MATH

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017BSCIT005

ASSIGMENTS

Set - 1

Group A

A

2)

a) $\frac{2x^2}{x^2-1}$

i) Domain $x^2 - 1 = 0$

$x = \pm 1$

$\therefore$ Domain $= (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

ii) x-intercept. $y = 0$, $x = 0$

Y-intercept $x = 0$ $y = 0$

$\therefore$ It lies betn origin (0,0)

iii) Asymptode

- vertical: $x^2 - 1 = 0$ $x = \pm 1$

$\therefore$ vertical asymptote is -1 & 1

- Horizontal asymptote

$$\lim_{x \to \infty} \frac{2x^2}{x^2 - 1} = 2$$

iv) symmetry

$$f(-x) = \frac{2(-x)^2}{(-x)^2 - 1} = \frac{2x^2}{x^2 - 1}$$ It lies in origin

$$f'(x) = \frac{(x^2 - 1)4x - 2x^2 \times 2x}{(x^2 - 1)^2} = \frac{4x^3 - 4x - 4x^3}{(x^2 - 1)^2} = \frac{-4x}{(x^2 - 1)^2}$$

$x = 0$, $x = \pm 1$

Interval	$-\infty, -1$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Sign of $f'(x)$	+ve	+ve	-ve	-ve
Nature	increasing	Increasing	Decreasing	Decreasing

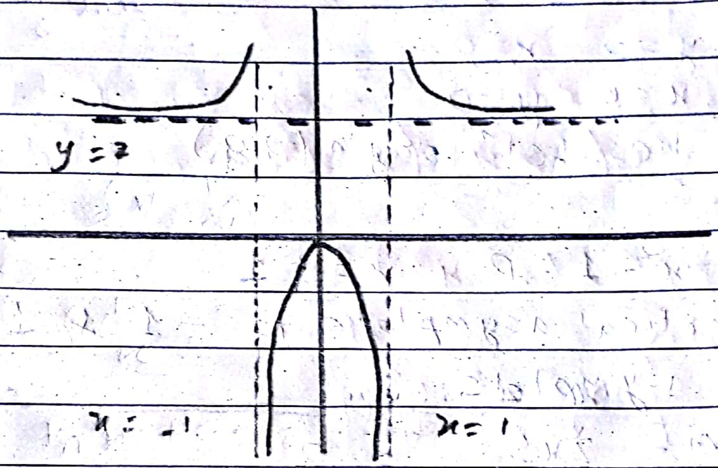
Sign is changed from +ve to -ve at $x = 0$ so Maxima ($x = 0$)

For concavity

$$f''(x) = \frac{(x^2-1)^2 \times -4 + 4x \cdot 2(x^2-1) \times 2x}{(x^2-1)^4} = \frac{12x^2+4}{(x^2-1)^3}$$

$f''(x) = 0$ $f''(x) = \infty$ we get $x = \pm 1$

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
sign of $f''(x)$	+ve	-ve	+ve
Nature	concave up	concave down	concave up



b) Here

$y = e^x$

Given interval is $x = 0$ & $x = 1$ $a = 0$ & $b = 1$

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{10} = 0.1$$

$f(a) = 0$

$f(a_1) = 0.1$ $f(a_5) = 0.5$

$f(a_2) = 0.2$ $f(a_6) = 0.6$ $f(a_9) = 0.9$

$f(a_3) = 0.3$ $f(a_7) = 0.7$ $f(b) = 1$

$f(a_4) = 0.4$ $f(a_8) = 0.8$

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$f(\bar{x}_1) = \frac{f(a) + f(a_1)}{2} = 0.05$

$f(\bar{x}_2) = 0.15$

$f(\bar{x}_3) = 0.25$

$f(\bar{x}_4) = 0.35$

$f(\bar{x}_5) = 0.45$

$f(\bar{x}_6) = 0.55$

$f(\bar{x}_7) = 0.65$

$f(\bar{x}_8) = 0.75$

$f(\bar{x}_9) = 0.85$

$f(x_{10}) = 0.95$

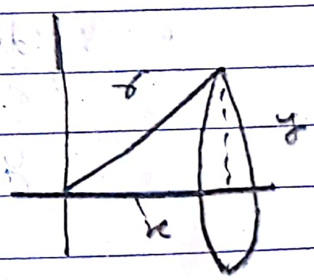
$$\int_a^b f(x) = \Delta x \left[f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + f(\bar{x}_4) + f(\bar{x}_5) + f(\bar{x}_6) + f(\bar{x}_7) + f(\bar{x}_8) + f(\bar{x}_9) + f(x_{10}) \right]$$

$$= 0.1 [1.05 + 1.16 + 1.28 + 1.41 + 1.56 + 1.73 + 1.91 + 2.11 + 2.33 + 2.5]$$

$$= 0.1 \times 17.01$$

$$= 1.71 \; //$$

3) a) here



From Pythagoras theorm $y^2 = r^2 - x^2$

now area $(A) = \pi r^2 = \pi (r^2 - x^2)$

$$\text{Volume} = \int_{-r}^{r} A(x)\, dx = 2 \int_0^r \pi (r^2 - x^2)\, dx$$

$$= 2\pi \int_0^r (r^2 - x^2)\, dx$$

$$= 2\pi \left[r^3 - \frac{r^3}{3} \right] - 0$$

$$= 2\pi \frac{2r^3}{3} = \frac{4\pi r^3}{3}$$

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b) Solve $x^2y'' + xy' = 1 \quad x>0 \quad y'(1)=0 \quad \& \quad y(1)=0$

Dividing by x^2

$$y'' + \frac{y'}{x} = \frac{1}{x^2} \quad — (1)$$

which is similar to $\frac{dy}{dx} + Py = Q$

$$If = e^{\int 1/x \, dx}$$
$$= e^{\ln x}$$
$$= x$$

$$x \times y = \int x \cdot 1/x^2$$

$$xy = \int \frac{1}{x}$$

$$xy = \ln x + c \quad — (11)$$

$$\cancel{\frac{dy}{dx} = \int \frac{\ln x}{x} + \int \frac{1}{x} + k}$$

$$\cancel{y = \frac{\ln x^2}{2} + \ln}$$

when $x = 1 \quad y = 0$

Eq

$$0 = \ln 1 + c$$
$$c = 0$$

$$xy = \ln x + 0$$
$$y = \frac{\ln x}{x}$$

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Group B

7)

⇒ If a function $y = f(x)$ is
i) continuous in close interval $[a,b]$
ii) differentiable for open interval (a,b)
iii) $f(a) = f(b)$
Then there exist a point on (a,b), $f'(c) = 0$

$f(x) = x^3 - x^2 - 6x + 2$

$f(0) = 0 - 0 - 0 + 2 = 2$

$f(3) = 3^3 - 3^2 - 3 \times 6 + 2$
$= 2$

$\therefore f(0) = f(3)$ $[\because f(a) = f(b)]$

which lies in interval $[0,3]$
Rolle's theorm is verified

8) $f(x) = x^3 - 6x - 5$ $f'(x) = 3x^2 - 6$

From Newton's Method

$$f_{n+1} = f_n - \frac{f(x)}{f'(x)}$$

$x_0 = 2$

$$x_1 = 2 - \frac{2^3 - 6 \times 2 - 5}{3 \times 4 - 6}$$

$= 3.5$

$x_1 = 3.5$

$$x_2 = x_1 - \frac{3.5^2 - 6 \times 3.5 - 5}{3 \times 2.5 \times 3.5 - 6} = 2.951$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.802$$ //

9) $\int_{-\infty}^{0} x e^{x} dx$

$\lim_{t \to -\infty} \int_{-t}^{0} x e^{x} dx$

$\lim_{t \to -\infty} \left[[x e^{x}]_{t}^{0} - \int_{t}^{0} e^{x} dx \right]$

$\lim_{t \to -\infty} (-t e^{t} - 1 + e^{t})$

$= 0 - 1 + 0$

$= -1$

10) Given

$y = x^2$

$y = x$

now by cylindrical shell

when $x = 0$ $y = 0$

$x = 1$ $y = 1$

now

volume $= \int_{0}^{1} 2\pi x (x - x^2) dx$

$= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_{0}^{1}$

$= 2\pi \left(\frac{1}{3} - \frac{1}{4} \right)$

$= \pi/6$ //

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21) $y'' + y' - 6y = 0$, $x > 0$, $y'(0) = 0$ & $y(0) = 1$

The auxiliary Eq is

$$m^2 + m - 6 = 0$$

$$m^2 + (3-2)m - 6 = 0$$

$$m^2 + 3m - 2m - 6 = 0$$

$$m(m+3) - 2(m+3) = 0$$

$$(m+3)(m-2) = 0$$

$$m_1 = 2 , \quad m_2 = -3$$

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$y = c_1 e^{2x} + c_2 e^{-3x}$$

21) $y'' + y' - 6y = 0$, $x > 0$, $y'(0) = 0$ & $y(0) = 1$

The auxiliary eq is

$$m^2 + m - 6 = 0$$
$$m^2 + (3-2)m - 6 = 0$$
$$m^2 + 3m - 2m - 6 = 0$$
$$m(m+3) - 2(m+3) = 0$$
$$(m+3)(m-2) = 0$$
$$m_1 = 2, \quad m_2 = -3$$

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$
$$y = c_1 e^{2x} + c_2 e^{-3x} - ①$$

$$\cancel{y(0) = c_1 + c_2 =}$$

Diff Eq ①

$$y' = 2c_1 e^{2x} - 3c_2 e^{-3x}$$

$$y(0) = c_1 + c_2 = 1$$

$$y'(0) = 2c_1 - 3c_2 = 0$$

$$c_2 = \tfrac{2}{3} c_1$$

$$c_1 + \frac{3}{2} c_1 = 1 \qquad c_1 = 3/5 \qquad c_2 = \frac{2}{5}$$

$$\therefore \; y = \frac{3}{5} e^{2x} + \frac{2}{5} e^{-3x}$$

13) $\sum_{n=0}^{\infty} \frac{1}{1+n^2}$

let $a_n = \frac{1}{1+n^2}$

$b_n = \frac{1}{n^2}$

it is seen $a_n < b_n$ for all $n \in Z^+$

$\sum a_n < \sum b_n = \sum \frac{1}{1+n^2} \quad \sum \frac{1}{n^2}$

The series $\sum \frac{1}{n^2}$ is convergent by test $P = 2$

so direct comparision test $\sum_{n=1}^{\infty} \frac{1}{1+n^2}$

is convergent

$\sum_{n=0}^{\infty} \frac{1}{1+n^2} = 1 + \sum_{n=0} \frac{1}{1+n^2}$.

$= 1 + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} \cdots \infty$

Hence it converges

14) $\iint_R f(x,y)\, dA = f(x,y) = x^2y - 2xy$

$R: -2 \leq x \leq 0$

$0 \leq y \leq 3$

$\int_0^3 \int_{-2}^{0} x^2y - 2xy \, dx\, dy$

$\int_0^3 \int_{-2}^{0} (x^2 y - 2xy\, dx)\, dy$

$$\int_0^3 \left[\frac{x^3}{3} y - \frac{2x^2}{2} y \right]_{-2}^{0}$$

$$- \int_0^3 \left[-\frac{8}{3} y - 4y \right] dy$$

$$= \left[\frac{8}{3} \frac{y^2}{2} + \frac{4y^2}{2} \right]_0^3$$

$= 12 + 18$

$= 30$

$$\int_{-2}^{0} \int_0^3 (x^2 y - 2xy)\, dy$$

$$\int_{-2}^{0} \left[\frac{x^2 y^2}{2} - \frac{2xy^2}{2} \right]_0^3 dx$$

$$\int_{-2}^{0} \left[\frac{x^2 \cdot 9}{2} - 9x \right] - 0 \, dx$$

$$\left[\frac{9x^3}{2 \times 3} - \frac{9x^2}{2} \right]_{-2}^{0} = \left[\frac{3 \times (-2)^3}{-2} - \frac{9 \times (-2)^2}{2} \right]$$

$= 12 - (-18)$

$= 30$ //

15)

$F(x,y) = x^3 + x^2 y^3 - 2y^2$ at $(2,1)$

$\frac{df}{dx} = \frac{d}{dx}(x^3 + x^2y^3 - 2y^2) = 3x^2 + 12xy^3$

$\left.\frac{df}{dx}\right|_{(2,1)} = 2(2)^2 + 2(2)1^3 = 8 + 4 = 12$

$F(2,1) = \left.\frac{df}{dx}\right|_{(2,1)} = 12$

$\frac{df}{dy} = 3y^2x^2 - 4y$

$\left.\frac{df}{dy}\right|_{(2,1)} = 3(1)^2(2)^2 - 4(1)$
$= 3\times 4 - 4$
$= 12 - 4 = 8$

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6) A function $F(x,y)$ is continuous at point (x_0,y_0) if F is defined at (x_0,y_0) $\lim_{(x,y)\to(x_0,y_0)} F(x,y)$ exist and $\lim_{(x,y)\to(x_0,y_0)} F(x,y) = F(x_0,y_0)$

$F(x) = 1 - \sqrt{1-x^2}$

$F(1) = 1 - \sqrt{0}$ $\quad F(-1) = 1 - \sqrt{1-(-1)^2}$
$= 1$ $\quad = 1 - 0 = 1$

since $F(1) = F(-1)$

It is continuous on interval $[-1, 1]$

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5)

a) $f(x) = x^5 + x$

so

$f(-x) = (-x)^5 + (-x)$

$= -x^5 - x$

$= -(x^5 + x)$

$= -f(x)$

It is ~~even~~ odd

b) $y = 1 - x^4$

$f(-x) = 1 - (-x)^4$

$= 1 - x^4$

$= f(x)$

$\therefore$ It is even

c) $y = 2x - x^2$

$f(-x) = 2 \cdot (-x) - (-x)^2$

$= -2 - x^2$

$\cancel{= -(2 + x^2)}$

$\therefore$ It is neither odd nor even

1) a) GrpA

$$f(x) = \begin{cases} 1-x & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$$

Since

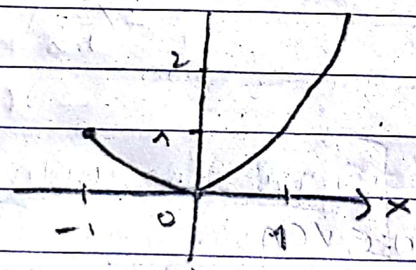
$-2 \le -1$ we have $f(-2) = 1-(-2) = 3$

$-1 \le -1$ we have $f(-1) = 1-(-1) = 2$

since, $0 > -1$ $f(0) = 0^2 = 0$

If $x \le -1$ then $f(x) = 1-x$ so part of graph of f that lies to left of vertical line $x=-1$ must coincide with line $y = 1-x$ which has slope -1 & y intercept 1.

If $x > -1$ then $f(x) = x^2$ so part of graph f that lies to right of line $x=-1$ must coin with graph $y = x^2$ parabola



b)

$$\lim_{x\to 0^+} \frac{|x|}{x} = \lim_{x\to 0^+} \frac{x}{x} = \lim_{x\to 0^+} 1 = 1$$

$$\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^-} \frac{-x}{x} = \lim_{x\to 0} -1 = -1$$

Sin RHL & LHL are different. From theorm it doesn't exist

Set 2

Group A

2)

a) Sketch the curve $f(x) = \frac{x^2}{\sqrt{x+1}}$

- domain

$x+1 \geq 0$

$x \geq -1$ So domain is $[-1, \infty)$.

- Intercept: x-intercept $x=0$ $y=0$

y-intercept $y=0$ $x=0$

$\therefore$ Curve meet at $(0,0)$

- Symmetry: It is not symmetry
- Asymptote

No horizontal

$\lim_{x \to -1^+} \frac{x^2}{\sqrt{x+1}} = \infty$ $x=-1$ is vertical asymptote

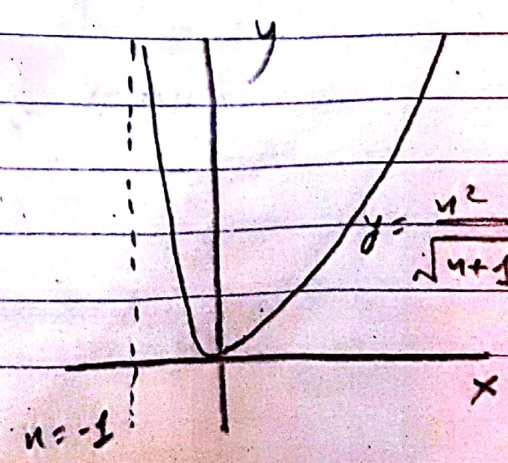
- $f'(x) = \frac{x(3x+4)}{2(x+1)^{3/2}}$ $x=0, -1$

Interval	$(-1, 0)$	$(0, \infty)$
sign of $f'(x)$	$-ve$	$+ve$
Nature of $f(x)$	Decreasing	Increasing

minima at $x=0$

- $f''(x) = \frac{3x^2+8x+8}{4(x+1)^{5/2}}$ $x=-1$

Interval	$(-1, \infty)$
sign of $f''(x)$	$+ve$
Nature of $f''(x)$	concave upward



b) Here

the curve $y = x^2$

now $a = 0,\ b = 1$

$$\Delta x = \frac{b-a}{n} = \frac{1}{10} = 0.1$$

$f(a) = 0$, $f(b) = 1$, $f(a_1) = 0.1$ $f(a_2) = 0.2$ $f(a_3) = 0.3$ $f(a_4) = 0.4$
$f(a_5) = 0.5$ $f(a_6) = 0.6$ $f(a_7) = 0.7$ $f(a_8) = 0.8$ $f(a_9) = 0.9$

$f(\bar{x}_1) = 0.05$ $f(\bar{x}_2) = 0.15$ $f(\bar{x}_3) = 0.25$ $f(\bar{x}_4) = 0.35$ $f(\bar{x}_5) = 0.45$
$f(\bar{x}_6) = 0.55$ $f(\bar{x}_7) = 0.65$ $f(\bar{x}_8) = 0.75$ $f(\bar{x}_9) = 0.85$ $f(\bar{x}_{10}) = 0.95$

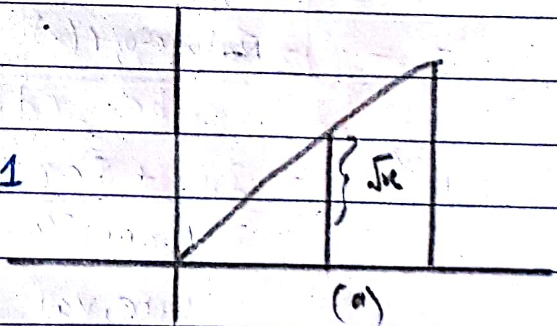
$$\int_a^b f(x) = \Delta x \left[0.0025 + 0.0225 + 0.0625 + 0.1225 + 0.2025 + 0.3025 + 0.4225 + 0.5625 + 0.7225 + 0.9025 \right]$$
$$= 0.1 \times 3.325$$
$$= 0.3325$$

3) a) $A(x) = \pi (\sqrt{x})^2$
$= \pi x$

& volume of approximation

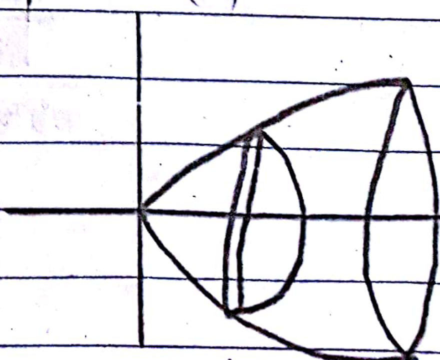
$A(x)\,\Delta x = \pi x\,\Delta x$

Solid lies betn $x = 0$ & $x = 1$



(a)

$$V = \int_0^1 A(x)\,dx$$
$$\int_0^1 \pi x\,dx$$
$$= \pi \left[\frac{x^2}{2}\right]_0^1 = \frac{\pi}{2}$$


(b)

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1)

a) $f(x) = \begin{cases} 1+x & \text{if } x \le 1 \\ x^2 & \text{if } x > 1 \end{cases}$

$f(3)$, $f(1)$ & $f(0)$

$f(3) = 3^2 = 9 \quad : \quad 3 > 1$

$f(0) = 1+0 = 1 \quad 0 \le 1$

$f(1) = 1+1 = 2 \quad 1 \le 1$

b) $\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x}$

$= \lim_{x \to 0^+} 1$

$= 1$

$\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} = -1$

Since RHL $\neq$ LHL. So from theorm it doesn't exist

Group B

5)

a) $y = x^5 + x$

$F(x) = x^5 + x$

$F(-x) = -x^5 - x$

$= -(x^5 + x)$

$= -F(x)$ ∴ It is odd

b) $y = 1 - x^4$

$f(x) = 1 - x^4$

$f(-x) = 1 - (-x)^4$

$= 1 - x^4$

$= F(x)$

∴ It is even.

c) $y = 2x - x^2$

$f(-x) = 2(-x) - x^2$

$= -2x - x^2$

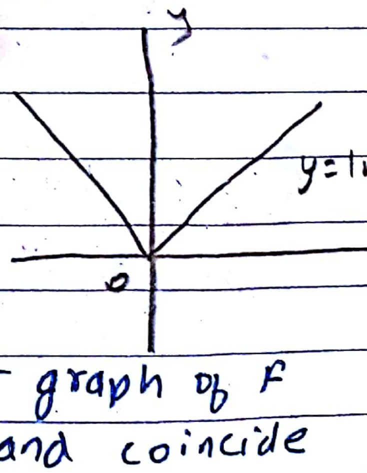
which is not $\neq f(x)$ & $\neq -F(x)$

It is neither odd nor even

7)

From the discussion we know

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



using the same method, we see that graph of F coincide with line $y = x$ to right of y-axis and coincide with $y = -x$ to left of y-axis. [when we keep values

8) $x_1 = 2$, $x_3 = ?$

$f(x) = x^3 - 2x - 5$

$f'(x) = 3x^2 - 2$

now

$$f_{n+1} = f_n - \frac{f(x)}{f'(x)}$$

$x_1 = 2$

$$x_2 = 2 - \frac{x^3 - 2x - 5}{3x^2 - 2} = 2 - \frac{8 - 4 - 5}{12 - 2}$$

$$= 2 - \frac{(-1)}{10} = \frac{20 + 1}{10} = \frac{21}{10} = 2.1$$

$$x_3 = 2.1 - \frac{(2.1)^3 - 2(2.1) - 5}{3(2.1)^2 - 2}$$

$$= 2.1 - \frac{9.261 - 4.2 - 5}{3 \times 4.41 - 2}$$

$$= \frac{9.03 + 0.061}{11.23} = \frac{9.091}{11.23} = 0.809$$

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10) $y^2 = x^3$ betn point $(1,1)$ & $(4,8)$

$y = x^{3/2}$ $\frac{dy}{dx} = \frac{3}{2} x^{1/2}$

So arc length

$$L = \int_1^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \int_1^4 \sqrt{1 + \frac{9}{4} x}\, dx$$

we substitute

$u = 1 + \frac{9}{4} x$, $du = \frac{9}{4} dx$

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when $x=1$, $u=13/4$, when $x=4$, $u=10$

$$\therefore L = \frac{4}{9}\int_{13/4}^{10} \sqrt{u}\,du = \left[\frac{4}{9}\cdot\frac{2}{3}u^{3/2}\right]_{13/4}^{10}$$

$$= \frac{8}{27}\left[10^{3/2} - \left(\frac{13}{4}\right)^{3/2}\right] = \frac{1}{27}(80\sqrt{10} - 13\sqrt{13})$$

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11) $P(x)\frac{d^2y}{dx^2} + Q(x)\frac{dy}{dx} + R(x)\,y = G(x)$ — (1) where P, Q, R & G are continuous function is called second order linear diff Eq.

If $G(x)=0$ for all x, then $P(x)\frac{d^2y}{dx^2} + Q\frac{dy}{dx} + R(x) = 0$ — (2) is homogeneous seco ord d.

$$y'' - y = 0, \quad x>0, \quad y'(0)=3 \;\&\; y(0)=2$$

$$m^2 - 1 = 0$$

$$m = +1$$

$$m = \pm 1$$

$$y = c_1 e^{x} + c_2 e^{-x}$$

6) let $f(x,y) = 2x^2 + y^2$

$f_x(x,y) = 4x \qquad f_y(x,y) = 2y$

$f_x(1,1) = 4 \qquad f_y(1,1) = 2$

Then

$z = 4(x-1) + 2(y-1)$

$z = 4x - 4 + 2y - 2$

$z = 4x + 2y - 6$ //

14) $\cancel{\iint_R f(x,y)\,dA \text{ for } R = f(x,y) = x^2y - 2xy}$

$\int_0^3 \int_1^2 x^2 y \, dy\, dx$ and $\int_1^2 \int_0^3 x^2 y \, dx\, dy$

$\cancel{\int_0^3 \left[\frac{x^3}{3}\right]_1^2 y\,d}$

$\int_0^3 x^2 \left[\frac{y^2}{2}\right]_1^2 dx \qquad \int_1^2 \left[\frac{x^3}{3}\right]_0^3 dx\, dy$

$= \int_0^3 \frac{3}{2} x^2\, dx \qquad = \int_1^2 9y\, dy$

$= \frac{3}{2}\left[\frac{x^3}{3}\right]_0^3 \qquad = 9\left[\frac{y^2}{2}\right]_1^2$

$= \frac{3}{2}[9] \qquad = 9 \times \frac{3}{2}$

$= \frac{27}{2} \qquad = \frac{27}{2}$ //

22) P

the vector $\vec{PQ} \times \vec{PR}$ is $\perp$ to both $\vec{PQ}$ & $\vec{PR}$ and is therefore perpendicular to the plane P, Q & R

$\vec{PQ} = -3\vec{i} + \vec{j} - 7\vec{k}$

$\vec{PR} = (1-1)\vec{i} + (-1-4)\vec{j} + (1-6)k = -5\vec{j} - 5\vec{k}$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ -3 & 1 & -7 \\ 0 & -5 & -5 \end{vmatrix}$$

$= (-5-35)\vec{i} - (15-0)\vec{j} + (15-0)\vec{k}$

$= -40\vec{i} - 15\vec{j} + 15K$

So $(-40, -15, 15)$ is $\perp$ to any given plane

$(-8, -3, 3)$.

14) $\sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3}$

$a_n = \frac{5}{2n^2+4n+3} = \frac{5}{n^2\left(2+\frac{4}{n}+\frac{3}{n^2}\right)}$

$b_n = \frac{1}{n^2}$

now,

$\lim_{n\to\infty} \frac{a_n}{b_n}$ $\lim_{n\to\infty} \frac{\frac{5}{2n^2+4n+3}}{\frac{1}{n^2}}$ $\lim_{n\to\infty} \frac{5}{2+\frac{4}{n}+\frac{3}{n^2}}$

$\frac{5}{2+0+0} = \frac{5}{2}$ as $\frac{a_n}{b_n} = \frac{5}{2} > 0$

& b_n converges from P series test $P>1$ so the series converges from limit comparison test

9)

a) $s(4) - s(1) = \int_1^4 v(t)\,dt = \int_1^4 (t^2 - t - 6)\,dt$

$= \left[\frac{t^3}{3} - \frac{t^2}{2} - 6t\right]_1^4 = -\frac{9}{2}$

This mean particle moved 4.5m to ~~be~~ward left

b) $v(t) = t^2 - t - 6 = (t-3)(t+2)$ so $v(t) \le 0$ on interval $[1, 3]$

$\int_1^4 |v(t)|\,dt = \int_1^3 -v(t)\,dt + \int_3^4 v(t)\,dt$

$= \int_1^3 (-t^2 + t + 6)\,dt + \int_3^4 (t^2 - t - 6)\,dt$

$= \left[-\frac{t^3}{3} + \frac{t^2}{2} + 6t\right]_1^3 + \left[\frac{t^3}{3} - \frac{t^2}{2} - 6t\right]_3^4$

$= \frac{61}{6} = 10.17\text{m}$

Set

a)

Set 3

2)

a) xe^x

A. Domain is $\mathbb{R}$

B. The x-intercep & y-intercept are both 0.

C. Symmetry: None

D. Because x & e^x become large $x \to \infty$

we have $xe^x = \infty$ As $x \to -\infty$ however $e^x \to 0$

$$\lim_{x\to\infty} xe^x = \lim_{x\to\infty} \frac{x}{e^{-x}} = 0$$

Thus x-axis is horizontal asymptote

E. Since e^x is always positive we see that $f'(x) > 0$ when $x+1<0$. So f is increasing on $(-1, \infty)$ and decres on $(-\infty, -1)$

$$f'(x) = xe^x + e^x = (x+1)e^x$$

F. $f(-1) = -e^{-1}$ is local minimum

G. $f''(x) = (x+2)e^x$

Since $f''(x) > 0$ if $x > -2$ $f''(x) = 0$.

concave upward on	$(-2, \infty)$
concave downward on	$(-\infty, -2)$

b) $y = e^x$

diff wrt x we get

$$\frac{dy}{dx} = e^x$$

using formal sol as

$$S = \int_0^1 2\pi e^x \sqrt{1+(e^x)^2}\, dx$$

Put $e^x = t$

$$\frac{de^x}{dx} = \frac{dt}{dx}$$

$$e^x\,dx = dt$$

when $x = 0,\ t = 1$

$x = 1;\ t = 0$

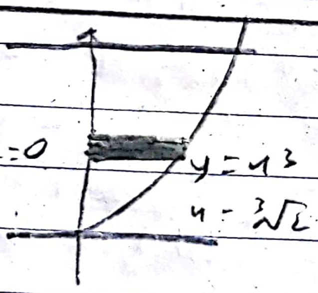
$$S = \int_1^e 2\pi\sqrt{1+t^2}\,dt$$

$$= 2\pi\left[\frac{t\sqrt{1+t^2}}{2} + \frac{1}{2}\ln\left(t + \sqrt{1+t^2}\right)\right]_1^e$$

$$= \pi\left[e\sqrt{1+e^2} + \ln\left(e + \sqrt{1+e^2}\right) - \sqrt{2} - \ln(1+$$

$$= \pi\left[e\sqrt{1+e^2} - \sqrt{2} + \ln\left(\frac{e+\sqrt{1+e^2}}{1+\sqrt{2}}\right)\right]$$

3) a)



b

$$\text{Area}(y) = \pi x^2 = \pi\left(\sqrt[3]{y}\right)^2 = \pi y^{2/3}$$

& volume of approximating cylinder is

$$A(y)\,\Delta y = \pi y^{2/3}\,\Delta y$$

since the solid lies betn $y = 0$ & $y = 8$

$$V = \int_0^8 A(y)\,dy = \int_0^8 \pi y^{2/3}\,dy = \pi\left[\frac{3}{5}y^{5/3}\right]_0^8$$

$$= \frac{96}{5}\pi$$

b)

$$\frac{dy}{dx} = \frac{x^2}{y^2}$$

$$y^2 dy = x^2 dx$$

$$\frac{y^3}{3} = \frac{x^3}{3} + c$$

where c is arbitrary constant

now

$$y = \sqrt[3]{x^3 + K} \qquad \boxed{3c = K}$$

If we put $x = 0$ in general solⁿ part a we get $y(0) = \sqrt[3]{K}$. To satisfy the initial $y(0) = 2$ we must $\sqrt[3]{K} = 2$ & so $K = 8$ the solⁿ will be

$$y = \sqrt{x^2 + 8}$$

b) $a = \vec{i} + 2\vec{j} - 3K \qquad b = 4\vec{i} + 7\vec{K}$

$$2\vec{a} = 2\vec{i} + 4\vec{j} - 6\vec{K} \qquad 3\vec{b} = 12\vec{i} + 21\vec{K}$$

$$2\vec{a} + 2\vec{b} = 2\vec{i} + 4\vec{j} - 6\vec{K} + 12\vec{i} + 21\vec{K}$$

$$= 14\vec{i} + 4\vec{j} + 15\vec{K}$$

$$\frac{14\vec{i} + 4\vec{j} + 15\vec{K}}{\sqrt{14^2 + 4^2 + 15^2}} = \frac{14\vec{i} + 4\vec{j} + 15K}{\sqrt{196 + 16 + 225}} = \frac{\vec{i}}{\sqrt{437}}$$

$$= \frac{14}{\sqrt{437}}\vec{i} + \frac{4\vec{j}}{\sqrt{437}} + \frac{15\vec{K}}{\sqrt{432}}$$

4) a) $\vec{i}+4\vec{j}-2\vec{k}$ .n $(5,1,3)$

$\vec{OP_0} = (5,1,3)$

$\vec{OP} = (x,y,z)$

$\vec{P_0P} = (x-5, y-1, z-3)$

$\vec{P_0P} = t\vec{v}$ $[\because \vec{P_0P} \parallel \vec{v}]$

$(x-5, y-1, z-3) = t(1,4,-2)$

$x-5 = t$ $x = 5+t$

$y-1 = 4t$ $y = 1+4t$

$z-3 = -2t$ $z = 3-2t$

– parametric form

vector form

$(x,y,z) = (5,1,3) + t(1,4,-2)$

Group B

5)

a) $y = x^5 + x$

$f(-x) = (-x)^5 - x$

$= -(x^5 + x)$

$= -f(x)$

$\therefore$ gt is odd

b) $y = 1 - x^4$

$f(-x) = 1-(-x)^4$

$= 1 - x^4$

$\therefore$ gt is even

c) $y = 2x - x^2$

$f(-x) = -2x - x^2$

$= -(2x + x^2)$

$\therefore$ gt is neither even nor odd

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6) $f(x) = \dfrac{\sqrt{2x^2+1}}{3x-5}$

vertical

$3x-5=0$

$x = 5/3$

$\therefore$ vertical asymptote is $5/3$

Horizontal

$$\lim_{x\to\infty} \frac{(2x^2+1)^{1/2}}{3x-5}$$

$= \infty$

$\therefore$ No Horizontal asymptote

9) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$

$I_1 = \displaystyle\int_{-\infty}^{1} \frac{dx}{1+x^2} = \lim_{h\to-\infty} \int_{h}^{1} \frac{dx}{1+x^2}$

$= \displaystyle\lim_{h\to\infty} [\tan^{-1}x]_h^1$

$= \displaystyle\lim_{h\to\infty} [\tan^{-1}1 - \tan^{-1}h]$

$= \dfrac{\pi}{4} + \dfrac{\pi}{2}$

$I_2 = \displaystyle\int_{1}^{\infty} \frac{dx}{1+x^2} = \lim_{h\to\infty} \int_{1}^{h} \frac{dx}{1+x^2}$

$= \displaystyle\lim_{h\to\infty} [\tan^{-1}h]_1^h$

$= \displaystyle\lim_{h\to\infty} (\tan^{-1}h - \tan^{-1}1)$

$= \pi/2 - \pi/4$

$I_1 + I_2 = \dfrac{\pi}{4} + \pi/4 + \pi/2 - \pi/4 = \pi$

11) $y'' + y' - 2y = x^2$ —(1)

complementary soln (y_c) $am^2 + bm + c = 0$

$m^2 + m - 2 = 0$

$m^2 + 2m - 1m - 2 = 0$

$(m-1)(m+2) = 0$

$m_1 = 1 \quad m_2 = -2$

$y_c = c_1 e^x + c_2 e^{-2x}$

For particular soln (y_p)

$y_p = ax^2 + bx + c$

$y' = 2ax + b$

$y'' = 2a$

Then Eq (1) will be

$2a + 2ax + b - 2(ax^2 + bx + c) = x^3$

$2a + 2ax + b - 2ax^2 - 2bx - 2c = x^2$

$-2ax^2 + 2ax - 2bx + 2a + b - 2c = x^2 + 0x + 0$

~~Equating coeff~~

$-2a = 1 \qquad 2a - 2b = x \qquad 2a + b - 2c = 0$

$a = -1/2 \qquad b = -1/2 \qquad c = -3/4$

$y_p = 1/2 x^2 + 1/2x - 3/4$

Finally

$y(x) = y_c + y_p$

$= c_1 e^x + c_2 e^{2x} - 1/2 x^2 - 1/2 x^2 - 3/4$

$\frac{1}{2}$

) Find the length of arc of parabola

$$\cancel{y^2 = x}$$
$$\cancel{y = x^{1/2}}$$
$$\cancel{\frac{dy}{dx} = \frac{1}{2} x^{-1/2} = \frac{1}{2x^{1/2}}}$$

$$y^2 = x$$
$$x = \frac{dy}{dx} = 2y$$

$$S = \int 2\pi x \, ds$$
$$= \int_0^1 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}$$
$$= \int_0^1 2\pi x \sqrt{1 + 4y^2} \, dx$$

$u = 1 + 4y^2$ wehow $du = 8y\,dy$

when $y = 1$, $u = 5$ $y=0$ $u=1$

$$S = \frac{\pi}{4} \int_1^5 \sqrt{u} \, du$$
$$= \frac{\pi}{4} \left[\frac{2}{3} u^{3/2} \right]_1^5 = \frac{\pi}{6} \left(5\sqrt{5} - 1\sqrt{1} \right)$$

---

2) ~~Mat~~ Maclaurin Expression

$$f(x) = e^x$$
$$f'(x) = e^x = f''(x) = f'''(x) = \dots$$

At $x = 0$

$$f(0) = e^0 = 1 = f'(0) = f''(0) = f'''(0) = \dots$$

$$f(x) = 1 + x.1 + \frac{x^2}{2!} \cdot 1 + \frac{x^3}{3!} + 1 + \dots$$
$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

14) $\iint_R y\sin(xy)\,dA$ where $R=[1,2]\times[0,\pi]$

$$\int_0^{\pi}\int_1^{2} y\sin(xy)\,dx\,dy = \int_0^{\pi}\left[-\cos(xy)\right]_1^2 dy$$

$$= \int_0^{\pi}\left[-\cos 2y+\cos y\right)dy$$

$$= \left[-\frac{\sin 2y}{2}+\sin y\right]_0^{\pi}$$

$$= \left[-\frac{\sin 2\pi}{2}+\sin\pi\right]-\left[\frac{\sin 0}{2}+\sin 0\right]$$

$$= 0$$

13) Here

Given $10x+2y-2z=5$ — (I)

$5x+y-z=1$ — (II)

From (I), $(10,2,-2)$ & (II) $\Rightarrow (5,1,-1)$ are parallel

when we keep $y=z=0$ in Eq. (I) we get $10x=5$ & so $(1/2,0,0)$ is point in plane

distance betn $(1/2,0,0)$ & plane $5x+y-z-1=0$ is

$$D=\frac{\left|5(1/2)+1(0)-1(0)-1\right|}{\sqrt{5^2+1^2+(-1)^2}}=\frac{3/2}{3\sqrt{3}}=\frac{\sqrt{3}}{6}$$

So distance betn plane is $\sqrt{3}/6$

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8)

Given

$y = x^2$

At points $(0,0)$, $(1,1)$ & $(2,4)$

Since

$y' = 2x \quad y'' = 2,$

$$K(x) = \frac{|y''|}{\left[1+(y')^2\right]^{3/2}} = \frac{2}{(1+4x^2)^{3/2}}$$

The curvature at $(0,0) = 2$

At $(1,1)$ $2/5^{3/2} = 0.18$

At $(2,4) = K(2) = \frac{2}{17^{3/2}} = 0.03$

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15)

Here

$a = (1, 4, -7)$

$b = (2, -1, 4)$

$c = (0, -9, 18)$

$$a \cdot (b \times c) = \begin{vmatrix} 1 & 4 & -7 \\ 2 & -1 & 4 \\ 0 & -9 & 18 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & 4 \\ -9 & 18 \end{vmatrix} - 4\begin{vmatrix} 2 & 4 \\ 0 & 18 \end{vmatrix} - 7\begin{vmatrix} 2 & -1 \\ 0 & -9 \end{vmatrix}$$

$$= 1(18) - 4(36) - 7(-18)$$

$$= 0$$

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Set 4

2)

a) $f(x) = \dfrac{\cos x}{2+\sin x}$

- Domain is $\mathbb{R}$
- y intercept $f(0) = 1/2$

x intercept $\cos x = 0$

- f is neither odd nor even but $f(x+2\pi) = f(x)$ for all x & so f is periodic & has period 2π.
- Asymptote:

VA: $2+\sin x = 0$ $\quad \sin x = -2$ ... NO VA

HA NO HA

- $f'(x) = \dfrac{(2+\sin x)(-\sin x) - \cos x(\cos x)}{(2+\sin x)^2} = -\dfrac{2\sin x + 1}{(2+\sin x)^2}$

$f'(x) > 0$ when $2\sin x + 1 < 0$

$\sin x < 1/2$

$\frac{7\pi}{6} < x < 11\pi/6$

f is increasin on $(7\pi/6, 11\pi/6)$

Decreasing $(0, 7\pi/6)$ & $(11\pi/6, 2\pi)$

local maxima value is $1\sqrt{3}$

$f''(x) = \dfrac{2\cos x(1-\sin x)}{(2+\sin x)^3}$

$(2+\sin x)^3 > 0$

$1 + \sin x \geq 0$

$f''(x) > 0$ when $\cos x < 0$ $\quad \pi/2 < x < 3\pi/2$

$(0, 2\pi/2)$	$(\pi/2, 3\pi/2)$	$(3\pi/2, 2\pi)$
concave upward	upward	downward

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$1.17 + 1.052\}$

b) $y = 1/x$

$\Delta x = 0.1$

$$\int_0^1 \frac{1}{x}\,dx = \frac{0.1}{2}\Big[F(0) + 2F(0.1) + 2F(0.2) + 2F(0.3) \ldots \ldots 2F(0.8) + 2F(0.9) + F(1)\Big]$$

$$= 0.05\left[\frac{2}{0.1} + \frac{2}{0.2} + \frac{2}{0.3} + \frac{2}{0.4} + \frac{2}{0.5} + \frac{2}{0.6} + \frac{2}{0.7} + \frac{2}{0.8} + \frac{b}{0.}\right.$$

$$= 0.05\left[20 + 10 + 6.67 + 5 + 4 + 3.3 + 2.8 + 2.5 + 2.22 + 1\right]$$

$$= 2.8745$$

a) Given Eq

$$y = x$$
$$y = x^2$$

$$x = x^2$$
$$x - x^2 = 0$$
$$x(1-x) = 0$$
$$x = 1$$
$$x = 0$$

so

$$V = \int_0^1 (2\pi x)(x - x^2)\,dx$$

$$= 2\pi \int_0^1 x(x - x^2)\,dx$$

$$= 2\pi \left[\frac{x^2}{2} + \frac{x^4}{4} \right]_0^1$$

$$= \pi/6$$

---

b) $x = ky^2$

If we diff we get

$$1 = 2ky\frac{dy}{dx}$$

or $\frac{dy}{dx} = \frac{1}{2ky}$

It depend on k we we need an Eq that is valid for all value of k simultaneously. To eliminate we note that from the eq of given parabola $x = ky^2$. $k = x/y^2$ & so

$$\frac{dy}{dx} = \frac{1}{2ky} = \frac{1}{2\frac{x}{y^2}\cdot y}$$

$$\frac{dy}{dx} = \frac{y}{2x} \quad \text{--- (ii)}$$

So slope of tangent is

$$\frac{dy}{dx} = -\frac{2x}{y}$$

the diff eq is seperable

$$\int y\,dy = -\int 2x\,dx$$

$$\frac{y^2}{2} = -x^2 + c$$

$$\boxed{x^2 + \frac{y^2}{2} = c}$$

4) a)

$A = (2, 4, -3) = (x_1, y_1, z_1)$

$B (3, -1, 1) = (x_2, y_2, z_2)$

In symetric form

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\frac{x-2}{3-2} = \frac{y-4}{-1-4} = \frac{z+3}{1+3}$$

$$\left.\frac{x-2}{1} = \frac{y-4}{-5} = \frac{z+3}{4}\right] \text{ symmetric form}$$

$$\frac{x-2}{1} = \frac{y-4}{-5} = \frac{z+3}{4} = t$$

$x - 2 = t$ $\qquad$ $x = 2 + t$

$y - 4 = -5t$ $\qquad$ $y = 4 - 5t$ $\Big\}$ in parametric form

$z + 3 = 4t$ $\qquad$ $z = 4t - 3$

for XY plane $z = 0$

$t = 3/4$. $\therefore$ the p $\left(2 + 3/4,\ 4 - 5 \times 3/4,\ 4 \times \frac{3}{4} \times 3\right) = \left(\frac{11}{4}, \frac{1}{4}, 0\right)$

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b) $\vec{a} = (1,2,3)$

$|a| = \sqrt{14}$

$\cos\alpha = \frac{1}{\sqrt{14}}$ $\cos\beta = \frac{2}{\sqrt{14}}$ $\cos\gamma = \frac{3}{\sqrt{14}}$

$\alpha = 74^\circ$ $\beta = 58^\circ$ $\gamma = 37^\circ$

Group B

5)

a) $F(x) = x^5 + x$

$F(-x) = (-x)^5 + (-x)$

$= -F(x)$

$\therefore$ It is odd

b) $y = 1 - x^4$

$f(-x) = 1 - (-x)^4$

$= 1 - x^4$

$= F(x)$ $\therefore$ It is even

c) $= y = 2x - x|$

$= -2 - x^2$

$\therefore$ Neither o nor eve

7) $\lim_{x \to 0} \frac{\tan x - x}{x^3}$

$\lim_{x \to 0} \frac{\tan x - x}{x^3} = \lim_{x \to 0} \frac{\sec^2 x - 1}{3x^2}$

Since limit on RHS is still indeterminate of type $\frac{0}{0}$ using L Hospital again

$\lim_{x \to 0} \frac{\sec^2 x - 1}{3x^2} = \lim_{x \to 0} \frac{2\sec^2 x \tan x}{6x}$

$= \frac{1}{3} \lim \sec^2 x \lim_{x \to 0} \frac{\tan x}{x} = \frac{1}{3} \lim_{x \to 0} \frac{\tan x}{x}$

$\lim_{x \to 0} \frac{2\sec^2 x \tan x}{6x} = 1/3$

9) $\int_0^1 \ln x \, dx$

$\lim_{a \to 0^+} \int_a^1 \ln x \, dx$

$\lim_{a \to 0^+} \left[\frac{1}{x}\right]_a^1$

$\lim_{a \to 0^+} \left[\frac{1}{1} - \frac{1}{a}\right]$

$\left[1 - \frac{1}{0}\right]$

$= \infty$

)

$y'' - 4y = xe^x + \cos 2x$

complementary sol^n

$m^2 - 4 = 0$

$m = \pm 2$

$CF = C_1 e^{2x} + C_2 e^{-2x}$

Particular sol^n $\quad G(x) = G_1(x) + G_2(x)$

$G_1(x) = xe^x$

$y_{p_1} = (Ax + B) e^x$ $\qquad A = -1/3 \qquad B = -2/9$

$y' = A.e^x + (Ax+B)e^x$

$y'' = Ae^x + Ae^x + (Ax+B)e^x$

$G_2(x) = \cos 2x$, $y_{p_2} = C\cos 2x + D\sin 2x$

$y' = -2C\sin 2x + 2D\cos 2x$

$y'' = -4\cos 2x - 4D\sin 2x$

$y'' - 4y = \cos 2x$

$-8C = 1$ $\qquad D = 0 \qquad y_{p_2} = -1/8 \cos 2x$

$C = -1/8$

$y = C_1 e^{2x} + C_2 e^{-2x} + \left(-\frac{1}{3}x - \frac{2}{9}\right)e^x - \frac{1}{8}\cos 2x$

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12) $F(x) = x\sin x$ 10)

we know $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots$

$x\sin x = x\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots\right.$

$= x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \dots + (-1)^n \frac{x^{2n+2}}{(2n+1)!} + \dots$

15) $F(x,y) = x^3 + x^2y^3 - 2y^2$

$\frac{df}{dx} = \frac{d}{dx}(x^3 + x^2y^3 - 2y^2)$

$= 3x^2 + 2xy^3$

$\frac{df}{dy} = 3y^2x^2 - 4y$

14) ~~3x - 2y - 6z = 1~~ $x + y + z = 1$ — (i)

$x - 3y + 2z = 1$ — (ii)

$A_1 = 1,\ B_1 = 1,\ C_1 = 1 \qquad A_2 = 1,\ B_2 = -3,\ C_2 = 2$

$\cos\theta = \frac{1\times 1 + (1\times -3) + 2\times 1}{\sqrt{1+1+1}\cdot\sqrt{1+9+4}}$

$= \frac{1-3+2}{\sqrt{3}\cdot\sqrt{14}} = \frac{0}{\sqrt{42}} = 0 = \cos 90^\circ$

10) $f(x) = x^2 - \frac{1}{8}\ln x$

$f'(x) = 2x - \frac{1}{8x}$

$1 + [f'(x)]^2 = 1 + \left(2x - \frac{1}{8x}\right)^2$

$= 1 + 4x^2 - \frac{1}{2} + \frac{1}{64x^2}$

$\sqrt{[1+f'(x)]^2} = 4x^2 + \frac{1}{2} + \frac{1}{64x^2} = \left(2x + \frac{1}{8x}\right)^2$

Arc length

$s(x) = \int_1^x \sqrt{1+f'(t)^2}\,dt$

$= x^2 + \frac{1}{8}\ln x - 1$

For $(1,1)$ to $(3, f(3))$

$s(3) = 3^2 + \frac{1}{8}\ln 3 - 1 = 8 + \frac{\ln 3}{8} = 8.1373$

---

5) A function f is continuous ~~on~~ in function from right at a no a if $\lim\limits_{x \to a^+} f(x) = f(a)$ & f is continuous from left at a if $\lim\limits_{x \to a^-} f(x) = f(a)$

$f(x) = 1 - \sqrt{1 - x^2}$

$f(1) = 1 - \sqrt{1 - 1} = 1 - 0 = 1$

$f(-1) = 1 - 1 - (-1)^2 = 1 - 0 = 1$

$\therefore f(1) = f(-1)$

$\therefore$ They are continuous on interval $[-1, 1]$

8) $A = 2\pi r^2 + 2\pi r h$

volume $= 1L$

$= 1000 cm^3$

$\pi r^2 h - 1000$

$h = 1000/(\pi r^2)$

$$A = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$$

$$= 2\pi r^2 + \frac{2000}{r}$$

$\therefore$ The funct. we want

$$A(r) = 2\pi r^2 + \frac{2000}{r} \quad r > 0$$

to find critical no, we differentiate

$$A(r) = 4\pi r - \frac{2000}{r^2}$$

$$= \frac{4(\pi r^3 - 500)}{r^2}$$

set 5

2)
a) $f(x) = \ln(4-x^2)$

A) Domain $4-x^2 > 0$

ie $x^2 - 4 < 0$

$(x-2)(x+2) < 0$

$\therefore$ Domain is $(-2, 2)$

B) x-intercept $y = 0$

$\ln(4-x^2) = 0$

$4 - x^2 = 1$

$x = \pm\sqrt{3}$

y-intercept $x = 0, y = \ln 4$

x-inter: $(\sqrt{3}, 0), (-\sqrt{3}, 0)$

y- $(0, \ln 4)$

c) Symmetry:

$f(-x) = f(x)$

$[\because x^2$ so $(-x)^2 = x]$

So it is symmetry abt y-axis

D) VA: $\lim_{x \to 2^-} \ln(4-x^2) = -\infty$

$\lim_{x \to 2^+} \ln(4-x^2) = -\infty$

$\therefore$ VA is $(-2, 2)$

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E) $f'(x) = \frac{-2x}{4-x^2}$

$x = 0, 2 \,\&\, (-2)$

$(-2, 0)$ $(0, 2)$

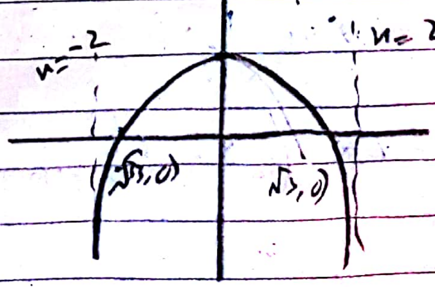
Increasing   Decreasing

F) Maximum point $(0, \ln 4)$

G) $f''(x) = -\frac{8 - 2x^2}{(4-x^2)^2}$

$x = -2, 2$

It is conca downwa



a)

b) $\int_0^1 e^{x^2}$.

$\Delta x = \frac{1-0}{10} = 0.1$

$\int_0^1 e^{x^2} = \frac{\Delta x}{3}\left[f(0) + 4f(0.1) + 2f(0.2) + \dots 2f(0.8) + 4f(0.9) + \right.$

$= \frac{0.1}{3}\left[e^0 + 4e^{0.01} + 2e^{0.04} + 4e^{0.09} + 2e^{0.16} + 4e^{0.25} + 2\right.$
$+ 4e^{0.49} + 2e^{0.64} + 4e^{0.81}$

$= \cancel{\frac{0.1}{3}} \cdot 1.462681$

3) a) $f(x) = 2x^3 - x^3$

$2x^3 - x^3 = 0$

$x^2(2-x) = 0$

$x^2 = 0 \qquad 2-x=0$

$x = 0 \qquad x = 2$

They are limit

$V = \int_0^2 (2\pi x)(2x^2 - x^3)$

$= 2\pi\left(\frac{1}{2}x^4 - \frac{1}{5}x^5\right]_0^2$

$= 2\pi\left(8 - \frac{32}{5}\right)$

$= \frac{16}{5}\pi$

a) $x = 1 + t$ $y = -2 + 3t$ $z = 4 - t$

$x = 2s$ $y = 3 + s$ $z = -3 + 4s$

Put $t = 0$ in L_1 the L_1 is given by

$(1, -2, 4)$

Put $s = 0$ L_2 is $(0, 3, -3)$

L_1 & L_2 are not parallel

$1 + t = 2s$ — (I)

$-2 + 3t = 3 + s$ — (II)

$4 - t = -3 + 4s$ — (III)

solving (I) & (II)

$2s - t + 1 = 0$

$s - 3t + 5 = 0$

$$\frac{s}{-5-3} = \frac{t}{-1-10} = \frac{1}{-6+1}$$

$$\frac{s}{-8} = \frac{t}{-11} = \frac{1}{-5}$$

$s = \frac{8}{11}$ $t = 11/5$

LHS of (3)

$4 - t$

$= 4 - 11/5 = 9/5$

RHS of (3) $= -3 + 4 \times s = 17/5$

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b) $\vec{b} = (1,1,2) = \vec{v}$

$\vec{a} = (-2,3,1) = \vec{u}$

$\text{Proj}_{\vec{v}}\,\vec{u} = \left(\dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\right)\vec{v} = \dfrac{-2+3+2}{1+1+4}\,(\vec{i}+\vec{j}+2\vec{k})$

$= \dfrac{1}{2}\,(\vec{i}+\vec{j}+2\vec{k})$

For scalar

$|\vec{u}|\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|} = -2\vec{i}+3\vec{j}+\vec{k}\cdot\left(\dfrac{1}{\sqrt{2}}\vec{i}+\dfrac{1}{\sqrt{2}}\vec{j}+\dfrac{2}{\sqrt{2}}k\right)$

$= \dfrac{-2}{\sqrt{2}}+\dfrac{3}{\sqrt{2}}+\dfrac{1}{\sqrt{2}}$

$= \dfrac{-2+3+1}{\sqrt{2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

Group B

5)

a) $f(x) = x^5 + x$

$f(-x) = (-x)^5 - x$

$= -(x^5 + x)$

$= -f(x)$

$\therefore$ gt is odd

b) $y = 1 - x^4$

$f(-x) = 1 - (-x)^4$

$= 1 - x^4$

$= f(x)$.

$\therefore$ gt is even

c) $y = 2x - x^2$

$f(-x) = 2.(-x) - (-x)^2$

$= -2 - x^2$

$\therefore$ Neither odd nor even

7) $P(1,3,2)$, $Q(3,-1,6)$ & $R(5,2,0)$

$\vec{PQ} = (1,3,2)-(3,-1,6) = (-2,4,-4)$

$\vec{PR} = (1,3,2)-(5,2,0) = (-4,1,2)$

$-4 \cdot 4$

$$\vec{PQ}\times\vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 4 & -1 \\ -4 & 1 & 2 \end{vmatrix} = 9\vec{i} \cancel{+} -8\vec{j} - 14\vec{k}$$

we can take $(1,3,2)$

$9(x-1) \cancel{+} -8(y-3) + -14(z-2) = 0$

$9x-9-8y+24-14z+28=0$

$9x-8y-14z = 52-9 = 43$

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8) $A = xy$

$2x+y = 2400$

$y = 2400-2x$

$A = x(2400-2x)$

$A = 2400x-2x^2$

$A' = 2400-4x$

For A is max

$A' = 0$

$2400 = 4x$

$x = 600$

$A'' = -4 < 0$

A is maximum when $x = 600$ft

& $y = 2400 - 2\times 600$

$= 1200$ft

$\therefore$ length of rectangular field is 1200ft & width is 600ft

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9)

$$f(x) = \frac{1}{x^p} \quad [p \text{ is constant}]$$

case I $p > 1$ $f(x) = \frac{1}{x^p}$ is positive decreasing fun of x.

$$\int_1^\infty \frac{1}{x^p} = \lim_{k\to\infty} \int_1^k \left(\frac{1}{x^p}\right) dx = \lim_{k\to\infty} \left[\frac{x^{-p+1}}{-p+1}\right]_1^k$$

$$= \frac{1}{1-p}(0-1)$$

$$= \frac{1}{p-1}$$

It converges for $p > 1$

case II If $p < 1$

$$\int_1^\infty \frac{1}{x^p} dx = \left(\frac{1}{1-p}\right) \lim_{k\to\infty} \left[x^{-p+1}\right]_1^k$$

$$= \left(\frac{1}{1-p}\right) \lim_{k\to\infty} \left[k^{1-p} - 1\right]$$

$$= \infty$$

It divergent for $p < 1$

case III If $p = 1$, then the series reduces to $\sum_{n=1}^{\infty} \frac{1}{n}$ which is harmonic series

10)

we have

$$y = \sqrt{4-x^2} \quad -1 \le x \le 1$$

diff wrt x

$$\frac{dy}{dx} = \frac{d(4-x)^{1/2}}{d(4-x)} \frac{d(4-x^2)}{dx}$$

$$= \frac{1}{2}(4-x^2)^{-1/2}(-2x)$$

$$= \frac{-x}{\sqrt{4-x^2}}$$

so ~~...~~ of the surface area is

$$S = \int_y^2 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= 2\pi \int_{-1}^{1} \sqrt{4-x^2} \sqrt{1+\frac{x^2}{4-x^2}}\, dx$$

$$= 2\pi \int_{-1}^{1} 2\, dx$$

$$= 4\pi(2)$$

$$= 8\pi$$

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b

11)

$$y'' - y = x^3 - x$$

complementary sol'

$$m^2 - 1 = 0$$

$$m = \pm 1$$

$$y_c = c_1 e^{x} + c_2 e^{-x}$$

second particular sol'

$$y_p = ax^3 + bx^2 + cx + d$$

$$y' = 3ax^2 + 2bx + c$$

$$y'' = 6ax + 2b$$

$$6ax + 2b - ax^3 - bx^2 - cx - d = x^3 - x$$

$a = -1$ $b = 0$ $c = -5$ $d = 0$

$$y = c_1 e^{x} + c_2 e^{-x} - x^3 - 5x^3$$

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13)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$S_k = \sum_{n=1}^{k} \frac{1}{n(n+1)} = \sum_{n=1}^{k} \frac{1}{n} - \frac{1}{n+1}$$

$$S_k = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{k} - \frac{1}{k+1}\right)$$

$$= 1 - \frac{1}{k+1}$$

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Then

$$\lim_{k\to\infty} S_k = 1 - \lim_{k\to\infty} \frac{1}{k+1} = 1$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$$

It convergent & sum is 1

Putting $a=2$, $b=3$, $c=4$, $x_0=2$, $y_0=4$ & $z_0 = -1$

$$2(x-2)+3(y-4)+4(z+1)=0$$

$$2x+3y+4z=12$$

to find x-intercept $y=z=0$ $x=6$

for y, $x=z=0$ $y=3$

for z $x=y=0$, $z=3$

12) $x = 2+3t$

$y = -4t$

$z = 5+t$

plane Eq $4x+5y-2z=18$ — 1

we substitute the val of (x,y,z) in Eq 1

$4(2+3t) + 5(-4t) - 2(5+t) = 18$

$-10t = 20$

so $t = -2$

So point of intersection when $t = -2$

$x = 2 + (3)(-2) = -4$

$y = -(4)(-2) = 8$

$z = 3$

So point of intersect a $(-4, 8, 3)$

$\lim_{x \to \infty} x^3$

when x become large x^3 become larger

$\lim_{x \to \infty} x^3 = \infty$

when x is large negative so x^3 then

$\lim_{x \to \infty} x^3 = -\infty$

Let f(x) be a function of x. Make x sufficiently close to a, if the value of f(x) is obtained greater than any pre assigned number however we say that limit of f(x) is infinit x tend to a

$\lim_{x \to a} f(x) = \infty$

12/2