Introduction to digital image processing

processing of images which are digital
in nature by a computer.

* why do we need image processing:-
  * Improvement of pictorial information for better human perception
  * efficient storage and transmission.

Applications:-

1. Human perception:-
   Employ methods capable of enhancing pictorial information
   for human interpretation and analysis.

   Typical applications:-

   (i) Noise filtering:- Images that we get are very noisy,
   so we use some techniques for better human perception.

   (ii) Content enhancement:-
   ↳ Contrast enhancement.
   ↳ deblurring (may be because of camera settings, lenses issues, moving objects)

   (iii) Remote sensing

   (iv) Medical imaging.

→ Machine vision applications:-

(1) Industrial Machine vision for product assembly and inspection.

(2) Automated target detection and tracking.

(3) Finger print recognition.

⋮

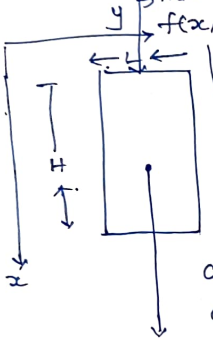
Image representation :- How image is represented in digital computer

→ An image is a 2D-Light intensity function $f(x,y)$ a
↳ This image.

$f(x,y) = r(x,y) \cdot I(x,y)$ → 0 to ∞,

↓ intensity of incident light

reflectivity of the object on the point surface.



giving an intensity at a particular point

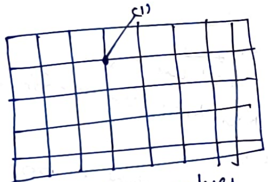
$0 \le x \le H$.
$0 \le y \le H$.

* if it is a analog image, How many points on image?
  → Infinite number of points
  ↓
  at every point, We have intensity value, Minimum value = 0; Maximum value = ∞

→ Can we store such an image in a digital computer?

answer :- NO

solution :- We have to go for some processing ~~of image~~



instead of storing values
↓
We try to take value by imposing grid points.

(1) spatial discretization by grids ↳ sampling.

(2) This value can be any thing
⇓
Discretization of intensity values ↳ Quantization

$$I = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ f(2,0) & f(2,1) & \cdots & f(2,N-1) \\ \vdots & & & \\ f(M-1,0) & f(M,1) & \cdots & f(M-1,N-1) \end{bmatrix}$$

↓ pixel or pel

image size :- 256×256.

Quantization :- 8 bit for black and white

Typical applications

steps in Image processing systems :

(1) Low Level processing :-

* These operations are directly applied to raw images and produce another image as o/p. They involve basic functions such as.

i/p image → [ ] → image
↓
operation on

(i) image acquisition — capture the image using sensors (or cameras).

preprocessing :- Removing noise, adjust contrast/brightness and H.E.

image enhancement :- Sharpening, smoothing

image restoration :- Correct geometric distortions (or blurring

* i/p and o/p are images.

(2) Intermediate Level processing :-

→ At this level, the system intrepets image data to extract features (or relevant information.
feature vector

i/p image → [system] → (or relevant information

* **Segmentation:-**
Dividing the image into Meaningful regions on Objects.
Tumour detection.

Edge detection
↳ identifying object boundaries.

feature extraction:-
↳ Detecting shapes, textures and key points.

Morphological processing:-
↳ structuring and modifying image shapes.

i/p image ; output :- attributes on regions.

3. High level processing:-
→ This level involves understanding and interpreting the image content to make decisions.

i/p image → $\boxed{\text{system}}$ → decision:-
classification
object recognition.

* **Key stages in Digital image processing:-**

(i) image acquisition

*talk about stereoscopic pipeline.

Problem Domain → $\boxed{\text{Image acquisition}}$ → $I(x,y)$
↓
be the i/p image
on
original image.

② image enhancement:-
processing on image to enhance certain features of image.

$$I'(x,y) = T(I(x,y))$$
↑
T-Transformation function

T → can be point operations
on
spatial operations
on
frequency domain

image restoration

↳ process of recovering an original image from degraded version using a Mathematical Model / appropriate restoration Technique.

$$g(x,y) = H(x,y) * I(x,y) + \eta(x,y)$$

H ↑ degradation function — blur

η ↑ noise — transmission noise (sensor Noise — AWGN)

objective of Image restoration

$$\hat{I}(x,y) = R[g(x,y)]$$

$R(\cdot)$ ↓ restoration operator. may be inverse filtering, Weiner filtering

③ Morphological processing:-

↳ structuring and modifying image shapes.

boundaries of foreground (white) regions

Dilation:- Expands the boundaries of foreground.

Erosion:- shrinks the boundaries } Biometric

opening:- Removes small objects from the foreground.

closing:- Fills small holes in the foreground.

④ image segmentation:-

↳ process of dividing an image into distinct regions That corresponds to Meaningful parts such as objects, boundaries on textures.

$I(x,y)$ ← i/p image. over domain D

Goal:- partition 'D' into non-overlapping regions R1, R2.... Rn.

⑤ object recognition/description

↓

process of detecting and Identifying objects in an image and assigning them semantic labels

eg:- car, person, tree

Mathematical

—x—x—

$I(x,y) \leftarrow$ I/p image.

i/p image $\rightarrow$ $\boxed{\text{system}}$ $\rightarrow$ $f = [f_1, f_2, \ldots f_n]$

$\uparrow$ feature vector

$C = \{c_1, c_2 \ldots c_k\}$

$\uparrow$ set of possible object classes.

$\phi(\cdot)$: classification

$$\hat{c} = \phi(f) = \underset{c_i \in C}{\arg\max}\ P(c_i | f)$$

c_i ↑ class, f ↑ feature

probability that feature f belongs to class c_i

<u>Object description</u>:-

↳ represent the recognized object using attributes such as shape, size, texture, color for further analysis

Brain tumour → identified — size of the tumour.

<u>image compression</u>:-

↳ process of reducing the number of bits required to represent an image by removing redundancy while preserving essential visual information

$$\hat{I}(x,y) = D(C(I(x,y)))$$

reconstructed image ← $\hat{I}(x,y)$; D ↳ decompression / reconstruction factor; C ↳ compression factor

<u>Goal</u>:-

Minimize the storage size of $C(I)$ while keeping $\hat{I} \approx I$ perceptually)

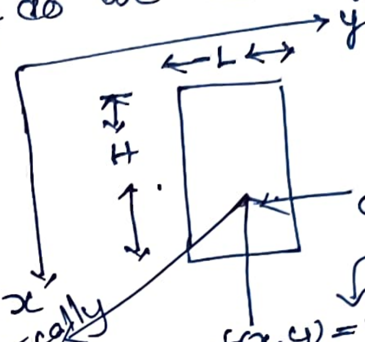
→ lossless compression:- Medical Images.

Color image processing:-

↳ involves analyzing, modifying or interpreting the color content of an image depending upon applications.

image digitization:-

(1) Why do we need digitization?



$0 \le x \le H$
$0 \le y \le L$ } properties of an image

consider any point in image.

Theoretically point ↓ can have any value 0 to ∞

multiplication of

$f(x,y) = r(x,y) . I(x,y)$

↳ intensity of light that is falling on objective surface

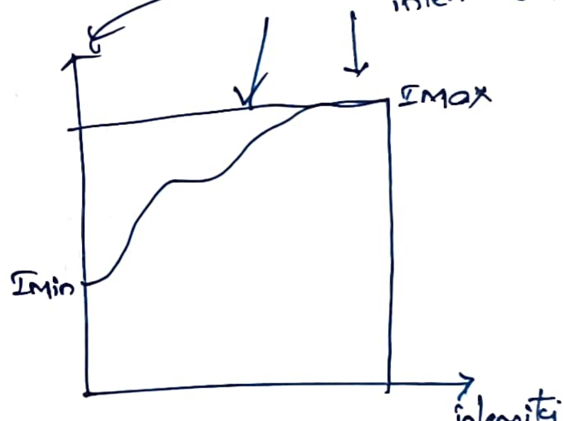
But practically ↳ $I_{min} \le I(x,y) \le I_{max}$
↓ 0 ∞

→ practically → $I_{min} \le I(x,y) \le I_{max}$

horizontal line in an image

for this intensity profile.

intensity profile



infinite number of points.

↑ such representation is not possible in computer

What is desired?

image with finite number of points

$$\begin{bmatrix} f(0,0) & \cdots & f(0,N-1) \\ \vdots & & \\ f(M-1,0) & \cdots & f(M-1,N-1) \end{bmatrix}$$

Digizitation includes
↳ sampling
↳ Quantization.

Sampling:- image representation by 2-D finite matrix
Each matrix element represented by one of the finite set of discrete values. - Quantization.

$\longrightarrow$ sampler $\longrightarrow$ Quantization — Digital computer
Analog image

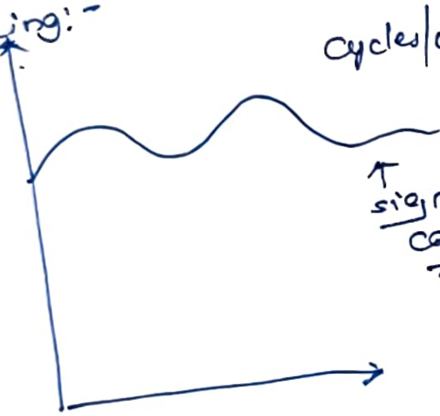
Digital computer $\longrightarrow$ Digital to Analog converter $\longrightarrow$ Display.

sampling:- Before two dimensional

1-D sampling:-

cycles/unit time.

x(t)

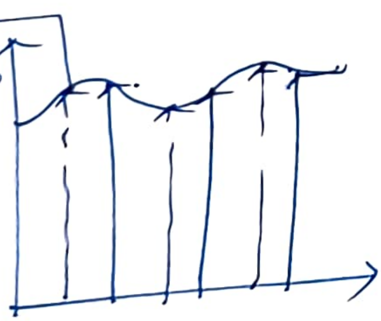


signal cannot be represented in a computer

t →
time

instead of considering the signal values of 't', consider at discrete intervals of ΔT
↓ sampling
⇓
sampling

But much of the information is missing



Δt_s t.
sampling interval

sampling frequency $f_s = \frac{1}{\Delta T_s}$

reduce the sampling interval

$$\Delta t_s' = \frac{\Delta t_s}{2}$$

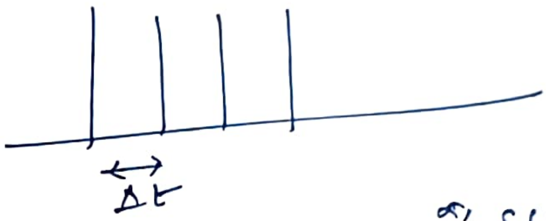
$$f_s' = \frac{1}{\Delta t_s'} = \frac{2}{\Delta t_s} = 2f_s$$

increase the sampling frequency, more information can be retained?

? whether there is an theoritical background

How to decide the sampling frequency?

sampling function → 1-D array of Dirac delta function situated at regular spacing of Δt



$$\delta(t) = \begin{cases} 1 ; & t=0 \\ 0 ; & \text{otherwise} \end{cases}$$

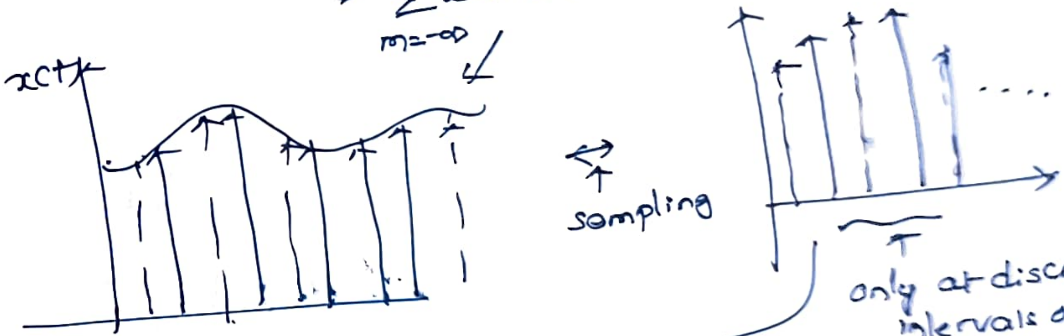
$$comb(t; \Delta t) = \sum_{m=-\infty}^{\infty} \delta(t - m\Delta t)$$

$x(t)\,\delta(t-\Delta t)$

$\Rightarrow x(\Delta t)\,\delta(t-\Delta t)$

$$x_s(t) = x(t) \cdot comb(t, \Delta t)$$

$$\Rightarrow \sum_{m=-\infty}^{\infty} x(m\Delta t)\,\delta(t - m\Delta t)$$



only at discrete intervals of time, we have amplitude

this sampling will be proper if we reconstruct from the original signal.

→ if we have continuous time signal, it can be represented as:-

$x(t)$

$$F[x(t)] = X(\omega) = \int^{\infty} x(t)\, e^{-j\omega t}\, dt$$

this is aperiodic signal

frequency components

if the signal is periodic

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}.$$

c_n → fourier coefficients.

$$c_n = \frac{1}{T_0}\int_{T_0} v(t)\, e^{-jn\omega_0 t}\, dt.$$

↑ fourier coefficient

in our case:- $v(t)$ is periodic

$$v(t) = \begin{cases} 1; & t=0 \\ 0; & \text{otherwise} \end{cases}$$

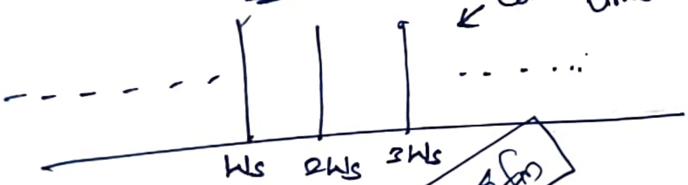
$T_0 = \Delta T_s$; $v(t$

$$c_n = \frac{1}{\Delta T_s}$$

$$\Rightarrow \boxed{v(t) = \frac{1}{\Delta T_s} \sum_{n=-\infty}^{\infty} e^{j n\omega_0 t}}$$

$n\omega_0$ → discrete

$v(t)$



← continuous time domain

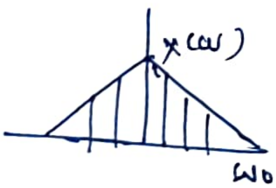
Convolution in time domain

$x(t) * h(t) = X(\omega)\cdot H(\omega)$

↑ multiplication in frequency

$x_s(t) \Rightarrow x(t).\text{comb}(t, \Delta t)$

$X(\omega) * H(\omega) = x(t)\cdot h(t)$



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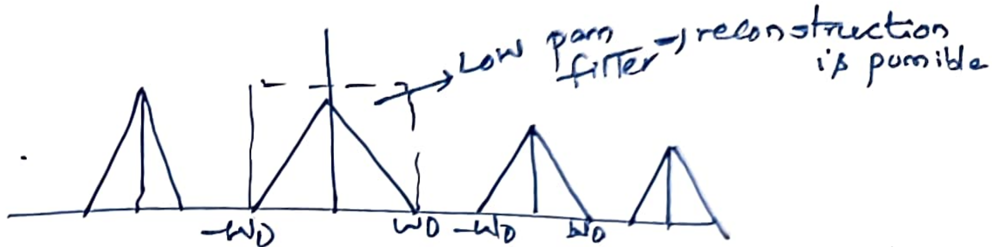


The replicated spectra must be spaced far enough apart in frequency. The minimum distance should be

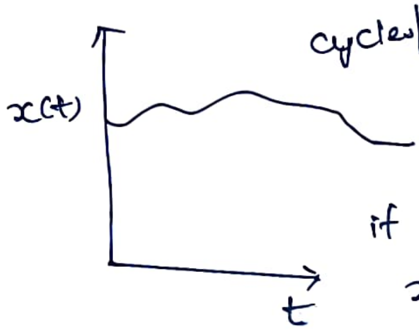
$$\frac{1}{\Delta t_s} - W_D > W_D$$

$$\Rightarrow \frac{1}{\Delta t_s} > 2W_D$$

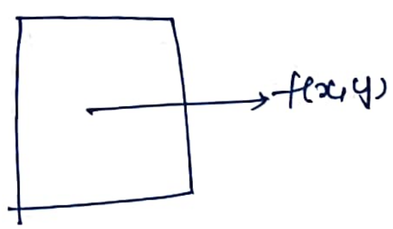
$\boxed{f_s > 2W_D}$ → sampling theorem

→ overlapping ⇒ aliasing

2D-sampling



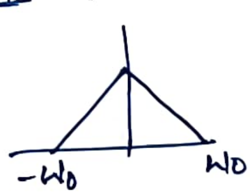
if t is a time
$x(t)$ is a signal carries with time measured in Hertz (cycles/unit time)



dimension of image is represented
5CM × 5CM
(or)
10CM × 10 CM

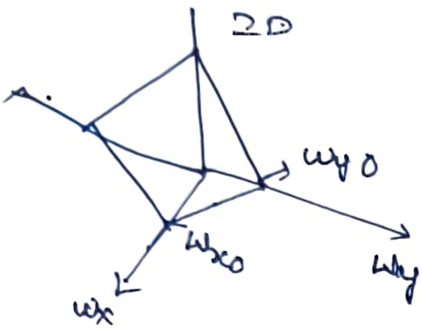
units:- cycles/unit length

1D sampling



Band limited ⇒ $X(\omega) = 0$; for $|W| > W_D$

2D -



Bandlimited :-

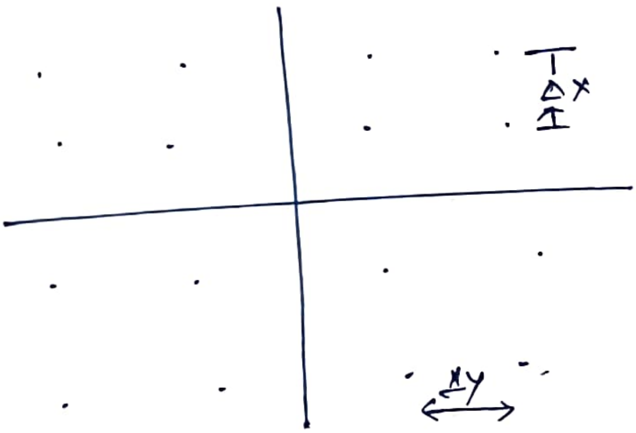
$F(\omega_x, \omega_y) = 0$

for $|\omega_x| > \omega_{x0}$

$|\omega_y| > \omega_{y0}$

$f_s(x,y) = f(x,y)\, comb(x, y ; \Delta x, \Delta y)$

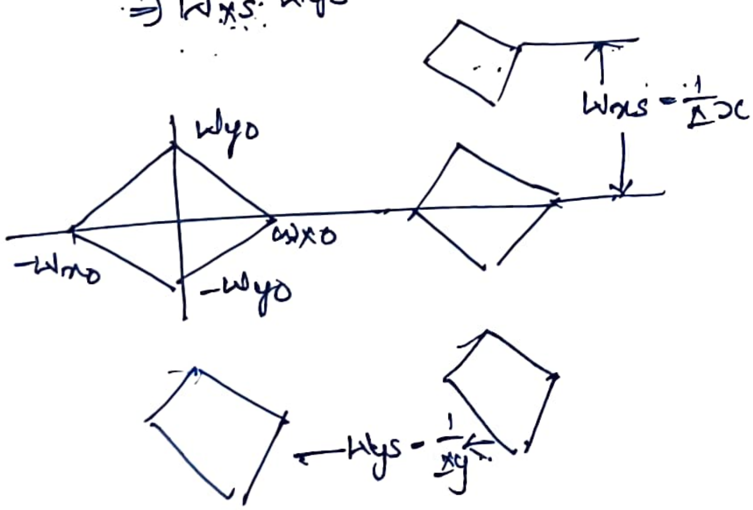
$$\Rightarrow \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(m\Delta x, n\Delta y) \,.\, \delta(x - m\Delta x, y - n\Delta y)$$



$F_s(\omega_x, \omega_y) \Rightarrow F(\omega_x, \omega_y) \circledast \cancel{comb}\; COMB(\omega_x, \omega_y)$

$COMB(x,y) = \mathcal{F}(comb(x,y, \Delta x, \Delta y))$

$\Rightarrow \omega_{xs} \cdot \omega_{ys} \cdot comb\left(\omega_x, \omega_y, \frac{1}{\Delta x}, \frac{1}{\Delta y}\right)$



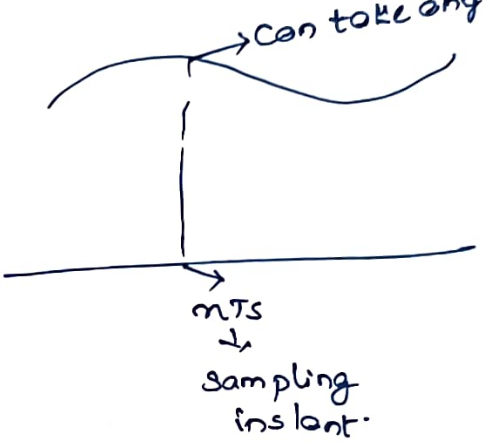
2D image can be reconstructed

$\omega_{xs} > 2\omega_{x0}$ and $\omega_{ys} > 2\omega_{y0}$

Recovery by a low pass filter with response

$$H(\omega_x, \omega_y) = \begin{cases} \dfrac{1}{\omega_{xs} \cdot \omega_{ys}} & ; (\omega_x, \omega_y) \in R \\ 0 & \text{otherwise} \end{cases}$$

Image Quantization:-



Mapping these samples to a set of discrete values is termed as Quantization.

↓

purpose of these quantization is to convert these samples to information bits

↓

can either be stored or transmitted over the channel

Goal:- $V = g(m)$

↳ set of samples

↑ function

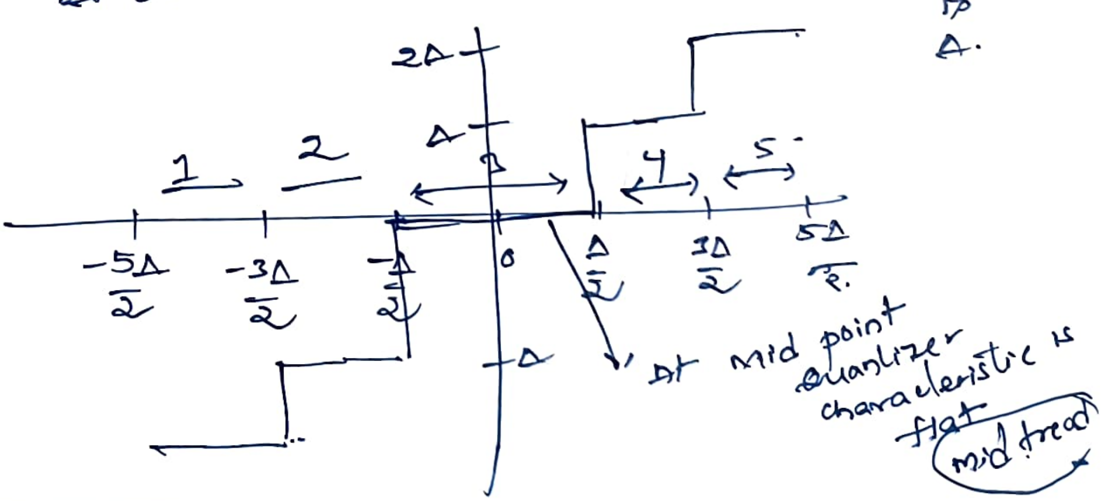
quantized value.

* uniform quantization

(i) Quantization levels are uniformly distributed.

↓ odd number of quantization levels.

* each quantization interval width is Δ.



In each Quantization interval, the Quantization levels is the midpoint of the interval

if $\frac{3\Delta}{2} \le m \le \frac{5\Delta}{2}$; $g(m) = 2\Delta$.

$\frac{\Delta}{2} \le m \le \frac{3\Delta}{2}$; $g(m) = \Delta$.

$-\frac{\Delta}{2} \le m \le \frac{\Delta}{2}$; $g(m) = 0$

$-\frac{3\Delta}{2} \le m \le \frac{\Delta}{2}$; $g(m) = -\Delta$

$-\frac{5\Delta}{2} \le m \le -\frac{3\Delta}{2}$; $g(m) = -2\Delta$.

$\Rightarrow$ all values from $\underbrace{\frac{\Delta}{2} \le m \le \frac{3\Delta}{2}}_{\text{Many values}} = \Delta$.

are mapping to single value

4)9(2.25
$\frac{8}{10}$
$\frac{8}{\ 2}$

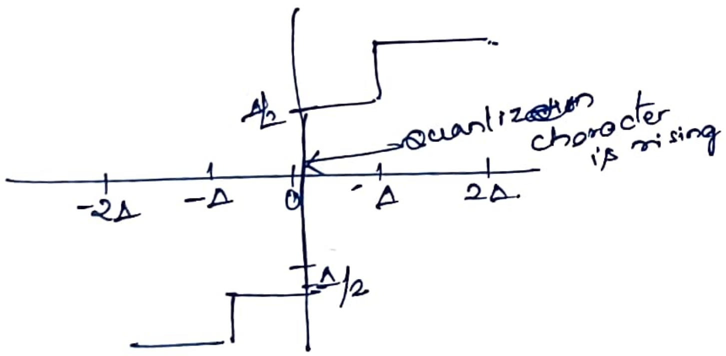
$\rightarrow$ if $m = \frac{9\Delta}{4} \Rightarrow$ is mapped to 2Δ.

$\underset{\uparrow \text{ original}}{\frac{9\Delta}{4}} - \underset{\uparrow \text{ mapped}}{2\Delta} = \frac{\Delta}{4}$

Quantization error will be in the range $\left(-\frac{\Delta}{2}, \frac{\Delta}{2}\right)$

Even number of Quantization levels:-

consider '4' Level, Quantization interval of width Δ



* In a uniform Quantizer, with even number of Q.L's the Quantizer character rises from

$-\frac{\Delta}{2}$ to $\frac{\Delta}{2}$. at $m=0$

↑

Mid rise Quantizer

Δ = step size.

↑

Quantization interval length

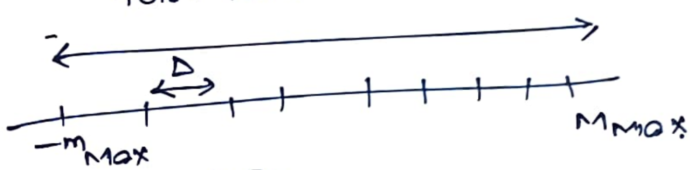
$\Rightarrow$ $m \in [-m_{max}, m_{max}]$

↑ signal

↑ peak -ve amplitude

↑ peak positive. amplitude

total Quantization interval = $m_{max} + m_{max}$



\# of. Quantizations Levels $\frac{L}{\uparrow} = \frac{2m_{max}}{\Delta}$.

$\boxed{L = 2^n}$

↓

encoding ~~How~~ number of bits we