EXAMINATION 2060
REGULAR/BACK

Time: 3 hrs.
Full marks: 60
Pass marks: 24

Candidates are required to give their answers in their own words as far as practicable. Attempt All questions.

1. **What is satellite? Derive an expression for the orbital velocity of an artificial satellite and hence derive its time period.**

Ans: An artificial body placed in orbit round the earth or another planet in order to collect information or for communication is known as satellite. *For example;* Moon is a satellite of the earth.

Expression for orbital velocity

Let a satellite of mass m moves around the earth in an orbit of radius 'r'. If 'M' is the mass of the earth, then to orbit around the earth necessary centripetal force is provided by gravitational force between the earth and the satellite.

i.e., centripetal force = gravitational force between earth and satellite

or, $\frac{mv^2}{r} = G\frac{Mm}{r^2}$

$$\therefore \quad v^2 = G\frac{Mm}{r} \qquad (1)$$

If g_1 is the acceleration due to gravity at the position of the satellite, then,

$$mg_1 = G\frac{Mm}{r^2}$$

$$\text{or,} \quad GM = g_1 r^2 \qquad (2)$$

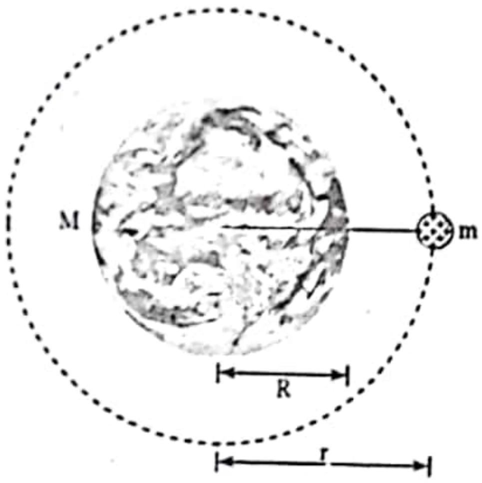
Substituting the value of GM from equation (2) to (1); we get,

$$v^2 = \frac{g_1 r^2}{r} = g_1 r$$

or, $v = \sqrt{g_1 r}$

On the orbit very close to the surface of earth, taking $g_1 = g$, relation becomes,

$\therefore \quad v = \sqrt{gr}$



But, we know,

$g = 10\ ms^{-2}$

and, $r = 6.4 \times 10^6$ m

$$\therefore \quad v = \sqrt{gr} = \sqrt{10 \times 6.4 \times 10^6} = 8 \times 10^3\ ms^{-1} = 8\ kms^{-1}$$

i.e., The orbital velocity of satellite is 8 kms^{-1}.

Period of satellite

The satellite moving with the velocity 'v' on an orbit around the earth is given by;

$$T = \frac{\text{Circumference of satellite}}{\text{Orbital velocity}} = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{g_1 r}} = 2\pi\sqrt{\frac{r}{g}}$$

or, $T = 2 \times 3.14 \sqrt{\frac{6.4 \times 10^6}{10}} = 83.73$ min.

i.e., The time period of the satellite is 83.73 minutes or 1 hour 37 minut...

Or

Define moment of inertia and discuss its significance. Derive a relatio... between moment of inertia and rotational kinetic energy of a body.

Ans; **Moment of inertia**

The moment of inertia is defined as the sum of the products of the mas... and the square of the distance of the different particles of the body from t... axis of rotation. It is represented by 'I'.

Mathematically,

$$I = \sum mr^2$$

It is the measure of an object's resistance to a change in the object's angu... acceleration due to the action of a torque.

The kinetic energy of a rotational motion is;

$$\frac{1}{2}I\omega^2$$

The kinetic energy of a body in linear motion is;

$$\frac{1}{2}mv^2$$

Significance of moment of inertia

On comparing relations (1) and (2); we find that moment of inertia is t... counter part of the mass and force is torque in rotational motion.

The mass of a flywheel is made concentrated on its rim to increase th... moment of inertia. Then its opposition to the change in uniform rotator... motion is large. Thus the flywheel of large moment of inertia when used ... engine makes them to run smoothly and steadily. The moment of inert... decreases with increase in mass. That is the reason a swimmer can jump in... water from height is able to make loop in air.

Relation between moment of inertia and rotational kinetic energy

Suppose a body rotates about an axis XY with an angular velocity 'ω'. If a body contains large number of particles, the particles on it also have the same angular velocity 'ω', but as particles are at different distances from the axis of rotation, their linear velocities will be different. Suppose a linear velocities of the particles of masses m_1, m_2, ... at distances r_1, r_2 from the axis of rotation are v_1, v_2 and v_3 etc.

Then, kinetic energy of the particles is;

$$\frac{1}{2}m_1v_1^2, \frac{1}{2}m_2v_2^2, \ldots\ldots$$

Total kinetic energy of the body is equal to the sum of the kinetic energies of various particles and is given by;

$$\text{Total kinetic energy} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \ldots\ldots$$

Since, $v_1 = \omega r_1$ and $v_2 = \omega r_2$ and so on.

$\therefore$ Total kinetic energy $= \frac{1}{2}m_1\omega^2r_1^2 + \frac{1}{2}m_2\omega^2r_2^2 + \ldots\ldots$

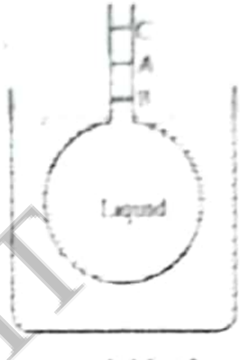
$= \frac{1}{2}\left(\sum mr^2\right)\omega^2 = \frac{1}{2}I\omega^2$

Kinetic energy of rotation $= \frac{1}{2}I\omega^2$

This is the required relation.

2. **Establish a relation between coefficients of real and apparent expansion of liquid. Write an expression for the variation of the density of a liquid with temperature.**

Ans: Let us consider a large glass flask. Fit to it a cork through which a glass tube with a narrow bore. Let the flask and glass tube contains liquid up to the mark 'A' at initial temperature. Now, the flask is immersed into a bath of hot Water. The liquid in the glass tube sinks at first to a level 'B' the reason is that the flask has expanded due to heat but no heat has yet entered the liquid. As a result the capacity of vessel increases and the level of liquid fall. The heat passes into the liquid which ultimately attains the temperature of the bath. As the liquid is more expandable than glass the liquid level increases fastly moves above the initial level 'A'. Let the level raises up to the level 'C'.



From the figure, 'A' is the initial level of liquid in the tube at lower temperature. B is the position, which the level would occupy if the vessel alone expanded to the final temperature of bath without any expansion of the liquid. Similarly, 'C' is the final position of the level when the liquid is heated to the temperature of bath.

Apparent expansion of the liquid is from level 'A' to 'C' or AC but true or real expansion of the liquid is from 'B' to 'C' or BC. Similarly AB gives the expansion of the volume of the vessel or container.

From the figure, we have,

$$BC = AB + AC \quad (1)$$

i.e., Real Expansion = Apparent expansion + Expansion of vessel

Let, a vessel contains V_0 volume of liquid at temperature 0°C, if the temperature of the liquid increases from 0°C to t°C, then,

Real increase in volume of liquid $= \gamma_a V_0 t$

and, Increase in volume of vessel $= \gamma_g V_0 t$

where, γ_a is the coefficient of cubical expansion of the containing vessel.

Substituting the values in relation (1); we get,

$$\gamma_r V_0 t = \gamma_a V_0 t + \gamma_g V_0 t$$

or, $\gamma_r V_0 t = V_0 t(\gamma_a + \gamma_g)$

or, $\gamma_r = \gamma_a + \gamma_g$

This is the required relation between coefficient of real and apparent expansion of liquid.

The expression for the variation of the density of a liquid with rise in temperature is;

$$\rho_t = \rho_0(1 - \gamma t)$$

where, ρ_0 is the density of liquid at 0°C and ρ_t is the density at t°C.

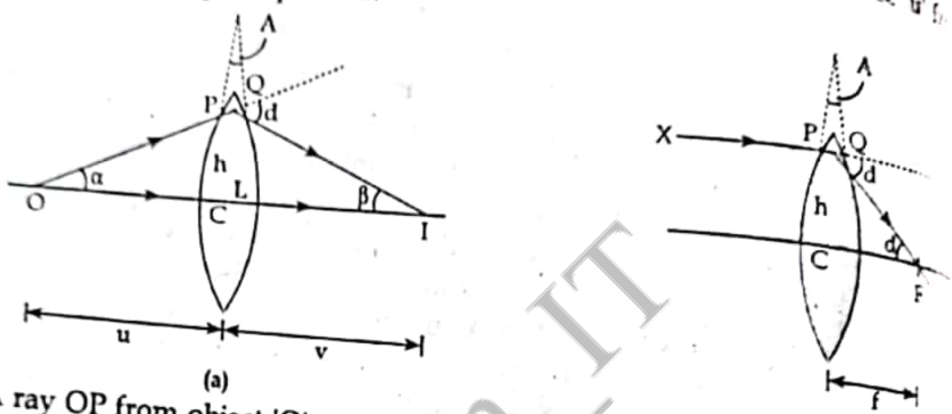
3. **Stating the sign convention deduces the relation between the focal length, the object distance and the image distance in case of a convex lens.**

Ans: The sign conventions used for convex lenses are as follows;

i) Focal length is taken as positive.
ii) Distances for real objects and images are taken as positive.
iii) Image distances for virtual images are taken as negative.
iv) Radius of curvature and focal length of convex lens are positive.

Relation between the f, u and v of a convex lens

Consider a convex lens and a point object 'O' is placed at a distance 'u' from the lens on the principle axis.



(a) (b)

A ray OP from object 'O' strikes the lens and refracted along PQ on the lens and then passes through QI. Similarly a ray from 'O' strikes the lens normally and moves straight. Then rays QI and LI meet at 'I'. Hence, 'I' is the image of the point object 'O' in next side of the lens. Let 'h' is the height of the point 'P' from the principle axis and 'α' is the angle made by the incident ray with principle axis and 'β', the angle made by the ray QI. If 'd' be the angle of deviation produced by the light during refraction.

We have; from the figure,

$$d = \alpha + \beta$$

From the right angled triangle POL; we have,

$$\tan\alpha = \frac{PL}{OL}$$

If angle 'α' is small, then,

$$\tan\alpha \approx \alpha = \frac{PL}{OL} = \frac{h}{u}$$

Similarly, from triangle PIL; we have,

$$\tan\beta = \frac{PL}{IL} = \frac{h}{v}$$

If 'β' is small then,

$$\tan\beta \approx \beta$$

Now, substituting the values in equation (1); we get,

$$d = \tan\alpha + \tan\beta = \frac{h}{u} + \frac{h}{v}$$

Again, if a parallel ray striking the lens at point 'P' as shown in the figure (b) refracted along the focus 'F'. Then,

$$\tan d = \frac{h}{f}$$

For a small angle of deviation,

$$\tan d \approx d = \frac{h}{f}$$

or, $d = \frac{h}{u} + \frac{h}{v}$

or, $\frac{h}{f} = \frac{h}{u} + \frac{h}{v}$

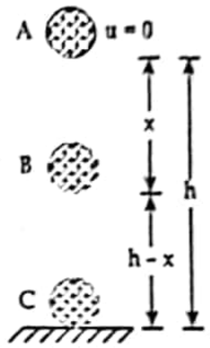
or, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

This is the required relation between focal length, object distance and image distance in case of a convex lens.

4. **Answer any six questions:**

a) **Prove the validity of the law of conservation of energy in an example of a freely falling body.**

Ans: Consider a body of mass 'm' at certain height 'h' above the ground at position 'A'. At this position total energy of is potential energy and equal to mgh. Let a body is allowed to fall freely. At certain instant of time 't' it reaches point 'B' and falls through height 'x'. Finally it falls to the ground level to point 'C'. The total energy at 'A', 'B' and 'C' can be calculated as follows;



At the position A

The height 'h' above the ground the potential energy possesses is given by;

$$P.E. = mgh$$

Since, initial velocity is zero hence,

$$K.E. = 0$$

$\therefore$ Total energy = P.E. + K.E. = mgh + 0 = mgh

At the position B

The height dropped from the position A is x, the height from the ground will be (h – x).The potential energy is given as;

$$P.E. = mg(h - x)$$

Let 'v' be the velocity of the body when it reaches the point 'B', hence kinetic energy is given by,

$$K.E. = \frac{1}{2}mv^2$$

From the equation of motion; we have,

$$v^2 = u^2 + 2gx$$

or, $v^2 = 2gx$ $\quad [\because u = 0]$

or, $K.E. = \frac{1}{2} \times 2mgx = mgx$

$\therefore$ Total energy = P.E. + K.E. = mg(h – x) + mgx = mgh

At position C

$$P.E. = 0 \quad [\because h = 0]$$

or, $K.E. = \frac{1}{2}mv^2$

where, 'v' is the velocity of the body at position 'C'.

Now, from the equation of motion we have,

$v^2 = u^2 + 2gh$

or, $v^2 = 2gh$

Hence,

$K.E. = \frac{1}{2} \times 2mgh = mgh$

$\therefore$ Total energy = P.E. + K.E. = 0 + mgh = mgh

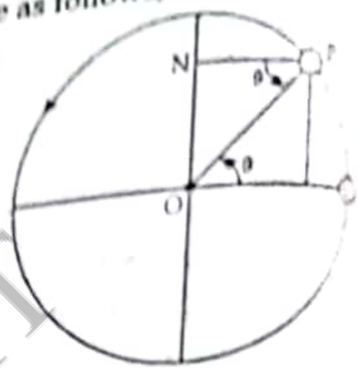
This shows that

Total energy at A = Total energy at B = Total energy at C

Hence, for a freely falling body law of conservation of energy is true.

b) **Write down the characteristics to simple harmonic motion.**

Ans: The characteristics to simple harmonic motion are as follows;



It is a to and fro motion in a straight line such that its acceleration is:

i) always directed towards a fixed point on a straight line.

ii) a magnitude of acceleration varies directly as the distance from the fixed position.

For example:

i) Motion of bob of a pendulum through a small angle.

ii) The motion of prong's of a sounding tuning fork.

Simple harmonic motion is closely associated with circular motion. Magnitude for S.H.M. is calculated by projecting circular motion into a diameter of a uniform circular motion.

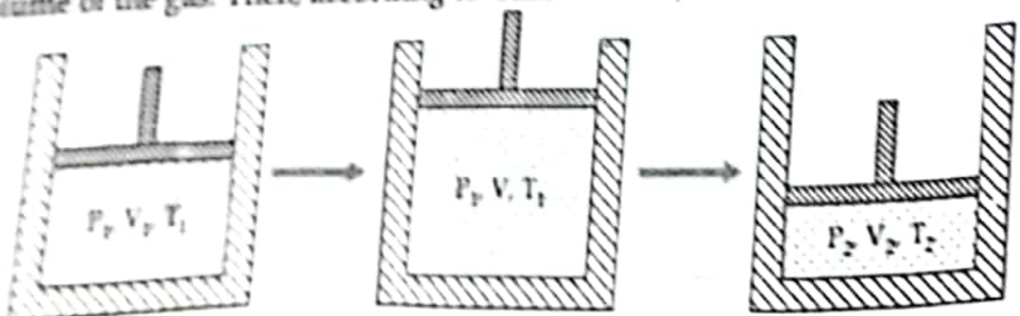
In mathematical word simple harmonic motion is defined as the projection of a uniform circular motion in a diameter of circular path.

c) **Explain how Boyle's law and Charles's law can be companied to give equation of state of ideal gas.**

Ans: A gas that obey completely both Boyle's and Charles's laws are called an ideal gas. The combination of Boyle's and Charles's Laws gives an important relation among pressure, volume and temperature of ideal gas. The relation is known as equation of state of ideal gas.

For the determination of equation consider a system of gas enclosed inside a cylinder with piston as shown in the figure.

Let initial pressure and volume of a given mass of gas are P_1, V_1 and temperature T_1. If temperature is changed to T_2 keeping pressure P_1 constant then, from Charles's law the volume must be increased. Let V' is the final volume of the gas. Then, according to Charles's law,



Expansion and compression of gases at different conditions.

Now, from the equation of motion we have,

$$v^2 = u^2 + 2gh$$

or, $v^2 = 2gh$

Hence,

$$K.E. = \frac{1}{2} \times 2mgh = mgh$$

$\therefore$ Total energy = P.E. + K.E. = 0 + mgh = mgh

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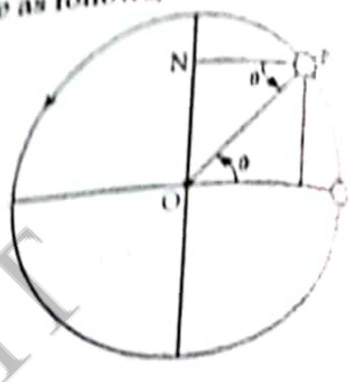

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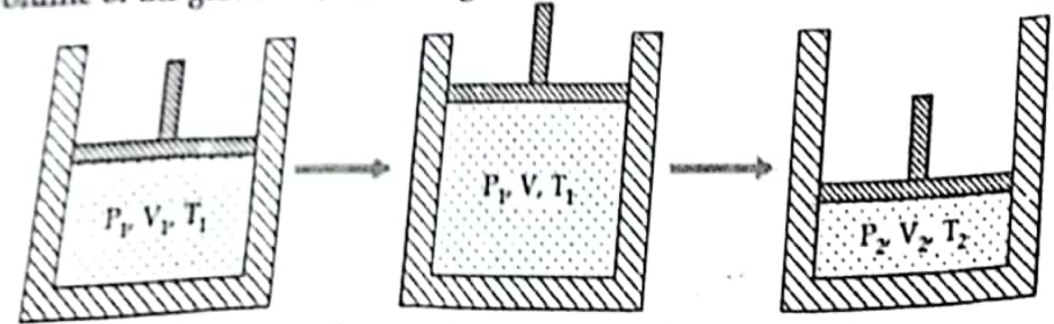


Expansion and compression of gases at different conditions.

$$\frac{V_1}{T_1} = \frac{V'}{T_2}$$

or, $\frac{V'}{V_1} = \frac{T_2}{T_1}$

or, $V' = \frac{T_2}{T_1} V_1$ (1)

Now, keeping temperature constant at T_2 changes its pressure from P_1 to P_2 so that volume changes from V' to V_2. Then, from Boyle's law,

$$\frac{V_2}{V'} = \frac{P_1}{T_2}$$

or, $V' = \frac{P_2}{P_1} V_2$ (2

Substituting the value of V' from equation (1) to (2); we get,

$$\frac{T_2}{T_1} V_1 = \frac{P_2}{P_1} V_2$$

or, $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \text{Constant}$

i.e., generalizing; we get,

$$\frac{PV}{T} = \text{Constant}$$

For one mole of gas, gas constant is represented by 'R'.

$\therefore \quad \frac{PV}{T} = R$

The constant 'R' is the called universal gas constant and is equal $8.31\ J^{-1}mol^{-1}K^{-1}$. Hence,

$$PV = RT$$

This is the equation of state of an ideal gas.

For n mole of gas, the equation becomes;

$$PV = nRT$$

d) State and explain Stefan's law of black body radiation.

Ans: Stefan's law states that, "heat radiation radiated per second per unit area of a bl body is directly proportional to the fourth power of its absolute temperature".

i.e., $E \propto T^4$

or, $E = \sigma T^4$

where, 'E' is the energy radiated per sec per unit area.

'T' is the temperature in absolute scale.

'σ' is Stefan's constant and is equal to $5.7 \times 10^{-8}\ Wm^{-2}K^{-4}$.

But, the law holds true when;

i) radiating body is a perfect black body and
ii) it receives no heat from surroundings.

If a black body is at temperature 'T' and is enclosed inside an enclo having temperature T_0, then the radiation energy radiated by black body second per unit area 'E' is related as follow;

If $T > T_0$; the relation is,

$$E = \sigma(T^4 - T_0^4)$$

If $T_0 > T$; then,

$$E = \sigma(T_0^4 - T^4)$$

But, if the body is not perfect black body the relation becomes;

$$E = e\sigma(T^4 - T_0^4)$$

where, 'e' is known as radiation emissivity of the surface and is different for the different materials.

e) What is critical angle? How is it related to the refractive index?

Ans: If the light travels from denser to rear medium then particular angle of incidence in denser medium when angle of refraction becomes 90° is called critical angle for the two given media. It is generally represented by 'C'.

Relation between critical angle and refractive index

If light travels from denser medium to rarer medium the refractive index 'η' is given by;

$$\eta = \frac{\sin r}{\sin i}$$

or, $$\frac{1}{\eta} = \frac{\sin i}{\sin r}$$

For critical angle 'C'; angle of refraction r = 90°

i.e., $$\frac{1}{\eta} = \frac{\sin C}{\sin 90^\circ}$$

∴ $$\eta = \frac{1}{\sin C}$$

i.e., the refractive index (η) is equal to the reciprocal of sine of the critical angle 'C'.

f) Explain ferromagnetism on the basis of domain theory.

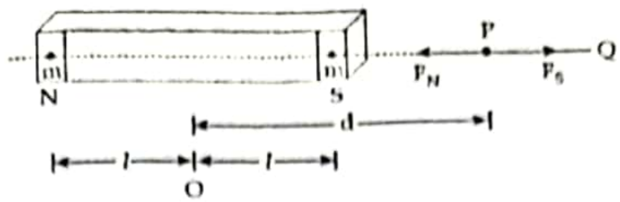
Ans: Modern theory of ferro-magnetism was developed by Weiss and later by Heisenberg. According to the theory a ferromagnetic substance like iron, is made from stable small groups called **magnetic domains**. The shapes of domain are odd and very smaller but fit together as in a mosaic. All the elements in a domain are arranged with their axes parallel to each other in absence of the external magnetic field. Every substance, there are one or more domains. This arrangement is of random orientation in un-magnetized condition.

Magnetic domains

When a magnetic field is applied, the elements within a domain may suddenly swing around and line up with the external field or aligned domain may grow in size. In other words, a common boundary between two adjacent domains in the same crystal will move. As the field grows stronger, more and more domains flip around suddenly to line up while others grow in size until all the domains are aligned. If a ferromagnetic specimen is heated above a certain temperature called **Curie point**, the exchange coupling disappears and the substance becomes paramagnetic.

g) Derive an expression for the magnetic intensity at a point on the axial line of a bar magnet.

Let us consider a bar magnet. If 'N' and 'S' be the poles of the bar magnet of length 2*l* and pole strength 'm' with centre 'O'.



Let a point P lies outside the magnet along its axial line at distant d from the centre of the magnet.

Let, OP = d

and, ON = OS = l

so that,

$$NP = (OP + ON) = (d + l)$$

and, $SP = (OP - OS) = (d - l)$

Now, magnetic intensity at 'P' due to the 'N' pole of the magnet is;

$$F_N = \frac{\mu_0}{4\pi}\frac{m \times l}{(NP)^2} = \frac{\mu_0}{4\pi}\frac{m}{(d+l)^2}; \text{ acts along the direction NP.}$$

Magnetic intensity at 'P' due to the 'S' pole of the magnet is;

$$F_S = \frac{\mu_0}{4\pi}\frac{m \times l}{(SP)^2} = \frac{\mu_0}{4\pi}\frac{m}{(d-l)^2}; \text{ acts along the direction PS.}$$

∴ Resultant magnetic intensity at 'P' is;

$$F = F_S - F_N = \frac{\mu_0}{4\pi}\frac{m}{(d-l)^2} - \frac{\mu_0}{4\pi}\frac{m}{(d+l)^2}; \text{ acts along the direction SN.}$$

∴ Magnetic intensity at 'P' is;

$$F = \frac{\mu_0}{4\pi}\left[\frac{m(d+l)^2 - m(d-l)^2}{(d^2-l^2)^2}\right] = \frac{\mu_0 m}{4\pi}\left[\frac{(d+l)^2-(d-l)^2}{(d^2-l^2)^2}\right]$$

$$= \frac{\mu_0}{4\pi}\frac{2m \cdot 2l \cdot d}{\left(d^2-l^2\right)^2}$$

∴ $F = \frac{\mu_0 m}{4\pi}\frac{2Md}{(d^2-l^2)^2}$; along the direction SN. [∵ $2ml = M$]

This is the required relation for magnetic intensity at a point.

5. Answer any six questions.

a) **A slab of mass 10 kg is lying on a plane inclined at 30° to the horizontal. Find the least force which will pull the slab upward. Given coefficient of friction is 0.2 and $g = 9.8\ ms^{-2}$.**

Solution:

Given that;

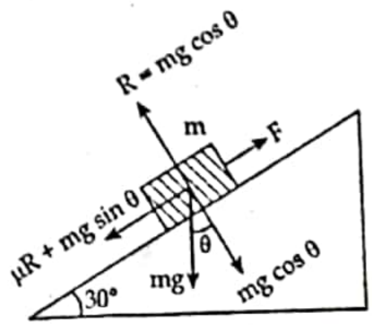
Mass of slab (m) = 10 kg

Angle of inclination (θ) = 30°

Coefficient of friction (μ) = 0.2

$g = 9.8\ ms^{-2}$

Least force required to pull (F) = ?



We have,

$$F = mg\sin\theta + \mu R$$

or, $F = mg\sin\theta + \mu mg\cos\theta$

or, $F = mg(\sin\theta + \mu\cos\theta)$

$\therefore$ $F = 10 \times 9.8(\sin 30° + 0.2 \times \cos 30°) = 65.97$ N

The least force that will pull the slab upward is 65.97 N.

b) A cyclist moves on a bent road of radius 5 m with a speed of 18 kmhr^{-1}. Calculate angle of inclination of cyclist with vertical. Take g = 9.8 ms^{-2}

Solution:

Given that;

Radius (r) = 5 m

Speed (v) = 18 kmhr^{-1} = $\frac{18 \times 1000}{60 \times 60} = 5$ ms^{-1}

Angle of inclination of the cyclist (θ) = ?

g = 9.8 ms^{-2}

We have,

$$\tan\theta = \frac{v^2}{rg} = \frac{(5)^2}{5 \times 9.8} = 0.51$$

$\therefore$ $\theta = \tan^{-1}(0.51) = 27.02°$

The angle of inclination of the cyclist with the vertical is 27.02°.

c) Calculate the amount of heat required to convert 1 kg of ice at −5°C to water at 100°C. Given; Specific heat capacity of ice = 2100 Jkg^{-1}K^{-1}, specific heat capacity of water = 4200 Jkg^{-1}K^{-1} and specific latent heat of fusion of ice = 3.34 × 10^5 Jkg^{-1}.

Solution:

Given that;

Mass of ice (m) = 1 kg

Latent heat of ice (L_i) = 3.34 × 10^5 Jkg^{-1}

Specific heat capacity of ice (s_i) = 2100 Jkg^{-1}K^{-1}

Specific heat of water (s_w) = 4200 Jkg^{-1}K^{-1}

Temperature of ice (t) = −5°C

To convert ice at −5°C to steam at 100°C heat will be gained be ice in following steps.

i) Heat taken be ice to raise its temperature from −5°C to 0°C is;

$Q_1 = ms_i(0 - t) = 1 \times 2100[0 - (-5)] = 0.105 \times 10^5$ J

ii) Heat taken by ice just to change its state from ice at 0°C to water at 0°C is;

$Q_2 = mL_i = 1 \times 3.34 \times 10^5 = 3.34 \times 10^5$ J

iii) Heat taken by water formed to raise its temperature from 0°C to 100°C is;

$Q_3 = ms_w(0 - t) = 1 \times 4200[0 - (-100)] = 4.2 \times 10^5$ J

$\therefore$ Total amount of heat needed $= Q_1 + Q_2 + Q_3$

$= (0.105 + 3.34 + 4.2) \times 10^5$ J

$= 7.645 \times 10^5$ J

Therefore, the amount of heat required is 7.645 × 10^5 J.

d) **A Carnot engine absorbs 1000 joules of heat energy from a reservoir at 127°C and rejects 600 joules of heat energy during each cycle. Calculate the temperature of the sink.**

Solution:

Given that;

Heat taken from reservoir (Q_1) = 1000 Joule

Heat rejected (Q_2) = 600 Joule

Temperature of reservoir (T_1) = 127°C = 127 + 273 = 400 K

Temperature of sink (T_2) = ?

We know,

Thermal efficiency (η) of a Carnot engine is given by;

$$\eta = 1 - \frac{Q_2}{Q_1} \qquad (1)$$

and, $$\eta = 1 - \frac{T_2}{T_1} \qquad (2)$$

By equating equation (1) and (2); we get,

$$\frac{T_2}{T_1} = \frac{Q_2}{Q_1}$$

$$\therefore \quad T_2 = \frac{Q_2 T_1}{Q_1} = \frac{600 \times 400}{1000} = 240\text{ K}$$

Therefore, temperature of the sink is 240 K.

e) **The image obtained by a concave mirror is erect and three times the size of the object. The focal length of the mirror is 20 cm. Calculate the object and image distances.**

Solution:

Given that;

Magnification (m) $= \frac{I}{O} = 3$

Focal length (f) = 20 cm

Object distance (u) = ?

Image distance (v) = ?

We know,

$$m = \frac{I}{O} = \frac{v}{u} = 3$$

or, $v = -3u$ (negative sign for virtual and erect image) (1)

From general mirror formula; we have,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or, $$\frac{1}{20} = \frac{1}{u} - \frac{1}{3u} = \frac{3-1}{3u} = \frac{2}{3u}$$

9+6

or, $3u = 40$

$\therefore \quad u = 13.33$ cm

Again, substituting the value of 'u' in equation (1); we get,

$$v = -3u = -3 \times 13.33 = -40\text{ cm}$$

Hence, object distance is 13.33 cm and image distance is 40 cm.

f) **A prism of angle 60° is made of glass of refractive index 1.5. Calculate the angle of minimum deviation.**

Solution:

Given that;

Angle of prism (A) = 60°

Refractive index (μ) = 1.5

Angle of minimum deviation (δ_m) = ?

Now; we have from the relation,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

or, $$\mu = \frac{\sin\left(\frac{60^\circ+\delta_m}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}$$

or, $$1.5 = \frac{\sin\left(\frac{60^\circ+\delta_m}{2}\right)}{\sin 30^\circ}$$

or, $$1.5 \sin 30^\circ = \sin\left(\frac{60^\circ+\delta_m}{2}\right)$$

or, $$\sin\left(\frac{60^\circ+\delta_m}{2}\right) = 0.75$$ $\sin^{-1} 0.75 \times 2 =$

or, $$\sin\left(\frac{60^\circ+\delta_m}{2}\right) = \sin 48.59^\circ$$

or, $$60^\circ + \delta_m = 97.18^\circ$$

or, $$\delta_m = 37.18^\circ$$

Hence, the angle of minimum deviation of the prism is 37.18°.

g) **A lift moves up with a constant acceleration 2 ms^{-2}. Calculate the reaction of the floor on a man of mass 50 kg standing in the lift (Take g = 9.8 ms^{-2}).**

Solution:

Given that;

Mass of the man (m) = 50 kg

Constant acceleration (a) = 2 ms^{-2}

g = 9.8 ms^{-2}

$\therefore$ $W_1 - mg = ma$

or, $W_1 = m(a+g)$

When lift moves up with acceleration the reaction on the floor is:

$$W_1 = m(g + a) = 30(9.8 + 2) = 590\ N$$

The reaction of the floor on a man is 590 N.

b) **The vertical and horizontal components of the earth's magnetic field at the place are equal. What is the angle of dip at the place?**

Solution:

Given that

Vertical component = V

Horizontal component = H

Angle of dip (θ) = ?

We know,

$V = H$

But, $\tan\theta = \frac{V}{H} = \frac{H}{H} = 1$

so, $\tan\theta = \tan 45°$

$\therefore$ $\theta = 45°$

vertical component = v

horizontal compant = v

angle of dip θ = ?

we know

v = h

θ = $\frac{v}{h}$

But θ = $\frac{v}{h}$ = 1

also Tanθ = 45°

θ = 45°

When lift moves up with acceleration the reaction on the floor is;

$$W_1 = m(g + a) = 50(9.8 + 2) = 590\text{ N}$$

The reaction of the floor on a man is 590 N.

h) **The vertical and horizontal components of the earth's magnetic fiel[d] at the place are equal. What is the angle of dip at the place?**

Solution:

Given that;

Vertical component = V

Horizontal component = H

Angle of dip (θ) = ?

We know,

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But, $\tan\theta = \frac{V}{H} = \frac{H}{H} = 1$

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$\therefore \theta = 45°$

vertical componand = V

Horizontal compont = V

angle of dip θ = ?

we know

$V = h$

$\theta = \frac{v}{h}$

But $\theta = \frac{v}{h} = 1$

also $\tan\theta = 45^0$

$\theta = 45^0$

EXAMINATION 2060
BACK

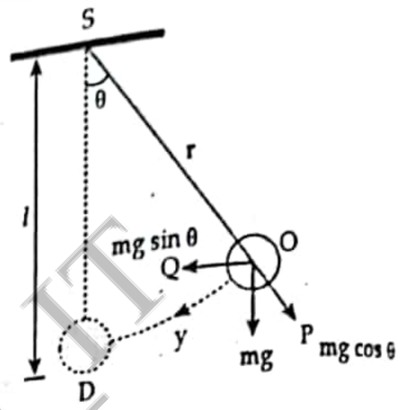
Full marks: 60
Pass marks: 24

Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.
Attempt All questions.

1. **What is simple pendulum? Show that motion of a simple pendulum is simple harmonic and calculate its time period.**

Ans: A heavy bob suspended from a light, extensible and flexible string forms a simple pendulum. When pendulum bob is taken to one side and let free it moves to and fro. Let the angle made by the string with vertical is less than 4° then the motion of the pendulum will be in simple harmonic motion. It can be shown as follow.



Suppose the bob is displaced making an angle θ from its mean position to 'O' and released. Then, weight (mg) acts vertically downwards as shown.

Resolving mg into two components $mg\sin\theta$ along OQ and $mg\cos\theta$ along OP. The component $mg\cos\theta$ balances the tension 'T' in the string and $mg\sin\theta$ provides necessary forces to return bob to its mean or original position.

∴ Force acting on the bob $= -mg\sin\theta$

or, $ma = -mg\sin\theta$

∴ $a = -g\sin\theta$

When θ is very small then $\sin\theta \approx \theta$

$$\therefore \quad \theta = -\frac{a}{g} \qquad (1)$$

Also,

$$\theta = \frac{\text{Arc OD}}{l} = \frac{y}{l}$$

Substituting the value of θ in the equation (1); we get,

$$a = -g\frac{y}{l} \qquad (2)$$

Since, $\frac{g}{l}$ is constant quantity, hence,

$$a \propto y$$

i.e., the acceleration 'a' is directly proportional to the displacement 'y'. This is according to the statement of S.H.M. Hence, motion of simple pendulum must be in S.H.M.

Time period

For a particle in S.H.M. the acceleration 'a' is;

$$a = -\omega^2 y \qquad (3)$$

Now, comparing equations (2) and (3); we get,

$$\omega^2 = \frac{g}{l}$$

$\therefore \quad \omega = \sqrt{\frac{g}{l}}$

Again, time period 'T' for particle in S.H.M. is given by;

$$T = \frac{2\pi}{\omega}$$

$\therefore \quad$ Time period (T) $= 2\pi\sqrt{\frac{l}{g}}$

This is the required relation for the time period of simple pendulum.

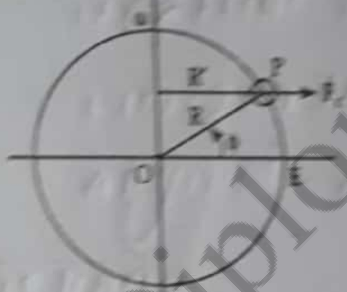
Or

Derive the relation acceleration due to gravity on the surface of the earth and show that acceleration due to gravity decreases due to the rotation of earth. What happens if the rotation of earth suddenly stopped?

Ans: The value of acceleration due to gravity is constant at the same place, but varies with distance from the centre of the earth above and below its surface. It also changes with rotation of earth.

Due to the rotation of the earth in its own axis

The earth is a sphere of radius 'R' and mass 'M' rotating on its axis with an angular velocity ω. If a body of mass 'm' at point 'P' such that OP makes an angle of ϕ with OE. Here, ϕ is the latitude of the place. The particle is moving in a circle of radius $R' = R\cos\phi$.



The net force pulling the particle towards the center of the earth is;

$$F_N = F_G - F_C \quad (1)$$

Here, F_N is the total force acting on the particle, F_G is the force due to gravity and F_C is the centrifugal force. Then, centrifugal force is given by;

$$F_C = m\omega^2 R' = m\omega^2 R\cos\phi \quad (2)$$

Substituting the value of F_C in equation (1); we obtain,

$$F_N = \frac{GmM}{R^2} - m\omega^2 R\cos\phi \quad (3)$$

If g' represents the gravitational acceleration, then,

$$F_N = mg'$$

or, $mg' = \frac{GmM}{R^2} - m\omega^2 R\cos\phi$

or, $mg' = mg - m\omega^2 R\cos^2\phi$

$\therefore \quad g' = g - \omega^2 R\cos^2\phi$

The relation shows that the acceleration due to gravity decreases with rotation of the earth.

The calculation shows that, value of 'g' decreases by 0.034 ms^{-2} due to rotation of the earth at equator but at the poles, value of 'g' remains unchanged.

$\therefore \quad \omega = \sqrt{\frac{g}{l}}$

Again, time period 'T' for particle in S.H.M. is given by;

$$T = \frac{2\pi}{\omega}$$

$\therefore \quad$ Time period (T) $= 2\pi\sqrt{\frac{l}{g}}$

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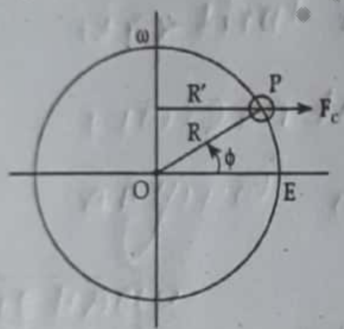

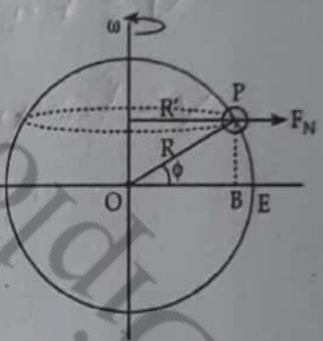

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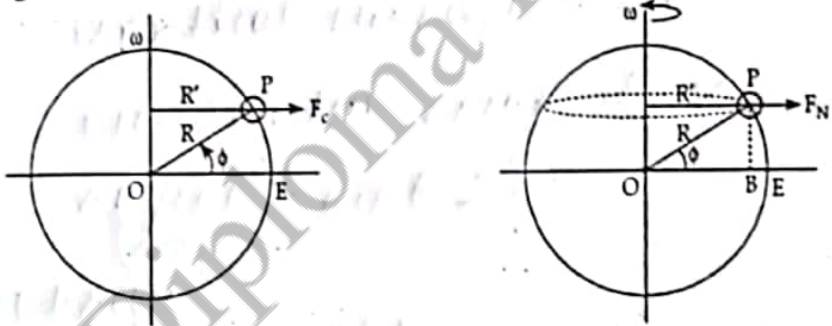

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The calculation shows that, value of 'g' decreases by 0.034 ms^{-2} due to rotation of the earth at equator but at the poles, value of 'g' remains unchanged.

If the rotation of earth suddenly stopped value of 'g' changes by 0.034 ms^{-2} at the equator but remains unchanged at the poles.

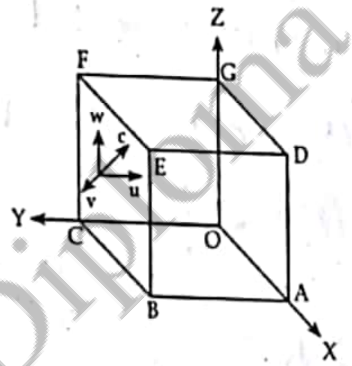
2. Write basic postulates of kinetic theory of gases. Using these derive an expression for the pressure exerted by an ideal gas.

Ans: The basic postulates of kinetic theory of gases are;

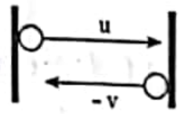
i) Every system of gas consists of large numbers of small particles called molecules.
ii) Molecules of a gas are alike but different from other gas molecules.
iii) The molecules of gases are continuously in motion with random velocity.
iv) The volume of a gas molecule is negligibly small with respect to the container.
v) All the collisions are perfectly elastic in nature.
vi) The molecules do not exert any force except during collisions and duration of collisions is negligibly small.
vii) Molecules travel straight path between two collisions and distance covered during two collisions is called the free path of the molecule.

Expression for the pressure exerted by an ideal gas

Consider a system of gas enclosed inside a cubical vessel of sides 'l'. Let the edges OA, OG and OC of the vessel are parallel to x, y and z-axis.



(a)



(b)

If 'N' is the number of gas molecules inside the vessel and 'm' is the mass of each gas molecule. 'N' molecules of gases are moving with velocities $c_1, c_2, \ldots\ldots\ldots c_n$ respectively. The velocity c_1 of a gas molecule can be resolved into three components, u_1, v_1 and w_1 along x, y and z-axis respectively. Then,

$$c_1^2 = u_1^2 + v_1^2 + w_1^2$$

Similarly, for other gas molecules have similar values given by;

$$c_2^2 = u_2^2 + v_2^2 + w_2^2$$
$$c_3^2 = u_3^2 + v_3^2 + w_3^2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$c_N^2 = u_N^2 + v_N^2 + w_N^2$$

Consider collision of a single molecule with the wall ABED as shown in the figure (b). The molecules strike the wall with momentum mu_1. As the collision is perfectly elastic the molecules rebound with same velocity in opposite direction. Thus, momentum of molecules after rebound is $-mu_1$.

Change in momentum $= mu_1 - (-mu_1) = 2mu_1$

In order to strike the same face again the molecule has to cover the distance $2l$. Therefore, time 't' between successive collisions is given by;

$$S = vt$$

or, $2l = u_1 t$

or, $t = \frac{2l}{u_1}$

$\therefore$ Change in momentum of first molecule along x-axis $= \frac{2mu_1}{t}$

$$= \frac{2mu_1}{\frac{2l}{u_1}}$$

$$= \frac{mu_1^2}{l}$$

i.e., Average force exerted by single molecule in the wall ABED $= \frac{mu_1^2}{l}$

Hence,

$$\text{Total force (F)} = \frac{mu_1^2}{l} + \frac{mu_2^2}{l} + \ldots\ldots\ldots + \frac{mu_N^2}{l}$$

$\therefore$ Pressure P_X on the same wall due to N-molecules is;

$$P_X = \frac{F_X}{l^2} = \frac{m}{l^3}(u_1^2 + u_2^2 + \ldots\ldots\ldots + u_N^2)$$

Similarly,

$$P_Y = \frac{F_Y}{l^2} = \frac{m}{l^3}(v_1^2 + v_2^2 + \ldots\ldots\ldots + v_N^2)$$

and, $P_Z = \frac{F_Z}{l^2} = \frac{m}{l^3}(w_1^2 + w_2^2 + \ldots\ldots\ldots + w_N^2)$

The average pressure 'P' due to N-molecules on the vessel is given by;

$$P = \frac{P_X + P_X + P_X}{3l^3}$$

$$= \frac{m}{3l^3}[(u_1^2 + u_2^2 + \ldots + u_N^2) + (v_1^2 + v_2^2 + \ldots + v_N^2) + (w_1^2 + w_2^2 + \ldots + w_N^2)]$$

$$= \frac{m}{3l^3}[(u_1^2 + v_1^2 + w_1^2) + (u_2^2 + v_2^2 + w_2^2) + \ldots\ldots\ldots]$$

$$= \frac{m}{3l^3}(c_1^2 + c_2^2 + \ldots\ldots\ldots + c_N^2)$$

$$= \frac{m}{3V}(c_1^2 + c_2^2 + \ldots\ldots\ldots + c_N^2) \; [\because l^3 = V] \quad (1)$$

Let 'c' is the root mean square velocity. Then,

$$c^2 = \frac{c_1^2 + c_2^2 + \ldots\ldots\ldots + c_N^2}{N}$$

$\therefore$ $c_1^2 + c_2^2 + \ldots\ldots\ldots + c_N^2 = Nc^2$

Now, substituting the value of $c_1^2 + c_2^2 + \ldots\ldots\ldots + c_N^2$ in relation (1); we get,

$$P = \frac{mNc^2}{3V} = \frac{2}{3} \times \frac{1}{2}mNc^2 = \frac{2}{3} \times \frac{1}{2}Mc^2$$

where, M = mN (*i.e.*, total mass of the gas).

$\therefore$ $P = \frac{2}{3}(\text{K.E. of a gas per unit volume})$

Therefore, pressure exerted by a gas is equal to the two thirds of the K.E. per unit volume of the gas.

3. **Define intensity of magnetization. Derive the expression for the intensity of magnetic field at a point on axial line and perpendicular bisector of a bar magnet.**

Ans: Intensity of magnetization

It is defined as the magnetic moment developed per unit volume of the substance. It is generally represented by 'I'.

i.e., $I = \frac{M}{V}$

where, M is the magnetic moment of piece of material.

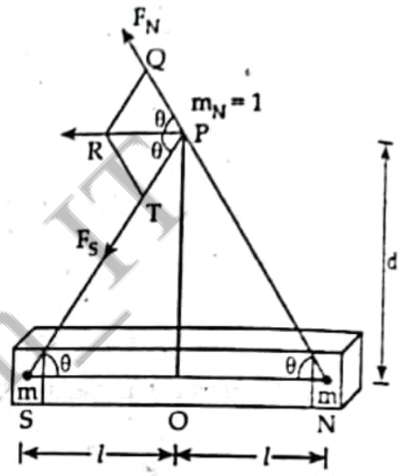
V is the volume in m^3 of the material.

Expression for the intensity of magnetic field at a point on axial line

See the solution of Q. no. 4 (g) of 2060 on page no. 8

Intensity of magnetic field at point perpendicular bisector of bar magnet

Let NS be a magnet of length $2l$ and magnetic moment 'M' and 'P' be a point at a distance 'd' from its centre, on its equatorial line as shown in the figure. Now, from the figure; we have,



$$OP = d$$
$$ON = OS = l$$
$$\therefore \quad NP = SP$$
$$= \sqrt{d^2 + l^2}$$
$$= (d^2 + l^2)^{\frac{1}{2}}$$

Now,

$$\text{Field at 'P' due to N-pole of magnet} = \frac{\mu_0}{4\pi}\frac{m \times l}{(NP)^2}$$
$$= \frac{\mu_0}{4\pi}\frac{m}{\left(d^2 + l^2\right)^2} \text{ along PQ}$$

Similarly,

$$\text{Field at 'P' due to S-pole of the magnet} = \frac{\mu_0}{4\pi}\frac{m \times l}{(SP)^2}$$

Let it is represented in the magnitude and direction by PT equals in the length to PQ. Thus, the two forces acting simultaneously at P due to N-pole and S-pole of the magnet respectively are represented in magnitude and direction by the straight lines PQ and PT respectively. Let resultant is represented in magnitude and direction, by the diagonal PR of the parallelogram PQRT, with PQ and PT as adjacent sides. In other words, PR represents the field at 'P' due to the magnet, in magnitude as well as direction.

Now, from two similar triangles, PQR and NPS; we have,

$$\frac{PR}{NS} = \frac{PQ}{NP}$$

$$\text{or,} \quad \frac{PR}{2l} = \frac{\mu_0}{4\pi}\frac{\frac{m}{\left(d^2+l^2\right)^2}}{\left(d^2+l^2\right)^{\frac{1}{2}}} = \frac{\mu_0}{4\pi}\frac{m}{\left(d^2+l^2\right)^{\frac{3}{2}}}$$

or, $PR = \frac{\mu_0}{4\pi}\frac{m \times 2l}{(d^2 + l^2)^{\frac{3}{2}}}$

$$= \frac{\mu_0}{4\pi}\frac{m \times 2l}{(d^2 + l^2)^{\frac{3}{2}}} = \frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{\frac{3}{2}}}$$

Therefore, field at 'P' due to the magnet is $\frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{\frac{3}{2}}}$ along PR and acts along the direction, parallel to the axis of the magnet.

4. **Answer any six questions:**

a) **State centripetal force and explain why it is necessary to reduce the speed on turning.**

Ans: A constant force acting on a body in circular motion directing towards a centre of a circular path and that keeps a body in circular motion is called centripetal force. Mathematically;

$$\frac{mv^2}{r} = m\omega^2 r$$

To reduce the speed on turning

When a car moves in a turn it is moving along a part of circular path. In order to provide necessary centripetal force by a component of weight of vehicles the roads are banked. During motion the relation between the speed of car and the banked angle θ is given by;

$$\tan\theta = \frac{v^2}{rg}$$

$$\therefore \quad v = \sqrt{rg\tan\theta}$$

A car moving with velocity 'v' successfully round the curve, as 'v' increases value of angle θ also increases. But for a road, angle θ is fixed. Therefore to move without skidding from turn of the car, the speed of car must be reduced. If not, a car will skid and accident is inevitable.

b) **Define angular momentum and state law of conservation of angular momentum with an example.**

Ans: The angular momentum of a rigid body is defined as the product of the moment of inertia and the angular velocity.

i.e., Angular momentum (L) = Iω

where, 'I' is the moment of inertia.

ω is the angular velocity

S.I. unit for angular momentum are Nms or kgm^2s^{-1}.

Law of Conservation of Angular Momentum

The law states that, "in the absence of external torque, the angular momentum of a rotating rigid body or a system of bodies is always conserved." *For example;* An ice skater on ice draws her arms inwards to decreases her moment of inertia. To obey the principle of conservation of angular momentum the angular momentum of the skater must remain constant. In order to keep angular momentum constant the speed of the skater increases and immediately starts to rotate faster.

c) **Define friction, force of kinetic friction, and force of static friction with expressions.**

Ans: **Friction**

Friction is a force that opposes the motion of a body. The friction is due to interlocking of the surfaces due to roughness of the sliding surfaces. Interlocking of the surfaces drag against each other and is the friction. According to the laws of friction the force of friction is directly proportional to the normal reaction for two sliding surfaces in contact

i.e., $F \propto R$

or, $F = \mu R$

The friction is usually divided into static friction and kinetic friction.

Force of kinetic friction

The frictional force that appears between two sliding surfaces when a body is in motion is called force of kinetic friction. The force of kinetic friction is given by;

$$F_K = \mu_K mg$$

where, μ_K is the coefficient of kinetic fiction.

Force of static friction

The frictional force that appears to oppose a body at rest into motion is called force of static friction. Static friction increase to prevent any relative motion to a limit where motion occurs and is also called limiting friction. The relation for force of static friction is given by;

$$F = \mu_S R$$

where, μ_S is the coefficient of static friction.

The coefficient of static friction is larger than coefficient of kinetic friction.

d) **State first law of thermodynamics and explain that this law leads conservation law of energy.**

Ans: First law of thermodynamics states that, "wherever heat energy is transferred into any other kind of energy or vice versa the quantity of energy which disappears in one form is exactly equivalent to the quantity produced in the other form."

Mathematically; first law of thermodynamics is given by,

$$dQ = dU + dW$$

where, dU is the increase in the internal energy of the system.

dQ is the amount of heat added to the system.

dW is the amount of external work done on the system.

First law of thermodynamics and conservation of energy

The law is actually a restatement of principle of conservation of energy applied specially to heat. The law gives the relation between work, heat and mechanical energy. First law of thermodynamics is the application of the conservation of energy principle to heat and thermodynamic processes.

From the relation,

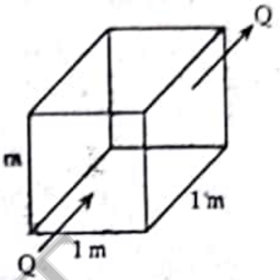
$$dQ = dU + dW$$

Here, dQ is the amount of heat energy given to a system. Among dQ certain portion of heat is converted to external work dW and remaining portion is converted to the internal energy dU of the system. It shows that during the process energy is neither created nor destroyed, converted from one form to another. This is according to the principle of conservation of energy. Therefore, first law of thermodynamics leads to conservation of energy.

e) Obtain a relation for the thermal conductivity of a conductor.

Ans: The quantity of heat Q flowing across two opposite faces of a conductor maintained at a constant high and low temperature depends;

i) directly on the cross sectional area A
ii) temperature difference $(\theta_2 - \theta_1)$ between its two ends
iii) time 't for which the heat is allowed to flow
iv) inversely to the distance 'l' between two ends



i.e., $Q \propto \frac{A(\theta_2 - \theta_1)}{l}$

or, $Q = \frac{KA(\theta_2 - \theta_1)t}{l}$

$\therefore \quad K = \frac{Q \times l}{A(\theta_2 - \theta_1)t}$

where, K is a constant called the thermal conductivity of a conductor.

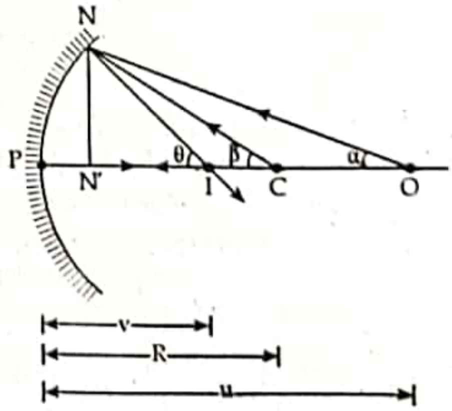
Now, if $A = 1$ unit$[1\ cm^2$ or $1\ m^2]$
$R = 1$ unit $[1$ cm or 1 m$]$
$\theta_2 - \theta_1 = 1°C$ and
$t = 1$ sec.

Then; we get,
$Q = K$

Thus, the thermal conductivity is defined as, "the amount of heat that flows normally per second across the two opposite faces of a unit cube in the steady state when they are maintained at a temperature difference of one degree centigrade."

f) Prove that $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ for a concave mirror.

Ans: Let us consider a concave mirror of focal length 'f' of small aperture as shown in the figure. A luminous point object 'O' is kept at distance 'u' from the mirror on the principle axis of the mirror. A light ray from 'O' moves along the path OIP and strikes on the point 'P' along the principle axis which reflects back along PIO.



Similarly, another ray travelled along ON incidents at the point 'N' and reflected back along 'NI, CN is the normal at N'. After reflection NI and OP meets at a point 'I' and is the image of 'O'. Now, OP is object distance 'u' and PI is image distance 'v'. CP is the radius of curvature 'R'.

Now, from ΔONC; we get,

$\beta = i + \alpha$

or, $\quad i = \beta - \alpha \qquad (1)$

Similarly, from ΔCNI; we have,

$\theta = \beta + r = \beta + i \qquad [\because i = r]$

or, $i = \theta - \beta$

From equations (1) and (2); we get,

$$\beta - \alpha = \theta - \beta$$

or, $2\beta = \alpha + \theta$

For small aperture mirror the angles α, β and θ are small so taking;

$$\alpha = \tan\alpha$$

$$\beta = \tan\beta$$

and, $\theta = \tan\theta$

Substituting the values of α, β and θ in the equation (3); we get,

$$2\tan\beta = \tan\alpha + \tan\theta$$

or, $2\frac{PN}{CP} = \frac{PN}{OP} + \frac{PN}{PI}$

or, $\frac{2}{CP} = \frac{1}{OP} + \frac{1}{PI}$

or, $\frac{2}{R} = \frac{1}{u} + \frac{1}{v}$

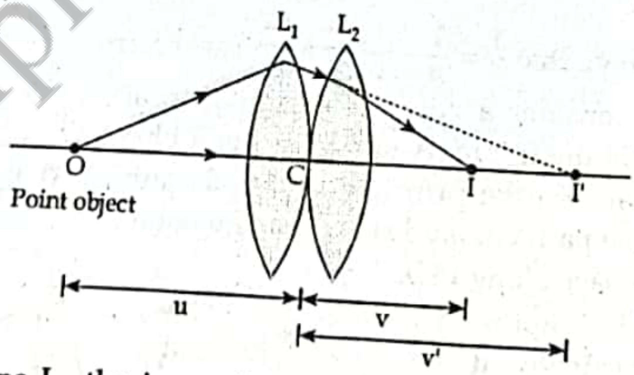
or, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ [$\because$ R = 2

The relation is proved.

g) **Derive equivalent focal length for a combination of two thin lense in contact.**

Ans: Let two thin lenses L_1 and L_2 are in contact as shown below. A point objec 'O' is kept on the common axis at distance 'u'. The light rays travels refracte from first lens L_1 and forms the image of the object at a distance v'. If 'u' i the object distance and f_1 is the focal length then,

$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v'} \quad (1$$



For the second lens L_2 the image formed by first lens L_1 acts as a virtual object. Then, lens L_2 forms the real image at 'I' at distance 'v' from the combination. For second lens L_2 the object distance will be $-v'$ as the object 'I' is virtual. Then,

$$\frac{1}{f_2} = \frac{1}{-v'} + \frac{1}{v} \quad (2)$$

Now adding equations (1) and (2); we have,

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v'} - \frac{1}{v'} + \frac{1}{v} = \frac{1}{u} + \frac{1}{v} \quad (3)$$

If the combination, form the image at 'I'. If 'F' is the focal length of the combination, then from the general lens formula; we have,

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{F} \quad (4)$$

From equation (3) and (4); we have,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

i.e., the reciprocal of the combined focal length is equal to the algebraic sum of the reciprocals of the focal length of the individual lenses.

5. **Answer any six questions.**
   a) **A stone is dropped from the top of a tall cliff and one second later a second stone is thrown vertically downward with a velocity of 20 ms^{-1}. How far below the top of the cliff will second stone pass the first?**

Solution:

Given that;

For the first stone

Initial velocity (u) = 0

Acceleration due to gravity $(g) = 10\ ms^{-2}$

Let height from the top where second stone pass the first is 'x', then,

Height (h) = x

From the equation of motion; we have,

$$h = ut + \frac{1}{2}gt^2$$

or, $x = 0 \times t + \frac{1}{2} \times 10 \times t^2 = 5t^2$ (1)

For the second stone

Initial vertical velocity (u) = 20 ms^{-1}

Acceleration due to gravity $(g) = 10\ ms^{-2}$

The height dropped (h) = x

Time taken (t) = (t − 1) sec.

From the equation of motion; we have,

$$h = ut + \frac{1}{2}gt^2$$

or, $x = 20 \times (t-1) + \frac{1}{2} \times 10 \times (t-1)^2 = 5t^2 + 10t - 15$ (2)

Now, from the equation (1) and (2); we get,

$5t^2 = 5t^2 + 10t - 15$

or, $t = 1.5$ sec.

Substituting the value of 't' in equation (1); we get,

$x = 5t^2 = 5 \times (1.5)^2 = 11.25$ m

Therefore, height from the top where second stone pass the first is 11.25 m.

   b) **An orbital satellite is revolving around the earth in a circular orbit at height of 30 km above the surface of earth. Find the orbital velocity and time period of revolution. Given; g = 10 ms^{-2}, R = 6370 km**

Solution:

Given that;

Height above the surface of earth (h) = 30 km

$g = 9.8\ ms^{-2}$

Radius of the earth (R) = 6370 km

Radius of the orbit of the satellite (r) = (R + h) = (6370 + 30)
$= 6400$ km $= 6.4 \times 10^6$ m

Orbital velocity (v) = ?
Time period of revolution (T) = ?

We know that;

$$\text{Orbital velocity } (v) = \sqrt{rg}$$
$$= \sqrt{6.4 \times 10^6 \times 9.8} = 7919.56\ ms^{-1}$$
$$= 7.919\ kms^{-1}$$

i.e., the orbital velocity of satellite to move around the earth is 7.919 kms^{-1}.

Now,

$$\text{Time period of a satellite } (T) = 2\pi\sqrt{\frac{r}{g}}$$
$$= 2\pi\sqrt{\frac{6.4 \times 10^6}{9.8}} = 5075 \text{ sec.}$$
$$= 1 \text{ hr. } 24.58 \text{ min.}$$

c) **The density of a substance is 9.82 $gmcm^{-3}$ at 0°C and 8.80 $gmcm^{-3}$ at 80°C. Calculate its coefficient of cubical expansion.**

Solution:

Given that;

Density of a substance at 0°C (ρ_0) = 9.82 $gmcm^{-3}$
Density of a substance at 80°C (ρ_t) = 8.80 $gmcm^{-3}$
Coefficient of cubical expansion (γ) = ?

We know,

$$\frac{\rho_0}{\rho_t} = 1 + \gamma t$$

$$\therefore \quad \gamma = \frac{\rho_0 - \rho_t}{\rho_t t} = \frac{9.82 - 8.80}{8.80 \times 80} = 1.45 \times 10^{-3}\ °C^{-1}$$

Therefore, coefficient of cubical expansion of substance is $1.45 \times 10^{-3}\ °C^{-1}$.

d) **The refractive index of water is $\frac{4}{3}$ and glass is $\frac{3}{2}$. A ray of light travelling through glass is incident on a glass-water surface at angle of 30°. Find the angle of refraction through water.**

Solution:

Given that;

Refractive index of water $({}_a\mu_w) = \frac{4}{3} = 1.33$

Refractive index of glass $({}_a\mu_g) = \frac{3}{2} = 1.5$

Angle of incidence on glass water surface $(i_g) = 30°$

Angle of refraction from glass water surface (i_w) = ?

We have,

$${}_g\mu_w = \frac{{}_a\mu_w}{{}_a\mu_g} = \frac{1.33}{1.5} = 0.88$$

Again, we know,

$$_g\mu_w = \frac{\sin(i_g)}{\sin(r_w)}$$

or, $\sin(r_w) = \frac{\sin(i_g)}{_g\mu_w} = \frac{\sin 30°}{0.88} = 0.568$

or, $r_w = \sin^{-1}(0.568) = 34.62°$

∴ Angle of refraction through water is 34.62°.

e) A source of light and screen are placed 90 cm apart. Where a convex lens of focal length 20 cm should be placed in order to form a real image of source on the screen?

Solution:

Given that;

Distance between screen and object (d) = 90 cm

Focal length (f) = 20

Let real image is formed on the screen at distance x from the lens.

$$\text{Distance } (x) = \frac{d \pm \sqrt{d^2 - 4df}}{2a} = \frac{90 \pm \sqrt{(90)^2 - 4 \times 90 \times 20}}{2 \times 1} = \frac{90 \pm 30}{2}$$

Taking positive; we get,

$x = 60$ cm

and, taking negative; we get,

$x = 30$ cm

Hence, the lens should be placed at distance 60 or 30 cm from the screen.

f) The vertical and horizontal component of the flux density of earth's magnetic field at a place are 0.2×10^{-5} T and 0.34×10^{-5} T respectively. Calculate angle of dip and total flux density at that place.

Solution:

Given that;

Vertical component (V) = 0.2×10^{-5} T

Horizontal component (H) = 0.34×10^{-5} T

Angle of dip (δ) = ?

Total flux density (I) = ?

The angle of dip 'δ' is given by,

$$\tan\delta = \frac{V}{H} = \frac{0.2 \times 10^{-5}}{0.34 \times 10^{-5}} = 0.59$$

∴ $\delta = \tan^{-1}(0.59) = 30.54°$

Again, from $H = I\cos\delta$; we have,

∴ $I = \frac{H}{\cos\delta} = \frac{0.34 \times 10^{-5}}{\cos 30.54°} = 3.95 \times 10^{-6}$ T

g) A bullet of mass 0.5 kg and moving with a velocity 100 ms⁻¹ is stopped within 0.1 m of target. Find the average distance offered by the target.

Solution:

Given that;

Mass of bullet (m) = 0.5 kg

Initial velocity (u) = 100 ms^{-1}

Final velocity (v) = 0

Distance stopped (S) = 0.1 m

Now, from the relation; we have,

$$v^2 = u^2 + 2aS$$

or, $(0)^2 = (100)^2 + 2a \times 0.1 = 10000 + 0.2a$

or, $a = -\frac{10000}{0.2} = -50000 \text{ ms}^{-2}$

Let 'v' be the common velocity and bullet together and then from the principle of conservation of momentum; we have,

Initial momentum of bullet = Final momentum of block and bullet

i.e., $mu = (m + M)v$

or, $v = \frac{mu}{m + M} = \frac{0.02 \times 100}{0.02 + 1} = 1.96 \text{ ms}^{-1}$

Now,

$$v^2 = u^2 + 2aS$$

or, $S = \frac{v^2 - u^2}{2a} = \frac{-(1.96)^2}{2 \times (-50000)} = 0.0006272 \text{ m}$

Hence, the average distance offered by the target 0.63 mm.

EXAMINATION 2061
REGULAR/BACK

Full marks: 60
Pass marks: 24

Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.
Attempt All questions.

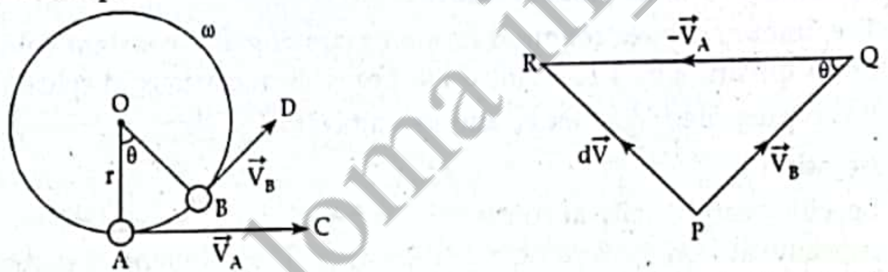
1. **Explain the term centripetal force. Derive an expression of centripetal force in terms of angular velocity when an object moves along the circumference of a circle with uniform speed.**

Ans: **Centripetal force**

According to the Newton's first law of motion, to keep a body in circular motion at every point of motion a constant force directing towards a fixed point must be applied. The force keeps body in circular motion and is called centripetal force. It must act when body is in circular motion. If not according to the first law of motion body moves in a straight path.

Expression of centripetal force

Let a body of mass m moving along a circular path of radius r with uniform speed 'v'. At time 't', the particle is at point 'A' and at time t + dt, let the particle be at point 'B'.



At point 'A', the velocity of the particle acts along tangent AC. Let at the point 'A', $\vec{v}_A$ be the velocity. Similarly at point 'B', $\vec{v}_B$ is the velocity along the tangent BD. As the particle is moving with uniform speed the magnitudes of velocities at 'A' and 'B' is same *i.e.,* $v_A = v_B = v$, however their directions are different *i.e.,* $\vec{v}_A \neq \vec{v}_B$.

Thus, change in velocity occurred from 'A' to 'B' is;

$$\vec{v}_B - \vec{v}_A = \vec{v}_B + (-\vec{v}_A)$$

i.e., $d\vec{v} = PR = \vec{v}_B + (-\vec{v}_A)$

For small interval of time dt, particle covers a small angle dθ, then,

$$\measuredangle PQR = d\theta$$

$$\text{Change in velocity or acceleration (a)} = \frac{\vec{v}_A - \vec{v}_B}{dt} = \frac{PR}{dt} \qquad (1)$$

But, from ΔPQR; we have,

$$\frac{PR}{v} = \sin\theta$$

Since, dθ is very small, taking $\sin d\theta = d\theta$; we have,

$$PR = v \sin d\theta = v\, d\theta$$

From relation (1); we get,

$$\therefore \quad a = \frac{v\, d\theta}{dt}$$

But, we know,

$$\frac{d\theta}{dt} = \omega \qquad \left[\because v = \omega r;\ \omega = \frac{v}{r}\right]$$

$\therefore \quad a = v\omega$

or, $\quad a = \omega^2 r = \frac{v^2}{r}$

For a particle of mass 'm' to move in a circular path the necessary centripetal force is;

$$F = ma = m\omega^2 r = \frac{mv^2}{r}$$

Or

Define simple pendulum. Show that the motion of a simple pendulum is simple harmonic for small angular oscillations.

Ans: See the solution of Q. no. 1 of 2060 Back on page no. 24

2. **Define specific heat capacities of a gas constant pressure and at constant volume and deduce a relation between them.**

Ans: **Specific heat capacity at constant pressure**

The amount of heat required for unit mass of gas at constant pressure to rise its temperature by 1 K is called its specific heat capacity at constant pressure. It is represented by letter c_P and its unit is $JKg^{-1}K^{-1}$.

Specific heat at constant volume

The amount of heat required for unit mass of gas at constant volume to rise its temperature by 1 K is called its specific heat capacity at constant volume. It is represented by letter c_V and its unit is $JKg^{-1}K^{-1}$.

Relation

Specific heat capacity at constant pressure is always greater than specific heat capacity at constant volume *i.e.,* $c_P > c_V$. The difference between them is always equal to amount of work done during expansion at constant pressure.

i.e., $\quad c_P - c_V = dW \qquad (1)$

Consider 1 kg of gas enclosed inside a cylinder fitted with smooth piston as shown in the figure. During heating at constant pressure piston has to be kept free. So, the gas expands and pushes the piston outward. Let change in volume from V_1 to V_2 due to expansion is dV.

i.e., $\quad dV = (V_2 - V_1)$

Thus, amount of work done is given by;

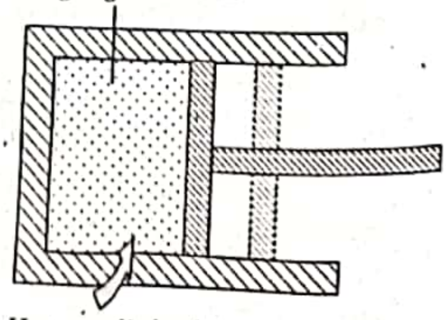
$$dW = P\,dV = P(V_2 - V_1) \qquad (2)$$

From equation (1); we have,

$$c_P - c_V = dW$$

or, $\quad c_P = c_V + P(V_2 - V_1)$

$\qquad = c_V + PV_2 - PV_1$



But, for 1 kg gas; we have,

$$PV = rT$$

so, $\quad PV_1 = rT_1$

and, $\quad PV_2 = rT_2$

As we know,

$$V_2 - V_1 = dV$$

and, $PV_2 - PV_1 = r(T_2 - T_1) = r\,dT$

since, $dT = T_2 - T_1 = 1°C$

Hence,

$$c_P = c_V + r\,dT = c_V + r \times 1 = c_V + r$$

$$\therefore \quad c_P - c_V = r$$

This is the relation between two specific heats of gas at constant pressure and at constant volume.

3. **Explain clearly the term minimum deviation as applied to a prism. Derive an expression connection the refractive index of the material of the prism with minimum deviation.**

Ans: **Minimum deviation to a prism**

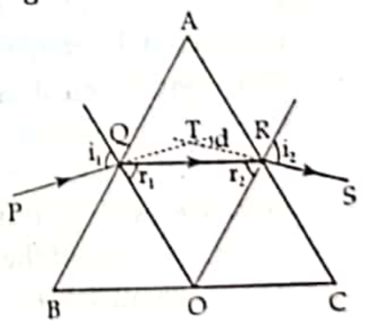
When light ray is made to strike on a prism then ray of light will emerges out as shown in the figure. The angle between incident ray and the emergent ray produces is the angle of deviation. It is represented by the letter 'd'. The deviation from a prism depends on the angles of incidence and emergence. For a prism there is a stationary value for minimum deviation. The deviation becomes minimum when the light ray traverses prism symmetrically. In this condition the angles of incidence and emergence are equal and the angle of deviation produced by a prism becomes minimum and is called angle of minimum deviation. The relation between the minimum deviation δ_m, the angle of the prism 'i', and the refractive index 'n' is given by;

$$n = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$$

Expression for minimum deviation

When light ray passes through a refracting surface of prism the light ray is deviated. The amount of deviation produced by the prism depends upon the angle of prism 'A', the refractive index 'n' and the angle of incidence 'i'.

Suppose ABC is the section of a prism of material having refractive index 'n'. Let a ray of light PQ is incident on the refracting face AB of the prism at point 'Q'. According to the law of refraction, it will refract along QR again suffers refraction at point 'R' on the second refracting surface AC and emerges out from the prism along RS. Produce SR to meet PQ in point 'T' then PQ, QR and RS will be incident ray, refracted ray and emergent ray respectively.



From quadrilateral ARO and AQR; we have,

$$\angle AQO + \angle ARO = 180° \qquad (1)$$

and, $\angle AQR + \angle QAR + \angle ARQ = 180° \qquad (2)$

Subtracting equation (2) from (1); we get,

$$(\angle AQO - \angle AQR) + (\angle ARO - \angle ARQ) - \angle QAR = 0$$

or, $\angle OQR - \angle ORQ = \angle QAR$

or, $r_1 + r_2 = A \qquad (3)$

But, $d = x + x' = (i_1 - r_1) + (i_2 - r_2) = i_1 + i_2 - (r_1 + r_2) = i_1 + i_2 - A \qquad (4)$

But, for deviation 'd' to be minimum,

$i_1 = i_2$

and, $r_1 = r_2$

Hence, from equation (3), putting $r_1 = r_2 = r$; we get,

$A = 2r$

or, $r = \frac{A}{2}$

Again, from equation (4); we get,

$2i = \delta_m + A$ [taking $d = \delta_m$ for minimum deviation]

or, $i = \frac{\delta_m + A}{2}$

$\therefore$ Refractive index (n) $= \frac{\sin i}{\sin r}$

$$\therefore \quad n = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}}$$

This is the required expression.

4. Attempt any six questions:

a) **Prove that total mechanical energy of a falling body remains constant.**

Ans: See the solution of Q. no. 4 (a) of 2060 R/B on page no. 15

b) **Define angular momentum and derive its expression for a rigid body.**

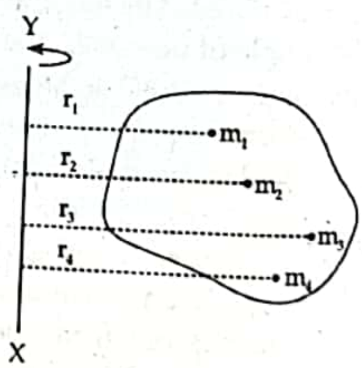
Ans: **Angular momentum**

The sum of the moments of momentum of all the particles of a rotating rigid body about the axis of rotation is known as angular momentum. Numerically, it is measured by product of moment of inertia 'I' and angular velocity ω. It is generally represented by 'L'.

i.e., Angular momentum (L) = Iω

Expression for angular momentum of a rigid body

Let a rigid body rotates about an axis XY with an angular velocity ω. The body consists of small particles at different distances as shown.



The particles on it also have the same angular velocity ω, but as particles are at different distances from the axis of rotation, their linear velocities will be different. Let us suppose the linear velocities of the particles of masses m_1, m_2, m_3,……… at distances r_1, r_2, r_3, ……… from the axis of rotation are v_1, v_2, v_3, ……… etc. Then,

Angular momentum of a particle (L) = mvr = $m\omega r^2$

The total angular moments of the particles is the sum of angular moments of all particles and is given by;

$$\text{Total angular momentum (L)} = m_1\omega r_1^2 + m_2\omega r_2^2 + m_3\omega r_3^2 + \ldots\ldots\ldots$$
$$= \omega\left(\sum mr^2\right)$$

where, $I = \sum mr^2$ and it is the moment of inertia of a body about the axis XY.

$L = I\omega$

Angular momentum is a vector quantity. This is the expression for angular momentum of a rigid body.

c) **State Newton's laws of motion and show that second law of motion gives the measurement of force.**

Ans: Newton's laws of motion are stated as;

First law

An object at rest always tends to remain at rest and an object in motion always tends to remain in motion with constant speed along the same direction until acted upon by an external agent (force).

Second Law

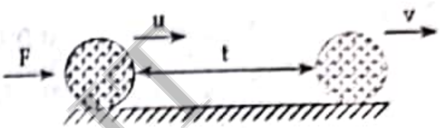
The rate of change of momentum of a body is directly proportional to the impressed force and takes place along the direction of the impressed force.

Third law

For every action, there is always equal and opposite reaction.

Second law of motion gives the measurement of force

Let a ball of mass 'm' is moving initially with velocity 'u' is acted by a force 'F', so that it is accelerated by 'a'. After time 't', the velocity of ball increases to 'v'. Then,



Initial momentum of the ball = mu

Final momentum of the ball = mv

∴ Change in momentum of the ball = mv − mu

The rate of change of momentum of the ball = $\frac{mv - mu}{t}$

According to the second law of motion; we have,

$$\text{Applied force (F)} \propto \frac{mv - mu}{t}$$

or, $$F = K\frac{mv - mu}{t} = Km\frac{v - u}{t} = Kma \quad (1)$$

Here, K is a constant and the value of the constant depends on the unit of force (F) chosen.

If, m = 1 unit [*i.e.*, 1 kg]

and, a = 1 unit [*i.e.*, 1 ms^{-2}]

Then,

F = 1 unit

Hence,

$$\text{Value of K} = \frac{F}{ma} = \frac{1}{1 \times 1} = 1$$

Therefore, the relation (1) becomes;

$$F = ma$$

This is the relation for the measurement of force. In this way, second law of motion gives measurement of force.

d) **Define coefficient of linear and superficial expansion of a solid and deduce a relation between them.**

Ans: **Coefficient of linear expansion**

It is defined as the increase in length per unit length per unit degree rise in temperature of the substance. It is represented by the symbol α and the unit is $°C^{-1}$ or K^{-1}.

i.e., $\alpha = \dfrac{\text{Increase in length}}{\text{Original length} \times \text{Rise in temperature}} = \dfrac{l_t - l_0}{l_0(t - t_0)}$

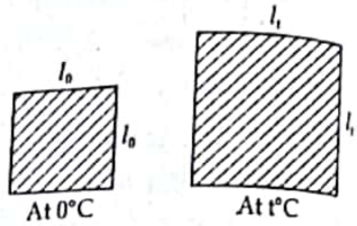
Coefficient of superficial expansion

It is defined as the increase in area per unit area per unit degree rise in temperature of the substance. It is represented by the symbol β and the unit of β is $°C^{-1}$ or K^{-1}.

i.e., $\beta = \dfrac{\text{Increase in area}}{\text{Original area} \times \text{Rise in temperature}} = \dfrac{A_t - A_0}{A_0(t - t_0)}$

Relation between α and β

Consider a square having sides l_0 at 0°C. Let the sides of the square increases to l_t when the temperature increases to t°C. Then, we get,



$$l_t = l_0(1 + \alpha t)$$

Also,

Area of square at 0°C $(A_0) = l_0^2$

and, Area of square at t°C $(A_t) = l_t^2 = l_0^2(1 + \alpha t)^2 = A_0(1 + 2\alpha t + \alpha^2 t^2)$

Since, the value of α is very small, so, $\alpha^2 t^2$ will be very small. Thus, the terms containing $\alpha^2 t^2$ is neglected.

$$\therefore \quad A_t = A_0(1 + 2\alpha t) \qquad (1)$$

But, from definition of the coefficient of superficial expansion; we have,

$$A_t = A_0(1 + \beta t) \qquad (2)$$

Comparing the relations (1) and (2); we get,

$$\beta = 2\alpha$$

Or

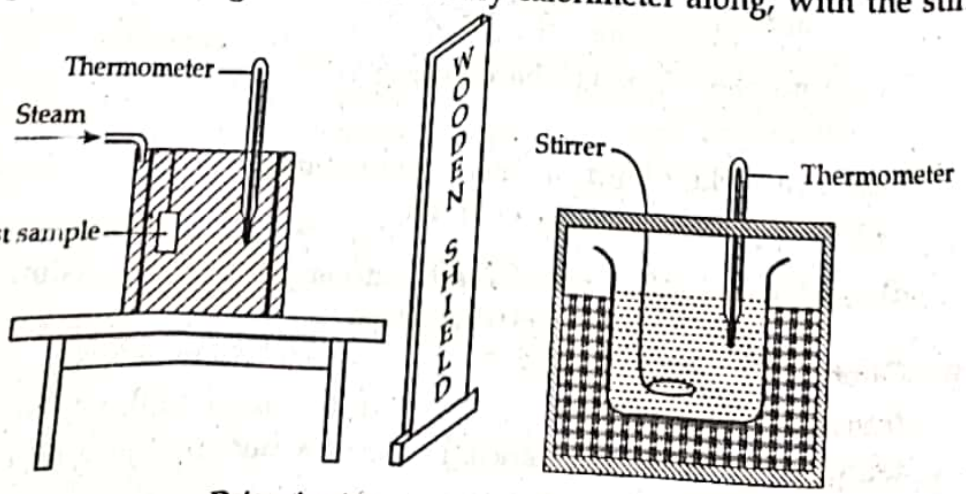
Define specific heat capacity of a substance. Describe the method of mixture to measure the specific heat capacity of a solid.

Ans: Specific heat capacity

It is defined as the amount of heat required to raise the temperature of unit mass of the substance by one degree. It is generally represented by 's'. The unit of specific heat capacity is $Jkg^{-1}K^{-1}$.

To determine specific heat of solid

First a solid for specific heat is to be determined is weighed in air. It is then hung inside a steam chamber and is heated by passing steam as shown in the figure. Then, weight a clean and dry calorimeter along, with the stirrer



Determination of specific heat of a solid

and again with ordinary cold water. It is then placed in wooden box and noted the temperature, then kept below the steam chamber.

Steam from the boiler is made to enter the jacket and heats the solid inside chamber. The temperature of the solid rises and finally attains a steady temperature. Solid is then quickly transferred into the calorimeter and the water in it is stirred continuously till the mixture attains a common temperature. Then, the calculations are made as follows;

Let, Mass of the calorimeter + stirrer = m_c

Mass of the water taken = m_w

Mass of the solid = m_s

Initial temperature of calorimeter + water = t_1

Initial temperature of hot solid = t_2

Final temperature of mixture = t

Specific heat of solid (s) = ?

Specific heat of water (s_w) = ?

Specific heat of calorimeter and stirrer = s_c

Now,

Heat lost by solid = Mass of solid × Specific heat of solid × decrease in temperature

$= m_s s(t_2 - t)$

Heat gained by calorimeter = $m_c s_c(t - t_1)$

Heat gained by water = $m_w s_w(t - t_1)$

Since; we have,

Heat lost by solid = Heat gained by calorimeter + Heat gained by water

or, $m_s s(t_2 - t) = m_c s_c(t - t_1) + m_w s_w(t - t_1)$

$$\therefore \quad s = \frac{m_c s_c(t - t_1) + m_w s_w(t - t_1)}{m_s s(t_2 - t)}$$

From the relation, specific heat capacity 's' of the solid is calculated.

e) Using pressure equations of a gas prove that average kinetic energy of a gas molecule is $\frac{3}{2}$KT where, K is Boltzmann constant.

Ans: See the solution of Q. no. 2 of 2060 Back on page no. 26

f) Derive a relation $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ for a concave mirror.

Ans: See the solution of Q. no. 4 (f) of 2060 Back on page no. 31

g) Explain the domain theory of ferromagnetism.

Ans: See the solution of Q. no. 4 (f) of 2060 R/B on page no. 18

5. Solve any six questions.

a) A body is sliding down a rough inclined plane which makes an angle of 30° with the horizontal. Calculate the acceleration. Coefficient of friction is 0.25, g = 9.8 ms^{-2}

Solution:

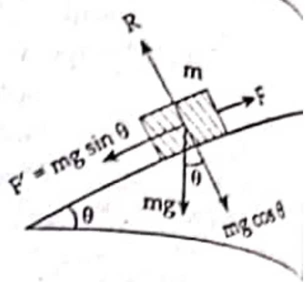
Given that;

Angle with the horizontal (θ) = 30°

Acceleration (a) = ?

Coefficient of friction (μ) = 0.25

$g = 9.8\ ms^{-2}$



In this case, the component of weight parallel to slope moves object down the slope.

$F' = mg \sin \theta$

The frictional force that opposes the motion of body is given by;

$f = \mu mg \cos \theta$

The resultant force that makes the body to slide on the plane is;

$F = F' - f = mg \sin \theta - \mu mg \cos \theta = mg(\sin \theta - \mu \cos \theta)$

or, $ma = mg(\sin \theta - \mu \cos \theta)$

$\therefore$ $a = g(\sin \theta - \mu \cos \theta) = 9.8(\sin 30° - 0.25 \times \cos 30°) = 2.78\ ms^{-2}$

b) An arrow of 100 gram mass is shot into a block of wood of mass 400 gram lying at rest on the smooth surface. If the arrow is traveling horizontally at the velocity 15 ms^{-1} before impact, calculate common velocity after impact.

Solution:

Given that;

Mass of arrow (m) = 100 g = 0.1 kg

Velocity of arrow (u) = 15 ms^{-1}

Mass of wooden block (M) = 400 g = 0.4 kg

Common velocity (V) = ?

From the principle of conservation of momentum; we have,

Initial total momentum = Total final momentum

or, $mu = (m + M)V$

or, $V = \dfrac{mu}{m + M} = \dfrac{0.1 \times 15}{0.1 + 0.4} = 3\ ms^{-1}$

Therefore, common velocity after impact is 3 ms^{-1}.

c) What is the result of mixing of 20 gram of water at 90°C with 10 gram of ice at −10°C? Given that; Specific heat of ice = 2100 Jkg^{-1}°C^{-1}, Latent heat of fusion of ice = 3.36×10^5 Jkg^{-1}

Solution:

Given that;

Mass of water taken (m_w) = 20 gm = 0.02 kg

Mass of ice (m_i) = 10 gm = 0.01 kg

Initial temperature of water (t_1) = 90°C

Initial temperature of ice (t_2) = −10°C

Temperature of the mixture = t (say)

Specific heat capacity of ice (s_i) = 2100 Jkg^{-1}°C^{-1}

Latent heat of ice (L) = 3.36×10^5 Jkg^{-1}

We have,

Total heat gain by ice at −10°C to become water at 0°C is;

$= m_i s_i \times 10 + m_i L = 0.01 \times 2100 \times 10 + 0.01 \times 3.36 \times 10^5 = 3570\ J$

Total heat gained by ice at −10°C to become water at t°C = Total heat lost by water at 90°C to become water at t°C or, $3570 + m_i s_w \times t = m_w s_w \times (90 - t)$

or, $3570 + 0.01 \times 4200 \times t = 0.02 \times 4200 \times (90 - t)$

or, $3570 + 42\,t = 7560 - 84\,t$

or, $126\,t = 3990$

or, $t = 31.7°C$

Hence, the temperature of the mixture is 31.7°C.

d) A diverging lens of 12 cm focal length produces a virtual image which linear dimensions are $\frac{1}{3}$ that of the object. Determine the position of object and image.

Solution:

Given that;

Focal length (f) = −12 cm

Magnification (m) = $-\frac{1}{3}$

We have,

$$m = \frac{v}{u}$$

or, $-\frac{1}{3} = \frac{v}{u}$

or, $v = -\frac{u}{3}$ (1)

Again, we have,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or, $-\frac{1}{12} = \frac{1}{u} - \frac{3}{u} = \frac{1-3}{u} = -\frac{2}{u}$

or, $u = 24$

Substituting the value of 'u' in equation (1); we get,

$$v = -\frac{u}{3} = -\frac{24}{3} = -8 \text{ cm}$$

Hence, required position of object and image are 24 cm and 8 cm from lens.

e) A bar magnet 20 cm long is placed in the magnetic meridian with its north pole pointing south. The neutral point is observed at a distance of 30 cm from one of its pole. Calculate the pole strength of magnet. [$H = 0.32 \times 10^{-4}$ T, $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}]

Solution:

Given that;

$2l = 20$ cm $= 0.2$ m

or, $l = 0.1$ m

$d = 0.3$ m $+ 0.1$ m $= 0.4$ m

$H = 0.32 \times 10^{-4}$ T

$\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}

When north pole points south, the neutral points are found on the axial line, then,

$$B = H = \frac{\mu_0}{4\pi} \times \frac{2Md}{(d^2 - l^2)^2}$$

or, $M = \frac{4\pi H(d^2 - l^2)^2}{2\mu_0 d} = \frac{4\pi \times 0.32 \times 10^{-4}[(0.4)^2 - (0.1)^2]^2}{2 \times 4\pi \times 10^{-7} \times 0.4} = 9\ Am^2$

Now,

$$\text{Pole strength} = \frac{M}{2l} = \frac{9}{2 \times 0.1} = 45\ Am$$

f) **The angle of dip at a place is 30°. Find the total intensity and vertical component. Given; $H = 3.2 \times 10^{-6}$ T**

Solution:

Given that;

Angle of dip (δ) = 30°

$H = 3.2 \times 10^{-6}$ T

Total Intensity (I) = ?

Vertical component (V) = ?

We have,

$$H = I \cos \delta$$

or, $I = \frac{H}{\cos \delta} = \frac{3.2 \times 10^{-6}}{\cos 30°} = 3.69 \times 10^{-6}$ T

Again, we know,

$$V = I \sin \delta = 3.69 \times 10^{-6} \times \sin 30° = 1.845 \times 10^{-6}\ T$$

g) **The refractive index of diamond is 2.47. Calculate the speed of light in diamond.**

Solution:

Given that;

Refractive index of diamond (μ) = 2.47

Speed of light in vacuum (c) = $3 \times 10^8\ ms^{-1}$

Speed of light in diamond (c_1) = ?

We have,

$$\text{Refractive index } (\mu) = \frac{c}{c_1}$$

or, $2.47 = \frac{3 \times 10^8}{c_1}$

or, $c_1 = 1.21 \times 10^8\ ms^{-1}$

Therefore, the speed of light in diamond is $1.21 \times 10^8\ ms^{-1}$.

EXAMINATION 2061
BACK

Time: 3 hrs.

Full marks: 60
Pass marks: 24

Candidates are required to give their answers in their own words as far as practicable.
Attempt All questions.

1. **Answer in brief any three:**

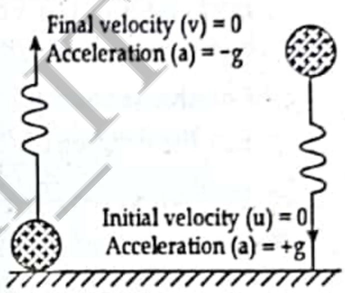
a) **Write the dimension of gravitation constant and pressure.**

Ans: The dimension of gravitational constant G is $[M^{-1}L^{3}T^{-2}]$

The dimension of pressure P is $[ML^{-1}T^{-2}]$

b) **Is it possible that the acceleration is not zero but the velocity zero? Give two examples.**

Ans: Yes, it is possible. Two examples of motion in which acceleration is non-zero and velocity is zero are the motion of particles in vertical direction.



First Motion

When a body is thrown vertically upward direction then the velocity of the body goes on increasing *i.e.,* accelerated. It continues until the body reaches the maximum height. When the body reaches the maximum height the velocity of the body becomes zero, but the acceleration is not zero and equal to the acceleration due to gravity *i.e.,* −g.

Second Motion

Similarly, the motion of a body when dropped from the height at initially the velocity of the body will be zero, but the acceleration of the body will be equal to +g.

c) **State and explain conservation law of energy.**

Ans: The principle of conservation of energy states that energy can neither be created nor be destroyed but can transform from one form to another form, hence total energy of a system always remains constant *i.e.,* conserved. *For example;* An electric generator used in a shop and different electric devices form a system. Then, suppose the electrical energy generated from the generator is 1000 joules of electrical energy.

Let the electrical devices consumes 850 joules of energy converts to heat and light. Similarly 140 joules of energy is converted to mechanical energy, 10 joules to sound energy. Hence,

Total electrical energy generated = 1000 Joules
Converted to heat and light energy = 850 joules
Converted to mechanical energy = 140 Joules
Converted to sound energy = 10 Joules

Hence,

Total electrical energy 1000 J = 850 J of heat and light + 140 J of mechanical energy + 10 Joules of sound energy = 1000 Joules

i.e., the total electrical energy remains constant.

This is according to the principle of conservation of energy. Hence, for a system the principle of conservation of energy is true.

d) **State the parallelogram law of vector.**

Ans: The parallelogram law of vector states that, "if two vectors acting simultaneously at the same point of a body can be represented in magnitude and direction by two adjacent sides of a parallelogram drawn from a point then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through the common point of intersection."

2. **Define S.H.M. show that the motion of simple pendulum is in simple harmonic. Hence, derive its time period.**

Ans: See the solution of Q. no. 1 of 2060 Back on page no. 24

Or

Explain what is meant by angular velocity. Show that acceleration of a body moving in circular path of radius 'r' with uniform angular velocity 'ω' is $\omega^2 r$

Ans: Angular velocity

The change in angle at the center of the circle of a circular path per unit time is called angular velocity of a particle. It is expressed in radian per second and denoted by symbol 'ω'.

For the second part

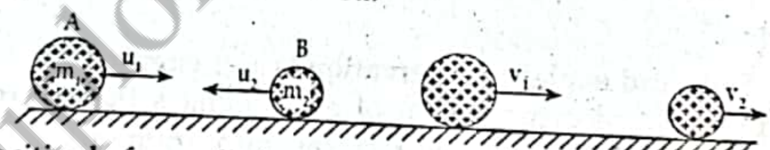
See the solution of Q. no. 1 of 2061 R/B on page no. 37

3. a) **State and prove the principle of conservation of linear momentum.**

Ans: The principle of conservation of linear momentum states that, "if no external force act the initial momentum on a colliding system of bodies is equal to the final momentum".

Verification of the principle from the laws of motion

Let two bodies 'A' and 'B' of masses m_1 and m_2 moving in a same straight path from opposite directions collided.



If initial velocities before collision are u_1 and u_2, suppose after collision they move in the same directions with the velocities v_1 and v_2. Therefore,

Initial momentum of A = m_1u_1

Final momentum of A = m_1v_1

Initial momentum of B = $m_2(-u_2) = -m_2u_2$

Final momentum of B = m_2v_2

∴ Change in momentum of A = $m_1v_1 - m_1u_1$

Change in momentum of B = $m_2v_2 - (-m_2u_2)$

$= m_2v_2 + m_2u_2$

According to the third law of motion, the force exerted by 'A' on 'B' is equal and opposite to the reaction exerted by 'B' on 'A'. Again, the time 't' during which the force acted on 'B' is equal to the time for which the reaction acted on A. Hence, impulse $F \cdot t$ on 'B' is equal and opposite to the magnitude of the impulse $R \cdot t$ on 'A'.

i.e., $F \cdot t = -R \cdot t$ (1)

From Newton's second law of motion; we have,

$F \cdot t$ = Change in momentum

For particle A;

$$-R \cdot t = -(m_1v_1 - m_1u_1) = m_1u_1 - m_1v_1$$

For particle B;

$$F \cdot t = m_2v_2 + m_2u_2$$

Substituting the values of $R \cdot t$ and $F \cdot t$ in equation (1); we get,

$$m_2v_2 + m_2u_2 = m_1u_1 - m_1v_1$$

or, $m_1u_1 - m_2u_2 = m_1v_1 + m_2v_2$

i.e., Total initial momentum = Total final momentum

This is according to the principle of conservation of linear momentum. Hence, principle is verified and followed according to laws of motion.

b) A lift moves up and down with an acceleration of $2\ ms^{-2}$ in each case. Calculate reaction of the floor on a man of mass 50 kg standing in the lift.

Solution:

Given that;

Acceleration (a) = $2\ ms^{-2}$

Mass of the man (m) = 50 kg

$g = 9.8\ ms^{-2}$

i) When lift moves up with acceleration the reaction on the floor is;

$$W_1 = m(g + a) = 50(9.8 + 2) = 590\ N$$

The reaction of the floor on a man is 590 N.

ii) When lift moves down with acceleration the reaction on the floor is;

$$W_2 = m(g - a) = 50(9.8 - 2) = 390\ N$$

The reaction of the floor on a man is 390 N.

4. Answer in brief any three.

a) State the Newton's law of cooling.

Ans: Newton's Law of cooling states that, "the rate of cooling of body is directly proportional to the temperature difference between the body and its surrounding provided that the temperature difference is small."

Newton stated the cooling law on the basic of experimental results with hot liquid contained in a vessel. When hot liquid is allowed to cool maintaining the temperature difference between the hot liquid and its surrounding small, the loss of heat is due to convection. Thus the law obeys when the convection plays the main role and that is possible only when the temperature difference is 30 - 35°C. If a liquid at temperature 't' losses a small amount of heat dQ in small interval of time dt and to be the temperature of surroundings. Then, according to the law,

$$\frac{dQ}{dt} \propto -(t - t_0)$$

The negative sign shows that heat given out as time passes.

$$\frac{dQ}{dt} = K(t - t_0) \qquad (1)$$

where, K is a constant whose value depends upon the nature and extent (*i.e.,* area) of the radiating surface.

If 'm' and 's' be mass and specific heat of the body; then, Rate of heat lost $\left(\frac{dQ}{dt}\right) = ms\frac{dt}{dt}$

From equation (1) and (2); we have,

$$ms\frac{dt}{dt} = -K(t - t_0)$$

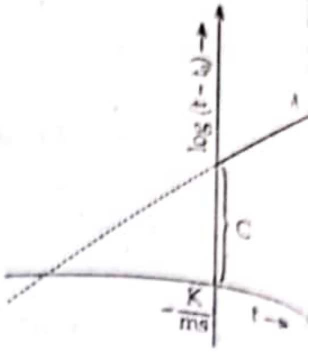
or, $\frac{dt}{dt} = -\frac{K}{ms}(t - t_0)$

or, $\frac{dt}{t - t_0} = -\frac{K}{ms}dt$

Integrating both sides; we get,

$$\int \frac{dt}{t - t_0} = -\frac{K}{ms}\int dt$$

or, $\log(t - t_0) = -\frac{K}{ms}t + c$



where, c is constant of integration.

This is an equation of form y = mx + c and represents straight line. The graph between log $(t - t_0)$ and 't' is shown in the figure.

The result shows that Newton's law of cooling is true.

b) **Can a body be heated without rising its temperature?**

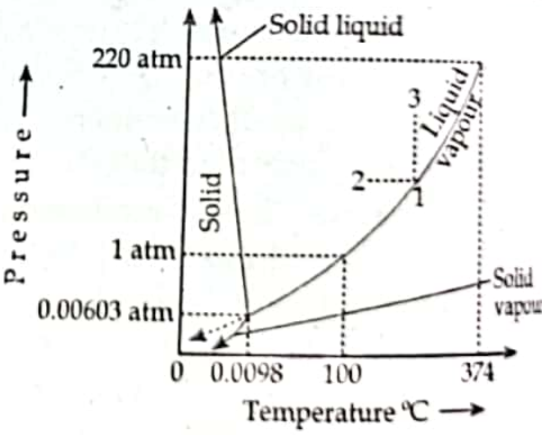
Ans: Yes, for instance if ice at 0°C is heated during its state change its temperature remains unchanged similarly water at 100°C can be heated without increasing its temperature.

c) **State the factors on which the thermal conduction depends.**

Ans: See the solution of Q. no. 4 (e) of 2060 Back on page no. 31

d) **Define triple point of water.**

Ans: The temperature and pressure at which the water can coexist in all three phases (solid, liquid and gas) in equilibrium is called triple point of water. For water there is only one pressure and temperature at which all three states can coexist. Triple point is represented by a diagram called phase diagram as shown in the figure.



The pressure and temperature corresponding to triple point of water is 4.56 mm of mercury and 0.0098°C respectively.

5. a) ***Explain why the specific heat capacity of a gas at constant pressure is greater than at constant volume. Prove that $C_P - C_V = R$ for one mole of gas.***

Ans: *During constant pressure piston is to leave free and pushes piston out doing work but at constant piston must not move. Hence at constant pressure heat supplied has to rise temperature by 1°C and external work. Hence, C_P is always greater than C_V.*

Proof

See the solution of Q. no. 2 of 2061 R/B on page no. 38

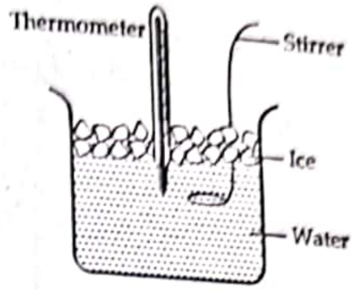
Or

Define latent heat of fusion of ice. How can you determine it by the method of mixture?

Ans: **Latent heat of fusion of ice**

The amount of heat required to convert unit mass of ice to water state without changing temperature is called latent heat of fusion of ice.

Determination by the method of mixtures



Take a dry and clean calorimeter and weight it empty. Fill it partly with water and weight again. The difference between these weights gives the weight of water taken let it is m_w. Keep a thermometer in it and record the initial temperature. Take small pieces of ice and soak away all water from it by means of blotting paper. Drop the ice into the calorimeter and stir the water until the whole of the ice is melted. The process is to be continuing as the temperature dropped about 10°C below the initial temperature of water. Then, note the lowest temperature reached and the temperature of the mixture. Finally, remove the thermometer from the calorimeter and weight to find mass of ice added.

Calculation

Let, Mass of the calorimeter + content = m_c

Mass of the water taken = m_w

Initial temperature of water and calorimeter = t_1°C

Initial temperature of ice and calorimeter = 0°C

Final temperature of the mixture = t_2°C

Mass of the ice added = m_i

From the principal of calorimetry; we have,

Heat gained by ice = heat used in melting the ice + heat used in warming melted ice from 0°C to t_2°C

$$= m_iL + m_is_w(t_2 - 0)$$

where, L is the latent heat of fusion of ice.

s_w is the specific heat of water.

Similarly,

Heat lost by calorimeter + water = $m_cs_c(t_1 - t_2) + m_ws_w(t_1 - t_2)$

Since; we have,

Heat gained by ice = heat lost by water and calorimeter

or, $m_iL + m_is_w(t_2 - 0) = m_cs_c(t_1 - t_2) + m_ws_w(t_1 - t_2)$

or, $m_iL = m_cs_c(t_1 - t_2) + m_ws_w(t_1 - t_2) - m_is_wt_2$

or, $L = \dfrac{m_cs_c(t_1 - t_2) + m_ws_w(t_1 - t_2) - m_is_wt_2}{m_i}$

or, $L = \dfrac{(t_1 - t_2)(m_cs_c + m_ws_w)}{m_i} - s_wt_2$

If thermal heat capacity of the calorimeter is "C" then,

$C = m_cs_c$

$\therefore$ Latent heat of ice (L) $= \dfrac{(t_1 - t_2)(C + m_ws_w)}{m_i} - s_wt_2$

Thus, knowing all the terms on the right hand side of the expression, value of 'L', the latent heat of ice can be determined.

b) Assuming the density of nitrogen at S.T.P. is to be 1.25 kgm^{-3}. Find the root mean square velocity of nitrogen molecules at 127°C.

Solution:

Given that;

Density of nitrogen (ρ) = 1.25 kgm^{-3}

P_0 = 760 mm of Hg = $760 \times 10^{-3} \times 13.6 \times 10^3 \times 9.8 = 101292.8$ Nm^{-2}

T_0 = 0°C = 273 K

T = 127°C = 400 K

r.m.s. velocity of nitrogen (c) = ?

From gas equation; we have,

$$P_0V_0 = RT_0$$

or, $$R = \frac{P_0V_0}{T_0} \quad (1)$$

But we know,

Molecular weight of nitrogen (M) = 28×10^{-3} kg

Volume of 1 mole of nitrogen at S.T.P. (V_0) $= \frac{M}{\rho} = \frac{28 \times 10^{-3}}{1.25}$

$= 22.4 \times 10^{-3}$ m^3

Substituting the value in equation (1); we get,

$$R = \frac{P_0V_0}{T_0} = \frac{101292.8 \times 22.4 \times 10^{-3}}{273} = 8.31 \text{ Jmol}^{-1}\text{K}^{-1}$$

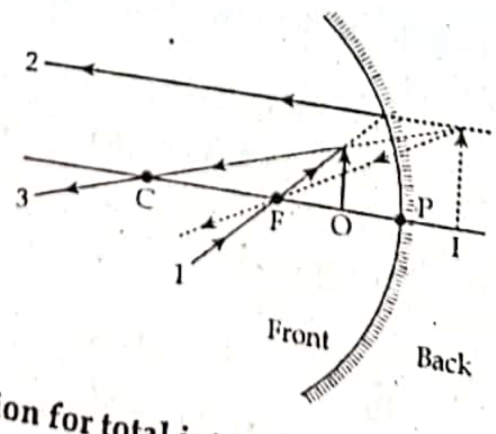
$\therefore$ r.m.s. velocity (c) $= \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 400}{28 \times 10^{-3}}} = 596.77$ ms^{-1}

$\therefore$ Required r.m.s. velocity of Nitrogen at 127°C is 596.77 ms^{-1}.

6. Answer in brief any two.

a) With a ray diagram draw the formation of virtual image in concave mirror.

Ans: A virtual image is formed whenever the object is inside the focus and lies always behind the mirror and erect in nature. The ray diagram for virtual image due to a concave mirror is shown in the figure. The image is virtual, erect and magnified in size.



b) What is the condition for total internal reflection?

Ans: The conditions for total internal reflection are as follows;

i) Light must passes from denser medium to rarer medium.

ii) The angle of incidence must be greater than the critical angle ... the mediums.

c) **A convex mirror is used as a rear view mirror in automobiles. Why?**

Ans: For rear view to a driver in an automobile requires a wide field of view even diminished image. That is possible from a convex mirror. Hence, convex mirror is used as a rear view mirror in automobiles.

7. a) **What is meant by power of lens?**

Derive lens maker's formula $\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$, where the symbols have their usual meanings.

Ans: Power of lens

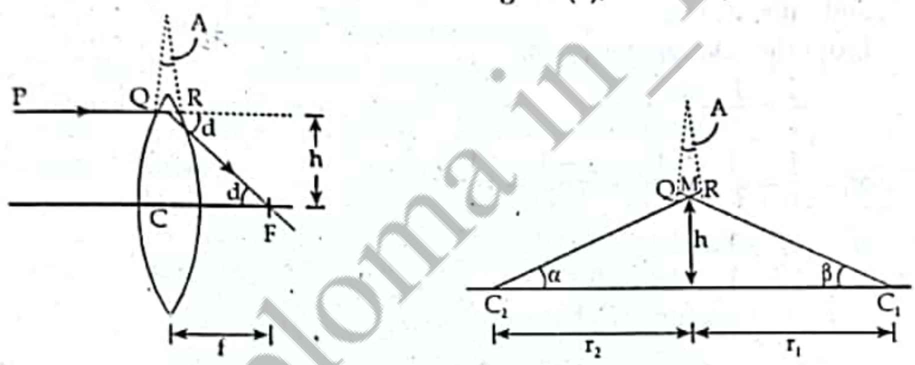
The reciprocal of focal length of a lens is called power. Mathematically;

$$\text{Power (P)} = \frac{1}{f}$$

If 'f' is in meter, the unit of power is called dioptre and represented by 'D'.

Lens maker's formula

Let us consider the lens as portion of a small angled prism as shown in the figure. If a ray PQ parallel to the principle axis strikes the lens at 'Q', and is refracted to the focus 'F'. If 'h' is the height of 'Q' from centre 'O' of the lens. If 'd' is deviation produced, then, from the figure (a); we have,



(a) (b)

$$\tan d = \frac{h}{f}$$

Since, lens is thin, so,

$$\tan d = d$$

$$\text{or,} \quad d = \frac{h}{f} \qquad (1)$$

We know for small angle prism the deviation produced is;

$$d = A(n-1) \qquad (2)$$

where, A is the angle of prism.

From equation (1) and (2); we get

$$\frac{h}{f} = A(n-1) \qquad (3)$$

Let, C_1Q and C_2R be the normal on 'Q' and 'R' as shown in the figure (b) on two faces of the lens. Then; we get,

$$\angle QMC_2 = A = \alpha + \beta = \frac{h}{r_1} + \frac{h}{r_2}$$

Substituting the value of A in the relation (3); we get,

$$\frac{h}{f} = \left(\frac{h}{r_1} + \frac{h}{r_2}\right)(n-1)$$

or, $\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

This is the lens maker's formula.

Or

Deduce the relation $\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$, where symbols have their usual meaning.

Ans: *See the solution of Q. no. 3 of 2061 R/B on page no. 39*

b) What is the power of the spectacles required by a hyperopic eye whose near point is 125 cm?

Solution:

Given that;

Near point (D) = 125 cm

Power of the spectacles (P) = ?

An object at a distance 25 cm must forms image at near point (*i.e.,* 125 cm) to get the correct vision.

∴ $v = -125$ cm

and, $u = 25$ cm

From the relation; we have,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or, $\frac{1}{f} = \frac{1}{25} - \frac{1}{125} = \frac{5-1}{125} = \frac{4}{125}$

or, $4f = 125$

∴ $f = 31.25 \text{ cm} = 0.3125 \text{ m}$

Using the relation; we have,

$$\text{Power (P)} = \frac{1}{0.3125} = 3.2 \text{ dioptre}$$

8. a) Define magnetic susceptibility. What do you mean by dia, para and ferromagnetism? Distinguish them.

Ans: Magnetic susceptibility of a magnetic material is defined as the ratio of intensity of magnetization, I induced in the material to the magnetizing field, H in which the material is placed.

i.e., $\chi = \frac{I}{H}$

Since, 'I' and 'H' are of the same dimensions, χ is dimensionless. On the basis of the behaviour of materials in a magnetizing field, the materials are generally classified into;

i) Diamagnetic substance

ii) Paramagnetic substance

iii) Ferromagnetic substance

The nature of behaviour is correspondingly called dia, para and ferromagnetism.

i) Diamagnetic Substance

The substances which when placed in a magnetic field if are feebly magnetized in the direction opposite to that of the magnetic field are classified as diamagnetic substances. *For example;* copper, silver, gold, water, helium, argon, etc.

ii) Paramagnetic Substance

The substances which when placed in a magnetic field if are feebly magnetized in the direction of the magnetizing field, then, the substances are classified as paramagnetic substances. *For example;* platinum, potassium, aluminium, oxygen, etc.

iii) Ferromagnetic Substance

Those substances which when placed in a magnetic field are strongly magnetized in the direction of the magnetizing field are classified as ferromagnetic substances. *For example;* iron, nickel, cobalt, alloy alnico, etc.

The main differences among dia, para and ferro magnetism are given below;

Dia magnetism	Para magnetism	Ferro magnetism
1. The net magnetic moment of atoms is zero.	Each atom or molecule has a net non-zero magnetic moment.	Each atom or molecule has a strong spontaneous net magnetic moment.
2. The susceptibility has a low negative value and is independent of temperature.	Susceptibility has a low positive value and is inversely proportional to absolute temperature.	The susceptibility is very large and is inversely proportional to the absolute temperature.
3. The relative permeability is slightly less than one. In a non uniform magnetic field they have a tendency to move away from the field.	The relative permeability is greater than one. When placed in a non uniform magnetic field, they have a tendency to move from weaker part to stronger part of field.	The relative permeability is very large. When suspended freely in uniform magnetic field, they set themselves parallel to the direction of magnetic field.
4. When suspended in a uniform magnetic field, they set themselves perpendicular to the direction of magnetic field.	When suspended in a uniform magnetic field, they set themselves parallel to the direction of magnetic field.	When placed in a non uniform magnetic field, they have a tendency to move from the weaker part to the stronger part of the field.
5. *For example;* Bi, Sb, Cu, Au, Hg, H_2O, H_2, etc.	*For example;* Al, Pt, Cr, O_2, Mn, $CuSO_4$, etc.	*For example;* Fe, Ni, Co and a number of their alloys.

Or

Obtain an expression for the intensity of the field due to a bar magnet at a point on its axial line.

Ans: See the solution of Q. no. 4 (g) of 2060 R/B on page no. 18

b) The total magnetic intensity at a place is 0.4 Oersted and the angle of dip is 30°. Calculate the horizontal and the vertical components.

Solution:

Given that;

Total magnetic Intensity (I) = 0.4 Oersted

Angle of dip (δ) = 30°

Vertical component (V) = ?

Horizontal component (H) = ?

The vertical component 'V' is given by,

$$V = I\sin\delta = 0.4 \times \sin 30° = 0.2 \text{ Oersted}$$

The horizontal component 'H' is given by,

$$H = I\cos\delta = 0.4 \times \cos 30° = 0.3464 \text{ Oersted}$$

Dividing both sides by 't'; we get,

$$\frac{s}{t} = r\frac{\theta}{t}$$

$\therefore \quad v = \omega r$

i.e., linear velocity is equal to the product of angular velocity and radius of the circular path.

c) State and explain the first law of thermodynamics.

Ans: See the solution of Q. no. 4 (d) of 2060 Back on page no. 30

d) Define dia, para and ferromagnetic materials.

Ans: See the solution of Q. no. 8 (a) of 2061 Back on page no. 54

e) Define luminous intensity and write its unit.

Ans: The luminous flux density per solid angle as measured in a given direction relative to the emitting source is called luminous intensity. The S.I. unit of luminous intensity is candela.

f) What are saturated and unsaturated vapours?

Ans: The maximum pressure that a vapour can exert at a given temperature is called the saturated vapour pressure. In short saturated vapour pressure is written as S.V.P. The pressure exerted by a vapour at any temperature is less than the maximum it can exert at that temperature and is called unsaturated vapour pressure. The saturated vapour pressure does not depend on the volume, but unsaturated vapour changes with the volume of the vapour. The saturated vapour pressure increases rapidly with temperature than ordinary gas but unsaturated vapour pressure varies as in perfect gas.

g) Define critical angle and write down the conditions of total internal reflection.

Ans: **Critical angle**

See the solution of Q. no. 4 (e) of 2060 R/B on page no. 18

Conditions of total internal reflection

See the solution of Q. no. 6 (b) of 2061 Back on page no. 52

h) Derive the relation between real and apparent depth.

Ans: Let us suppose an object is submerged in water at depth 't'. A light ray OM from 'O' incident normally from it passes undeviated into air along its own path OM to 'S'. Another ray ON is very close to 'M' is refracted at 'N' into the air away from the normal in a direction NT. An observer viewing 'O' directly above the object sees it in the position 'I', which is the point of intersection of SM and TN produced. Thus to an eye at 'S' therefore see the object appears to be at 'I'. Thus the depth of the glass appears as MI instead of MO.

Suppose 'n' be the refractive index of the glass medium then,

$$n = \frac{\sin r}{\sin i}$$

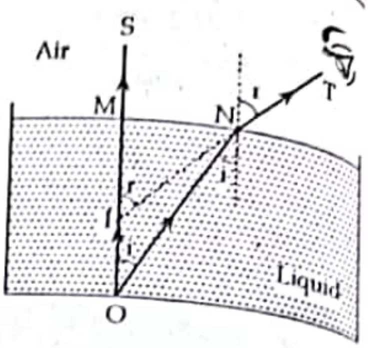
or, $n \sin i = \sin r$

From the figure; we have,

$i = \angle NOM$

and, $r = \angle MIN$

From ΔOMN and ΔMIN; we get,



$$\sin i = \frac{MN}{ON}$$

and, $\sin r = \frac{MN}{IN}$

$\therefore \quad n = \frac{\sin r}{\sin i} = \frac{MN}{IN} \times \frac{ON}{MN} = \frac{ON}{IN}$

Since, the pupil of the eye looking from the upper layer is small, the two points MS and NT must lie very close to each other.

$\therefore \quad ON = OM$

and, $IN = IM$

$\therefore \quad n = \frac{ON}{IN} = \frac{OM}{IM} = \frac{\text{Real depth}}{\text{Apparent depth}}$

Hence,

$$\text{Apparent depth} = \frac{\text{Real depth}}{\text{Refractive index (n)}}$$

5. **Answer any six questions:**

a) **A lift moves (i) up and (ii) down with an acceleration of 2 ms^{-2}. In each case, calculate the reaction of the floor on mans of mass 50 Kg standing in the lift.**

Solution: See the solution of Q. no. 3 (b) of 2061 Back on page no. 49

b) **What force is necessary to keep a mass of 0.8 kg revolving in a horizontal circle of radius 0.7 m with a period of 0.5 second?**

Solution:

Given that;

Mass (m) = 0.8 kg

Radius (r) = 0.7 m

Time period (T) = 0.5 sec.

We have,

$$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{0.5} = 12.56 \text{ rads}^{-1}$$

The required centripetal force (F) = $m\omega^2 r = 0.8 \times (12.56)^2 \times 0.7 = 88.342$ N

Therefore, the force necessary to keep a mass revolving is 88.342 N.

c) **An object is placed in front of a concave mirror and the distance between the object and image is 4 cm. If the magnification is 2, calculate the focal length.**

Solution:

Given that;

Object and image distance (d) = u + v = 4 cm

Magnification (m) = 2

Focal length (f) = ?

Now; we have,

$$m = \frac{v}{u} \qquad (1)$$

or, $2 = \frac{v}{u}$

$\therefore \quad v = 2u$

From the relation (1); we get,

so, $u + v = 4$

or, $u + 2u = 4$

or, $u = \frac{4}{3}$ cm

and, $v = 2u$

or, $v = 2 \times \frac{4}{3}$

∴ $v = \frac{8}{3}$ cm

Hence, from the relation; we have,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{3}{4} + \frac{3}{8} = \frac{6+3}{8} = \frac{9}{8}$$

or, $9f = 8$

∴ $f = 0.89$ cm

Therefore, the focal length of the mirror is 0.89 cm.

d) 25 gm of water at 100°C is mixed with 25 gm of ice at 0°C. Find the result of mixing. Given; Latent heat of ice = 80 $calg^{-1}$, Specific heat capacity of water = 1 $calg^{-1}{}^{\circ}C^{-1}$

Solution:

Given that;

Mass of water taken (m_w) = 25 gm

Mass of ice (m_i) = 25 gm

Initial temperature of water (t_1) = 100°C

Initial temperature of ice (t_2) = 0°C

Temperature of the mixture = t (say)

Specific heat of ice (s_i) = 0.5 cal $g^{-1}{}^{\circ}C^{-1}$

Specific heat of water (s_w) = 1 cal $g^{-1}{}^{\circ}C^{-1}$

Latent heat of ice (L) = 80 $calg^{-1}$

We know,

Heat lost by water $= m_w s_w (t_1 - t) = 25(100 - t) = 2500 - 25t$

Total heat gained by ice to raise its temperature from 0°C to t°C is;

$= m_i L + m_i s_w (t - 0) = 25 \times 80 + 25t = 2000 + 25t$

Using the principle of calorimeter; we have,

Heat lost by water = Heat gained by ice

or, $2500 - 25t = 2000 + 25t$

or, $50t = 500$

or, $t = 10°C$

Hence, the result of mixing attains common temperature of mixture to 10°C.

e) What is the apparent position of an object below a rectangular block of 4 cm thickness? $({}_a\mu_g = 1.5)$

Solution:

Given that;

Thickness of the block (t) = 4 cm

${}_a\mu_g = 1.5$

Apparent position of an object (d) = ?

We have,

$$\text{Displacement of object (d)} = t\left(1-\frac{1}{{}_a\mu_g}\right) = 4\left(1-\frac{1}{1.5}\right) = 1.33$$

Therefore, apparent position of the block is 1.33 cm above from the bottom.

f) **The vertical and horizontal component of earth's field is equal. What is the angle of dip at the place?**

Ans: See the solution of Q. no. 5 (h) of 2060 R/B on page no. 23

g) **A metal rod of length 0.2 m and diameter 0.02 m is ideally lagged. One end is maintained at 100°C while the other end is placed in ice at 0°C. It is found that 0.025 kg of ice melts in 5 minutes. Calculate the thermal conductivity of the rod. Given; $(L_f = 3.34 \times 10^5\ Jkg^{-1})$**

Solution:

Given that;

Length of rod (l) = 0.2 m

Diameter of rod (d) = 0.02 m

Area of cross section (A) $= \frac{\pi d^2}{4} = \frac{3.14 \times (0.02)^2}{4} = 3.14 \times 10^{-4}\ m^2$.

Time (t) = 5 min = 300 sec.

Temperature on one side (θ_1) = 0°C = 273 K

Temperature on other side (θ_2) = 100°C = 373 K

Mass of ice melt (m_i) = 0.025 kg

$L_f = 3.34 \times 10^5\ Jkg^{-1}$

Thermal conductivity of rod (K) = ?

Let, 'Q' be the required amount of heat conducted in 300 seconds through the rod. Then, we have,

$$Q = \frac{KA(\theta_2 - \theta_1)t}{l}$$

Since, it is used for melting the ice,

$$Q = m_i L_f$$

or, $$\frac{KA(\theta_2 - \theta_1)t}{l} = m_i L_f$$

or, $$K = \frac{m_i L_f l}{A(\theta_2 - \theta_1)t} = \frac{0.025 \times 3.34 \times 10^5 \times 0.2}{3.14 \times 10^{-4} \times (373 - 273) \times 300} = 177.28\ Wm^{-1}K^{-1}$$

Therefore, thermal conductivity is 177.28 $Wm^{-1}K^{-1}$.

EXAMINATION 2062
BACK

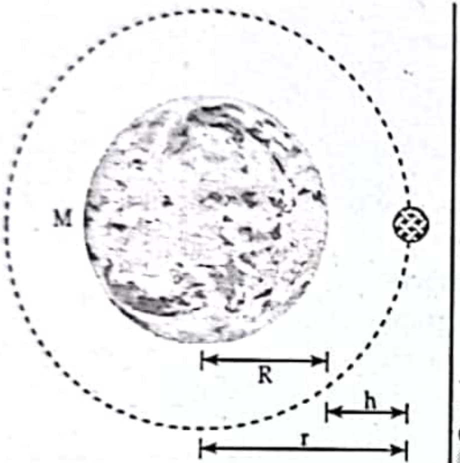
Time: 3 hrs. | Full marks: 60
Pass marks: 24

Candidates are required to give their answers in their own words as far as practicable. Attempt All questions.

1. **Prove that the value of acceleration due to gravity is higher at the surface of earth than at points above and below it.**

Ans: The acceleration produced by gravity is called acceleration due to gravity and is denoted by 'g'. The value of 'g' is constant at the same place, but varies with distance from the centre of the earth above and below its surface.

i) Height above the earth's surface



Consider a body at a height h above the surface of the earth. Let the radius of the earth is 'R' and its mass is 'M'.

The body is at a distance $(R + h)$ from the center of the earth. If g_h is the value of acceleration due to gravity at that point, then,

$$g_h = \frac{GM}{(R + h)^2} \qquad (1)$$

But acceleration due to gravity at the earth's surface

$$g = \frac{GM}{R^2} \qquad (2)$$

Dividing equation (1) by (2); we get,

$$\frac{g_h}{g} = \frac{R^2}{(R + h)^2}$$

Let, $r = R + h$ and as 'R' is constant.

Then,

$$g_h \propto \frac{1}{r^2} \qquad (3)$$

The acceleration due to gravity decreases as move height above the surface of the earth.

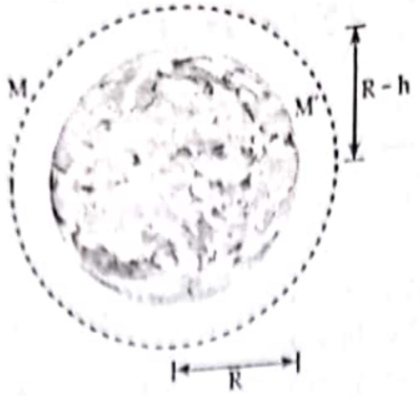
ii) Depth below the earth's surface

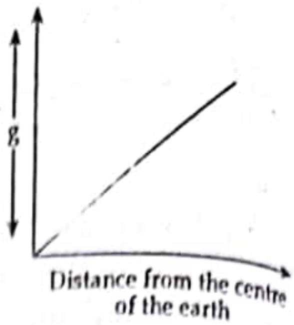
Consider a body taken to a depth 'h' below the earth's surface. The body will be attracted by the mass of the earth which is enclosed in a sphere of radius $(R - h)$. If the mass of this portion is M', the acceleration due to gravity at the point by g'_h and the density of the earth is ρ then,

$$g'_h = \frac{GM'}{(R - h)^2}$$

$$= \frac{4r(R - h)^3\rho}{3(R - h)^2} = G \times \frac{4}{3}\pi(R - h)\rho$$

Similarly,





'g' at earth's surface $= G \times \frac{4}{3}\pi R\rho$

Now,

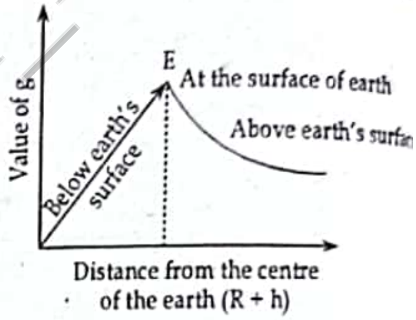
$$\frac{g'_h}{g} = \frac{R-h}{R}$$

Let, $(R-h) = b$

Then, we get,

$\therefore \quad g'_h \propto b$

Since, all other values are constants. The relations (3) and (4) show that the value of 'g' varies inversely as the height above the earth's surface and directly below the earth's surface. If graph between distance from the centre of the earth and 'g' is drawn gives a straight line passing through the origin as shown. Similarly, the graph above the earth's surface is an inverse square type.



From the graph, it is seen that the value of 'g' decreases either above or below the surface of the earth. Thus value of 'g' must be maximum at the surface of the earth.

Or

Define simple harmonic motion. Show that the motion of a simple pendulum is simple harmonic. Derive its time period.

Ans: Simple harmonic motion

Simple harmonic motion is a to and fro motion in a straight line such that its acceleration (i) is always directed towards a fixed point on a straight line and (ii) magnitude of acceleration varies directly as the distance from its mean position.

For the remaining part

See the solution of Q. no. 1 of 2060 Back on page no. 24

2. **Deduce the coefficient of linear and cubic expansion of a solid and deduce the relation between them.**

Ans: Coefficient of linear expansion

See the solution of Q. no. 4 (d) of 2061 R/B on page no. 41

Coefficient of cubic expansion

The increase in volume per unit original volume per unit degree rise in temperature of the substance is called coefficient of cubical expansion. It is represented by γ and unit is $°C^{-1}$ or K^{-1}.

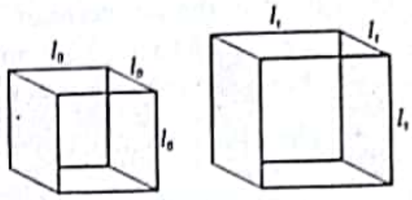
$$\text{Cubical expansivity } (\gamma) = \frac{\text{Increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

$$= \frac{V_t - V_0}{V_0(t - t_0)}$$

where, V_0 is the volume at a temperature 0°C of material of cubical expansivity γ, its volume V_t at the temperature t°C is given by,

Relation between α and γ

Let us consider a cube having sides l_0 at 0°C. If temperature of the cube increases from 0°C to t°C, then the sides of the cube increases to 'L', then, we have,



$$L = l_0(1 + \alpha t)$$

∴ Volume of cube at 0°C $(V_0) = l_0^2$

and, Volume of cube at t°C $(V_t) = L^3 = l_0^3(1 + \alpha t)^3$

$$= V_0(1 + 3\alpha t + 3\alpha^2 t^2 + \alpha^3 t^3)$$

Since, the value of α is very small so higher power of α will be very small. Thus, the terms containing higher power of α can be neglected.

∴ $V_t = V_0(1 + 3\alpha t)$ (1)

But, from the definition of the coefficient of cubical expansion; we have,

$V_t = V_0(1 + \gamma t)$ (2)

Hence, comparing the relation (1) and (2); we get,

$$\gamma = 3\alpha$$

i.e., the coefficient of cubical expansion of a solid is nearly equals to three times its coefficient of linear expansion or linear expansivity.

3. Deduce the relation $n = \dfrac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}}$ for a prism, where the symbols carry usual meaning.

Ans: See the solution of Q. no. 3 of 2061 R/B on page no. 39

4. **Answer any six questions:**

a) **Define linear momentum and impulse. Write down the unit and dimension of each.**

Ans: **Linear momentum**

The quantity of measurement of motion of a body in a straight path is called linear momentum. If a body moves faster and faster, then the momentum of the body is said to be increasing.

The linear momentum of a body is calculated by the product its mass and velocity. If a body has mass 'm' is moving with velocity 'v' then, the momentum is given by mv.

i.e., Linear or momentum of the body = Mass × Velocity = mv

By doubling the velocity, we double the momentum. The direction of the momentum is also same as that of the velocity of the body. Thus momentum is also a vector quantity having both magnitude and direction.

The unit of the momentum or linear momentum is $kgms^{-1}$. The dimension of momentum is $[MLT^{-1}]$.

g) State and explain Coulomb's law of magnetism and define unit pole.

Ans: The Coulomb's law of magnetism states that the attractive or repulsive force between any two poles of the magnets is directly proportional to the product of the pole strength of the poles and inversely proportional to the square of the distance between them.

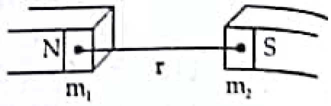
Let two poles of magnets having pole strengths m_1 and m_2 are separated by distance 'r', then,

$$F \propto m_1 m_2 \quad (1)$$

and, $$F \propto \frac{1}{r^2} \quad (2)$$

Combining equation (1) and (2); we get,

$$F \propto \frac{m_1 m_2}{r^2}$$



$$\therefore \quad F = K \cdot \frac{m_1 m_2}{r^2}$$

where, K is a constant and $K = \frac{\mu}{4\pi}$.

Here, μ is a constant called permeability of the medium between the poles.

$$\therefore \quad F = \frac{\mu}{4\pi} \cdot \frac{m_1 m_2}{r^2}$$

For vacuum; it is represented by μ_0.

The value of $\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$.

The relation for the attractive or repulsive force between poles in vacuum;

$$F = \frac{\mu_0}{4\pi} \cdot \frac{m_1 m_2}{r^2}$$

5. Answer any six questions.

a) An object moves in a circular path of radius 2 meters with a period of 5 seconds. Calculate its angular velocity and centripetal acceleration.

Solution:

Given that;

Radius of circular path (r) = 2 m

Time period (T) = 5 sec.

Angular velocity (ω) = ?

Centripetal acceleration (a) = ?

We know,

$$\text{Angular velocity } (\omega) = \frac{2\pi}{T} = \frac{2 \times 3.14}{5} = 1.256 \text{ rads}^{-1}$$

Again,

$$\text{Centripetal acceleration (a)} = \omega^2 r = (1.256)^2 \times 2 = 3.155 \text{ ms}^{-2}$$

Therefore, the required angular velocity is 1.256 $rads^{-1}$ and centripetal acceleration is 3.155 ms^{-2}.

b) A wheel has a moment of inertia of 182.5 kgm^2 about an axis of rotation. If the wheel rotates at the rate of 1000 r.p.m., find the rotational kinetic energy of the wheel.

Solution:

Given that;

Moment of inertia (I) = 182.5 kgm^2

Rotational kinetic energy (K.E.) = ?
The rate of rotation = 1000 r.p.m.

Hence,

Number of rotation per second (n) $= \frac{1000}{60} = 16.67$

Angular velocity (ω) $= 2\pi n = 2\pi \times 16.67 = 104.69$ rads^{-1}

K.E. of rotation $= \frac{1}{2} I\omega^2 = \frac{1}{2} \times 182.5 \times (104.69)^2 = 1 \times 10^6$ J

c) **A copper calorimeter of mass 300 gm contains 500 gm of water at a temperature 15°C. A 500 gm of block of aluminium at a temperature 100°C is dropped in calorimeter. If the temperature of mixture becomes 21.5°C, find specific heat capacity of aluminium. Given specific heats of copper and water are 910 Jkg^{-1}K^{-1} and 4200 Jkg^{-1}K^{-1} respectively.**

Solution:

Given that;

Mass of calorimeter (m_c) = 300 gm = 0.3 kg
Mass of water (m_w) = 500 gm = 0.5 kg
Mass of aluminium (m_s) = 500 gm = 0.5 kg
Initial temperature of calorimeter + water = 15°C
Initial temperature of hot solid (t_2) = 100°C
Final temperature of mixture (t) = 21.5°C
Specific heat of aluminium (s) = ?
Specific heat of water (s_w) = 4200 Jkg^{-1}K^{-1}
Specific heat of calorimeter (s_c) = 910 Jkg^{-1}K^{-1}

The specific heat of the solid is;

$$s = \frac{m_c s_c (t - t_1) + m_w s_w (t - t_1)}{m_s (t_2 - t)}$$

$$= \frac{0.3 \times 910 \times (21.5 - 15) + 0.5 \times 4200 \times (21.5 - 15)}{0.5 \times (100 - 21.5)} = 392.98 \text{ Jkg}^{-1}\text{K}^{-1}$$

d) **Calculate the r.m.s. speed for a gas at 0°C if its density at s.t.p. is 1.29 kgm^{-3}. Given; standard pressure is 1.03×10^5 Nm^{-2}**

Solution:

Given that;

r.m.s. speed of gas 0°C (C) = ?
Pressure (P) = 1.03×10^5 Nm^{-2}
Density of helium at S.T.P. (ρ) = 1.29 kgm^{-3}

We know,

$$C = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3 \times 1.03 \times 10^5}{1.29}} = 489.42 \text{ ms}^{-1}$$

i.e., r.m.s speed of gas 0°C is 489.42 ms^{-1}.

e) **A concave mirror produces a real image 3 times as big as the object placed on its principal axis. If the distance between the object and image is 16 cm, what is the focal length of the mirror?**

Solution:

Given that;

Magnification (m) = 3

$$\text{Refractive index of glass } (\mu) = \frac{\text{Real depth (t)}}{\text{Apparent depth (A)}}$$

$$= \frac{4.5}{3} = 1.5$$

∴ The refractive index of glass is 1.5.

h) **Calculate the horizontal component of the flux density of earth's magnetic field at a place where the resultant flux density is 10^{-5} T and angle of dip is 51°.**

Solution:

Given that;

$I = 10^{-5}$ T

$d = 51°$

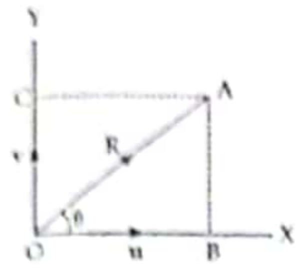
$H = ?$

The horizontal component 'H' is given by;

$$H = I \cos d = 10^{-5} \times \cos 51° = 6.293 \times 10^{-6} \text{ T}$$

i.e., the horizontal component of flux density is 6.293×10^{-6} T.

0.25



Now, draw perpendicular AB from 'A' along OX and AC along OY. Then, the components of 'R' along OX and OY are given by;

$$OB = u = R\cos\theta \quad (1)$$

$$OC = v = R\sin\theta \quad (2)$$

Here, $u = R\cos\theta$ is the resolved part or component of 'R' along OX and $v = R\sin\theta$ is component along OY.

c) **Derive a relation for thermal conductivity of a conductor.**

Ans: The quantity of heat 'Q' flowing across two opposite faces of conductor maintained at a constant high and low temperature depends;

i) directly on the cross-sectional area 'A'
ii) temperature difference $(\theta_2 - \theta_1)$
iii) time 't' for which the heat is allowed to flow and
iv) inversely to the distance 'l' between the two faces

i.e., $Q \propto \dfrac{A(\theta_2 - \theta_1)}{l}$

or, $Q = \dfrac{KA(\theta_2 - \theta_1)t}{l}$

where, K is a constant called thermal conductivity of conductor.

$$K = \frac{\frac{Q}{At}}{\frac{(\theta_2 - \theta_1)}{l}} = \frac{Jm^{-2}s^{-1}}{Km^{-1}} = Wm^{-1}K^{-1}$$

This is the required relation for thermal conductivity of a conductor.

d) **Explain about saturated and unsaturated vapour pressure.**

Ans: See the solution of Q. no. 4 (f) of 2062 R/B on page no. 57

e) **Distinguish between the molecular theory of magnetism and domain theory of magnetism.**

Ans: The molecular theory of magnetism assumes that all magnetic substances are composed of tiny molecular magnets. Any unmagnetized material has the magnetic forces of its molecular magnets neutralized by adjacent molecular magnets, thereby eliminating any magnetic effect. A magnetized material will have most of its molecular magnets lined up so that the north pole of each molecule points in one direction, and the south pole faces the opposite direction. A material with its molecules thus aligned will then have one effective north pole, and one effective south pole. According to the theory when a non magnetic substance is magnetized by stroking several times in the same direction by a magnet, then one types of poles of the molecular magnets align themselves to one side and next type of pole on next side. By any means if molecular magnets are disturbed it converted to magnetic substance only.

Domain theory

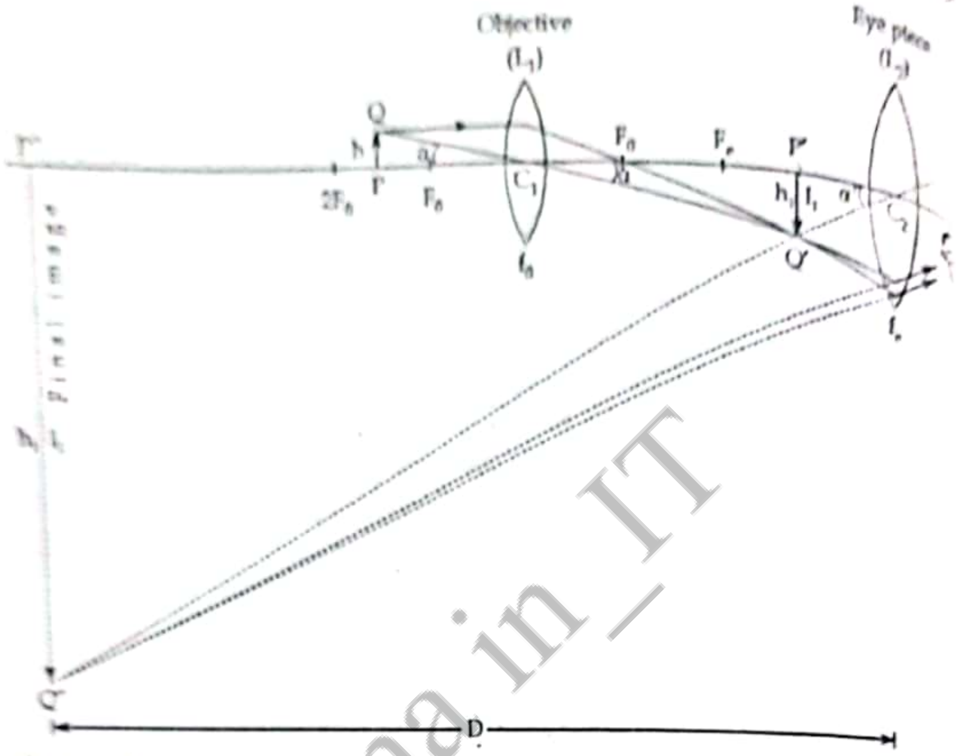
See the solution of Q. no. 4 (f) of 2060 R/B on page no. 18

f) **Draw neat and clean ray diagram of a compound microscope and derive the relation for angular magnification of compound microscope.**

Ans: When microscope is in normal adjustment the magnification 'M' is given by;

$$M = \frac{\alpha'}{\alpha}$$

where, α' is the angle subtended by final image I_2 at eye and α is the angle subtended by object at the eye.



But, we have,

$$\alpha' = \frac{h_2}{D}$$

and, $\alpha = \dfrac{h}{D}$

where, h_2 and 'h' are the heights of the final image I_2 and the object. 'D' is least distance of distinct vision.

$$\therefore \quad M = \frac{\alpha'}{\alpha} = \frac{\frac{h_2}{D}}{\frac{h}{D}} = \frac{h_2}{h} = \frac{h_2}{h_1} \times \frac{h_1}{h}$$

But, $M_0 = \dfrac{h_1}{h} = \dfrac{v}{f_1} - 1$

where, f_1 is the focal length of objective.

Similarly, the magnification due to eyepiece is given by;

$$M_e = \frac{h_2}{h_1} = -\left(\frac{D}{f_2} + 1\right)$$

where, f_2 is the focal length of eyepiece.

Hence,

$$\text{Total magnification (M)} = M_0 \times M_e = -\left(\frac{v}{f_1} - 1\right)\left(\frac{D}{f_2} + 1\right)$$

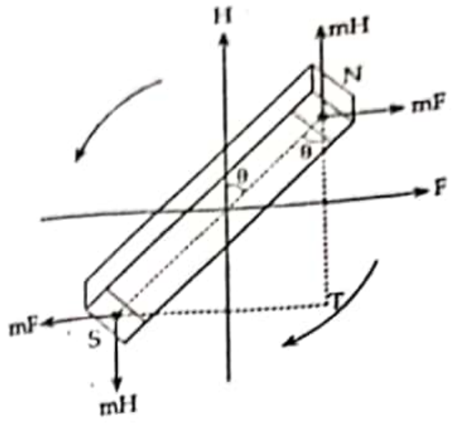
g) **State and explain tangent law of magnetism.**

Ans: **Tangent law**

The law states that, if a magnetic needle is suspended freely to two mutually perpendicular magnetic fields 'F' and 'H', then it rests along the resultant of these two fields, such that;

$$F = H \tan \theta$$

where, θ is the angle made by the axis of the needle with field 'H'.



Let, a magnet NS be suspended in between two magnetic fields 'H' and 'F' perpendicular to each other and come to rest at an angle θ with the field 'H'. Then clearly, forces mH and mF, due to the fields 'H' and 'F' respectively act upon the two poles of the magnet in the directions shown.

The moment of the couple due to field $H = mH \times ST$ tends to rotate the magnet in the anti-clockwise direction, and moment of the couple, due to field $F = mF \times NT$, tending to rotate the magnet in the clockwise direction.

From the principle of moments; we get,

$$mF \times NT = mH \times ST$$

or, $F = H\dfrac{ST}{NT} = H\tan\theta$

or, $F = H\tan\theta$

Since, 'H' is constant for the place.

$$F \propto \tan\theta$$

This is the tangent law.

5. **Solve any six questions.**

a) **A bullet of mass 6×10^{-3} kg traveling at $120\ ms^{-1}$ penetrates deeply into a fixed target is brought to rest in 0.01 second. Calculate:**

i) **Penetrating depth**

ii) **Average retarding force**

Solution:

Given that;

Mass of bullet (m) $= 6 \times 10^{-3}$ kg

Velocity of bullet (u) $= 120\ ms^{-1}$

Time to comes to rest (t) $= 0.01$ sec.

Penetrating depth (s) $= ?$

Retarding force (F) $= ?$

Final velocity (v) $= 0$

Now,

$$v = u + at$$

or, $a = \dfrac{v-u}{t} = \dfrac{0-120}{0.01} = -1200\ ms^{-2}$

Again,

$$v^2 = u^2 - 2aS$$

or, $S = \dfrac{v^2-u^2}{2a} = \dfrac{0-(120)^2}{2\times(-1200)} = 6\ m$

$\therefore$ Average force (F) $= ma = 6 \times 10^{-3} \times 1200 = 7.2$ N

Therefore, penetrating depth is 6 m and average retarding force is 7.2 N.

b) What is the work done by a man carrying luggage weighing 50 kg over his head when travels a distance of 20 m in (i) vertical direction (ii) horizontal direction.

Solution:

Given that;

Weight of luggage (m) = 50 kg

Distance travelled (S) = 20 m

Work done = $FS\cos\theta$

According to the value of $\cos\theta$, the amount of work done may vary.

i) Work done during horizontal motion (W_H) $= FS\cos\theta$

$= mg \times 20 \times \cos 90°$

$= 0$ J

ii) Work done during vertical motion (W_V) $= FS\cos\theta$

$= mg \times 20 \times \cos 0°$

$= 50 \times 9.8 \times 20 \times 1$

$= 9800$ J

Therefore, the work done is 0 and 10000 J respectively.

c) A simple pendulum whose length is 1.0 m oscillating 30 times per minute in certain place. What is acceleration due to gravity at that place?

Solution:

Given that;

Length of pendulum (l) = 1 m

Frequency of oscillation (f) = 30 times per minute = 0.5 times per sec.

∴ Time period (t) $= \frac{1}{f} = \frac{1}{0.5} = 2$ sec.

Acceleration due to gravity (g) = ?

We have,

Time period of a pendulum 't' is given by;

$$t = 2\pi\sqrt{\frac{l}{g}}$$

or, $$g = \frac{4\pi^2 l}{t^2} = \frac{4 \times (3.14)^2 \times 1}{(2)^2} = 9.86\ ms^{-2}$$

d) What power will be radiated from a spherical surface 10 cm in diameter if its temperature is 600°C? The emissivity of the surface is 0.04. Stefan's constant = $5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$

olution:

Given that;

Diameter of sphere (d) = 10 cm = 0.1 m

∴ Radius of sphere (r) = 0.05 m

Temperature of the sphere (T) = 600°C = 273 + 600 = 873 K

Stefan's constant (σ) = $5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$

Emissivity of the surface (ε) = 0.04

Power radiated (P) = ?

We have,

$$P = \varepsilon\sigma T^4 = 0.04 \times 5.67 \times 10^{-8} \times (873)^4 = 1317.34 \text{ Watt}$$

The power radiated from a spherical surface is 1317.34 Watt.

e) What is the apparent position of an object below a rectangular block of glass 6 cm thick? If a layer of water 4 cm thick is on top of the glass. What is the apparent position? (Given; μ for glass = 1.5, μ for water = 1.33)

Solution:

Given that;

Thickness of glass $(t_g) = 6$ cm

Thickness of water $(t_w) = 4$ cm

${}_a\mu_g = 1.5$

${}_a\mu_w = 1.33$

For glass; we have,

$$\text{Displacement } (d_1) = t_g\left(1 - \frac{1}{{}_a\mu_g}\right) = 6\left(1 - \frac{1}{1.5}\right) = 2 \text{ cm}$$

For water; we have,

$$\text{Displacement } (d_2) = t_w\left(1 - \frac{1}{{}_a\mu_w}\right) = 4\left(1 - \frac{1}{1.33}\right) = 0.99 \text{ cm}$$

$\therefore$ Total displacement $= d_1 + d_2 = 2 + 0.99 = 2.99$ cm

Thus, the apparent position of an object is 2.99 cm above from the bottom.

f) An object 10 cm in height is placed on the axis and 10 cm from a concave mirror of focal length 15 cm. Find the position, size and nature of the image.

Solution:

Given that;

Height or size of object (O) = 10 cm

Focal length (f) = 15 cm

Object distance (u) = 10 cm

Size of image (I) = ?

Image distance (v) = ?

Now, we have,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{15} - \frac{1}{10} = \frac{2-3}{30} = -\frac{1}{30}$

or, $v = -30$ cm

Again,

$$m = \frac{I}{O} = \frac{v}{u}$$

or, $-\frac{v}{u} = \frac{I}{O}$

or, $-\frac{30}{10} = \frac{I}{10}$

3

$\therefore$ $I = -30$ cm

Hence, image distance is 30 cm or three times the object; image is virtual and erect in nature.

g) A bar magnet has a length 8 cm. the magnetic field at the distance 5 cm from both ends of magnet is 4×10^{-6} T. Calculate pole strength of the magnet. $[\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1}]$

Solution:

Given that;

$2l = 8\text{ cm} = 0.08\text{ m}$

$\therefore \quad l = 0.04\text{ m}$

$\sqrt{d^2 + l^2} = 5\text{ cm} = 0.05\text{ m}$

$B = 4 \times 10^{-6}\ T$

$4\pi \times 10^{-7}\ TmA^{-1}$

Now, from broad side on position; we have,

$$B = \frac{\mu_0}{4\pi} \times \frac{M}{(d^2 + l^2)^{\frac{3}{2}}}$$

or, $$4 \times 10^{-6} = 10^{-7} \times \frac{M}{(0.05)^3}$$

or, $$M = \frac{4 \times 10^{-6} \times (0.05)^3}{10^{-7}}$$

or, $$M = 5 \times 10^{-3}\ Am^2$$

Hence,

$$\text{Pole strength (m)} = \frac{M}{2l} = \frac{5 \times 10^{-3}}{0.08} = 0.0625\ Am$$

EXAMINATION 2063
BACK

Time: 3 hrs. Full marks: 60
Pass marks: 24

Candidates are required to give their answers in their own words as far as practicable.
Attempt All questions.

1. **State Newton's law of motion and prove conservation law on motion from Newton's laws.**

Ans: The laws of motion are stated as;

First law

An object at rest always tends to remain at rest and an object in motion always tends to remain in motion with constant speed along the same direction until acted upon by an external agent (force).

Second law

The rate of change of momentum of a body is directly proportional to the impressed force and takes place along the direction of the impressed force

Third law

For every action, there is always equal and opposite reaction.

Principle of conservation of motion

During collision of particles always obeyed a principle called the principle of conservation of momentum or motion. The principle of conservation of motion states that "If no external force acts the initial measurement of motion (momentum) on a colliding system of bodies is equal to the final motion (momentum)".

Verification of the principle

See the solution of Q. no. 3 of 2061 Back on page no. 48

Or

Describe circular motion and prove $F = \frac{mv^2}{r}$, where symbols have their usual meaning.

Ans: See the solution of Q. no. 1 of 2061 R/B on page no. 37

2. **State and explain Stefan's law of black body radiation.**

Ans: See the solution of Q. no. 4 (d) of 2060 R/B on page no 17

Or

Define coefficient of linear and cubical expansion of a solid and derive the relation between them.

Ans: See the solution of Q. no. 2 of 2062 Back on page no. 62

3. **What is power of lens? Deduce lens maker formula for a convex lens.**

Ans: See the solution of Q. no. 7 (a) of 2061 Back on page no. 53

4. **Attempt (any six) questions from the following:**

a) **State moment of inertia and derive an expression for rotational kinetic energy of an object in terms of moment of inertia and angular velocity.**

Ans: See the solution of Q. no. 1 (or) of 2060 R/B on page no. 12

b) **Show that value of 'g' decreases as we go up from the surface of earth.**

Ans: See the solution of Q. no. 1 of 2062 Back on page no. 61

c) **What is efficiency of an engine? Why it is not 100 percent? Expla...**

Ans: Heat engine absorbs Q_1 amount of heat from the source and part of converted into mechanical work 'W'. The remaining heat will be rejected sink, let it be Q_2. The amount of heat energy converted into mechan... energy 'W' is given by;

$$W = Q_1 - Q_2$$

The amount of heat input in the device is Q_1.

$$\text{Efficiency of heat engine } (\eta) = \frac{\text{Energy output}}{\text{Energy input}} \times 100\%$$

$$= \frac{Q_1 - Q_2}{Q_1} \times 100\%$$

$$= -\left(\frac{Q_2}{Q_1}\right) \times 100\%$$

$$= 100\% \text{ if } Q_2 = 0;$$

which is independent of working substance used.

It shows that efficiency of a heat engine will be greater if Q_2 is sm... However, from experiences with a wide variety of system, it has been fo... that the value of Q_2 will never be possible to reduce to zero. Hence, ... certain that efficiency of any heat engine is always less than 1 or 100%... means every heat engine the heat taken from the source can never ... completely converted into mechanical energy.

d) **Deduce an expression for thermal conductivity of a conductor.**

Ans: See the solution of Q. no. 4 (e) of 2060 Back on page no. 31

e) **Show that when a mirror is rotated through certain angle, ... reflected ray is deviated through double the angle.**

Ans: See the solution of Q. no. 4 (f) of 2062 Back on page no. 65

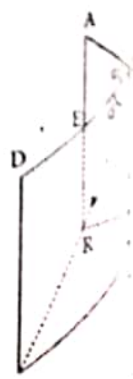
f) **What are the elements of earth magnetic field? Develop the relation $R = \sqrt{H^2 + V^2}$**

Ans: In order to specify completely the magnetic field of the earth at any place three quantities are chosen and are called the magnetic elements of the earth magnetic field at a place. They are as follows;

i) Declination or variation
ii) Dip or inclination
iii) Horizontal intensity

To derive the relation $R = \sqrt{H^2 + V^2}$

In the figure, the angle between BD and 'R' is the angle of dip at that place. The resultant 'R' at a place has been shown there to make an angle 'd' with the horizontal direction BD.



For a perfectly freely suspended needle at 'B' would not set in the horizontal direction BD but sets along the direction BR of the resultant magnetic intensity 'R'. If 'H' and 'V' are horizontal and vertical components of the earth's magnetic fields at a plane; then,

$$V = R\sin d \qquad (1)$$
$$H = R\cos d \qquad (2)$$

Dividing equation (1) by (2); we get,

$$\frac{V}{H} = \tan d \qquad (3)$$

Squaring equation (1) and (2) and adding; we get,

$$V^2 + H^2 = R^2\sin^2 d + R^2\cos^2 d = R^2(\sin^2 d + \cos^2 d) = R^2$$

or, $V^2 + H^2 = R^2$

$\therefore$ $R = \sqrt{H^2 + V^2}$

g) **What is tangent law of magnetism?**

Ans: See the solution of Q. no. 4 (g) of 2063 R/B on page no. 72

i. **Answer any six questions:**

a) **A ball mass 0.1 Kg moving with a velocity of 6 ms^{-1} collides directly with a ball of mass 0.2 Kg at rest. Calculate the common velocity of them when they stick together.**

Solution:

Given that;

Mass of first particle (m_1) = 0.1 kg

Initial velocity of first particle (u_1) = 6 ms^{-1}

Mass of second particle (m_2) = 0.2 kg

Velocity of second particle (u_2) = 0

Let 'V' be the common velocity of the balls after collision. According to the principle of conservation of momentum; we have,

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

or, $$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{0.1 \times 6 + 0.2 \times 0}{0.1 + 0.2} = 2\ ms^{-1}$$

i.e., the common velocity will be 2 ms^{-1}.

b) **What is work done by a man carrying luggage weighing 50 Kg over his head when travels a distance of 20 m in (a) vertical direction (b) horizontal direction.**

Solution: See the solution of Q. no. 5 (b) of 2063 R/B on page no. 74

c) **Surface temperature of sun is 6000 K considering such as a perfectly blackbody, calculate the power radiate from it. Given; Radius of sun = 6.9×10^9 m, $\sigma = 5.67 \times 10^{-8}$ Wattm^{-2}K^{-4}**

Solution:

Given that;

Sun's radius (r) = 6.9×10^9 m

Temperature of the sun (T) = 6000 K

Stefan constant (σ) = 5.67×10^{-8} Wattm^{-2}K^{-4}

Area of sun's surface (A) = $4\pi r^2$

Power radiated by the sun's surface (P) = ?

We have,

Power radiated by sun's surface (P) = σAT^4 ...

$= 5.67 \times 10^{-8} \times 4\pi \times (6.9 \times 10^{9})^{2} \times (6000)^{4}$

$= 4.4 \times 10^{28}$ Watt

d) A metal piece of mass 25 Kg at 80°C is dropped into 0.12 Kg water at 20°C. Find the final temperature measured. Specific heat of metal is 504 $Jkg^{-1}K^{-1}$

Solution:

Given that;

Mass of water taken (m_w) = 0.12 kg

Mass of metal (m_m) = 25 kg

Initial temperature of water (t_1) = 20°C = 293 K

Initial temperature of metal (t_2) = 80°C = 353 K

Temperature of the mixture (t) = ?

Specific heat of metal (s_m) = 504 $Jkg^{-1}K^{-1}$

Specific heat of water (s_w) = 4200 $Jkg^{-1}K^{-1}$

Using the principle of calorimeter; we have,

Heat lost by metal = Heat gained by water

or. $m_m s_m (t_2 - t) = m_w s_w (t - t_1)$

or, $25 \times 504(353 - t) = 0.12 \times 4200(t - 293)$

or, $4447800 - 12600t = 504t - 147672$

or, $t = 350.69$ K = 77.69°C

Hence, result of mixing attains common temperature of mixture to 77.69°C.

e) Find out the value of angle of dip at a place for which horizontal and vertical component of earth's magnetic fields are equal.

Solution: See the solution of Q. no. 5 (h) of 2060 R/B on page no. 23

f) A prism of angle 60° is made of glass of refractive index 1.5. Calculate the angle of minimum deviation.

Solution: See the solution of Q. no. 5 (f) of 2060 R/B on page no. 22

g) A lift moves up with a constant acceleration 1 ms^{-2}. Calculate the reaction of the floor when a man of mass 50 kg standing on the lift. $g = 9.8\ m/s^2$

Solution: See the solution of Q. no. 5 (g) of 2060 R/B on page no. 22

Hint

Use $a = 1\ ms^{-2}$ instead of $2\ ms^{-2}$.

EXAMINATION 2064
REGULAR/BACK

Full marks: 60
Pass marks: 24

Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.
Attempt All questions.

1. **How does the acceleration due to gravity vary with distance above and below from the surface of the earth? Where is its maximum value?**

Ans: See the solution of Q. no. 1 of 2062 Back on page no. 61

Or

State the principle of conservation of linear momentum. Show that the law follows from Newton's laws of motion.

Ans: See the solution of Q. no. 3 (a) of 2061 Back on page no. 48

2. **Stating the postulates of kinetic theory of gases, derive the relation $P = \frac{1}{3}\rho c^2$, where, the symbol has their usual meanings.**

Ans: See the solution of Q. no. 2 of 2060 Back on page no. 26

Then,

$$\therefore \quad \text{Pressure (P)} = \frac{mNc^2}{3V} = \frac{1}{3}\frac{Mc^2}{V}$$

where, M = mN.

Let, ρ is the density of the gas, then,

$$\rho = \frac{M}{V}$$

$$\therefore \quad P = \frac{1}{3}\rho c^2$$

3. **Define magnetic field. Derive magnetic field at a point on the equatorial line and at a point on axial line of a bar magnet.**

Ans: Magnetic field is the surface area around a magnet in which other magnet or magnetic substance is influenced.

Magnetic field due to bar magnet at a point in equatorial line of magnet

See the solution of Q. no. 3 of 2060 Back on page no. 28

Magnetic field due to a bar magnet on axial line

See the solution of Q. no. 4 (g) of 2060 R/B on page no. 18

4. **Answer any six questions:**
   a) **Show that the total mechanical energy of a freely falling body under gravity is constant.**

Ans: See the solution of Q. no. 4 (a) of 2060 R/B on page no. 15

   b) **Obtain an expression for the angular momentum of a rigid body. How is it related to the torque acting on the body?**

Ans: Let a rigid body rotates about an axis XY with an angular velocity ω. The particles on it also have the same angular velocity ω, but as particles are at different distances from the axis of rotation, their linear velocities will be different. Let us suppose the linear velocities of the particles of masses m_1, m_2, m_3,........., at distances r_1, r_2, r_3,........., from the axis of rotation are v_1, v_2, v_3, ..., etc.

Then, angular momentum of a particle is given by;

$$L = mvr = m\omega r^2$$

The total angular moments of the particles is the sum of angular moments of all particles and is given by,

$$\text{Total angular momentum (L)} = m_1\omega r_1^2 + m_2\omega r_2^2 + m_3\omega r_3^2 + \ldots\ldots$$

$$= \sum m\omega r^2$$

where, $I = \sum mr^2$ and is the moment of inertia of body about the axis XY.

$\therefore \quad L = I\omega$

Angular momentum is a vector quantity.

Relation between torque and the angular momentum

We have,

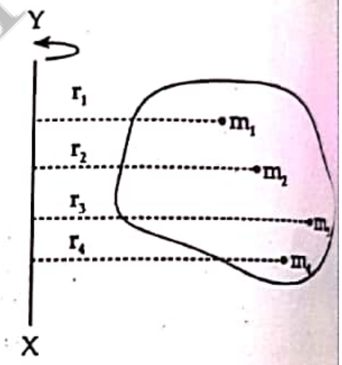
$$L = I\omega$$

Now, differentiating the relation; we get,

$$\frac{dL}{dt} = \frac{d}{dt}(I\omega)$$

For a fixed axis of rotation; 'I' remains unchanged, hence,

$$\frac{dL}{dt} = I\frac{d\omega}{dt}$$



But, $\frac{d\omega}{dt} = \alpha$

$$\therefore \quad \frac{dL}{dt} = I\alpha$$

Again, we know that,

$$I\alpha = \tau$$

$$\therefore \quad \frac{dL}{dt} = \tau$$

The torque is equal the rate of change of angular momentum of the body.

c) **Why are roads banked at the turnings? Derive a relation for the velocity of a car at the turning with the banking angle.**

Ans: Roads are slightly inclined outwards at the curves are called banked road. The angle made by the road with horizontal is called banking angle. Civil engineers generally bank curves on roads in such a manner that a vehicles going around the curve at the recommended speed does not have to rely on friction between its tires and the road surface in order to round the curve. The necessary centripetal force is provided by a component of normal reaction.

Relation for the velocity of a car at the turning

Suppose a car is moving on a banked road as shown in figure. The banking angle between the road and the horizontal is θ, the normal reaction on car is 'R' so, resolving 'R' into horizontal and vertical components.

In the vertical direction there is no acceleration, and component R cos θ balances weight of the car, hence,

$$R\cos\theta = mg \quad \text{(i)}$$

In the horizontal direction centripetal force (F) is provided by the component of the normal reactions is R sin θ.

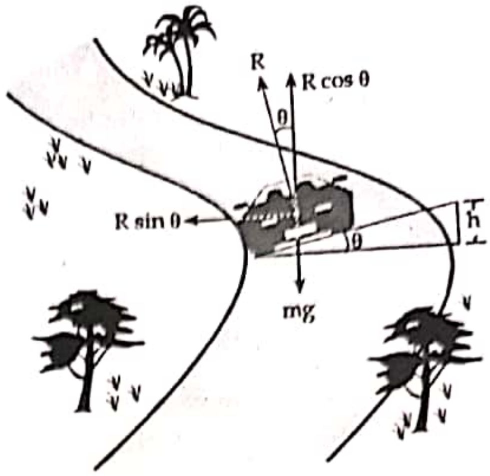
i.e., $\quad R\sin\theta = \frac{mv^2}{r} \quad$ (ii)

Dividing equation (2) by (1); we get,

$$\frac{\sin\theta}{\cos\theta} = \frac{mv^2}{r \cdot mg} = \frac{v^2}{rg}$$

or, $$\tan\theta = \frac{v^2}{rg}$$

$\therefore$ $$v = \sqrt{rg\tan\theta}$$



A car moving at velocity 'v' will successfully round the curve. As 'v' increases value of angle θ also is required to be increased. But, for a road, banking angle is fixed. For safety only possibility is to limit the speed.

d) **Derive an expression for the amount of heat that passes through a given length of a conductor and hence define coefficient of thermal conductivity. Also mention its unit.**

Ans: See the solution of Q. no. 4 (e) of 2060 Back on page no. 31

The unit of thermal conductivity 'K' is $Wm^{-1}K^{-1}$.

e) **Derive a relation for combined focal length of two thin lenses placed in contact.**

Ans: See the solution of Q. no. 4 (g) of 2060 Back on page no. 32

f) **Define critical angle and the total internal reflection of light. Derive a relation for this angle and the refraction index.**

Ans: **Critical angle**

See the solution of Q. no. 4 (e) of 2060 R/B on page no. 18

Total internal reflection

If angle of incidence in denser medium increases more than critical angle then, the ray of light reflected back to the same denser medium without refraction to rarer medium. The reflected ray obeys the laws of reflection. Such phenomenon of reflection without refraction is called total internal reflection.

Relation for critical angle and the refraction index

See the solution of Q. no. 4 (e) of 2060 R/B on page no. 18

g) **What is tangent law? Derive an expression for it.**

Ans: See the solution of Q. no. 4 (g) of 2063 R/B on page no. 72

5. **Answer any six questions.**

a) **A car of mass 1000 kg moves on a horizontal road at a constant speed of 20 ms^{-1}. If the frictional force of road on the car is 200 N, calculate the power developed by the engine. If the car now moves the inclined road at the same speed such that $\sin\theta = \frac{1}{20}$, where, θ is the angle of the inclination to the horizontal, calculate the new power developed by the engine. Suppose that frictional force is same.**

Solution:

Given that;

Mass of the vehicle (m) = 1000 kg

Speed of engine (v) = 20 ms^{-1}

Frictional resistance (R) = 200 N

Power developed by the car (P) = ?

Total force applied by engine (F) = 200 N

Now, to drive the car in horizontal direction is given by;

Power (P) = Fv = 200 × 20 = 4000 W

If the car now moves the inclined road with $\sin\theta = \frac{1}{20}$ then,

$$\text{Downward component of weight of vehicle} = mg\sin\theta = 1000 \times 9.8 \times \frac{1}{20} = 490\text{ N}$$

To drive up the incline the force 'F' that the engine of the vehicle must apply is given equal to the total force of $mg\sin\theta + R$.

i.e., $F = mg\sin\theta + R = 490 + 200 = 690$ N

∴ Power = Fv = 690 × 20 = 13800 W

b) An object moving with S.H.M. has amplitude of 0.02 m and frequency 20 Hz. Calculate; (i) acceleration at the middle and at the extremes of oscillation and (ii) velocities at the corresponding points.

Solution:

Given that;

Amplitude (r) = 0.02 m

Frequency (f) = 20 Hz

i) We have,

$$T = \frac{1}{f} = \frac{1}{20} = 0.05\text{ sec.}$$

ii) At mean position; we have,

$$\text{Maximum velocity } (V_{max}) = \omega r = 2\pi fr = 2\pi \times 20 \times 0.02 = 2.51\text{ ms}^{-1}$$

Minimum acceleration $(a_{min}) = 0$ $[\because a = -\omega^2 y \text{ and } y = 0]$

At extreme position; we have,

Minimum velocity $(V_{min}) = 0$ $[\because v = -\omega\sqrt{r^2 - y^2} \text{ and } r = y]$

$$\text{Maximum acceleration } (a_{max}) = \omega^2 r = (2\pi f)^2 r = 4\pi^2 f^2 r = 4 \times (3.14)^2 \times (20)^2 \times 0.02 = 315.8\text{ ms}^{-2}$$

c) A copper calorimeter of water equivalent 10 gm contains 200 gm of water and 50 gm of ice at 0°C. How much steam at 100°C must be passed into the calorimeter to raise the temperature of it and its content to 20°C? Given; Specific heat of ice $(s_i) = 0.5$ calg^{-1}°C^{-1}, Specific heat of water $(s_w) = 1$ calg^{-1}°C^{-1}, Latent heat of ice $(L_i) = 80$ calg^{-1}, Latent heat of steam $(L_s) = 540$ calg^{-1}

Solution:

Given that;

Mass of ice $(m_i) = 50$ g

Mass of water $(m_w) = 200$ g

Water equivalent of calorimeter $(W) = (m_c s_c) = 10$ cal°C^{-1}

Initial temperature of ice (t_1) = 0°C

Final temperature of mixture (t) = 20°C

Since, water equivalent is given in C.G.S. system it is required to solve in C.G.S. system. We have,

Specific heat of ice (s_i) = 0.5 $calg^{-1}°C^{-1}$

Specific heat of water (s_w) = 1 $calg^{-1}°C^{-1}$

Latent heat of ice (L_i) = 80 $calg^{-1}$

Latent heat of steam (L_s) = 540 $calg^{-1}$

Mass of steam required (m_s) = ?

The total heat lost by steam and to decrease temperature to 20°C is;

$= m_sL_s + m_ss_w(100 - 20) = m_s \times 540 + m_s \times 1 \times 80 = 620\ m_s$

The total heat gained by ice, calorimeter, and water is;

$= m_iL_i + m_is_w \times 20 + W \times 20 + m_ws_w \times 20$

$= 50 \times 80 + 50 \times 1 \times 20 + 10 \times 20 + 200 \times 1 \times 20 = 9200$ cal

∴ Total Heat gained = Total heat lost

or, $9200 = 620\ m_s$

or, $m_s = 14.84$ gm

Therefore, mass of steam to pass is 14.84 gm.

d) **The pendulum of a clock is made of brass. If the clock keeps the correct time at 15°C, how many seconds per day will it lose at 20°C? Given; Linear expansivity (α) = $19 \times 10^{-6}\ K^{-1}$**

Solution:

Given that;

Linear expansivity (α) = $19 \times 10^{-6}\ K^{-1}$

Let, Time period at 15°C (T_{15}) = 2 sec.

l_{15} and l_{20} be the length of the pendulum at 15°C and 20°C respectively.

Then, we have,

$$T_{15} = 2\pi\sqrt{\frac{l_{15}}{g}} \qquad (1)$$

$$T_{20} = 2\pi\sqrt{\frac{l_{20}}{g}} \qquad (2)$$

Dividing equation (2) by (1); we get,

$$\frac{T_{20}}{T_{15}} = \sqrt{\frac{l_{20}}{l_{15}}} = \sqrt{\frac{l_{15}(1 + 5\alpha)}{l_{15}}}$$

or, $T_{20} = T_{15}\left(\sqrt{1 + 5\alpha}\right) = 2\left(\sqrt{1 + 5 \times 19 \times 10^{-6}}\right)$

$= 2 \times 1.0000475 = 2.000095$

Since, time period for oscillation of the pendulum increase the time of clock.

Loss of time in 2 sec. = 2.000095 − 2 = 0.000095 = 95×10^{-6} sec.

∴ In 24 hours, lost time $= \dfrac{95 \times 10^{-6}}{2} \times 24 \times 60 \times 60 = 4.104$ sec.

e) **A small object placed in front of a concave spherical mirror gives an image that is real and four times the size of the object. When the object is moved 10 cm towards the mirror, a virtual image of same size is formed. Find the focal length of the mirror.**

Solution:

Given that;

Magnification (m) = 4

Now,

$$m = \frac{v}{u} = 4$$

$\therefore$ $v = 4u$

Hence, from general mirror formula; we have,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or, $$\frac{1}{f} = \frac{1}{u} + \frac{1}{4u} = \frac{4+1}{4u} = \frac{5}{4u} \quad (1)$$

Similarly, when object is moved towards the mirror by 10 cm, an object distance will be;

$$u - 10 = u_1 \text{ (say)}$$

We have,

$$m_1 = \frac{v_1}{u_1} = -4 \text{ [Since, same size and virtual image]}$$

$\therefore$ $v_1 = -4u_1 = -4(u - 10)$

Again using the general formula, we get,

$$\frac{1}{f} = \frac{1}{u_1} + \frac{1}{v_1}$$

or, $$\frac{1}{f} = \frac{1}{(u-10)} - \frac{1}{4(u-10)} = \frac{3}{4(u-10)} \quad (2)$$

Equating relations (1) and (2); we get,

$$\frac{5}{4u} = \frac{3}{4(u-10)}$$

$\therefore$ $u = 25$ cm

Substituting the value of 'u' in equation (1); we get,

$$\frac{1}{f} = \frac{5}{4 \times 25} = \frac{1}{20}$$

$\therefore$ $f = 20$ cm

Hence, the focal length of the mirror is 20 cm.

f) **The refractive index of glass prism is 1.62. If the angle of minimum deviation is 42°, what would be the angle of the prism?**

Solution:

Given that;

Refractive index (μ) = 1.62

Angle of minimum deviation (δ_m) = 42°

Angle of prism (A) = ?

Now, we have from the relation,

$$\mu = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

EXAMINATION 2064
BACK

Full marks: 60
Pass marks: 24

Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.
Attempt All questions.
Specific heat of water (s_w) = 4200 $Jkg^{-1}K^{-1}$
Specific heat capacity of aluminium = 910 $Jkg^{-1}K^{-1}$
Specific heat capacity of iron = 470 $Jkg^{-1}K^{-1}$
Horizontal component of the earth's magnetic field = 3.4×10^{-5} T
$\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$
Relative molecular mass of hydrogen = 2 gm
Relative molecular mass of oxygen = 32 gm
Assume suitable data if necessary

1. **How does the acceleration due to gravity vary with distance above and below from the surface of the earth? Where is the maximum value?**

Ans: See the solution of Q. no. 1 of 2062 Back on page no. 61

Or

What is the centrifugal force? Derive an expression for the magnitude of this force.

Ans: An outward force experienced by a body in circular path is called centrifugal force. The value depends on the mass of the object, the speed of rotation, and the distance from the center.

For every action there is an equal and opposite reaction. Centripetal force, an action, has the reaction of centrifugal force. The two forces are equal in magnitude and opposite in direction. Hence, the magnitude of the centrifugal force is also given by the relation as centripetal force.

Expression for the centrifugal force

See the solution of Q. no. 1 of 2061 R/B on page no. 37

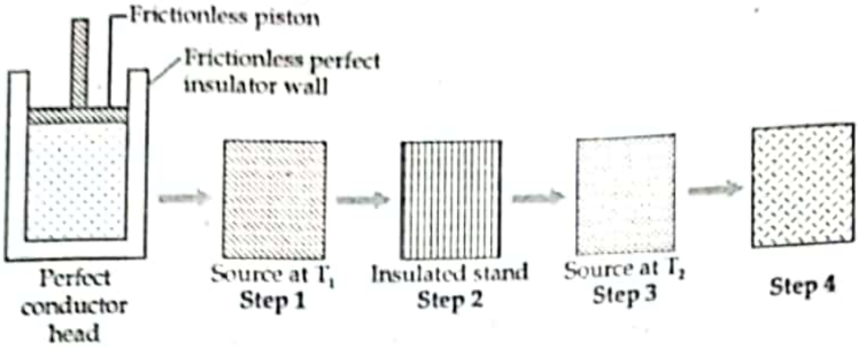
2. **What is Carnot engine? Derive an expression for efficiency of Carnot engine using PV diagram.**

Ans: Carnot engine

An ideal heat engine working in a Carnot cycle having maximum efficiency that is possible from any heat engine is called Carnot engine. The basic model for this engine was developed by Nicolas Leonard Sadi Carnot in 1824. It can operate between two specified temperatures.

Carnot cycle and efficiency

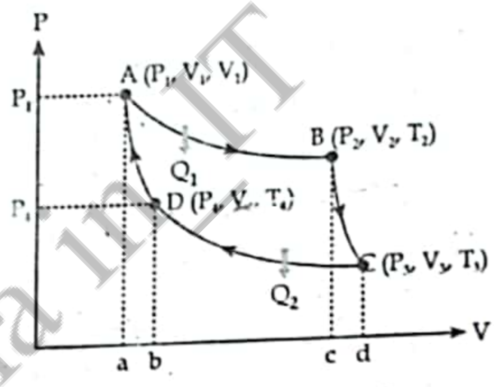
Reversible working cycle for Carnot engine is called Carnot cycle. In a Carnot cycle, work done is converted from portion of heat taken from reservoir at higher temperature and remaining portion Q_2 is rejected to sink. The Carnot cycle consists of two isothermal processes and two adiabatic processes. Carnot engine consists of a perfect non–conducting and frictionless piston fitted into a cylinder made from perfect conductor except its head. The head of the piston is made from perfect conductor. Again, heat source reservoir at a temperature T_1, another sink at a lower fixed temperature T_2 and an insulator stands is provided. The system uses a working substance as a gas or gas formed by ignition of fuel; the working substance undergoes a complete cycle of operation known as Carnot's cycle. The cycle takes place in four steps as follows:



P–V diagram

Step 1

Let the initial pressure volume and temperature at equilibrium is respectively P_1, V_1 and T_1 as shown by point 'A' on P–V diagram. Now, place the engine on the reservoir at temperature T_1 and allow the gas to expand slowly to P_2, V_2 and T_2 shown by point 'B'. During the process heat Q_1 is absorbed from the source to keep the temperature constant T_1 through the base. The expansion is isothermal at T_1 and gas does work raising the piston.



Step 2

Remove the engine from the source and put on the non-conducting stand and allow the gas to expand slowly to P_3, V_3 and T_3 represented by point 'C'. Hence, no heat can enter nor leave the system as it is completely surrounded by insulators. So, work is done by the expansion in adiabatic process from 'B' to 'C'. At 'C', temperature falls from T_1 to T_2.

Step 3

Now, place the engine on the sink at temperature T_2 and compress the gas to P_4, V_4 and T_2 as shown by point 'D' in the curve. During the process heat, Q_2 is transferred from the gas to the sink through the base. The work is done on the gas and the compression is isothermal at T_2. Finally the point 'D' is reached.

Step 4

Lastly, place the engine on the insulating stand, compress adiabatically to state A since no heat can enter nor leave the system. Work is done on the gas and its temperature rises to T_1 and is represented by the curve DA completing the cycle. The process represented by ABCDA in figure is the Carnot cycle.

Efficiency of ideal gas Carnot cycle

During each cycle Carnot engine takes Q_1 amount of heat from the source and Q_2 is transferred to sink, the remaining $Q_1 - Q_2$ is converted to useful work.

$$\therefore \quad \text{Efficiency of engine } (\eta) = \frac{\text{Useful work output (w)}}{\text{Input } (Q_1)} \times 100\%$$

$$= \frac{Q_1 - Q_2}{Q_1} \times 100\% = \left(1 - \frac{Q_2}{Q_1}\right) \times 100\%$$

Again,

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

The relation shows that the efficiency of heat engine is always less than 100% so long as heat is delivered to the sink *i.e.,* Q_2 is not zero, which is practically impossible. Therefore, efficiency of heat engine can never be cent percent efficient. Again, for η to be 100%, $T_2 = 0$, that is practically impossible.

3. **State and derive tangent law. Discuss how you use this law in deflection magnetometer to find the magnetic moment of a bar magnet.**

Ans: State and derive tangent law

See the solution of Q. no. 4 (g) of 2063 R/B on page no. 72

Use in deflection magnetometer

A magnetometer can be used to find magnetic moments. They are;

i) Tan A position
ii) Tan B on position

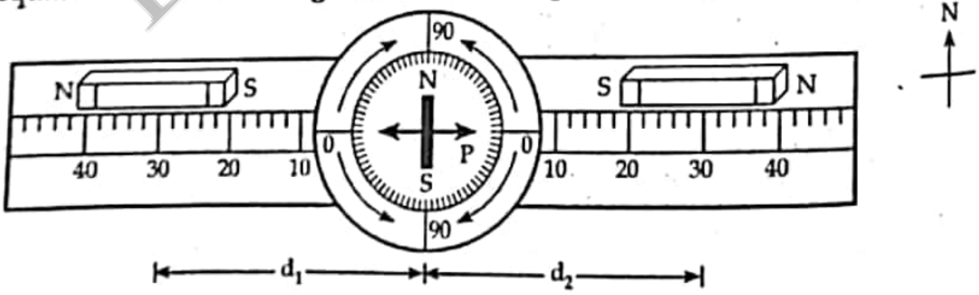
To find magnetic moment of a bar magnet

Magnetic moments of two magnets can be determined by using a deflection magnetometer in the Tangent A or Tangent B position as explained below.

Tangent A–Position of Magnetometers

i) Deflection Method

In this case the magnetometer is placed along east west direction or parallel to the needle. Then rotate the box until end of the pointer shows zero–zero readings. Now, take a magnet and placed in east arm pointing its north pole to east. The centre of the magnet is kept at a known distance 'd' from the centre of the needle, and noted the deflections for both the ends of the pointer. Reverse the poles of the magnet so that its north and south poles interchange their places and again noted both the ends of the pointer. Find the mean deflection θ of these readings. Since, the magnetic needle now lies on the equatorial line of the magnet then according to the tangent law,



$$F = H \tan\theta$$

But, $$F = \frac{\mu_0}{4\pi}\frac{2Md}{\left(d^2 - l^2\right)^2}$$

$$\therefore \quad \frac{\mu_0}{4\pi}\frac{2Md}{\left(d^2 - l^2\right)^2} = H \tan\theta$$

or, $$M = \frac{4\pi\left(d^2 - l^2\right)^2 H \tan\theta}{2\mu_0 d}$$

$$\therefore \quad H\tan\theta = \frac{2\mu_0 M}{4\pi d^3}$$

and, for small magnets; we have,

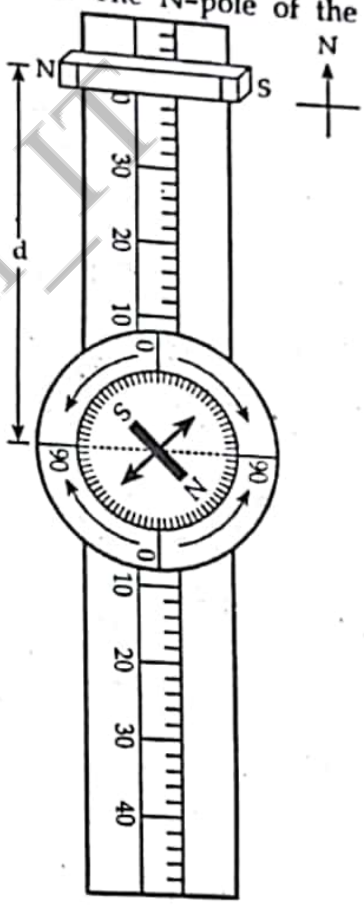
$$F = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

$$\text{or,} \quad M = \frac{4\pi d^3 H\tan\theta}{2\mu_0}$$

From the relation the magnetic moment is calculated.

Tangent B–Position

The magnetometer is set in north–south direction and the box is rotated until the needles shows 90°–90° deflection. Then take a magnet and is placed across north–arm so that the centre of the magnet is being placed at a particular distance d from the centre of the needle. The N–pole of the magnet pointing towards east direction and south pole towards west direction. The mean deflection θ is determined as above. Since, we have for this broad side on position;



From tangent law; we have,

$$F = H\tan\theta_1$$

In this case

$$F = \frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{\frac{3}{2}}}$$

$$\therefore \quad \frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{\frac{3}{2}}} = H\tan\theta_1$$

$$\text{or,} \quad M = \frac{4\pi(d^2 + l^2)^{\frac{3}{2}} H\tan\theta_1}{\mu_0}$$

For small magnets; we have,

$$F = \frac{\mu_0}{4\pi}\frac{M}{d^3} = H\tan\theta_1$$

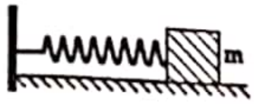
$$\therefore \quad M = \frac{4\pi d^3 H\tan\theta_1}{\mu_0}$$

From the relations 'M' can be calculated for long and short magnets as required by using deflection magnetometer.

4. **Answer any six questions:**

a) **Show that motion of a body on a horizontal frictionless table, which is attached at one end for a compressed spring and the other end attached at a rigid support, is simple harmonic motion. Find the period also.**

Ans: Let a mass attached with light horizontal spring slides over a horizontal frictionless surface. Now if the mass is pulled a little then released, it vibrates in horizontal direction.



Let 'x' is the extension produced in one side due to application of force 'F'. Then, mass experiences a restoring force given by Hooke's law. The magnitude of restoring force is directly proportional to the displacement of system from its initial or mean position

i.e., $F \propto x$

$\therefore \quad F = -Kx \quad (1)$

where, 'K' is the spring constant.

The negative sign indicates for restoring force.

According to Newton's second law; we have,

$F = ma \quad (2)$

From equation (1) and (2); we get,

$ma = -Kx$

or, $a = -\frac{Kx}{m}$

Since, $\frac{K}{m}$ is constant quantity.

$\therefore \quad a \propto x$

This shows acceleration 'a' is directly proportional to the displacement 'x' which is according to the statement of S.H.M. Hence, motion of is in S.H.M.

Time period

The time period 'T' is given by;

$T = \frac{2\pi}{\omega} \quad (1)$

For a particle in S.H.M.;

$a = \omega^2 y \quad (2)$

Comparing equation (1) and (2); we get,

$$\omega = \sqrt{\frac{K}{m}}$$

Hence,

$$T = 2\pi\sqrt{\frac{m}{K}}$$

Again,

$mg = Ke$

where, 'e' is the extension on spring.

$$\therefore \quad T = 2\pi\sqrt{\frac{e}{g}}$$

b) Define moment of inertia of a rigid body. Derive the kinetic energy of rotation of rigid body.

Ans: *See the solution of Q. no. 1 (or) of 2060 R/B on page no. 12*

c) Prove that the total mechanical energy for a freely falling body under the gravity is constant.

Ans: *See the solution of Q. no. 4 (a) of 2060 R/B on page no. 15*

d) State and derive the Stefan's law of black body radiation. If the body is not perfectly black. How do you change the law?

Ans: *See the solution of Q. no. 4 (d) of 2060 R/B on page no. 17*

e) Define coefficient of apparent and real expansion of liquid. Derive a relation between them.

Ans: *See the solution of Q. no. 2 of 2060 R/B on page no. 13*

f) What is apparent depth and why does it occur in transparent liquid and solid? Derive a relation for it.

Ans: **Apparent Depth**

An object submerged in transparent liquid appears closer depth than its real one and is called apparent depth. Apparent depth is due to the refraction of light. The light rays from the object inside the transparent medium leaving the transparent medium bends away from the normal. Thus, an observer sees the object position is slightly higher than its real depth.

Relation for apparent depth

See the solution of Q. no. 4 (h) of 2062 B/B on page no. 57

g) **Show that inverse of combined focal length of two thin lenses placed in contact is the sum of focal length of each lens.**

Ans: See the solution of Q. no. 4 (g) of 2060 Back on page no. 32

5. **Solve any six problems:**

a) **A bullet of mass 10 gm traveling horizontally with a velocity of 300 ms^{-1} strikes a block of mass 290 gm, which rest on the rough horizontal floor. After impact, the bock and the bullet move together and come to rest when the block has traveled a distance of 15 m. Calculate the coefficient of sliding friction between floor and block.**

Solution:

Given that

Mass of bullet (m) = 10 gm = 0.01 kg

Initial velocity of bullet (v) = 300 ms^{-1}

Mass of wooden block (M) = 290 gm = 0.29 kg

Co efficient of sliding friction (μ) = ?

If V be the final velocity of the block and the bullet together then from principle of conservation of momentum we have,

Initial momentum of bullet = Final momentum of block and bullet

i.e., $mv = (m + M)V$

or, $V = \frac{mv}{(m + M)} = \frac{0.01 \times 300}{(0.01 + 0.29)} = 10 \text{ ms}^{-1}$

The block and bullet first move together with the velocity 10 ms^{-1} and come to rest after traveling through a distance 15 m. Hence, for next case;

Distance traveled (S) = 15 cm

Initial velocity (u) = 10 ms^{-1}

Final velocity (v) = 0

We have,

$v^2 = u^2 - 2aS$ [Since, retardation]

or, $a = \frac{v^2 - u^2}{2S} = \frac{(10)^2 - (0)^2}{2 \times 15} = \frac{10}{3} \text{ ms}^{-2}$

Now,

Retarding frictional force (F) $= \text{Total mass} \times \text{Acceleration}$

$= (m + M) \times a$

$= (0.01 + 0.29) \times \frac{10}{3} = 1 \text{ N}$

Again,

Normal reaction (R) = (m + M)g = (0.01 + 0.29) × 9.8 = 2.94

∴ Co-efficient of sliding friction (μ) = $\frac{F}{R} = \frac{1}{2.94} = 0.34$

Hence, required value of coefficient of sliding friction between the block the floor is 0.34.

b) A rubber ball of mass 400 gm falls from a height of 5 m rebounds to a height of 4.5 m. Find the impulse and the average between the ball and the ground if the time during which they contact was 0.2 sec.

Solution:

Given that;

Mass of ball (m) = 400 gm = 0.4 kg

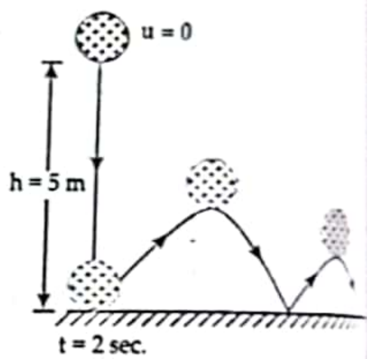
Initial velocity (u) = 0

Height dropped (h) = 5 m

Acceleration due to gravity (g) = 10 ms^{-2}

Now, from the equation of motion; we have,

$$v^2 = u^2 + 2gh$$
$$= (0)^2 + 2 \times 10 \times 5 = 100$$
$$\therefore \quad v = 10\ ms^{-1}$$



Again,

Ball rebounds to height (h′) = 4.5 m

Time of impact (t) = 0.2 sec

Initial velocity (u) = ?

Final velocity (v) = 0

We know,

$$v^2 = u^2 + 2gh$$
or, $$0 = u^2 - 2 \times 10 \times 4.5$$
or, $$u^2 = 90$$
$$\therefore \quad u = 9.49\ ms^{-1}$$

Now,

$$\text{Force of impact} = m\left(\frac{v-u}{t}\right) = 0.4 \times \left(\frac{10 - 9.49}{0.2}\right) = 1.02\ N$$

and, Impulse = Ft = 1.02 × 0.2 = 0.204 Ns^{-1}

c) The r.m.s. speed of hydrogen gas s.t.p. is 1840 ms^{-1}. Find the r.m.s. speed of oxygen molecule at 27°C.

Solution:

Given that;

r.m.s. speed of hydrogen at 0°C (C_1) = 1840 ms^{-1}

Pressure at S.T.P. (P_0) = 1.013 × 10^5 Nm^{-2}

Temperature (T_1) = 0°C = 273 K

r.m.s. speed of oxygen at 27°C (C_O) = ?

Temperature (T_2) = 27°C = 300 K

Relative molecular mass of hydrogen (M_H) = 2 gm

Relative molecular mass of oxygen (M_O) = 32 gm

We know that,

r.m.s. speed of a gas (C) $\propto \sqrt{T}$

i.e., $C_1 \propto \sqrt{T_1}$ (1)

and, $C_2 \propto \sqrt{T_2}$ (2)

where, C_1 and C_2 are r.m.s. speed of hydrogen at 0°C and 27°C respectively.

Dividing equation (2) by (1); we get,

$$\frac{C_2}{C_1} = \sqrt{\frac{T_2}{T_1}}$$

or, $C_2 = \sqrt{\frac{T_2}{T_1}} \times C_1$

$$= \sqrt{\frac{300}{273}} \times 1840 = 1928.84 \text{ ms}^{-1}$$

Again, we know at constant temperature,

$$C \propto \sqrt{\frac{1}{M}}$$

i.e., $\frac{C_O}{C_2} = \sqrt{\frac{M_H}{M_O}}$

or, $C_O = \sqrt{\frac{M_H}{M_O}} \times C_2 = \sqrt{\frac{2}{32}} \times 1928.84 = 482.21 \text{ ms}^{-1}$

Therefore, r.m.s. speed of oxygen molecule at 27°C is 482.21 ms^{-1}.

d) **An aluminium can of mass 500 gm contains 117.5 gm of water at a temperature of 20°C. A 200 gm block of iron at 75°C is dropped into the can. Find the final temperature of the mixture, assuming no heat loss to the surrounding.**

olution:

Given that;

Mass of the iron block (m_i) = 200 gm = 0.2 kg

Mass of the aluminium can (m_c) = 500 gm = 0.5 kg

Mass of the water taken (m_w) = 117.5 gm = 0.1175 kg

Initial temperature of can and water (t_1) = 20°C

Initial temperature of hot solid (t_2) = 75°C

Specific heat of water (s_w) = 4200 $\text{Jkg}^{-1}\text{K}^{-1}$

Specific heat capacity of aluminium calorimeter (s_c) = 910 $\text{Jkg}^{-1}\text{K}^{-1}$

Specific heat capacity of iron (s_i) = 470 $\text{Jkg}^{-1}\text{K}^{-1}$

Final temperature of mixture (t) = ?

Now, according to the given condition; we have,

Heat lost by iron = Heat gained by can and water

or, $m_i s_i (75 - t) = m_c s_c (t - 20) + m_w s_w (t - 20)$

or, $0.2 \times 470(75 - t) = 0.5 \times 910(t - 20) + 0.1175 \times 4200(t - 20)$

or, $1042.5t = 26020$

$\therefore$ $t = 24.96°C$

Therefore, final temperature of mixture is 24.96°C.

e) **A concave mirror forms, on a screen, an image of twice the linear ... of the object. The object and the screen are then moved until ... image is three times the size of the object. If the shift of the screen ... 20 cm, determine the focal length of the mirror.**

Solution:

Given that;

Magnification (m) = 2

We know when the image is twice the linear dimension of the object. ... magnification 'M' is given by;

$$m = \frac{v}{u} = 2$$

$$\therefore \quad u = \frac{v}{2}$$

From general mirror formula; we have,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{or,} \quad \frac{1}{f} = \frac{1}{\frac{v}{2}} + \frac{1}{v} = \frac{2+1}{v} = \frac{3}{v}$$

Similarly, when screen and object are moved to get magnification = 3.

We have,

$$m_1 = \frac{v_1}{u_1} = 3$$

$$\therefore \quad u_1 = \frac{v_1}{3}$$

Again using the general formula; we get,

$$\frac{1}{f} = \frac{1}{u_1} + \frac{1}{v_1}$$

$$\text{or,} \quad \frac{1}{f} = \frac{3}{v_1} + \frac{1}{v_1} = \frac{3+1}{v_1} = \frac{4}{v_1}$$

Equating relations **(1)** and **(2)**; we get,

$$\frac{3}{v} = \frac{4}{v_1}$$

But according to the question; we have,

$$v_1 = v + 20$$

$$\text{so,} \quad \frac{3}{v} = \frac{4}{v+20}$$

$$\text{or,} \quad 3v + 60 = 4v$$

$$\therefore \quad v = 60 \text{ cm}$$

Substituting the value of 'v' in equation **(1)**; we get,

$$\frac{1}{f} = \frac{3}{v} = \frac{3}{60}$$

$$\therefore \quad f = 20 \text{ cm}$$

Therefore, the focal length of the mirror is 20 cm.

f) **The refractive index of the material of a prism is 1.5. When the prism is placed in minimum deviation position, the angle of incidence is 48.6°. Calculate the angle of prism and the angle of minimum deviation.**

Solution:

Given that;

Refractive index (μ) = 1.5

For minimum deviation;

Angle of incidence ($\angle i$) = 48.6°
Minimum deviation (δ_m) = ?
Angle of prism (A) = ?

We have, angle of incidence at minimum deviation position will be given by;

$$\frac{\delta_m + A}{2} = i$$

or, $$\frac{\delta_m + A}{2} = 48.6°$$

Now, we have,

$$\mu = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$= \mu = \dfrac{\sin \frac{A + D_m}{2}}{\sin \frac{A}{2}}$

or, $$1.5 = \frac{\sin 48.6°}{\sin\left(\frac{A}{2}\right)}$$

or, $$1.5 = \frac{0.75}{\sin\left(\frac{A}{2}\right)}$$

or, $$\sin\left(\frac{A}{2}\right) = \frac{0.75}{1.5} = 0.5$$

$\therefore 1.5 = \dfrac{\sin \frac{A + 48.6}{2}}{\sin \frac{A}{2}}$

or, $$\sin\left(\frac{A}{2}\right) = \sin 30°$$

or, $$\frac{A}{2} = 30°$$

$\therefore$ A = 60°

or $1.5 \; \dfrac{0.75}{\sin\left(\frac{A}{2}\right)}$

or $\sin \frac{A}{2} = \frac{0.75}{1.5} = 0.5$

Again; we have,

$$\frac{\delta_m + A}{2} = 48.6°$$

or, $$\frac{\delta_m + 60°}{2} = 48.6°$$

or, $$\delta_m + 60° = 97.2°$$

$\therefore$ $\delta_m = 37.2°$

Hence, angle of prism and the angle of minimum deviation respectively are 60° and 37.2°.

g) **A bar magnet 6 cm long is kept its north pole pointing north. A neutral point is found at a distance of 25 cm from each pole. Calculate pole strength of the magnet.**

Solution: See the solution of Q. no. 5 (g) of 2064 R/B on page no. 87

Full marks
Pass marks

Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.
Attempt All questions.
$g = 10\ ms^{-2}$
Coefficient of linear expansion of brass = $0.000018\ K^{-1}$
$\mu = 4\pi \times 10^{-7}\ Hm^{-1}$
Horizontal component of the earth's magnetic field = $3.4 \times 10^{-5}\ T$

1. **Define linear momentum and derive its dimensions. State the law of conservation of linear momentum. When two bodies of masses m_1 and m_2 moving in same direction collide, prove that total momentum before collision is equal to total momentum after collision.**

Ans: **Linear momentum**

See the solution of Q. no. 4 (a) of 2062 Back on page no. 63

Law of conservation of linear momentum

The principle of conservation of linear momentum states that, "if no external force act on a system of collision, total momentum will be always conserved".

Verification of the principle from the laws of motion

See the solution of Q. no. 3 (a) of 2061 Back on page no. 48

Or

Define simple harmonic motion and frequency. Show that motion of bob of simple pendulum with small displacement is simple harmonic and derive its time period.

Ans: **Simple harmonic motion**

See the solution of Q. no. 1 (or) of 2062 Back on page no. 62

Frequency

The numbers of oscillations made per second is called the frequency. The frequency is measured in unit called hertz (Hz).

Motion of simple pendulum

See the solution of Q. no. 1 of 2060 Back on page no. 24

2. **What is lens maker's formula? Derive this formula with suitable assumptions.**

Ans: See the solution of Q. no. 7 (a) of 2061 Back on page no. 53

3. **What are assumptions in kinetic theory of gas? Derive $P = \frac{1}{3}\rho c^2$.**

Ans: See the solution of Q. no. 2 of 2060 Back on page no. 26

Add the following

We know that;

$$P = \frac{mNc^2}{3V} = \frac{1}{3}\frac{Mc^2}{V}$$

where, $M = mN$ (*i.e.*, total mass of the gas).

Let, ρ be the density of the gas, then,

$$\rho = \frac{M}{V}$$

$$\therefore \quad P = \frac{1}{3}\rho c^2$$

4. Answer any six questions:

a) **Define angular velocity and mention its S.I. units. Derive a formula to establish a relation between linear velocity and angular velocity in circular motion.**

Ans: See the solution of Q. no. 4 (b) of 2062 R/B on page no. 56
The S.I. unit for angular velocity is radians per second $(rads^{-1})$.

b) **Define moment of inertia of a rigid body. Establish a relation between kinetic energy and moment of inertia of rigid body.**

Ans: See the solution of Q. no. 1 (or) of 2060 R/B on page no. 12

c) **What is thermal conductivity? Derive formula for thermal conductivity.**

Ans: See the solution of Q. no. 4 (e) of 2060 Back on page no. 31

d) **State Newton's law of cooling and derive its differential equation.**

Ans: See the solution of Q. no. 4 (a) of 2061 Back on page no. 49

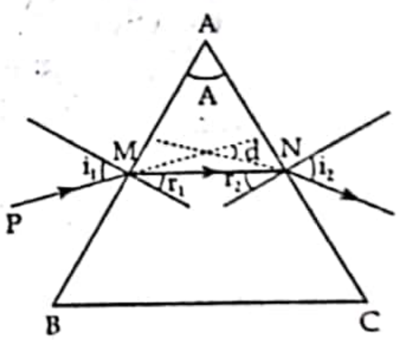
e) **Define first law of thermodynamics. On which principle does it depend? Write its formula.**

Ans: See the solution of Q. no. 4 (d) of 2060 Back on page no. 30

f) **Derive a relation for deviation produced by a small angled prism.**

Ans: If angle of a prism is very small then the type of prism is called a small angled prism. Let ABC be a small angle prism and an incident ray PM strikes the prism almost normally on the face AB. If 'A' is the angle of prism and i_1 be the small angle of incidence. Then, the angles r_1, r_2 and i_2 also will be small angle. Hence, we can consider,

$$\sin i_1 = i_1$$
$$\sin i_2 = i_2$$
$$\sin r_1 = r_1$$
$$\sin r_2 = r_2$$



Now, we have,

$$n = \frac{\sin i_1}{\sin r_1} = \frac{i_1}{r_1}$$

$$\therefore \quad i_1 = nr_1$$

and, $$n = \frac{\sin i_2}{\sin r_2} = \frac{i_2}{r_2}$$

$$\therefore \quad i_2 = nr_2$$

But, angle of deviation 'd', of the ray passing through the prism is given by;

$$d = (i_1 - r_1) + (i_2 - r_2) = (nr_1 - r_1) + (nr_2 - r_2)$$
$$= r_1(n - 1) + r_1(n - 1)$$
$$= (n - 1)(r_1 + r_2)$$

But, $r_1 + r_2 = A$

$$\therefore \quad d = (n - 1)A$$

For small angle prism the deviation 'δ' produced is independent of the angle of incidence and is given by $d = (n - 1)A$. It depends on the refractive index 'n' and angle of prism 'A'.

g) **State and derive tangent law in magnetism.**

Ans: See the solution of Q. no. 4 (g) of 2063 R/B on page no. 72

5. **Solve only six problems.**

a) **A truck of mass 3000 kg moves up an inclined surface at a constant speed 36 kmhr^{-1} against a frictional force of 200 N. The inclination such that it rises 1 m for every 15 m along the incline. Calculate the power developed by the truck engine.**

Solution:

Given that;

Mass of the truck (m) = 3000 kg

Inclination $(\sin\theta) = \frac{p}{h} = \frac{1}{15}$

Speed of the truck (v) $= 36\ \text{kmhr}^{-1} = \frac{36 \times 1000}{60 \times 60} = 10\ \text{ms}^{-1}$

Acceleration due to gravity $(g) = 10\ \text{ms}^{-2}$

Friction force (f) = 200 N

Now, resolving mg into two components $mg\cos\theta$ and $mg\sin\theta$.

The total force to be applied by truck is given by;

$$F = f + mg\sin\theta = 200 + 3000 \times 10 \times \frac{1}{15} = 2200\ \text{N}$$

Since, the truck moves with speed of 10 ms^{-1}; we have,

Power to be developed by truck $= Fv = 2200 \times 10 = 2.2 \times 10^4$ W

b) **Mount Everest is 8848 m above the sea level. Estimate the acceleration due to gravity at the height, give the mean g on the surface of earth is 9.8 ms^{-2} and mean radius of the earth 6.37×10^6 m. Ignore variation of 'g' due to the rotation of the earth.**

Solution:

Given that;

Height of the Mount Everest (h) = 8848 m

Acceleration due to gravity at the Mount Everest $(g_E) = ?$

Mean acceleration due to gravity $(g) = 9.8\ \text{ms}^{-2}$

Radius of the earth (R) $= 6.37 \times 10^6$ m

The distance from centre of the earth to the top of Mount Everest is (R + h).

The body is at a distance (R + h) from the center of the Earth the acceleration due to gravity at the Mount Everest is given by;

$$g_E = \frac{gR^2}{(R+h)^2} = \frac{9.8 \times (6.37 \times 10^6)^2}{(6.37 \times 10^6 + 8848)^2} = 9.77\ \text{ms}^{-2}$$

i.e., acceleration due to gravity on the Mount Everest will be 9.77 ms^{-2}.

c) **A brass disc at 20°C has diameter of 30 cm and a hole cut in the centre is 10 cm in diameter. Calculate the diameter of the hole when the temperature of the disc is raised to 50°C.**

Solution:

Given that;

Diameter of disc (D) = 30 cm

Diameter of hole (d) = 10 cm
Rise in temperature (t) = 50°C − 20°C = 30°C
Final diameter of the hole (d_t) = ?

When a metallic disk with hole is heated, all the dimensions, including the diameter of the hole increases. That is because during expansion with the rise in temperature, thermal expansion is linear expansion in every direction in same fraction. So, the hole expand same amount as the metal block around.

∴ Increase in diameter of disc = Increase in diameter of hole

i.e. $D\alpha t = d\alpha t$

Hence, final diameter is given by;

$$d_t = d + D\alpha t = d + d\alpha t$$

α is the linear expansivity of brass and is equal to $1.8 \times 10^{-5}\ °C^{-1}$.

$$d_t = d + d\alpha t = 10 + 10 \times 1.8 \times 10^{-5} \times 30 = 10.0054 \text{ cm}$$

∴ Diameter of the hole will be 10.0054 cm.

d) **A Carnot engine absorbs 1000 J of heat from a reservoir at 127°C and rejects 600 J of heat during each cycle. Calculate temperature of sink.**

Solution:

Given that;

Heat taken from reservoir (Q_1) = 1000 Joule
Heat rejected (Q_2) = 600 Joule
Temperature of reservoir (T_1) = 127°C = 127 + 273 = 400 K
Temperature of sink (T_2) = ?

We know,

Thermal efficiency (η) of a Carnot engine is given by;

$$\eta = 1 - \frac{Q_2}{Q_1} \quad (1)$$

and, $$\eta = 1 - \frac{T_2}{T_1} \quad (2)$$

By equating equation (1) and (2); we get,

$$\frac{T_2}{T_1} = \frac{Q_2}{Q_1}$$

$$\therefore \quad T_2 = \frac{Q_2 T_1}{Q_1} = \frac{600 \times 400}{1000} = 240 \text{ K}$$

Therefore, temperature of the sink is 240 K.

e) **A concave mirror forms a virtual image two times the size of the object placed 10 cm away from the pole of the mirror. Find the location of the image when the object is 30 cm away from the mirror.**

Solution:

Given that;

Magnification (m) = −2 (virtual image)
Object distance (u) = 10 cm

Now,

$$m = \frac{v}{u} = -2$$

$$\therefore \quad v = -2u = -2 \times 10 = -20 \text{ cm}$$

From general mirror formula; we have,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or, $$\frac{1}{f} = \frac{1}{10} - \frac{1}{20} = \frac{2-1}{20} = \frac{1}{20}$$

$\therefore$ $f = 20$ cm

Similarly, when object is placed at distance of 30 cm,

$$\frac{1}{f} = \frac{1}{u_1} + \frac{1}{v_1}$$

or, $$\frac{1}{v_1} = \frac{1}{f} - \frac{1}{u_1} = \frac{1}{20} - \frac{1}{30} = \frac{3-2}{60} = \frac{1}{60}$$

$\therefore$ $v_1 = 60$ cm

Therefore, the image distance is 60 cm.

f) The refractive index of glass and water are $\frac{3}{2}$ and $\frac{4}{3}$ respectively. Calculate the critical angle in glass-water interface.

Solution:

Given that;

Refractive index of glass $({}_a\mu_g) = \frac{3}{2} = 1.5$

Refractive index of water $({}_a\mu_w) = \frac{4}{3} = 1.33$

We have,

$$ {}_g\mu_w = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{1.5}{1.33} = 1.128$$

The critical angle in glass-water interface is given by;

$$\sin C = \frac{1}{{}_g\mu_w} = \frac{1}{1.128} = 0.886$$

$\therefore$ $C = \sin^{-1}(0.886) = 62.44°$

Therefore, the required critical angle in glass-water interface is 62.44°.

g) A magnetic dipole of moment 1.44 Am^2 is placed horizontally with the north pole pointing towards north. Find the position of the neutral point, assuming a very short magnet.

Solution:

Given that;

Magnetic moment (M) = 1.44 Am^2

Horizontal component of earth's field (H) = 0.35×10^{-4} T

$\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}

Distance to neutral point (d) = ?

This is the broadside on position of the magnet, at neutral point for small magnet; we have,

On the axial line; we have,

$$H = \frac{\mu_0}{4\pi} \times \frac{M}{d^3}$$

or, $$d = \left(\frac{\mu_0}{4\pi} \times \frac{M}{H}\right)^{\frac{3}{2}} = \left(\frac{4\pi \times 10^{-7}}{4\pi} \times \frac{1.44}{0.35 \times 10^{-4}}\right)^{\frac{3}{2}} = 2.63 \times 10^{-4} \text{ m}$$

i.e., position of neutral point is 2.63×10^{-4} m from the magnet.

EXAMINATION 2065
BACK

Full marks: 60
Pass marks: 24

Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.
Attempt All questions.

1. **State and explain the parallelogram law of force with special case.**

Ans: Parallelogram law of force states that, "if two forces acting simultaneously at the same point of a body or a particle can be represented in magnitude and direction by two adjacent sides of a parallelogram drawn from a point, then their sum or resultant is represented in magnitude and direction completely by diagonal of the parallelogram passing through their points of intersection."

In figure below F_1 and F_2 are two forces acting on a body, and 'O' is the point of intersection of their lines of action, if F_1 and F_2 are represented in magnitude and direction by the straight lines OA, OB respectively as shown in the figure.

Now, to find resultant of F_1 and F_2 completed the parallelogram OACB and drawn a diagonal OC through 'O'. The diagonal OC represents the resultant 'R' of the forces F_1 and F_2 in magnitude and direction.

If $\measuredangle AOB = \measuredangle CAD = \theta$

and, $\measuredangle AOC = \alpha$

We have from trigonometry,

$$OC^2 = OD^2 + CD^2$$

But, $OD = OA + AD$; $CD = F_2 \sin\theta$; and $AD = F_2 \cos\theta$

so, $OC^2 = (OA + AD)^2 + (F_2 \sin\theta)^2$

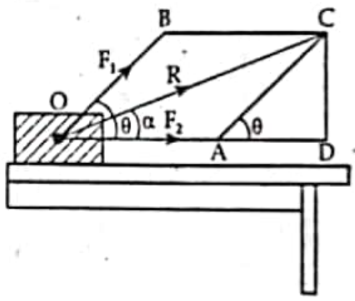
or, $R^2 = (F_1 + F_2\cos\theta)^2 + (F_2 \sin\theta)^2$

or, $R^2 = F_1^2 + 2F_1F_2\cos\theta + F_2^2\cos^2\theta + F_2^2\sin^2\theta$

or, $R^2 = F_1^2 + 2F_1F_2\cos\theta + F_2^2(\cos^2\theta + \sin^2\theta)$

or, $R^2 = F_1^2 + 2F_1F_2\cos\theta + F_2^2$

or, $R = \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2}$



This is the magnitude of the resultant.

The direction of resultant is given by angle α which is determined by relation;

$$\tan\alpha = \frac{CD}{OD} = \frac{F_2\sin\theta}{OA + AD} = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{F_2\sin\theta}{F_1 + F_2\cos\theta}\right)$$

Special cases

i) If two forces act in the same direction, then $\theta = 0°$ and $\cos\theta = 1, \sin\theta = 0$

Then,

$$R = \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2} = \sqrt{(F_1 + F_2)^2} = F_1 + F_2$$

and, $\tan\alpha = \dfrac{F_2\sin\theta}{F_1 + F_2\cos\theta} = 0$

$\therefore$ R acts along the direction of the forces.