Project Instruction Document: Portfolio VaR & Risk Metrics Analysis

Overview

In this project you will create three distinct portfolios and analyze their risk using various Value at Risk (VaR) methods and additional risk metrics. You will:

1. Construct three portfolios:
    - **Portfolio 1:** Equal-weighted portfolio of 10 value stocks.
    - **Portfolio 2:** Equal-weighted portfolio of 10 growth stocks.
    - **Portfolio 3:** Equal-weighted portfolio of 10 industrial ETF stocks.
2. Pull daily return data for each portfolio.
3. Compute VaR using 10 different methods and document the formulas and assumptions.
4. Compare VaR results across the three portfolios and across the different VaR methods.
5. List and test the underlying assumptions of each VaR method using statistical tests.
6. Explain the concept of a coherent risk measure and, using Portfolio 1 data, test whether each VaR method meets these criteria.
7. Calculate the Expected Shortfall (ES) for the three portfolios.
8. Calculate five additional risk metrics to evaluate the portfolios, including formulas and detailed explanations.

Detailed Instructions

Step 1: Construct Portfolios

1. **Select Stocks/ETFs:**

    - **Value Stocks:** Identify 10 well-known value stocks (e.g., JNJ, PG, KO, PEP, WMT, XOM, IBM, MCD, MMM, CVX).
    - **Growth Stocks:** Identify 10 well-known growth stocks (e.g., AAPL, MSFT, AMZN, GOOGL, NFLX, NVDA, ADBE, CRM, META, TSLA).
    - **Industrial ETFs:** Identify 10 industrial ETF stocks (e.g., XLI, IYJ, VIS, ITA, FXR, PIL, IGV, SOXX, PPA, FTEC). *(Please verify ticker suitability for "industrial ETF stocks" as per your research.)*
2. **Portfolio Construction:**

- For each portfolio, assign equal weights. For example, if there are 10 stocks, each weight = 1/10.
- Save the list of tickers for each portfolio in separate files or as separate Python lists.

Step 2: Pull Daily Return Data

1. **Data Source:**
   - Use a reliable financial API (e.g., Yahoo Finance via `yfinance` or another data provider).
2. **Data Extraction:**
   - Write Python code to download daily closing prices for each ticker from a specified start date to an end date.
   - Calculate daily returns using: [R_{t} = \frac{P_{t} - P_{t-1}}{P_{t-1}}] where (P_t) is the closing price at time (t).
3. **Portfolio Returns:**
   - For each portfolio, calculate the portfolio daily return as the weighted sum of individual stock returns: [R_{\text{portfolio}, t} = \sum_{i=1}^{N} w_i R_{i,t}] where (w_i = \frac{1}{10}).

Step 3: Calculate VaR Using 10 Different Methods

For each method below, include code comments with formulas and assumptions.

Method 1: Variance-Covariance (Parametric Normal) Method

- **Assumptions:** Returns are normally distributed.
- **Formula:** [\text{VaR} = -\left(\mu + z_{\alpha} \sigma\right)] where (\mu) is the mean return, (\sigma) is the standard deviation, and (z_{\alpha}) is the z-score for the confidence level.

Method 2: Historical Simulation Method

- **Assumptions:** Historical returns are representative of future risk.
- **Procedure:** Sort historical returns and pick the quantile corresponding to (1-\alpha).
- **Formula:** VaR is the ((1-\alpha)\times 100%) percentile of returns.

Method 3: Monte Carlo Simulation Method

- **Assumptions:** Return distribution is known (commonly normal); can simulate future returns.

- **Procedure:** Generate simulated returns, then calculate the quantile.

Method 4: Extreme Value Theory (EVT) Method

- **Assumptions:** Focuses on tail behavior of returns.
- **Procedure:** Fit a Generalized Pareto Distribution (GPD) to the tail data.
- **Formula:** [\text{VaR} = \text{threshold} - \frac{\sigma}{\xi}\left[\left(\frac{p}{\nu}\right)^{-\xi} - 1\right]] where (\xi) is the shape parameter, (\sigma) is the scale parameter, (\nu) is the tail fraction, and (p = 1-\text{confidence level}).

Method 5: Filtered Historical Simulation (FHS)

- **Assumptions:** Returns are filtered using a volatility model (e.g., GARCH).
- **Procedure:** Filter returns to remove volatility clustering, then apply historical simulation.
- **Formula:** Standardize returns by dividing by estimated volatility.

Method 6: GARCH-Based VaR

- **Assumptions:** Conditional volatility can be modeled with GARCH.
- **Procedure:** Estimate conditional variance using GARCH, then compute VaR.
- **Formula:** [\text{VaR} = -\left(\mu + z_{\alpha} \sqrt{h_t}\right)] where (h_t) is the forecasted variance.

Method 7: Cornish-Fisher Expansion

- **Assumptions:** Adjusts for skewness and kurtosis in return distribution.
- **Formula:** [z_{\text{adj}} = z + \frac{(z^2-1)\text{skew}}{6} + \frac{(z^3-3z)(\text{kurt}-3)}{24} - \frac{(2z^3-5z)(\text{skew}^2)}{36}] and [\text{VaR} = -\left(\mu + z_{\text{adj}} \sigma\right)]

Method 8: Bootstrapping Method

- **Assumptions:** Historical data can be resampled to create a distribution.
- **Procedure:** Resample returns with replacement and compute the VaR from the bootstrap distribution.

Method 9: Kernel Density Estimation (KDE)

- **Assumptions:** The distribution of returns can be estimated non-parametrically.
- **Procedure:** Estimate density using KDE and then integrate to get the CDF.

Method 10: Parametric VaR with t-Distribution

- **Assumptions:** Returns follow a Student's t-distribution with fat tails.
- **Formula:** [\text{VaR} = -\left(\mu + t_{\alpha, \nu} \sigma \sqrt{\frac{\nu-2}{\nu}}\right)] where (t_{\alpha, \nu}) is the t-distribution quantile with (\nu) degrees

of freedom.

Step 4: Comparative Analysis

1. **Between Portfolios:**

    - For each VaR method, compare the risk levels (VaR values) among the three portfolios.
    - Use visualizations (e.g., bar charts, box plots) to illustrate differences.
    - Provide commentary on why the portfolios might have different risk profiles.

2. **Across VaR Methods:**

    - Compare how each method's VaR estimates differ for the same portfolio.
    - Discuss the sensitivity of each method to assumptions (e.g., normality, tail behavior).

Step 5: List and Test Assumptions for Each VaR Method

1. **Document Assumptions:**
    - For each method, list its key assumptions (e.g., normality for the variance-covariance method, representativeness of historical data for historical simulation, etc.).
2. **Statistical Tests:**
    - **Normality:** Use tests like the Shapiro-Wilk or Kolmogorov-Smirnov test.
    - **Independence & Stationarity:** Use autocorrelation tests (e.g., Ljung-Box test).
    - **Tail Behavior:** Use tests such as the Anderson-Darling test for tail fit when using EVT.
3. **Reporting:**
    - Write a summary report of test results and state whether each portfolio's returns satisfy the method's assumptions.

Step 6: Coherent Risk Measure Analysis

1. **Definition:**
    - A risk measure is coherent if it satisfies:
        - **Monotonicity:** If portfolio A always has worse outcomes than B, then its risk should be higher.
        - **Sub-additivity:** Diversification should not increase risk.
        - **Positive Homogeneity:** Scaling the portfolio scales the risk measure proportionally.
        - **Translation Invariance:** Adding a risk-free asset decreases the risk measure by the same amount.

2. **Task:Using Portfolio 1 and 2's data, examine whether the VaR computed by each method meets these properties.**

Step 7: Calculate Expected Shortfall (ES)

1. **Definition:**
    - Expected Shortfall (also known as Conditional VaR) is the expected loss given that the loss exceeds the VaR threshold.
    - **Formula:** $\text{ES} = -E\left[R \mid R < -\text{VaR}\right]$
2. **Procedure:**
    - For each portfolio, calculate the ES at the same confidence level used for VaR.
    - Use historical data and/or simulation to compute ES.

Step 8: Calculate 5 Other Risk Metrics

Risk Metric 1: Standard Deviation (Volatility)

- **Definition:** Measures the dispersion of returns.
- **Formula:** $\sigma = \sqrt{\frac{1}{N-1}\sum_{t=1}^{N}(R_t - \mu)^2}$

Risk Metric 2: Sharpe Ratio

- **Definition:** Measures the excess return per unit of risk.
- **Formula:** $\text{Sharpe Ratio} = \frac{\mu - R_f}{\sigma}$ where R_f is the risk-free rate.

Risk Metric 3: Sortino Ratio

- **Definition:** Similar to Sharpe Ratio but penalizes only downside volatility.
- **Formula:** $\text{Sortino Ratio} = \frac{\mu - R_f}{\sigma_d}$ where σ_d is the standard deviation of negative returns.

Risk Metric 4: Maximum Drawdown (MDD)

- **Definition:** Measures the largest peak-to-trough loss.
- **Procedure:** Calculate cumulative returns and find the maximum drop from a peak.

Risk Metric 5: Conditional Drawdown at Risk (CDaR)

- **Definition:** The average of drawdowns that exceed a certain threshold.
- **Procedure:** Identify periods of drawdowns exceeding the threshold and compute the average.

Final Deliverables

1. **Python Scripts/Notebook:**
    - Scripts for data extraction, portfolio construction, and all risk measure calculations.
    - Clear comments and markdown cells explaining each section.
2. **Analysis Report:**
    - Comparative analysis of VaR results across portfolios and methods.
    - Statistical test results for assumptions.
    - Discussion on coherence properties and additional risk metrics.
3. **Visualizations:**
    - Graphs for return distributions, VaR comparisons, drawdown plots, etc.
4. **Documentation:**
    - A final project report detailing methodologies, assumptions, formulas, and conclusions.

Good luck!

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