MATH 569   HOMEWORK 2

Due: February 15^{th}, Wednesday, 11:59pm

How to submit: via Blackboard

If you have multiple files, upload a zipped file

Problem 1 Let $\hat{\beta}$ be the Least Square estimate of parameters β in the linear model $Y = X^T\beta + \epsilon$, with $\epsilon \sim \mathcal{N}(0, \sigma^2)$. Let $\theta = \alpha^T\beta$ be a linear combination of the parameters β.

1) First show that $\alpha^T\hat{\beta}$ is an unbiased estimate of θ.

2) Then show that for any other unbiased linear estimate $c^T y$ of θ, the variance of $\alpha^T\hat{\beta}$ is no bigger than the variance of $c^T y$.

Problem 2 Ex. 3.2 from the textbook. Submit your answers along with the code.

Problem 3 Ex. 3.16 from the textbook.

Problem 4 Consider a univariate model with no intercept, $Y = \beta_1 X + \epsilon$. First find the least square estimate of the coefficient β_1, then prove that for the training set $(\boldsymbol{X}, \boldsymbol{Y})$, the vector $(\boldsymbol{Y} - \hat{\boldsymbol{Y}})$ is orthogonal to the vector $\boldsymbol{X}$.

Extra Credit (10%) Consider the least square estimation of parameters β in the linear model $Y = X^T\beta + \epsilon$ with $\epsilon \sim \mathcal{N}(0, \sigma^2)$. Input matrix $\boldsymbol{X}$ is an $N \times (p+1)$ matrix. Show that $\frac{\text{SSE}}{\sigma^2}$ is distributed as a χ^2 random variable with $N - p - 1$ degrees of freedom. SSE is defined as:

$$\text{SSE} = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$