optimazation Assignment-1

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September 2022

Problem Statement **- Given $P(x) = x^4 + ax^3 + bx^2 + c^x + d$ such that x=0 is the only real root of P'(x)=0.If P(-1) less than P(1),then find the maximum and minimum values of P(x)in the interval [-1,1].**

Solution

$$P(x) = x^4 + ax^3 + bx^2 + cx + d \tag{0.0.1}$$

$$From P(-1) < P(1) \tag{0.0.2}$$
$$1 - a + b - c + d = 1 + a + b + c + d \tag{0.0.3}$$
$$\implies a > 0 \tag{0.0.4}$$

$P'(x) = 4x^3 + 3ax^2 + 2bx + c$
Given,
x=0 is a root of P'(x)
0= 0+0+c

$$\implies c = 0 \tag{0.0.5}$$

$P'(x) = x(4x^2 + 3ax + 2b)$since it has only one real root discerement of $4x^2 + 3ax + 2b$ is less than 0.
$D = b^2 - 4ac < 0$
$=(3a)^2 - 32b < 0$

$$\implies b > 0 \tag{0.0.6}$$

f(x) consists only minima,
Using gradient ascent method we can find its minima ,

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \tag{0.0.7}$$
$$\implies x_{n+1} = x_n - \alpha \left(8x_n - 4\right) \tag{0.0.8}$$

Using gradient descent method we can find its maxima ,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \tag{0.0.9}$$
$$\implies x_{n+1} = x_n + \alpha \left(8x_n - 4\right) \tag{0.0.10}$$

Taking $a = 1, b = 1, d = 2, x_0 = 0.1, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\boxed{\text{Minima} = 2.00000} \tag{0.0.11}$$
$$\boxed{\text{Minima Point} = 0} \tag{0.0.12}$$

$$\boxed{\text{Maxima} = 5.06} \tag{0.0.13}$$
$$\boxed{\text{Maxima Point} = 1} \tag{0.0.14}$$