AI

<@mit>
Jn-14

Informed Search:

(1) Give the general principle or informed search.

Ans:

An informed search strategy-

- uses problem-specific knowledge beyond the definition of the problem itself
- can find solutions more efficiently than an uninformed strategy

*General approach: Best-first search

→ uses two functions

(i) evaluation function: $f(n)$
 - estimate of 'desirability'
 - expand most desirable unexpanded node

(ii) heuristic function: $h(n)$

$h(n)$ = estimated cost of the cheapest path from node n to a goal node

(4) When does a heuristic become consistent? Explain.

Ans: A heuristic becomes consistent when the following condition is satisfied—

$$\boxed{h(n) \leq c(n, a, n') + h(n')}$$

where
$h(n)$ = a heuristic function
n = each node
n' = every successor of n
a = any action
$c(n, a, n')$ = step cost for n to n' for any action a

Explanation:

A heuristic $h(n)$ is consistent if for every node n & every successor n' of n generated by any action a, the estimated cost of reaching the goal from n is no greater than the step cost of getting to n' plus the estimated cost of reaching the goal from n'.

~~Q~~ Prove that, with admissible heuristic, A^* algorithm is optimal.

Ans: We can prove that for TREE-SEARCH A^* is optimal if $h(n)$ is an admissible heuristic - that is, provided that $h(n)$ never overestimates the cost to reach the goal.

Proof:

Let us suppose ~~a~~ that G is ~~the~~ optimal goal-node & G_2 is ~~the~~ sub-optimal goal node.

Let the cost of the optimal solution be C^* where

$$C^* = f(G) = g(G) + h(G)$$

$$\Rightarrow \boxed{C^* = g(G)} \quad [h(G) = 0]$$

here $g(G)$ = exact cost to reach G from source

$h(G)$ = heuristic function

Then, because G_2 is suboptimal & because $h(G_2) = 0$ (true for any goal node), we know

$$f(G_2) = g(G_2) + h(G_2) > C^*$$

$$\Rightarrow \boxed{f(G_2) = g(G_2) > C^*},$$

since we considered that G is the optimal goal-node.

Now let's consider a fringe node n that is on an optimal solution path. If $h(n)$ is admissible i.e doesn't overestimate the cost of completing the solution path, then

$$\boxed{f(n) = g(n) + h(n) \leq C^*}$$

Now we have shown that,

$$\boxed{f(n) \leq C^* < f(G_2)},$$

so G_2 will not be expanded & A* must return an optimal solution.

(Proved)

... from n to the goal node & $h(n)$ is admissible if: $0 \le h(n) \le h^*(n)$ ... optimal

Show that if a heuristic is consistent it must be admissible.

<u>Ans:</u> A heuristic is consistent iff, for every node n & every successor n' of n generated by any action a,

$$\boxed{h(n) \le c(n,a,n') + h(n')}$$

One simple proof is by induction on the number k of nodes on the shortest path to any goal from n. For $k=1$, let n' be the goal node, then

$$\boxed{h(n) \le c(n,a,n')}$$

For the inductive case assume n' is on the shortest path, k steps from the goal node & that $h(n')$ is admissible by hypothesis, then,

$$\boxed{h(n) \le c(n,a,n') + h(n') \le c(n,a,n') + h^*(n') = h^*(n)}$$

so $h(n)$ at $k+1$ steps from the goal is also admissible.

(Showed)

(v) Give reason for the need & function of admissible heuristics & consistent heuristics & comment on their inter-relationship.

Ans:

Admissible heuristics:

- Let $h^*(N)$ be the true cost of the optimal path from N to a goal node
- Heuristic $h(N)$ is admissible if:

$$0 \leq h(N) \leq h^*(N)$$

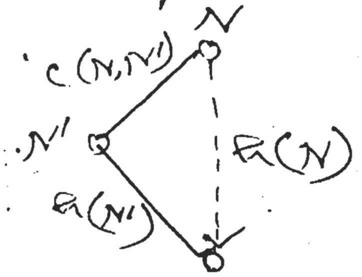
- An admissible heuristic is always optimal.

*Consistent heuristics:

The admissible heuristic h is consistent if for every node N & every successor N' of N:

$$h(N) \leq c(N, N') + h(N')$$

[Triangular inequality]



* Comment:

If a heuristic is consistent, it must be admissible.

(Q) Prove that uniform cost search is a special case of A* search.

Ans: Let us assume that

$f(n)$ = evaluation function

$g(n)$ = cost to reach n

$h(n)$ = heuristic function

We know, for A* search,

$$f(n) = g(n) + h(n) \quad \text{--- (1)}$$

& for uniform cost search

$$f(n) = g(n) \quad \text{--- (2)}$$

If we consider $h(n) = 0$ for all n

$$(1) = (2)$$

i.e., uniform cost search becomes a special case of A* search if $h(n) = 0$.

(Proved)