High school	degree	total
Children level	primary	0,04	0,01	0,00	0,05
	High school	0,06	0,24	0,05	0,35
	degree	0,05	0,30	0,25	0,60
	total	0,15	0,55	0,30	1,00

$$P(E_1|E_2)=\frac{P(E_1 \cap E_2)}{P(E_2)}$$

$$P(E_1 \cap E_2)=P(E_1|E_2)\cdot P(E_2)$$

$P(C_d \mid P_p)$= P[(child=degree) given (parent=primary)] = 0,05/0,15 = 0,33

$P(C_d \mid P_{hs})$= P[(child=degree) given (parent=high school)] = 0,30/0,55 = 0,54

$P(C_d \mid P_d)$= P[(child=degree) given (parent=degree)] =

Conditional probability

		Parent level of study			
		primary	High school	degree	total
Children level	primary	0,04	0,01	0,00	0,05
	High school	0,06	0,24	0,05	0,35
	degree	0,05	0,30	0,25	0,60
	total	0,15	0,55	0,30	1,00

$$P(E_1|E_2)=\frac{P(E_1 \cap E_2)}{P(E_2)}$$

$$P(E_1 \cap E_2)=P(E_1|E_2)\cdot P(E_2)$$

$P(C_d \mid P_p)$= P[(child=degree) given (parent=primary)] = 0,05/0,15 = 0,33
$P(C_d \mid P_{hs})$= P[(child=degree) given (parent=high school)] = 0,30/0,55 = 0,54
$P(C_d \mid P_d)$= P[(child=degree) given (parent=degree)] = 0,25/0,30 = 0,83

Independent event?

Two dice case

P(sum=10 | dice are equal)	=? P(sum=10)
P(sum=even | 2 dice are different)	=? P(sum = even)
P(2 equal dice | the first is even)	=? P(dice are equal)
P(the first is even | dice are equal)	=? P(first is even)

Independent event?

Two dice case

P(sum=10 | dice are equal) 1/6	=? P(sum=10) $\neq$ 3/36	False
P(sum=even | 2 dice are different) 12/30	=? P(sum = even) $\neq$ 18/36	False
P(2 equal dice | the first is even) 3/18	=? P(dice are equal) = 6/36	True
P(the first is even | dice are equal) 3/6	=? P(first is even) = 18/36	True

INDEPENDENT : P=Pa*Pb

DEPENDENT:P=Pa+Pb|Pa

Exercise

If A and B are conditionally independent

P(A,B) = P(A)P(B)

What about *not A* and *not B ?*:

P(notA, notB) ?=? P(notA)P(notB)

Hint: remember that P(not X)=1-P(X) and that not(A or B) = (notA) and (notB)

de morgan Law

Some useful measure: Odd ratio and log-odd score

A measure of the relative influence of A and B is

$$\mathbf{odd(A,B)=P(A,B) / P(A)P(B)}$$

$\boxed{\text{if A and B are independent odd(A,B) } \sim 1}$

alternatively **log(odd(A,B))** >> 0 or << 0

indicates strong dependence

Computing the joint probability

$$P(E_1 \cap E_2) = P(E_1 | E_2) P(E_2)$$

Hint:

assuming E_2 is a certain event we can compute **$P(E_1 | E_2)$.**

Then we can relax this assumption by multiplying the results by **$P(E_2)$**.

The product is the joint probability (intersection) of the 2 events

If E_1 and E_2 are independent the $P(E_1 | E_2) = P(E1)$

and this imply

$$P(E_1 \cap E_2) = P(E_1)\, P(E_2)$$

- Is it possible that two mutually exclusive events are also independent?

$\boxed{\text{NO}}$

- Is it possible that two independent events are also mutually exclusive?

$\boxed{\text{NO}}$

33 Pirates (zecchino d'oro)

- 11 "pirati nell'occhio hanno una benda" (sight problem)
- 11 "pirati son zoppi in una gamba" (leg problem)
- 11 "pirati non sentono la tromba" (hearing problem)

What is the probability of:

- Having all three injuries
- Having 2 injuries
- Having 1 injury
- No injury

33 Pirates (zecchino d'oro)

Suppose that the problems are independent

P(I_i)=1/3 (prob injury i), P(NI_I)=2/3 (prob not injury j)

- Having all three injuries

  P(S)*P(L)*P(H)=1/3*1/3*1/3=1/27

- Having 2 injuries

  P(i,j,not k) + P(j,k,not i) + P(k,i,not j) =3*2/3*1/3*1/3=6/27

- Having 1 injury

  P(i,not (j,k))+P(j, not (i,k))+P(k,not (i,j))= 3*2/3*2/3*1/3=12/27

- No injury

  P(not S)*P(not L)*P(not H)= 2/3*2/3*2/3 = 8/27

Some exercises

1. The Census Bureau has estimated the following survival probabilities for men: - probability that a man lives at least 70 years: 80%;
   - probability that a man lives at least 80 years: 50%.

   What is the conditional probability that a man lives at least 80 years given that he has just celebrated his 70th birthday?
2. $P(A)=0.4$, $P(B)=0.3$ and $P(A \cup B)=0.5$. Calculate: $P(A \cap B)$, $P(A|B)$, $P(B|A)$, $P(\text{not } A)$, $P(\text{not } B \cap A)$.

es1 P=50\80

es2 Pa+Pb-PU
Pint\Pb
Pint\Pa
1-Pa
Pa-Pint

Bayes Theorem

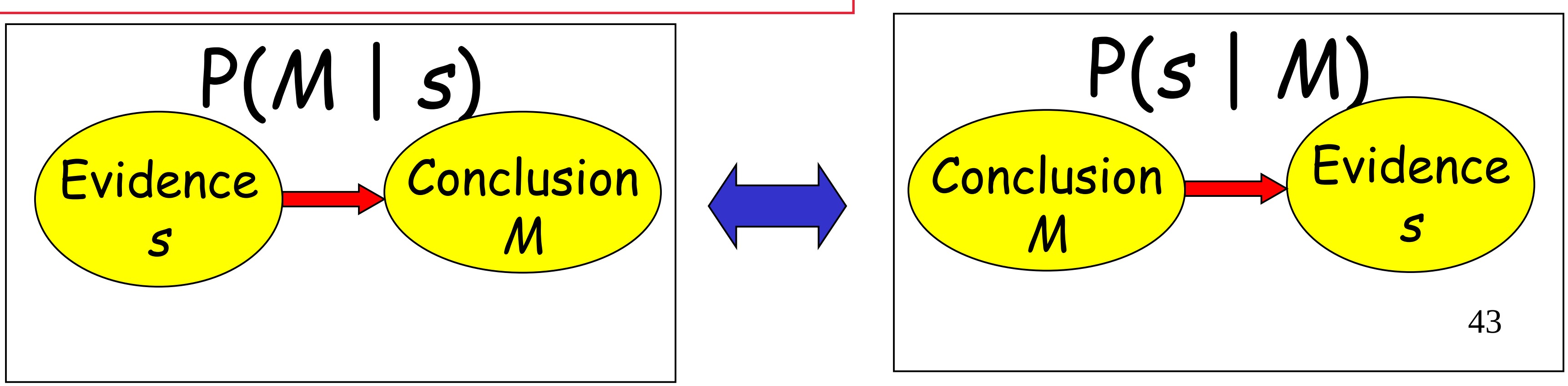
$$P(X,Y) = P(X \mid Y)\,P(Y) = P(Y \mid X)\,P(X)$$ Joint probability

$$P(Y \mid X) = \frac{P(X \mid Y)\,P(Y)}{P(X)}$$

So:

$$P(M \mid s) = \frac{P(s \mid M)\,P(M)}{P(s)}$$

A priori probabilities

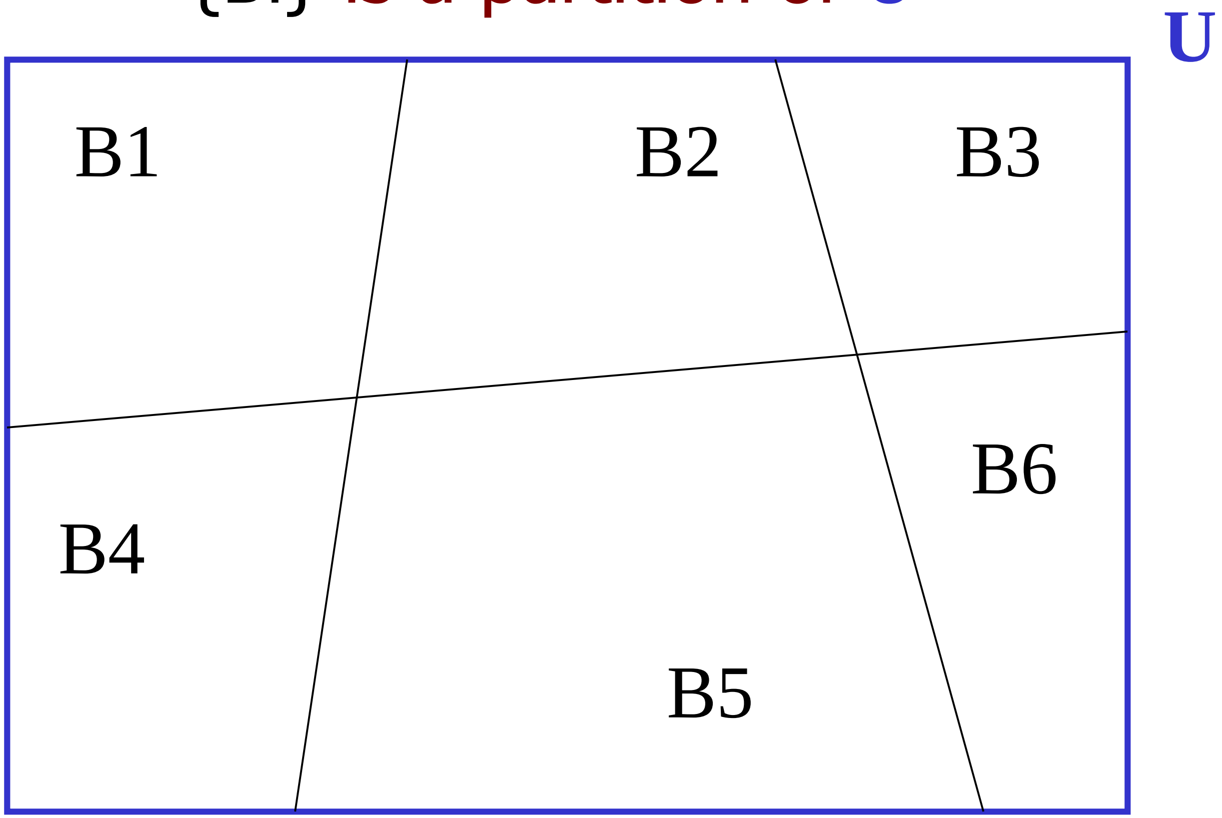


Partition

If $U = \cup_i Bi$

and $Bi \cap Bj = \varnothing$ for all $i \neq j$

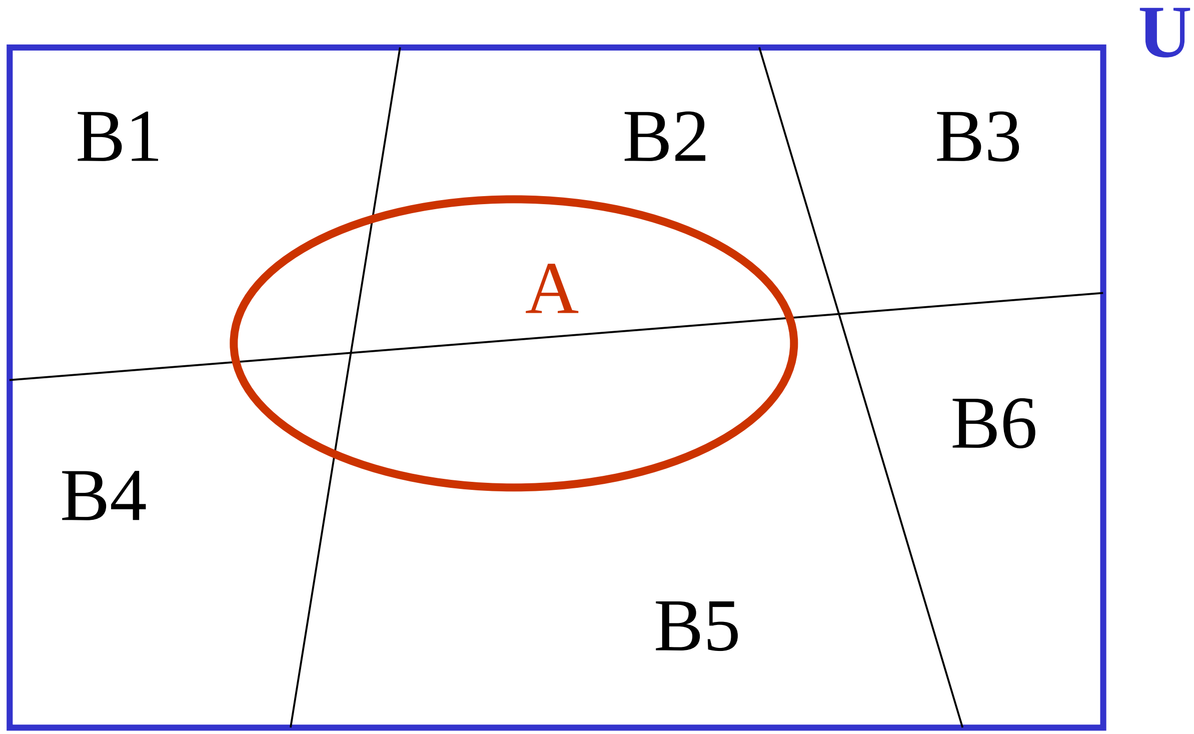
$\{Bi\}$ is a partition of U



Partition

If {Bi} is a partition of U

$P(A) = \sum_i P(A,Bi) = \sum_i P(A|Bi)P(Bi)$



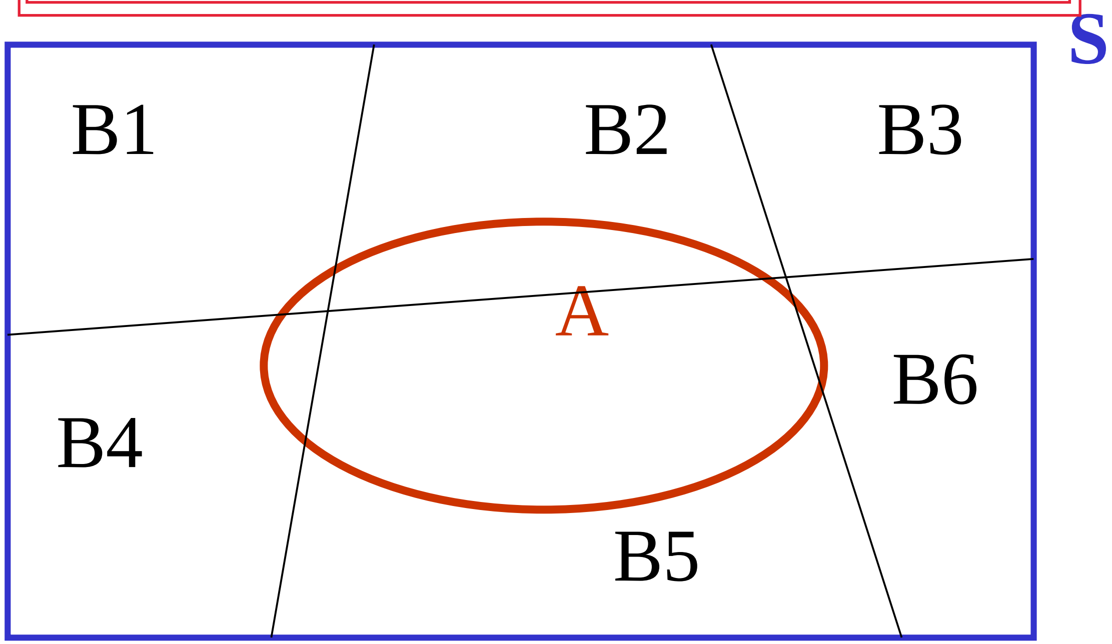
Bayes' Rule

- Suppose that B_1, B_2, … B_k form a partition of S:

$$B_i \bigcap B_j = \varnothing; \quad \bigcup_i B_i = S$$

Suppose that $\Pr(B_i) > 0$ and $\Pr(A) > 0$. Then

$$\boxed{\Pr(B_i \mid A) = \frac{\Pr(B_i A)}{\Pr(A)} = \frac{\Pr(A \mid B_i)\Pr(B_i)}{\Pr(A)}}$$



Bayes' rule: Example

- A rare disease affects 1 out of 100,000 people.
- A test shows positive
  - with probability 0.99 when applied to an *ill* person, and
  - with probability 0.01 when applied to a *healthy* person.
- You result positive to the test.
  ARE YOU ILL?

Bayes' rule: Example

$P(+|ill) = 0.99$ $\qquad$ $P(+|healthy) = 0.01$ $\qquad$ $P(ill) = 10^{-5}$

$$P(ill\,|\,+) = \frac{P(+\,|\,ill)P(ill)}{P(+\,|\,ill)P(ill) + P(+\,|\,healthy)P(healthy)}$$

$$= \frac{0.99\cdot 10^{-5}}{0.99\cdot 10^{-5} + 0.01\cdot(1-10^{-5})} = 9.89\cdot 10^{-4}$$

Happy End:

More likely the test is incorrect!!

Is the pope an alien?

Since the probability

$$P(\text{Pope}|\text{Human}) = 1/(6{,}000{,}000{,}000)$$

do this imply that

the Pope is not a human being?

Beck-Bornholdt HP, Dubben HH, Nature 381, 730 (1996)

THAT IS:

if Human → Pope is RARE, is Pope → Human RARE ?

(Human ~→ Not Pope) ➔? (Pope ~→ Not Human)

Aristotle investigated situations in which conclusions can be derived from premises. A well known example is that from the two premises (1) all humans are mortal, and (2) Socrates is human, it can be concluded (3) therefore Socrates is mortal. A necessary prerequisite for the validity of this type of syllogistic reasoning is that the premises are absolutely sure. Absolute certainty, however, is not the subject of statistics.

This reasoning becomes invalid when applied to probabilistic premises. If, for example, we randomly pick a human being, the probability that it is the Holy Father is extremely low — it is 1:6 billion = 0.00000000017. Therefore (1) if an individual is human, it is probably not the Pope ($P<0.00000000017$); (2) John Paul II is the Pope; (3) therefore, he is not a human being ($P<0.00000000017$). Which is obviously not sensible.

This example proves that the change from absolute certainty to probability makes the syllogistic reasoning false. Unfortunately, this is formally exactly the procedure that is applied in statistical hypothesis testing: (1) if the null hypothesis is true, these data are unlikely ($P<0.05$); (2) the data have occurred; (3) therefore the null hypothesis is wrong ($P<0.05$).

That this type of inference is wrong was noticed by Aristotle more than 2,000 years ago. It has also been sporadically discussed in the literature for several decades[5]. Nevertheless, this fallacy is still in use, probably because no alternatives are available. Does anybody know a way out of this dilemma?

Hans-Peter Beck-Bornholdt
Hans-Hermann Dubben
University of Hamburg,
Martinistrasse 52,
20246 Hamburg, Germany.

1. Amos, B. *Nature* **379**, 484 (1996).
2. Bulstrode, C. *Nature* **379**, 765 (1996).
3. Dunthorn, D. *Nature* **380**, 477 (1996).
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5. Cohen J. *Am. Psychologist* **49** ,997–1003 (1994).

P(Pope|Human) is not the same as P(Human|Pope)

$$P(Human \mid Pope) =$$

$$\frac{P(Pope \mid Human)P(Human)}{P(Pope \mid Human)P(Human) + P(Pope \mid Alien)P(Alien)}$$

but P(Alien) ~ 0

So

P(Human|Pope) ~ 1.0

The pope is (probably) not an alien

S Eddy and D McKay's answer

More examples of fallacious inference

Since most of sport accidents occur when playing soccer, *Stern* titled: "SOCCER IS THE MOST DANGEROUS SPORT"
(without considering that soccer is probably the most common sport)

Since a third of all fatal accidents in Germany occurs in private homes, *Die Welt* titled :"PRIVATE HOMES AS DANGER SPOTS"
(without considering that home is the place where people spend most of the time)

Since most of the cars entering in one-way streets in the wrong direction are driven by women, *Bild* titled:"WOMEN MORE DISORIENTED DRIVERS"
(without considering whether the samples of men and women drivers had the same size)

From: Kramer W, Gigerenzer G, Statistical Science 20:223-230 (2005)

es1
P(u1|red)=P(red|u1)P(u1)\p(red)
P(u1|red)=P(red|u1)P(u1)\p(red|u1)P(u1)+P(red|u2)P(u2)

1. There are two urns. The first urn contains 50 red balls and 50 blue balls. The second urn contains 30 red balls and 70 blue balls. One of the two urns is randomly chosen (both urns have probability 50% of being chosen) and then a ball is drawn at random from one of the two urns. If a red ball is drawn, what is the probability that it comes from the first urn?
2. An economics consulting firm has created a model to predict recessions. The model predicts a recession with probability 80% when a recession is indeed coming and with probability 10% when no recession is coming. The unconditional probability of falling into a recession is 20%. If the model predicts a recession, what is the probability that a recession will indeed come?
3. Alice has two coins in her pocket, a fair coin (head on one side and tail on the other side) and a two-headed coin. She picks one at random from her pocket, tosses it and obtains head. What is the probability that she flipped the fair coin?

es2
P(t|r)=0,8
P(t|n)=0,1
P(r)=0,2

P(r|t)=P(t|r)P(r)\P(t)
P(r|t)=P(t|r)P(r)\P(t|r)P(r)+P(t|n)P(n)

es3
P(c1|h)=P(h|c1)P(c1)\P(h|c1)P(c1)+P(h|c2)P(c2)

Game: 1 car and 2 sheep

- Two sheep and a care are hidden by three different doors

From “The Curious Incident of the Dog in the Night-Time” by Mark Haddon

Game: 1 car and 2 sheep

- **The game**: you select one door (ex. 1)

- From the remaining two one door with a sheep is shown to you (ex. 2)
- You may change your door (selecting 3) or you can keep the your first choice (1)

From “The Curious Incident of the Dog in the Night-Time” by Mark Haddon

Game: 1 car and 2 sheep

Question: Are the 2 choices

1. Equivalent
2. Better change opinion
3. Better keeping the first choice

From “The Curious Incident of the Dog in the Night-Time” by Mark Haddon

Game: 1 car and 2 sheep

Suppose you select x (y and z are the alternatives).

$P(x)=P(y)=P(z)=1/3$

$P(Sz)$=probability of showing z

$P(\textbf{\textit{first}}) = \mathbf{1/3}$

$P(\textbf{\textit{second}}) = P(y,Sz)+P(z,Sy)=$

$P(Sz|y)P(y)+P(Sy|z)P(z)= 1*1/3+1*1/3= \mathbf{2/3}$

From "The Curious Incident of the Dog in the Night-Time" by Mark Haddon