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Generalized subgraph-restricted matchings in graphs

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Abstract

For a graph property $\mathscr{P}$, we define a $\mathscr{P}$-matching as a set M of disjoint edges such that the subgraph induced by the vertices incident to M has property $\mathscr{P}$. Previous examples include strong/induced matchings and uniquely restricted matchings. We explore the general properties of $\mathscr{P}$-matchings, but especially the cases where $\mathscr{P}$ is the property of being acyclic or the property of being disconnected. We consider bounds on and the complexity of the maximum cardinality of a $\mathscr{P}$-matching and the minimum cardinality of a maximal $\mathscr{P}$-matching.

Keywords: Matchings; Graphs; Induced matchings

1. Introduction

Let $G=(V, E)$ be a graph. A set of edges $M \subseteq E$ is a *matching* if no two edges of M share a common vertex. Matchings have been researched extensively for many years. Indeed, Lovász and Plummer [16] have written a book devoted to matchings. Recently, several authors have considered restricting matchings based on some property of the matchings or of the subgraph induced by the vertices of the matching. This prompts the following general definition.

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If M is a matching, we use the notation $G[M]$ to denote the subgraph induced by the vertices of the edges of M (also known as the *saturated* vertices). We define a matching M to be a *$\mathscr{P}$-matching* if $G[M]$ has property $\mathscr{P}$, where $\mathscr{P}$ is some property of graphs. The *$\mathscr{P}$-matching number* $\beta_{\mathscr{P}}(G)$ is the maximum cardinality of a $\mathscr{P}$-matching. Furthermore, we define a *maximal* $\mathscr{P}$-matching as one which is not contained in a larger $\mathscr{P}$-matching, and define the *lower $\mathscr{P}$-matching number* $\beta^-_{\mathscr{P}}(G)$ as the minimum cardinality of a maximal $\mathscr{P}$-matching.

Matchings M with the property that the induced subgraph $G[M]$ has no other edges have been called strong or induced matchings [4,6–8,12,13,15,19,22,23]. Recently, uniquely restricted matchings, where $G[M]$ has no other matching of size $|M|$, have been studied by Golumbic et al. [11].

In this paper we consider several variations of matchings. These include connected, acyclic, isolatefree and disconnected matchings. We also establish some general results for $\mathscr{P}$-matchings.

Similar generalizations for vertex covers are given in [2] and for dominating and independent sets in [17,10]. Indeed, the largest $\mathscr{P}$-matching is limited by the largest subgraph with property $\mathscr{P}$. We will use the notation $I_{\mathscr{P}}(G)$ to denote the maximum order of a subgraph of G with property $\mathscr{P}$ (sometimes called the $\mathscr{P}$-independence number of the graph), though we will still use $\beta_0(G)$ to denote the ordinary independence number. It follows trivially that

$$\beta_{\mathscr{P}}(G) \leqslant I_{\mathscr{P}}(G)/2.$$

1.1. Examples of subgraph-restricted matchings

Specific examples are:

(1) If the property $\mathscr{P}$ is that of being a graph, then a $\mathscr{P}$-matching is an ordinary matching, and the $\mathscr{P}$-matching number is just the ordinary *matching number*, denoted by $\beta_1(G)$.
(2) A matching M is called *strong* or *induced* if no two edges of M are joined by an edge. That is, $G[M]$ is 1-regular. The *induced matching number* is denoted by $\beta_*(G)$.
(3) A matching M is called *uniquely restricted* if $G[M]$ has exactly one maximum matching, namely M. The *uniquely restricted matching number* is denoted by $\beta_{\mathrm{ur}}(G)$.
(4) A matching M is called *connected* if $G[M]$ is connected. The *connected matching number* is denoted by $\beta_{\mathrm{c}}(G)$. It turns out to be simply the maximum matching number of a component of G.
(5) A matching M is called *isolatefree* if $|M| = 1$ or $G[M]$ has no K_2 component. The *isolatefree matching number* is denoted by $\beta_{\mathrm{if}}(G)$. This parameter is also given directly by the matching numbers of the components of G.
(6) A matching M is called *acyclic* if $G[M]$ is acyclic. The *acyclic matching number* is denoted by $\beta_{\mathrm{ac}}(G)$.
(7) A matching M is called *disconnected* if $|M| = 1$ or $G[M]$ is disconnected. The *disconnected matching number* is denoted by $\beta_{\mathrm{dc}}(G)$.

Some of these parameters are related to parameters in the line graph $L(G)$. A matching in G of course corresponds to an independent set in the line graph and so $\beta_1(G) = \beta_0(L(G))$.

Similarly, a strong matching corresponds to a 2-packing in the line graph and so $\beta_*(G) = \rho(L(G))$.

1.2. Previous work

Induced or strong matchings are well studied. Cameron [4] introduced induced matchings. She showed that determining $\beta_*(G)$ is NP-hard for bipartite graphs, while Ko and Shepherd [15] showed this for cubic planar graphs. The maximum induced matching problem was shown to be polynomial for chordal graphs by Cameron [4] and for circular-arc graphs by Golumbic and Laskar [12], who introduced the terminology strong matching. Fricke and Laskar [8] gave a linear-time algorithm for trees. Further complexity results are given in [13,22,23]. El Maftouhi [6] considered the parameter $\beta_*^-(G)$ for a random G.

Uniquely restricted matchings, on the other hand, are relatively new, and were introduced by Golumbic et al. [11]. They were motivated by a problem in linear algebra [14].

One useful observation is the following:

Theorem 1 (*Golumbic [11]*). *A matching M is uniquely restricted iff $G[M]$ does not contain an alternating cycle w.r.t. M.*

So, for example, $\beta_{\rm ur}(G)=\beta_1(G)$ if G does not contain an even cycle. It follows that testing whether a given matching is uniquely restricted can be performed quickly. Golumbic et al. [11] showed that the uniquely restricted matching number can be computed in polynomial time for interval graphs inter alia, but is NP-hard for bipartite and chordal graphs.

For a general survey of complexity results related to matchings, see [18].

2. General results

It is clear that for two graph properties $\mathscr{P}$ and $\mathscr{Q}$ if $\mathscr{P} \subseteq \mathscr{Q}$, then $\beta_{\mathscr{P}} \leqslant \beta_{\mathscr{Q}}$. From the above discussion it is easy to prove the following inequalities:

$$\beta_* \leqslant \beta_{\rm ac} \leqslant \beta_{\rm ur} \leqslant \beta_1,$$
$$\beta_* \leqslant \beta_{\rm dc} \leqslant \beta_1,$$
$$\beta_{\rm c} \leqslant \beta_{\rm if} \leqslant \beta_1.$$

While these are all the relationships in general, for connected graphs the connected and isolatefree matching numbers equal the ordinary matching number.

Theorem 2. *If G is connected, then $\beta_{\rm c}(G) = \beta_{\rm if}(G) = \beta_1(G)$.*

Proof. Let M be a maximum matching such that the order of a largest component of $G[M]$ is a maximum. Suppose $G[M]$ is not connected; then let C be a largest component. Since G is connected, there exists a path from a vertex in C to a saturated vertex not in C. Let $P = a, b, c, \ldots, u$ be a shortest such path: then b is not saturated by M. Define a matching M' by adding to M the edge bc and removing the edge of M incident with c, if any. It follows that M' is a maximum matching, but the largest component of M' is larger than C, a contradiction. □

2.1. Special graphs

The $\mathscr{P}$-matching number of the complete graph $\beta_{\mathscr{P}}(K_n)$ is clearly half the maximum even cardinality of a clique that has property $\mathscr{P}$. Similarly, for a complete bipartite graph, $\beta_{\mathscr{P}}(K_{m,n})$ is half the maximum order of a balanced bipartite graph that has property $\mathscr{P}$. An *induced hereditary* property is one that is closed under vertex removal. Thus, such a $\mathscr{P}$ contains all complete graphs of at most some order, and all balanced bipartite graphs of at most some order.

If the property $\mathscr{P}$ is *additive* (closed under disjoint unions), then the $\mathscr{P}$-matching number of a graph is the sum of the $\mathscr{P}$-matching numbers of its components.

Additive-induced hereditary properties also provide simple formulas for $\beta_{\mathscr{P}}(P_n)$ for the path P_n. If $\mathscr{P}$ does not contain all paths, then there is a maximum path in $\mathscr{P}$, and any shorter path is in $\mathscr{P}$. Let $2z_{\mathscr{P}}$ be the maximum path length, rounded down to the nearest even number. (This was prompted by results in [10].)

Theorem 3. *Let $\mathscr{P}$ be an additive-induced hereditary property. If $\mathscr{P}$ includes all paths, then $\beta_{\mathscr{P}}(P_n) = \lfloor n/2 \rfloor$. Otherwise, $\beta_{\mathscr{P}}(P_n) = (n+1)z_P/(2z_P+1) + \mathrm{O}(1)$ with exact equality if $n+1$ is a multiple of $2z_P+1$.*

For example, $\beta_*(P_n) = n/3 + \mathrm{O}(1)$ and $\beta_{\mathscr{P}}(P_n) = n/2 - \mathrm{O}(1)$ for the other six parameters listed in Section 1.1.

Similar results hold for the cycles C_n.

2.2. Complexity

Edmonds [5] was the first to show that the matching number $\beta_1(G)$ can be found in polynomial time for any graph G. In contrast, as mentioned above, the induced matching number and the uniquely restricted matching number are both NP-hard to determine. We show that the latter behavior is typical.

Consider first the following decision problem:

INDUCED $\mathscr{P}$-SUBGRAPH
Input: Graph G and integer k
Question: Is $I_{\mathscr{P}}(G) \geqslant k$?

This problem is generally intractable. In particular, it is NP-hard for all nontrivial-induced hereditary properties; see [9, Problem GT21].

We are interested in the following decision problem:

$\mathscr{P}$-MATCHING
Input: Graph G and integer k
Question: Is $\beta_{\mathscr{P}}(G) \geqslant k$?

The proof of the following theorem mirrors that of Theorem 2.1 of [13].

Theorem 4. *Fix some property $\mathscr{P}$. If* INDUCED $\mathscr{P}$-SUBGRAPH *is* NP-*hard and $\mathscr{P}$ is closed under the addition and removal of endvertices, then* $\mathscr{P}$-MATCHING *is* NP-*hard*.

Proof. We reduce from INDUCED $\mathscr{P}$-SUBGRAPH. Given a graph $G = (V, E)$, its corona $H = G \circ K_1$ is obtained by introducing for each vertex $v \in V$ a new endvertex v' adjacent only to v. We claim that

$$\beta_{\mathscr{P}}(H) = I_{\mathscr{P}}(G).$$

Consider any maximum $\mathscr{P}$-matching M in H. If it uses an edge of G say uv, then one can replace uv with the corona edges uu' and vv' to form a larger matching M', and the graph induced by M' still has property $\mathscr{P}$, since $\mathscr{P}$ is closed under the addition of endvertices. Thus, M uses no edge of G.

Furthermore, the graph $H[M]$ restricted to V has property $\mathscr{P}$, since $\mathscr{P}$ is closed under the deletion of endvertices. On the other hand, any $\mathscr{P}$-subgraph of G can be extended to a $\mathscr{P}$-matching. It follows that the $\mathscr{P}$-matching number of H is obtained by starting with a maximum $\mathscr{P}$-subgraph of G, and matching each vertex with its corona partner. □

Thus, the ACYCLIC MATCHING Problem is NP-hard. The proof of Theorem 2.1 in [13] for strong matching used the same construction, except that it follows that

$$\beta_*(H) = \beta_0(G).$$

By Theorem 2, the connected and isolatefree matching numbers can be computed in polynomial time. Of the seven parameters from Section 1.1, this leaves unresolved only the complexity of the disconnected matching number.

3. Some specific results

3.1. The acyclic matching number

The parameter $I_{\mathrm{ac}}(G)$ has been studied by several people under various names. Most recently, it was investigated by Beineke and Vandell [1], who defined the *decycling number* of a graph as the minimum number of vertices whose removal destroys all cycles. From [1] it follows that:

Theorem 5. *For the grid $G_{n,n} = P_n \Box P_n$, it holds that $\beta_{\mathrm{ac}}(G_{n,n}) = n^2/3 \pm \mathrm{O}(n)$.*

Proof. Beineke and Vandell [1] showed that $I_{\mathrm{ac}}(G_{n,n}) = 2n^2/3 + \mathrm{O}(n)$, so that $\beta_{\mathrm{ac}}(G_{n,n}) \leqslant n^2/3 + \mathrm{O}(n)$. Then define an acyclic matching as follows: In odd rows, pair vertices 12, 45, 78, etc. In even rows, pair vertices 23, 56, 89, etc. This shows that $\beta_{\mathrm{ac}}(G_{n,n}) \geqslant n^2/3 - \mathrm{O}(n)$. □

3.2. The disconnected matching number

The disconnected matching number in a tree is close to the ordinary matching number.

Theorem 6. *For any tree T, it holds that $\beta_1(T) - 1 \leqslant \beta_{\mathrm{dc}}(T) \leqslant \beta_1(T)$.*

Proof. We need to show that $\beta_{\mathrm{dc}}(T) \geqslant \beta_1(T) - 1$. Clearly $\beta_1(T) \leqslant 1 + \beta_1(T - v)$ for any vertex v. If there is a vertex v whose removal creates two nontrivial components, then we are done (since for such a vertex $\beta_{\mathrm{dc}}(T - v) = \beta_1(T - v)$). But if no vertex creates two nontrivial components, then T is a star or double star and the theorem is still true. □

For an example of a tree with $\beta_{\mathrm{dc}}(T) = \beta_1(T) - 1$, consider an even path or a double star. For an example of a tree with $\beta_{\mathrm{dc}}(T) = \beta_1(T)$, consider a star or an odd path.

In general, a (vertex) cutset is said to be *nontrivial* if its removal leaves K_2 or creates at least two nontrivial components. Let $\kappa_n(G)$ denote the smallest cardinality of a nontrivial cutset. It holds that $\beta_{\mathrm{dc}}(G) \geqslant \beta_1(G) - \kappa_n(G)$.

The nontrivial cutsets are also the key to an algorithm for the disconnected matching number. For, if M is a disconnected matching, then the unsaturated vertices form a nontrivial cutset and contain a minimal nontrivial cutset. Thus, if the minimal nontrivial cutsets of the graph can be enumerated in polynomial time, then the disconnected matching number can be calculated in polynomial time. For, it suffices to consider each minimal nontrivial cutset S in turn, calculate the matching number of $G - S$, after which take the maximum value found.

One example is the family of interval graphs: these have $\mathrm{O}(n)$ minimal nontrivial cutsets, which can be generated efficiently. But all chordal graphs have $\mathrm{O}(n)$ minimal cutsets, and these can be efficiently determined (see [3,20]) (and this can be adapted for nontrivial cutsets). Similarly, circular-arc graphs have $\mathrm{O}(n^2)$ minimal nontrivial cutsets, and these can be enumerated efficiently.

4. The lower parameters

For simple classes of graphs, the lower matching number can easily be calculated for each of the seven versions listed earlier. For example, the following gives the values for the path P_n on n vertices:

β_*^-	β_{ac}^-, β_1^-, β_{dc}^-, β_{ur}^-	β_{if}^-	β_{c}^-
$n/5 \pm \mathrm{O}(1)$	$n/3 \pm \mathrm{O}(1)$	$2n/5 \pm \mathrm{O}(1)$	$n/2 - \mathrm{O}(1)$

We note that if two properties $\mathscr{P}$ and $\mathscr{Q}$ are such that $\mathscr{P} \subseteq \mathscr{Q}$, there is not necessarily a relationship between $\beta_{\mathscr{P}}^-$ and $\beta_{\mathscr{Q}}^-$. For example, there are graphs where $\beta_*^-(G) \gg \beta_{\mathrm{ac}}^-(G)$, and graphs where $\beta_{\mathrm{c}}^-(G) \ll \beta_{\mathscr{P}}^-(G)$ for the other six properties $\mathscr{P}$. Consider for example, the graph H_m formed by taking four disjoint copies of the graph $K_1 + mK_2$ (m triangles identified at one vertex), and then adding three edges so that the high-degree vertices induce a P_4. Then $\beta_*^-(H_m) = m + 2$ (one cannot saturate all the high-degree vertices) while $\beta_{\mathrm{ac}}^-(H_m) = 2$ (saturate the four high-degree vertices). On the other hand, if a graph G contains K_2 as a component, then $\beta_{\mathrm{c}}^-(G) = 1$, but the other lower parameters can be arbitrarily large.

Nevertheless, there are examples of comparable parameters. The following result is straightforward.

Theorem 7. *If G is connected, then every maximal connected matching is a maximal isolatefree matching and every maximal isolatefree matching is a maximal matching. Thus*

$$\beta_1^-(G) \leqslant \beta_{\mathrm{if}}^-(G) \leqslant \beta_{\mathrm{c}}^-(G).$$

Proof. We show that if M is a maximal connected matching or a maximal isolatefree matching then it is a maximal matching. Since a nontrivial connected graph is always isolatefree, this establishes the result.

If M is not a maximal matching, then there exists an edge that can be added; if that edge is adjacent to a saturated vertex then we have a contradiction. But even otherwise consider a shortest path from that edge to a saturated vertex, and consider the third-to-last and second-to-last vertex on that path; that edge can be added to the matching, and will maintain the property of being connected or isolatefree, a contradiction. Hence, M is a maximal matching. □

For the ordinary matching number, the lower and upper parameters are at most a factor of two apart. Such a result does not hold in general for an arbitrary property $\mathscr{P}$. It does not hold even for connected matchings (consider the union $K_2 \cup K_{2m}$, where the lower and upper connected matching numbers are 1 and m, respectively). But if G is required to be connected, then $\beta_1^-(G) \leqslant \beta_{\mathrm{if}}^-(G) \leqslant \beta_{\mathrm{c}}^-(G) \leqslant \beta_{\mathrm{c}}(G) = \beta_{\mathrm{if}}(G) = \beta_1(G)$; hence for isolatefree and connected matchings the lower and upper parameters are at most a factor of 2 apart in connected graphs.

But suppose a property $\mathscr{P}$ is additive and *clique-bounded* (meaning that there is a value $k_{\mathscr{P}}$ such that no graph with clique number larger than $k_{\mathscr{P}}$ is in $\mathscr{P}$ and every complete graph of order at most $k_{\mathscr{P}}$ is in $\mathscr{P}$). Then take the join $K_{k_{\mathscr{P}}} + mK_2$. The $\mathscr{P}$-matching number is at least m, and the lower $\mathscr{P}$-matching number is $\lfloor k_{\mathscr{P}}/2 \rfloor$.

4.1. Hardness

The lower parameter β_1^- is NP-hard (see for example [19]). From this it follows that the lower connected and disconnected matching numbers are NP-hard. For the former, join a K_2; for the latter union a K_2. We now give the proof for acyclic matching.

Theorem 8. *The parameter $\beta_{\mathrm{ac}}^-(G)$ is* NP-*hard.*

Proof. For any graph $G = (V, E)$ define a graph H_G as follows. Take G and for each vertex $v \in V$ introduce a copy of $2K_2$ (say $a_v b_v$ and $c_v d_v$) and make v adjacent to three of the new vertices (say a_v, b_v and c_v).

We claim that the lower acyclic matching number of H_G is determined by the maximum cardinality of an acyclic subgraph of G:

$$\beta_{\mathrm{ac}}^-(H_G) = 2|V| - I_{\mathrm{ac}}(G).$$

From this the theorem follows.

Take a maximum acyclic set $A \subseteq V$. For each $v \in A$, pair v with c_v. For each vertex $v \notin A$, take both $a_v b_v$ and $c_v d_v$. This is a maximal acyclic matching with $2|V| - |A|$ edges.

On the other hand, consider any maximal acyclic matching M of H_G. Define A as the vertices of G that are saturated. Clearly A induces an acyclic subgraph. For each vertex not in A, both $a_v b_v$ and $c_v d_v$ must be in M. If $v \in A$, then if matched with a vertex in V then $c_v d_v$ must be in M. So the smallest maximal acyclic matching is achieved by matching $v \in A$ with c_v. $\square$

4.2. Trees

Standard techniques provide linear-time algorithms for the lower parameters on trees. For example, we give here the details of the algorithm for the lower-induced matching number β_*^-. This uses the approach developed by Wimer (see for example [21]).

Consider a rooted tree T. For a vertex v and subset C of its children, we define T_v^C as the subtree consisting of v, C, and all of C's descendants. If C is all the children, we will write simply T_v.

Consider a maximal-induced matching (MIM). We define a *quasi-MIM* of T_v^C as the set S of saturated vertices in a restriction of a MIM to T_v^C. (In this we consider all possible trees with T_v^C as a subtree and all possible MIMs of those trees.) We identify five types of quasi-MIMs (where $N(S)$ denotes the vertices adjacent to S and $N[S] = S \cup N(S)$):

1. $v \notin N[S]$ but all of C is in $N[S]$,
2. $v \notin N[S]$ and at least one of C is not in $N[S]$,
3. $v \in S$ but none of C is in S,
4. $v \in S$ and one of C is in S,
5. $v \in N(S) - S$.

We define $f_i(T_v^C)$ as the minimum cardinality of a quasi-MIM of type i.

Now the algorithm proceeds by following a postorder traversal of the edges. Visiting an edge e has the following meaning: Say $e = xy$, where x is the parent of y, and C is those children of x whose edge to x has already been visited. Then in visiting the edge e we use the vectors $[f_i(T_x^C)]$ and $[f_i(T_y)]$ to calculate $f_i(T_x^{C\cup y})$.

The following table gives the type of a quasi-MIM of $T_x^{C\cup y}$ formed by the union of quasi-MIMs in T_x^C and T_y. A cross means that the combination does not produce a quasi-MIM.

Type in T_y \ Type in T_x^C	1	2	3	4	5
1	2	2	3	4	5
2	×	×	3	4	×
3	×	×	4	×	×
4	5	5	×	×	5
5	1	2	3	4	5

From this, one can then determine a formula for the optimum quasi-MIM of each type. For example,

$$f_2(T_x^{C\cup y}) = \min\{f_1(T_x^C) + f_1(T_y),\ f_2(T_x^C) + f_1(T_y),\ f_2(T_x^C) + f_5(T_y)\}.$$

To start the process, we need to define these parameters for a leaf v. A quasi-MIM in a leaf is either type 1 ($f_1 = 0$) or type 3 ($f_3 = 1$). To finish the process, we need to consider which quasi-MIMs are indeed MIMs: these are types 1, 4 and 5. Thus the overall value is

$$\min\{f_1(T_r),\ f_4(T_r),\ f_5(T_r)\}/2,$$

where r is the root.

Since each visit takes constant time, the overall algorithm runs in linear time.

5. Open questions and future work

We consider some open questions.

1. What is the complexity of the disconnected matching number?
2. Find fast algorithms for acyclic matching number on special graphs (e.g. interval graphs).

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