Optimizing Production and Distribution in Widget Manufacturing: A Linear Programming Approach
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Abstract

This study tackles a tricky production planning puzzle for NU Industries, a company making Widgets, Gadgets, and Flugels across two factories. We used Python's PuLP library to build a linear programming model and determine the best way to maximize their profit over five periods. Our model takes into account a variety of things, like how much to make, how much to keep in stock, how to use raw materials and labor (including overtime), and even how to spend their marketing budget wisely.

The solution we came up with shows how NU Industries can really optimize its operations. Their second factory is a real workhorse, putting in some serious overtime. Meanwhile, the first factory focuses on making Gadgets and storing most of the inventory. The model suggests they go all-in on marketing Flugels in the final stretch. By following our plan, NU Industries can expect a robust profit of $7,571,549.66. This shows how powerful linear programming can be for businesses dealing with complex decisions.

Keywords

Linear Programming, Optimization, Production Planning, Distribution, Inventory Management, Supply Chain

1. Introduction

The manufacturing world is a wild ride, and companies like NU Industries are always looking for ways to stay ahead of the curve. NU Industries makes Widgets, Gadgets, and Flugels, and they have a juggling act going on. They need to figure out how to make the right amount of each product at their two different factories, keep just the right amount of inventory, and get those products to customers without overspending on shipping. On top of that, they need to decide how to best spend their marketing budget to drum up more demand.

It is a tricky balancing act. They do not want to make too much product and have it sitting around collecting dust, but they also do not want to run out of it and negatively impact their customers. And, of course, they need to get those products where they need to go without spending a fortune on gas. Plus, they need to make sure their marketing is hitting the right notes to bring in those sales.

Here is the breakdown of what NU Industries is dealing with:

- Production Power: It is not just about making products; it is about making the right products at the right time. They need to think about things like how much raw material they have, how many workers are on the clock, and what each factory is good at making.
- Inventory: They need to find that sweet spot where they have enough to keep customers happy but not so much that it starts costing them exorbitant amounts.
- Delivery: Getting those products from the factory to the customer is not free. They need to figure out the cheapest and fastest way to ship things so they do not end up spending more on delivery than they make on the products themselves.
- Marketing: NU Industries needs to be strategic about its marketing efforts. How can they spend their marketing budget to generate the most buzz and bring in more customers? Which products should they focus on, and when is the best time to ramp up their marketing campaigns?

This research tries to answer some burning questions:

- Can we find the magic formula that helps NU Industries make the right amount of products, keep the right amount in stock, ship it all in the most efficient way possible, and make savvy marketing decisions?
- What happens if things suddenly change? For example, what if the price of raw materials goes through the roof or shipping costs skyrocket? How can they adapt?

In the manufacturing environment, if you snooze, you lose. NU Industries needs to be on top of its game to keep customers happy, run a tight ship, and stay ahead of the competition. If they do not plan carefully, they could end up wasting money, piling up inventory they do not need, and taking the scenic route when shipping their products. And if their marketing is not on point, they might miss out on valuable opportunities to boost sales.

This research dives deep into how companies can use data to make smarter choices about their manufacturing and supply chain. We have built a model similar to a super-powered planner, helping businesses figure out the best way to make a product, store it, and ship it, all while factoring in the impact of marketing. We also looked at how unexpected changes can negatively impact things and how companies can roll with the punches. We are giving manufacturers the tools they need to be data-driven fighters and level up their supply chain game.

2. Literature Review

Comparator Paper 1: "A multi-period multiple parts mixed integer linear programming model for AM adoption in the spare parts supply chain" (Mecheter et al. 2023)

Key Findings: The paper employs a mixed-integer linear programming (MILP) optimization model. This approach is used to analyze trade-offs in spare parts supply over multiple periods, considering various parts.

Mixed-integer Linear Programming (MILP) is a type of optimization problem that involves both continuous and integer variables, subject to linear constraints and a linear objective function (Williams 2020). The problems we have solved throughout this Decision Analytics course show that these problems are prevalent in various industries, including automotive, chemical, energy, finance, healthcare, and telecommunications.

Methodologies: This paper uses MILP to figure out the best way to manage spare parts when we have 3D printing (additive manufacturing, or AM) in the mix. They look at things like how much to produce, when to use 3D printing, and how much inventory to keep in stock. The model is smart enough to handle these decisions over time and for different kinds of parts. It is like a super-powered planner for companies with 3D printers.

Limitations: Even though this paper is helpful for figuring out spare parts and 3D printing, it has some limitations. For example, it assumes things stay predictable. Demand, costs, and all those variables are assumed to be set in stone. But in the real world, things can change fast. Also, they do not dive into the granular details of how 3D printing is done. Different materials, different printers, all those elements can make a big difference. Therefore, while this gives us a good starting point, we cannot just take it and run with it without thinking about those real-world curveballs.

Comparison: Both our project and this paper use this mixed-integer linear programming (MILP), but they are tackling different problems. Mecheter et al. are all about figuring out how to use 3D printing in a smart way for spare parts. They want to know when it makes sense to print a part versus just making it the old-fashioned way. Our problem is more about juggling production and inventory at a couple of different factories. We are dealing with things like workers putting in overtime, the cost of shipping inventory around, and how to keep customers happy. Therefore, Mecheter et al. are more focused on the tech side of 3D printing, while we are trying to untangle the challenge of running a couple of busy factories. Both projects are helpful in their own way, showing how you can use this MILP tool to make better decisions, even if the problems are pretty different.

Comparator Paper 2: “Optimization of Production Planning Using Linear Programming” (Adriantantri and Indriani 2021)

Key Findings: This paper by Adriantantri and Indriani (2021) shows how a company used some clever math to get their production game on point. They built this model to figure out the best way to make products, like how many widgets to output, how much raw material they will need to achieve that, and how to use their workers effectively. By using this model, the company found the sweet spot for making their products while keeping costs down.

Methodologies: In this paper, the authors show how a company can use some math magic to make their production process super efficient. They start by gathering all the important details about how the company makes products, like what they are producing, what resources they have (raw materials and workers), and how much people want their products. Then, they turn this information into a mathematical model that is like a blueprint of the whole production process. This model is designed to help the company make the most of what they have while keeping costs low. They use a special tool that crunches the numbers and spits out the best possible way to run the whole operation.

The paper points out that you have to know what is coming to make good production decisions. They are basically saying, "If you want to make the right product, you have to have a decent idea of how much people are going to want it". Additionally, they mention some ways to guesstimate future demand, like using moving averages or exponential smoothing.

They also throw in that simulations can be useful for reviewing different production plans and what-if scenarios. It gives us the idea that we can run some simulations to see if it can handle the chaos. This helps make sure your plan is not just good in theory but can also roll with the punches in the real world.

Limitations: While this paper gives us some good ideas on how to use linear programming for production, this study is not perfect. First off, it is too straightforward. The authors focus too much on saving money, but businesses often have other things to worry about, like making as much profit as possible or being eco-friendly. Also, they do not really think about unexpected events, like if demand suddenly drops or if there is a hiccup in the supply chain. Those things can really derail an operation. Lastly, their model is similar to a custom-made suit for one particular company. It might not fit other businesses that have different ways of doing things or face different challenges.

Comparison: Our project and this paper both use linear programming to figure out the best way to make something, but our project seems more complicated and advanced. They are focused on helping one company save money, while we are trying to make big money across two whole factories. We have a lot more to juggle, like paying overtime, moving products around, and even throwing some cash at marketing. Their model is like a basic calculator, while ours is more like a spreadsheet that can handle way more complexity.

3. Methodology

Statement of Objectives

Our main goal is to help NU Industries make as much money as possible over the next five planning periods. This means figuring out the best way to produce their Widgets, Gadgets, and Flugels at their two factories while also keeping track of inventory, raw materials, labor, shipping costs, and even marketing efforts.

Proposed Methodology

We are using a method called linear programming (LP) to tackle this problem. It is an organized way to model all the different costs, limitations, and choices involved in this whole production puzzle. We are using a handy tool called the PuLP library in Python to build and solve our LP model.

Model formulation

Decision Variables: The model defines decision variables for key operational factors, including how much of each product to produce at each plant during each period, inventory levels, raw material consumption, workforce allocation (both regular and overtime), and marketing expenses.

Objective Function: The goal is to maximize profit, which is calculated by subtracting total costs, such as production, storage, labor, transportation, raw materials, and marketing, from total revenue.

Constraints: The model includes several restrictions to ensure realistic and efficient operations:

- Demand Constraints: Production must meet customer demand in each period.
- Production Capacity Constraints**:** Each plant has a set production limit that cannot be exceeded.
- Inventory Capacity Constraints: Storage space is limited, so inventory levels must stay within capacity.
- Labor Constraints: Workforce availability is restricted, both in terms of regular shifts and overtime.
- Raw Material Constraints: Production depends on the availability of raw materials.
- Marketing Budget Constraint: There is a set limit on how much can be spent on marketing.

Evaluation measures for performance

To see how well the model works, we look at a few key factors. These help us check if the optimization is doing a good job of balancing production, inventory, resources, and demand while staying within the set limits. Here’s what we focus on:

- Overall Profit: The main success indicator is the highest possible profit the model can achieve.
- Production Levels: The ideal amount of each product to be manufactured at each plant during each period.
- Stock Levels: The best inventory amounts to maintain at each plant throughout each period.
- Resource Allocation: How raw materials, labor, and overtime are utilized efficiently.
- Additional Demand: The extra demand created as a result of marketing efforts.
- Constraint Compliance: Ensuring that all operational limits and restrictions are satisfied.

Modeling assumptions

- Linear Relationships Only: We are keeping things simple. If production doubles, so do the costs. There are no complicated curves or unpredictable changes.
- Demand is Known: We assume we have a clear idea of how much customers want for each product, except when marketing efforts come into play to increase demand.
- Fixed Costs: No matter how many units we produce, the cost per item remains constant.
- Instant Production: The moment we decide to make something, it is ready. That means no delays or waiting for production lines to catch up.
- No Backorders: We cannot delay orders if we do not have enough stock to meet demand. We either fulfill them immediately or rely on available inventory.
- Marketing Affects Next Period: Any marketing done in this period will impact sales in the following period, not immediately.
- High-Quality Materials: We assume that all raw materials are perfect and ready to use. That means no defects or unusable stock.
- Flat Shipping Costs: The cost of moving each item is fixed, with no variations based on distance or quantity.
- Storage Limits Stay Constant: The amount of space available at each location remains unchanged over time.

Parameter Generation, Verification, and Validation

Data Source: All the numbers we used, like demand, production costs, and available raw materials, came directly from the problem description.

Checking for Accuracy: We did not just accept the numbers as they were. We went back and verified that everything matched the given information, ensuring consistency and avoiding mistakes.

Validation: After running the model, we did not just accept the results at face value. We took a step back to see if they made sense. We checked that all the constraints were followed and that the production and inventory levels were reasonable.

Key Observations from the Output

Plant B is Carrying the Load: It is clear that Plant B is handling most of the production. They are making significantly more than Plant A and relying heavily on overtime to keep up.

Gadgets and Flugels Stay in Their Lanes: Gadgets are produced only at Plant A, while Flugels are the specialty of Plant B. Each plant is sticking to what it does best.

Marketing Push: The entire marketing budget was used to increase demand for Flugels in the final period, leading to an extra 44.69 Flugels being sold.

Inventory is at Plant A: All stored products are kept at Plant A, while Plant B is strictly focused on production.

Raw Material 2 Belongs to B: Only Plant B requires Raw Material 2, while Plant A does not use it at all.

4. Computational Experiment and Results

A sensitivity analysis was performed in Python to identify potential ways to improve the profitability of the operations.

Sensitivity Analysis - Binding Constraints

Variable	Plant	Period 1	Period 2	Period 3	Period 4	Period 5
Widget Demand	A	yes	yes	yes	yes	yes
	B	yes	yes	yes	yes	yes
Gadget Demand	A	yes	yes	yes	yes	yes
	B	yes	yes	yes	yes	yes
Labor Demand	A	yes	yes	no	no	yes
	B	no	no	yes	no	no
Labor Supply	A	no	no	no	no	yes
	B	yes	yes	yes	yes	yes
Raw Material 1 - Supply	N/A	yes	yes	yes	yes	yes
Raw Material 2 - Supply	N/A	yes	no	no	no	no
Raw Material 1 - Demand	A	yes	yes	yes	yes	no

	B	no	no	no	no	no
Raw Material 2 - Demand	A	yes	yes	yes	no	no
	B	no	no	no	no	no

A subset of the corresponding shadow prices was also examined and ranked in descending order.

Shadow Prices - in descending order for positive values:

1. Flugel Demand - $180
2. Raw Material 1 Demand - $16 (average)
3. Raw Material 1 Supply - $14 (average)
4. Labor Demand - $14 (average)
5. Plant B Labor Supply - $5 (average)

Focussing on the supply side of the optimization and including an option to drive more demand using more of the marketing budget, the model was used to run a number of ‘what if’ scenarios, with the following results.

Scenarios to Increase Objective Function (Maximize Profit)

Scenario	Description	Max Profit (in millions)
Baseline	Model with constraints as given	$7.571
Marketing Spend	Force spend of 100% of marketing budget	$7.501
Increase RM Supply	Increase both RM supplies by 25%	$9.750
Increase Labor Supply	Increase labor supply by 25% across the board	$7.595
All 3 of the above	Increase RM and labor supply by 25%, force marketing spend to budget	$9.761

The scenarios illustrate that the main constraint holding back the profitability of the operations is the availability of the raw materials.

5. Discussion and Conclusions

We built this model to help NU Industries figure out how to maximize profits, and it gave us some great insights. Using a smart mathematical approach, we analyzed how they could improve production, storage, shipping, and marketing strategies.

One of the biggest takeaways is how much work Plant B is handling. It is producing the most and relying heavily on overtime, which raises the question of whether it should expand or find ways to improve efficiency. Meanwhile, Plant A specializes in making Gadgets and storing inventory, showing the value of having each plant focus on what it does best.

The model also confirmed that putting all their marketing efforts into boosting Flugel sales in the last period was a smart choice. It highlights how data-driven decisions can make marketing dollars more effective.

A deeper look revealed that raw material availability is the biggest constraint holding NU Industries back. If they can secure more Raw Material 1, they could significantly increase profits. This is a bigger factor than just spending more on labor or marketing.

We also examined shadow prices, which help show where loosening constraints would have the biggest impact. The high shadow price for Flugel demand suggests that selling more Flugels would bring in a lot more revenue. Similarly, the shadow prices for raw materials and labor reinforce how critical those resources are.

Overall, this model gives NU Industries a clear plan to improve profitability. Their focus should be on securing more raw materials, optimizing plant operations, and making data-driven marketing choices. Looking ahead, it could be useful to introduce some variability into the model to see how they would handle unexpected demand changes or take a closer look at their production process for further efficiencies.

6. References

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Appendix

Refer to the 'Group_1_Final_Project.ipynb' file submitted with this paper for the corresponding Python code.