

$$\begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ 0 & & & \sigma_n \end{bmatrix}$$

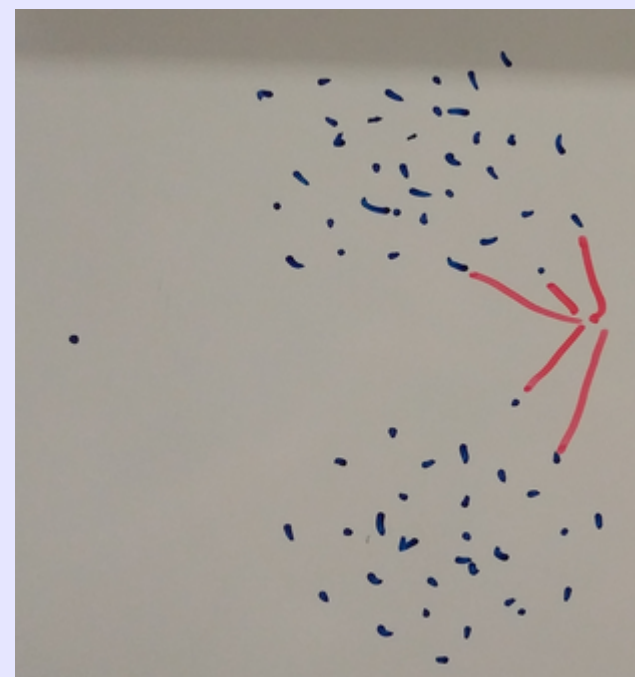
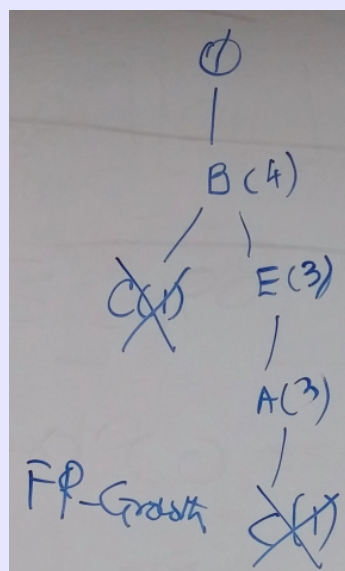
$$X = \sum_{i=1}^{\text{rank}(X)} \sigma_i u_i v_i^T = U \Sigma V^T$$

i^{th} singular value of X → Captures the patterns among attributes
 i^{th} left singular value of X (i^{th} column of U) → Captures the patterns among the objects
 i^{th} right singular vector of X (i^{th} column of V^T)

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

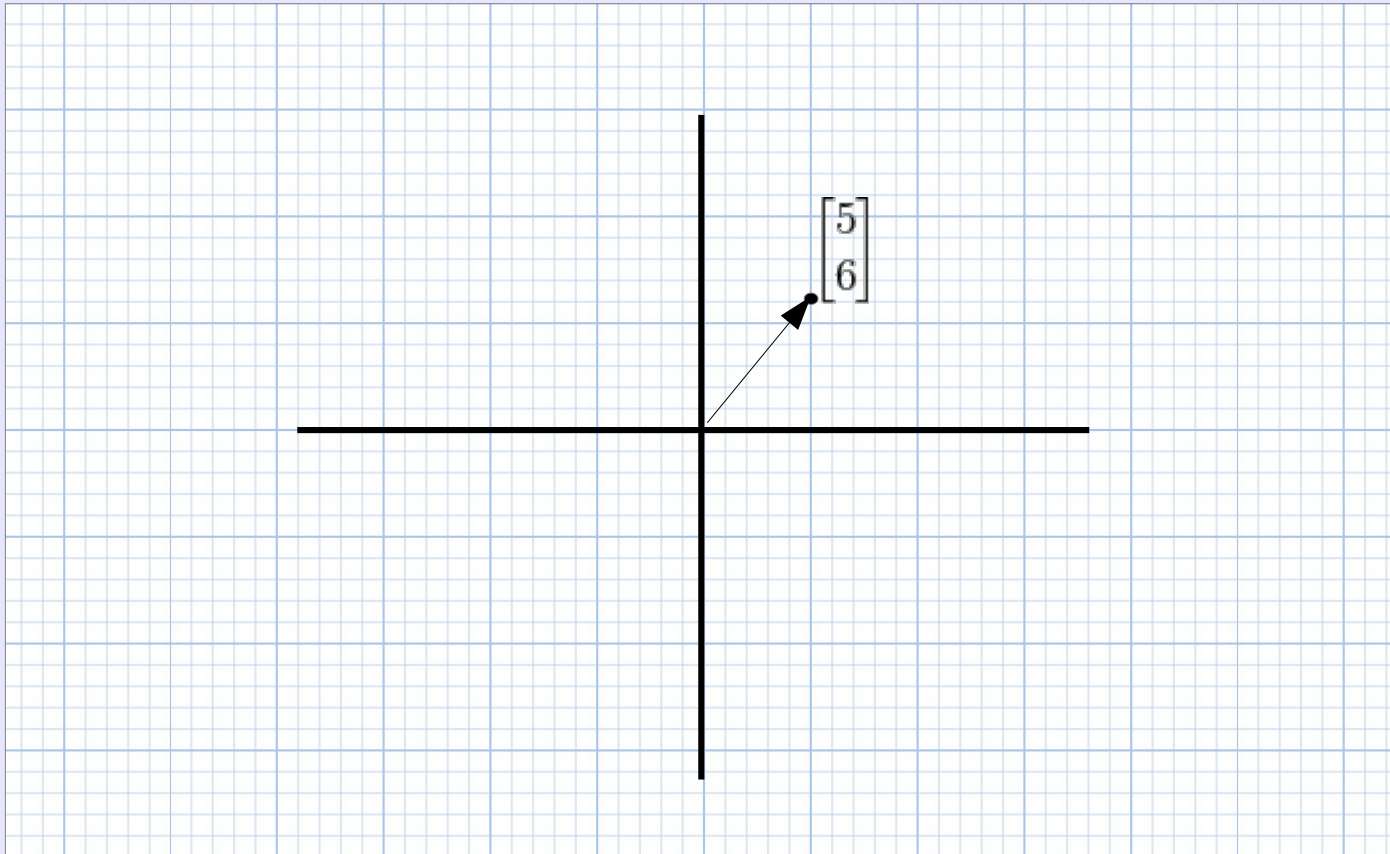
CS 422: Data Mining
 Vijay K. Gurbani, Ph.D.,
 Illinois Institute of Technology

Lecture: **The Perceptron**



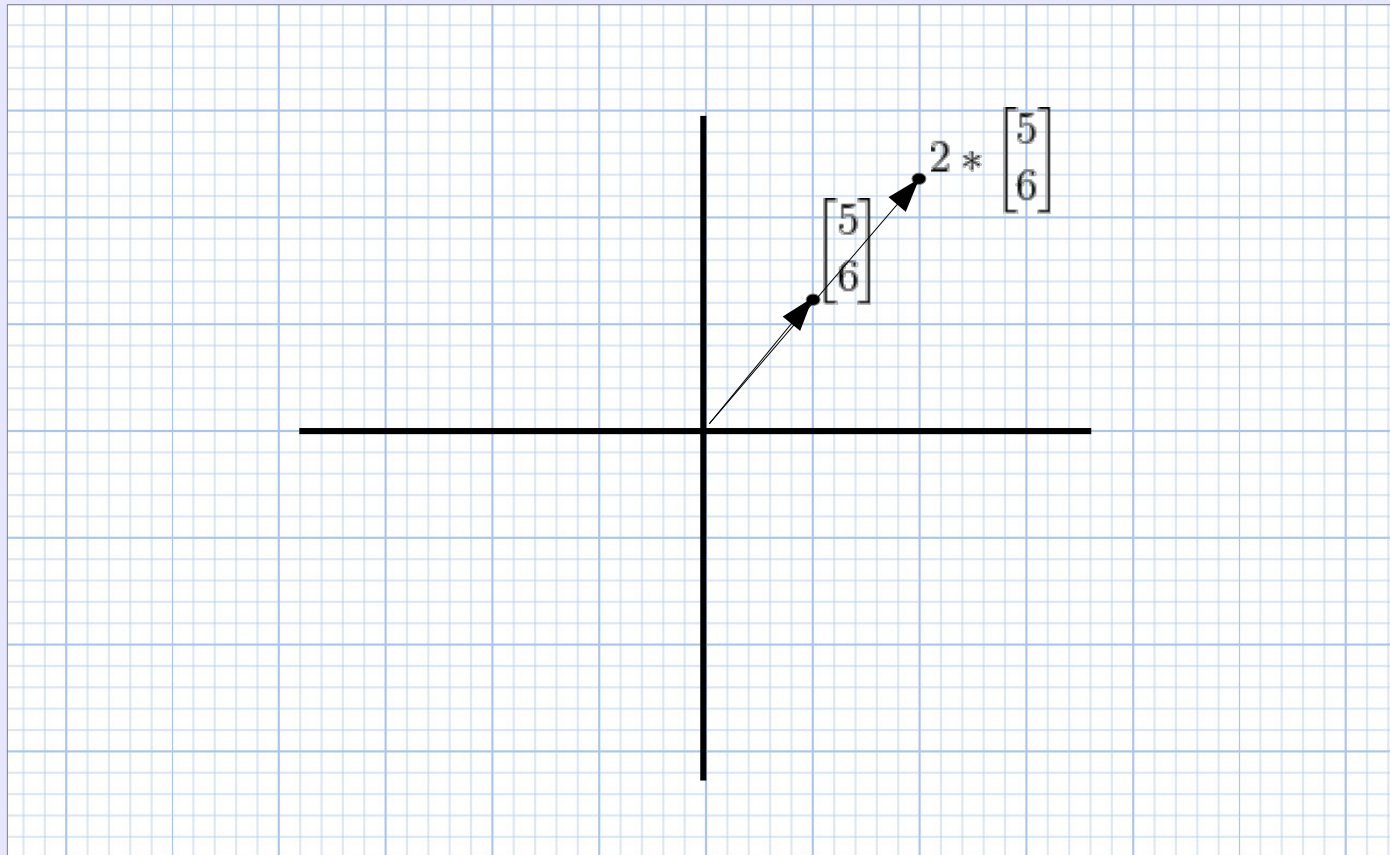
Recap: Matrices (Vector arithmetic)

Scalar multiplication with vector



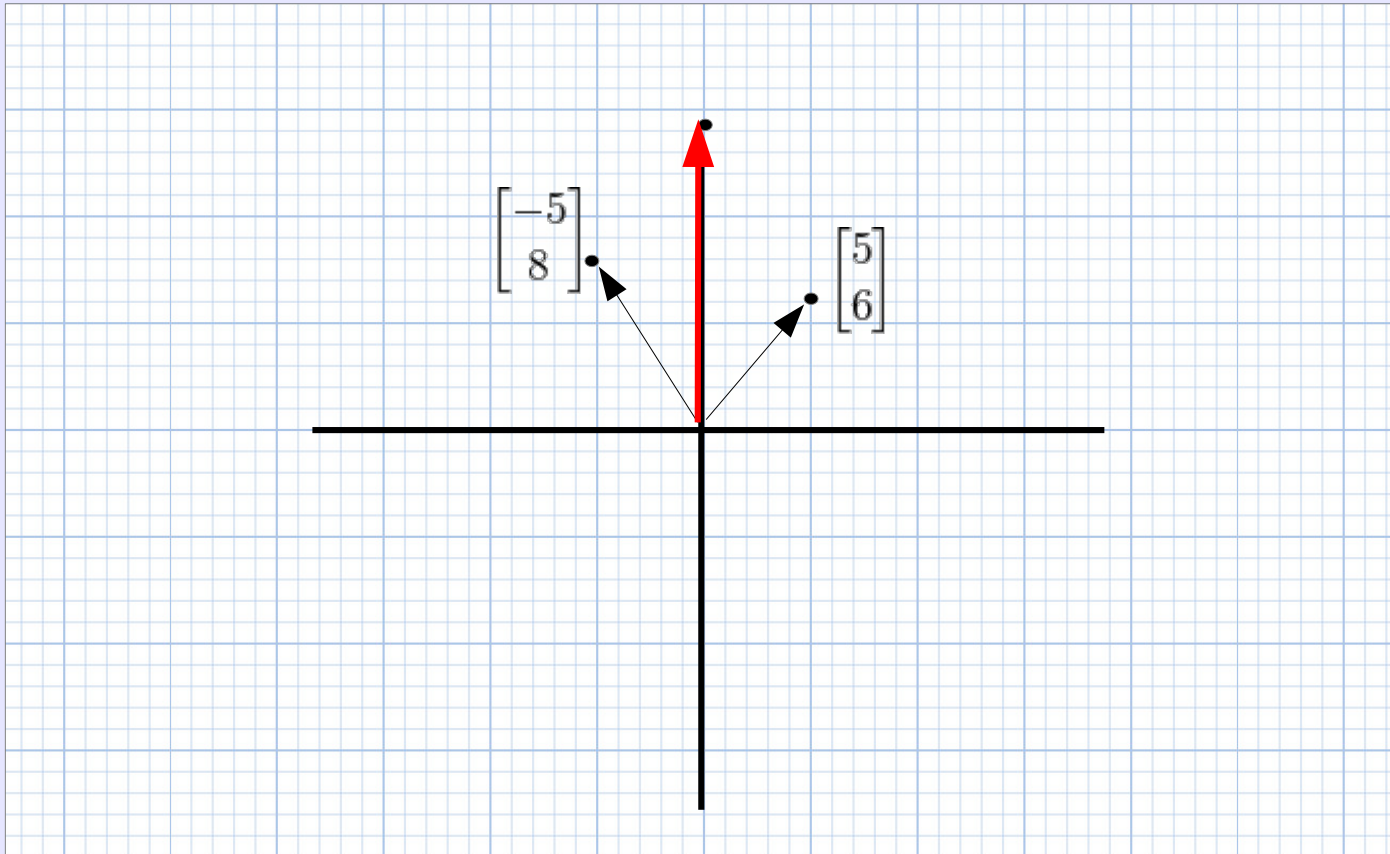
Recap: Matrices (Vector arithmetic)

Scalar multiplication with vector



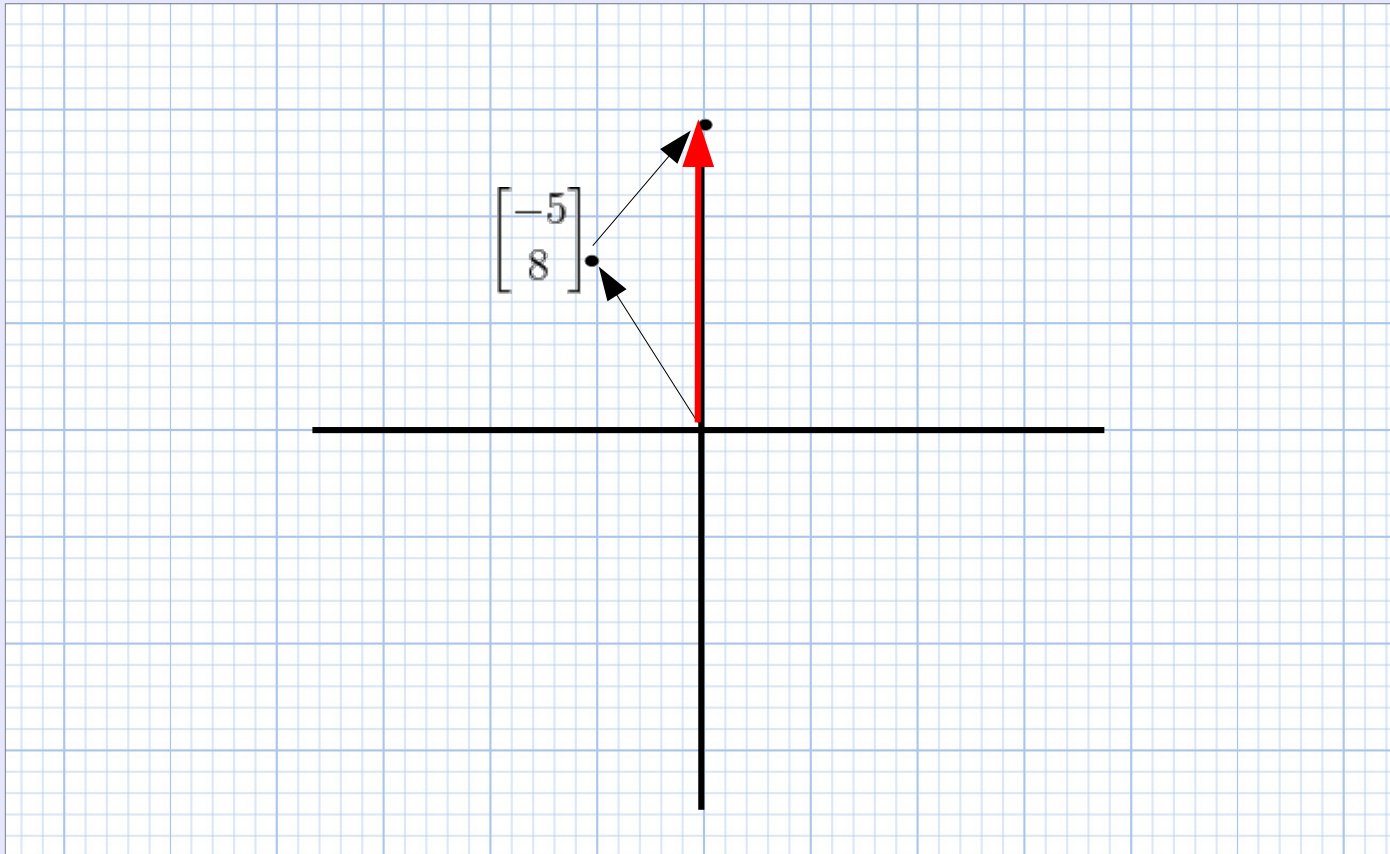
Recap: Matrices (Vector arithmetic)

Adding vectors $\begin{bmatrix} -5 \\ 8 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 14 \end{bmatrix}$



Recap: Matrices (Vector arithmetic)

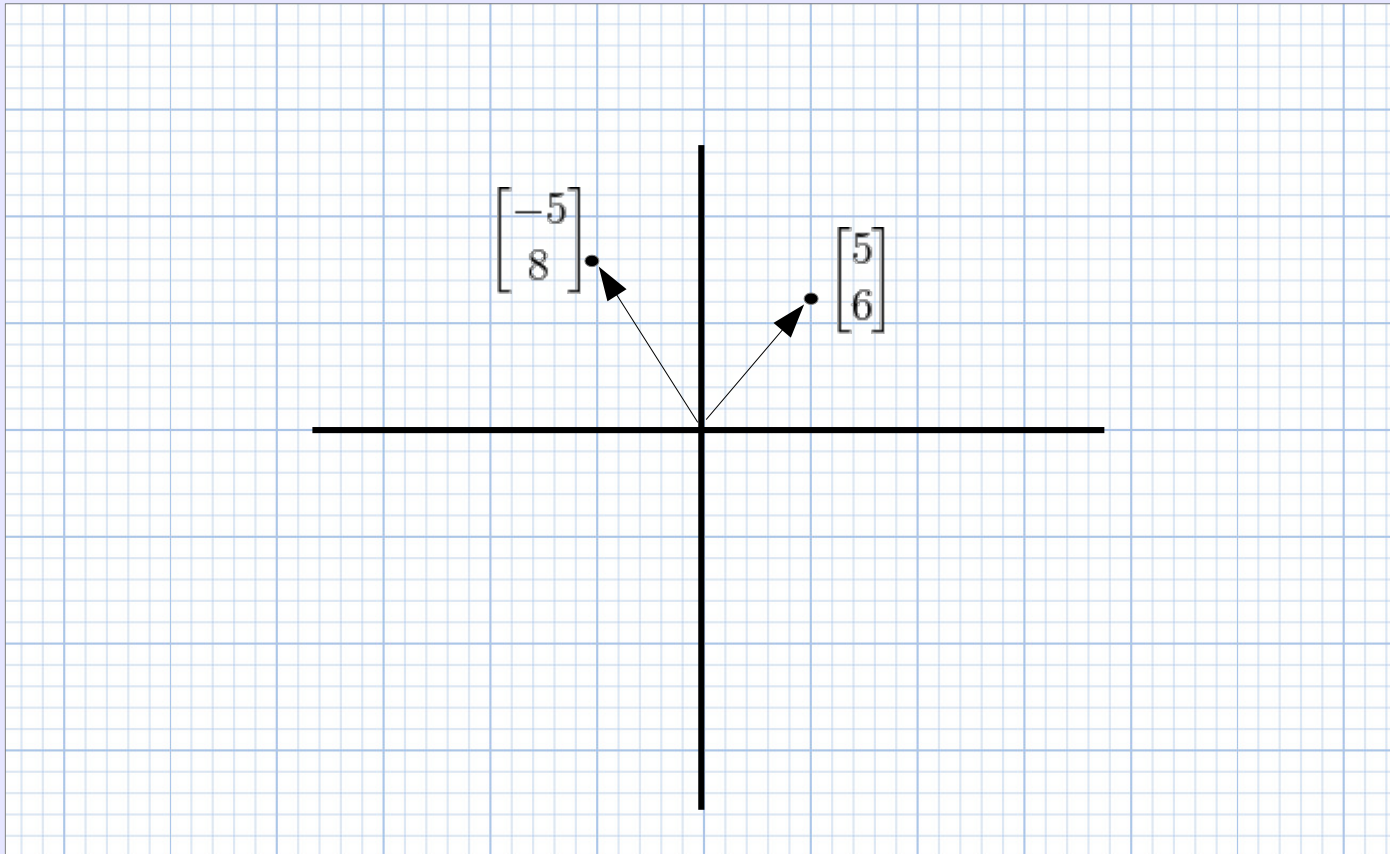
Adding vectors $\begin{bmatrix} -5 \\ 8 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 14 \end{bmatrix}$



Recap: Matrices (Vector arithmetic)

Subtracting
vectors

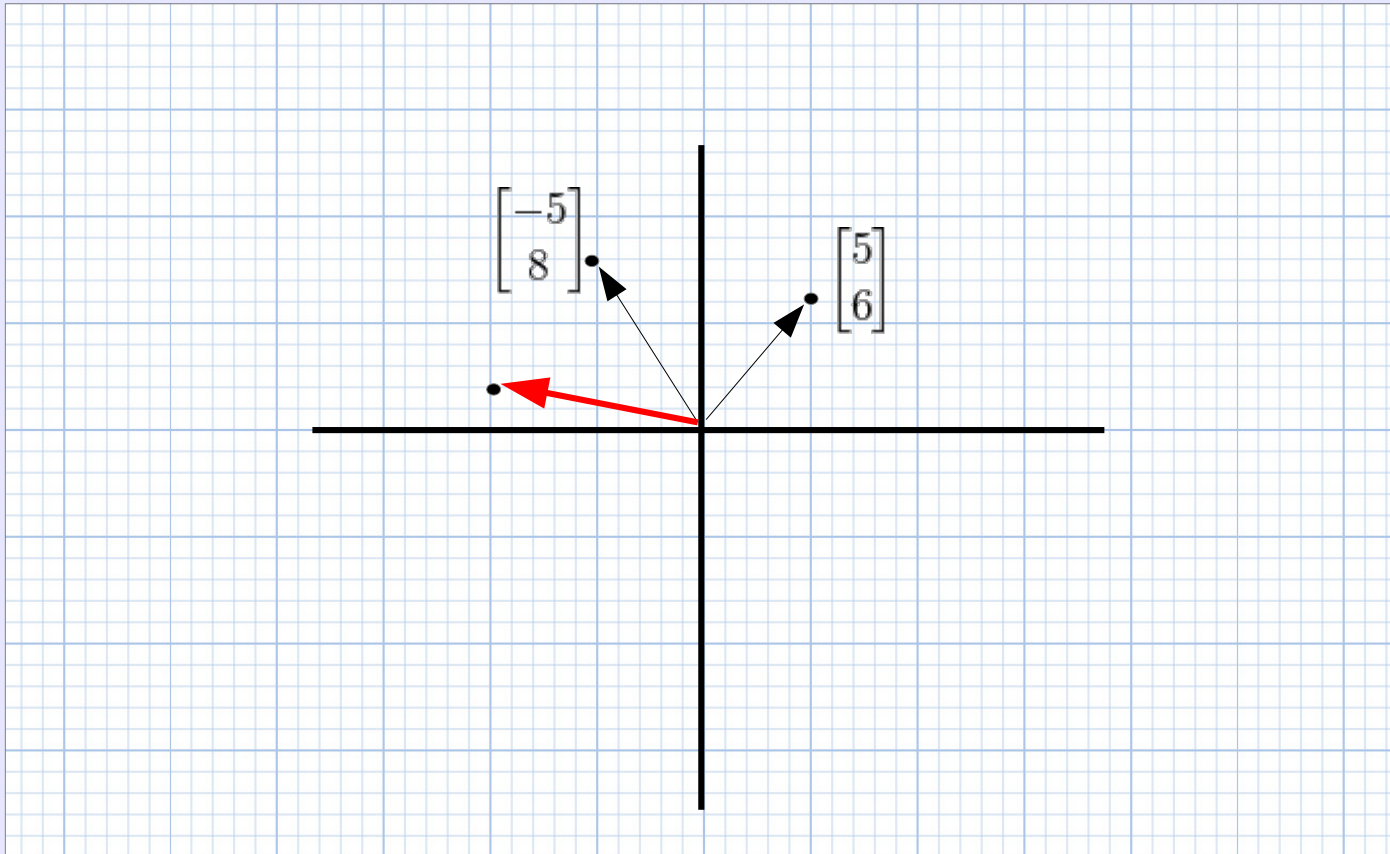
$$\begin{bmatrix} -5 \\ 8 \end{bmatrix} - \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$



Recap: Matrices (Vector arithmetic)

Subtracting
vectors

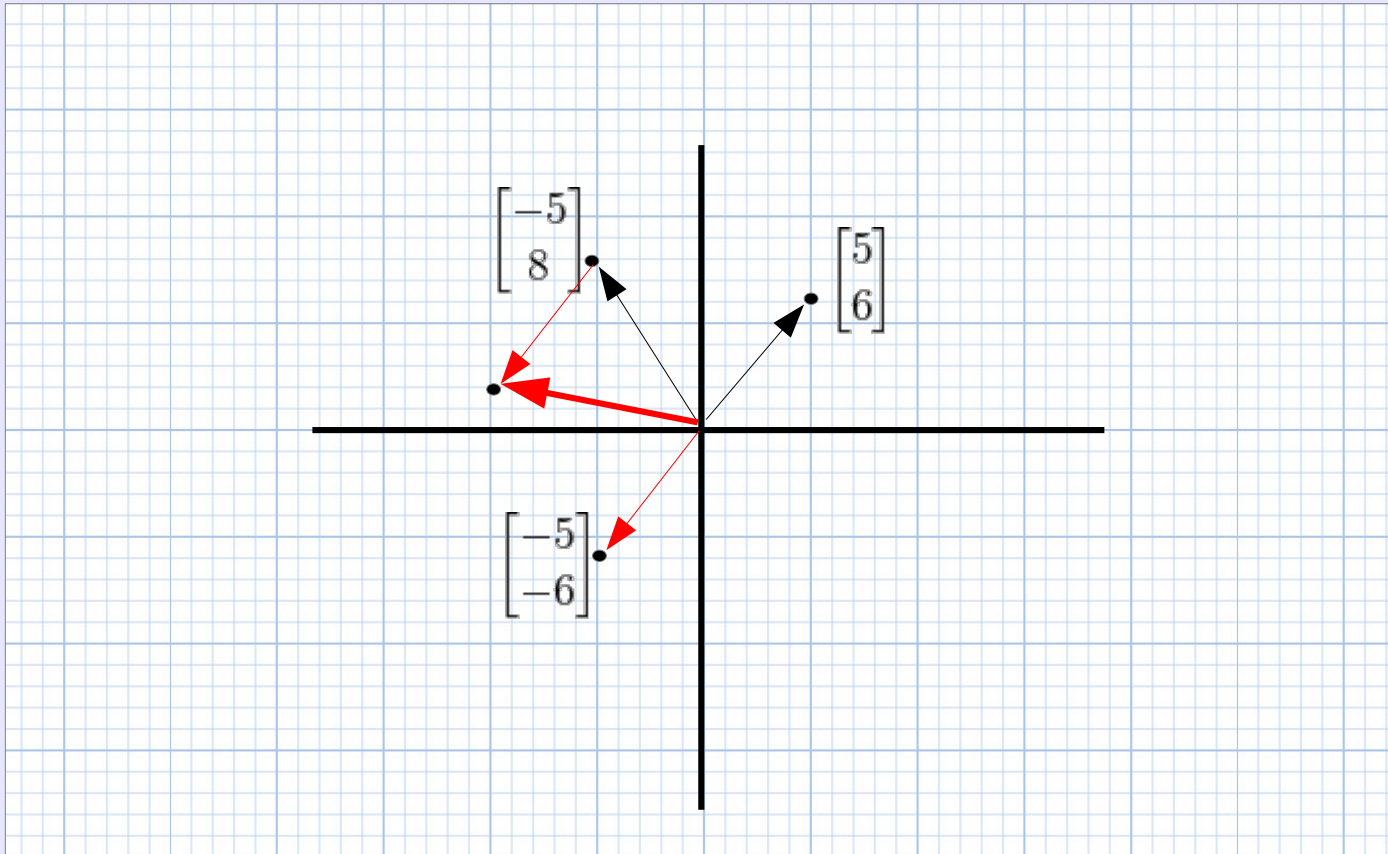
$$\begin{bmatrix} -5 \\ 8 \end{bmatrix} - \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} -10 \\ 2 \end{bmatrix}$$



Recap: Matrices (Vector arithmetic)

Subtracting
vectors

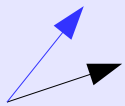
$$\begin{bmatrix} -5 \\ 8 \end{bmatrix} - \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} -5 \\ 8 \end{bmatrix} + \left(-1 * \begin{bmatrix} 5 \\ 6 \end{bmatrix} \right) = \begin{bmatrix} -5 \\ 8 \end{bmatrix} + \begin{bmatrix} -5 \\ -6 \end{bmatrix} = \begin{bmatrix} -10 \\ 2 \end{bmatrix}$$



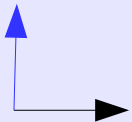
Recap: Matrices (Dot Product)

The dot product of two vectors $\vec{a} \cdot \vec{b}$ is the projection of \vec{a} onto \vec{b} .

We want to see how much of \vec{a} is pointing in the same direction as \vec{b} .



Dot product is positive if vectors are pointing in the same direction (geometrically, acute angle),



... 0 if perpendicular (right angle),



... negative if vectors pointing in nearly opposite direction (obtuse angle).

Mathematical interpretation of dot product: $\vec{a} \cdot \vec{b} = \sum_{i=1}^n a_i b_i + a_2 b_2 + \dots a_n b_n$
(also called an “inner product”)

Geometrical interpretation of the dot product: $\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos(\theta)$

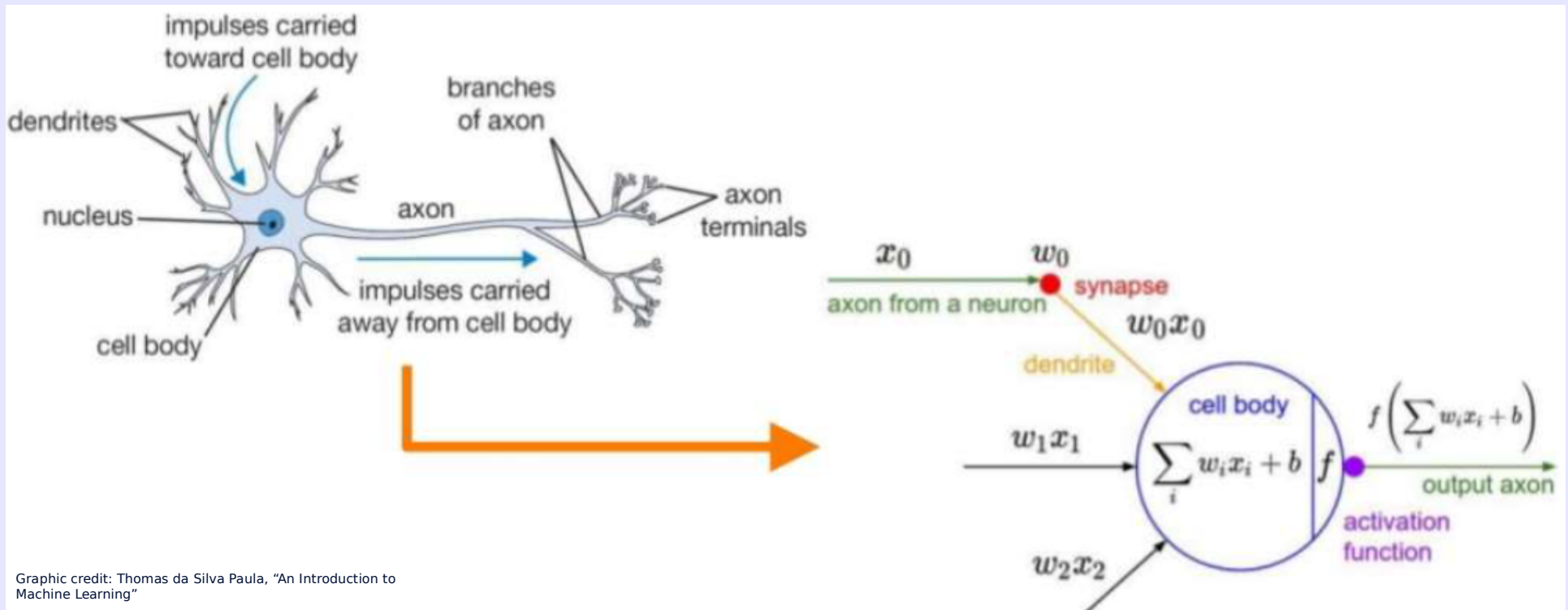
Ex: $\vec{a} = \langle 1, 3, 2 \rangle$; $\vec{b} = \langle 5, 1, 8 \rangle$

$$\vec{a} \cdot \vec{b} = 24, \|\vec{a}\| = \sqrt{1^2 + 3^2 + 2^2} = \sqrt{13}, \|\vec{b}\| = \sqrt{90}$$

$$\theta = \cos^{-1} \left(\frac{24}{\sqrt{13}\sqrt{90}} \right)$$

Introduction

- Neural networks: If computers are to think, why not model them after the human brain?



Introduction

- ... Except, of course, the human brain is in a league of its own.

Organism	Number of neurons
Jellyfish	5,600
Fruit fly	250,000
Frog	16,000,000
Cat	760,000,000
Humans	