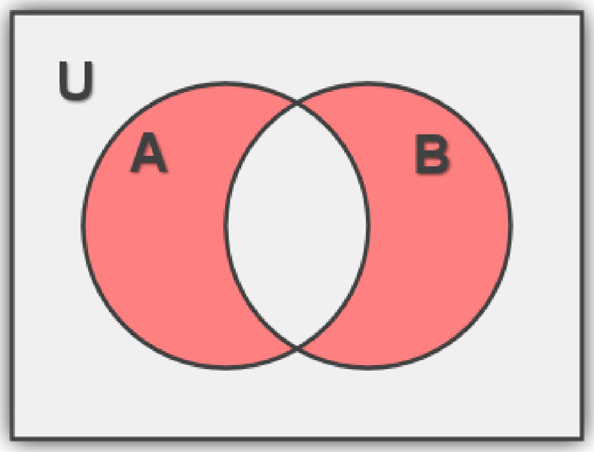
IT 1002 – Mathematics for Computing

Sets & Basic Set Operations

Symmetric Difference

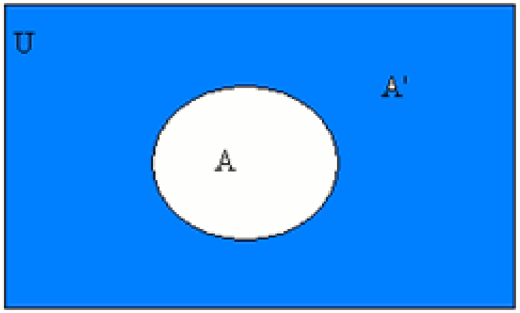
- The symmetric difference of the sets A & B consist of those elements which belong to A or B but not to both A & B
- This is denoted by $A \oplus B$ or $A \Delta B$



- $A \oplus B = (A \cup B) \setminus (A \cap B)$

Compliment

- The complement of set A is the set of elements which belongs to the universal set but which do not belongs to A
- This is denoted by A^c /$\bar{A}$ /A'



- $\bar{A} = \{ x: x \notin A , x \in U \}$

Laws of algebra of sets

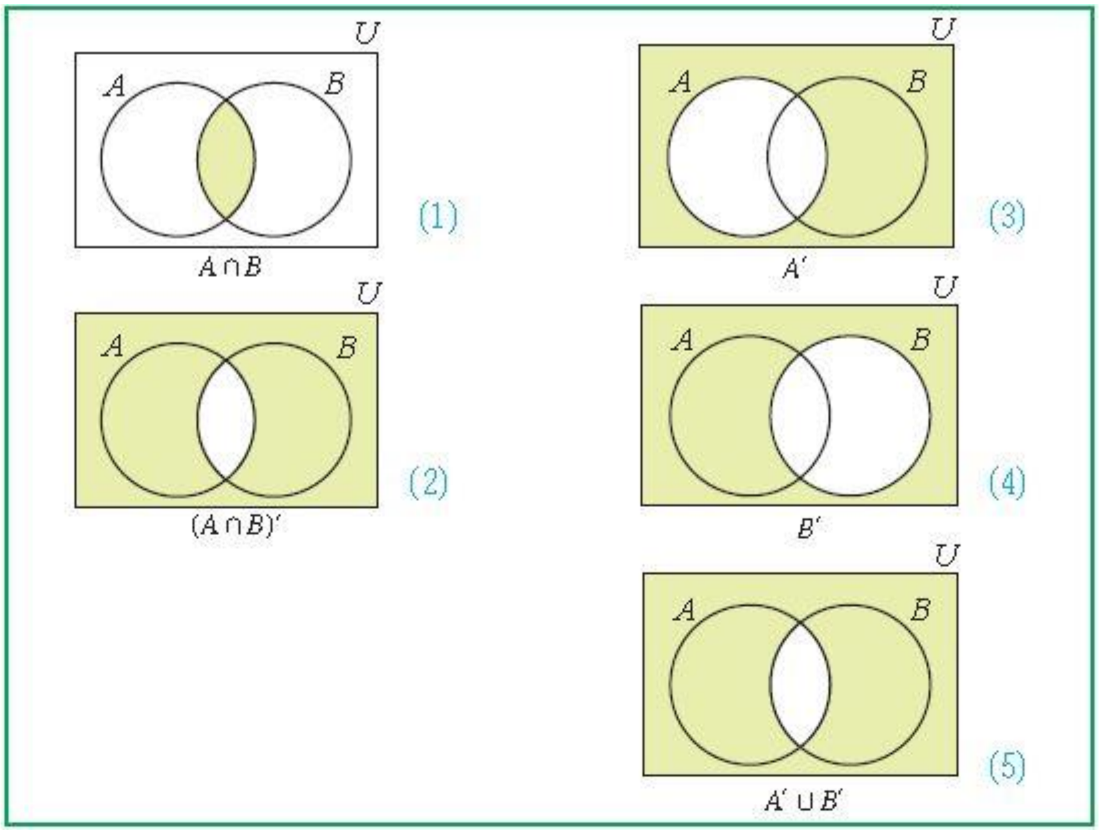
- Idempotent laws
  - $A \cap A = A$
  - $A \cup A = A$
- Assosiative laws
  - $(A \cap B) \cap C = A \cap (B \cap C)$
  - $(A \cup B) \cup C = A \cup (B \cup C)$
- Commutative laws
  - $A \cap B = B \cap A$
  - $A \cup B = B \cup A$

- Distributive laws
  - $A \cap (B \cup \mathrm{C}) = (A \cap \mathrm{B}) \cup (A \cap \mathrm{C})$
  - $A \cup (B \cap \mathrm{C}) = (A \cup \mathrm{B}) \cap (A \cup \mathrm{C})$
- Identity laws
  - $A \cup \emptyset = A$
  - $A \cap \mathbf{U} = A$
  - $A \cup \mathbf{U} = \mathbf{U}$
  - $A \cap \emptyset = \emptyset$

- Involution law
  - $(A^c)^c = A$
- Complement laws
  - $A \cup A' = \mathbf{U}$
  - $A \cap A' = \emptyset$
  - $\mathbf{U}' = \emptyset$
  - $\emptyset' = \mathbf{U}$
- De morgan's laws
  - $(A \cap B)' = A' \cup B'$
  - $(A \cup B)' = A' \cap B'$

Proof of De Morgan’s law

- Using Venn Diagram
  - $(A \cap B)' = A' \cup B'$



- Algebraic method

(i) Let p be an arbitrary element of $(A \cup B)'$.

Then $p \in (A \cup B)'$

$\Rightarrow p \notin (A \cup B)$

$\Rightarrow p \notin A$ and $p \notin B$

$\Rightarrow p \in A'$ and $p \notin B'$

$\Rightarrow p \in A' \cap B'$

$\therefore (A \cup B)' \subseteq A' \cap B' \qquad (1)$

Let q be an arbitrary element of $A' \cap B'$.

Then $q \in (A' \cap B')$

$\Rightarrow q \in A'$ and $q \in B'$

$\Rightarrow q \notin A$ and $q \notin B$

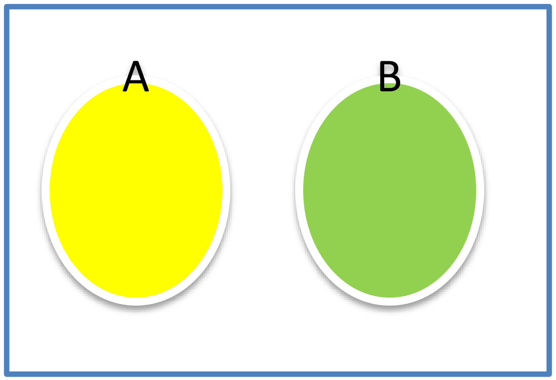
$\Rightarrow q \notin A \cup B$

$\Rightarrow q \in (A \cup B)'$

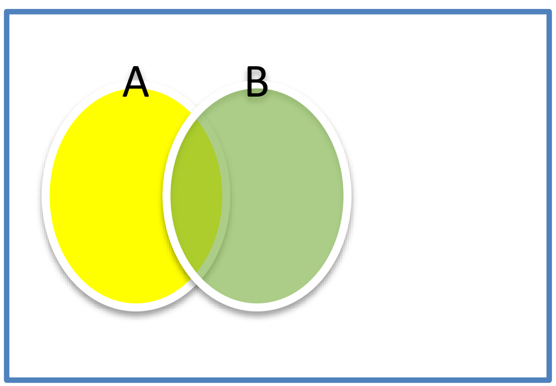
$\therefore A' \cap B' \subseteq (A \cup B)' \qquad (2)$

From (1) and (2), we get $(A \cup B)' = A' \cap B'$

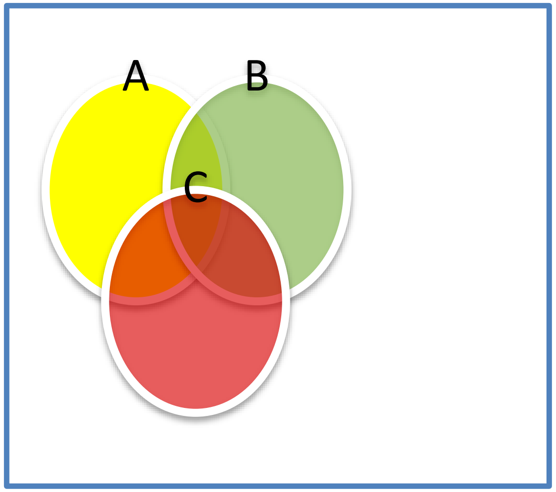
Counting Principles



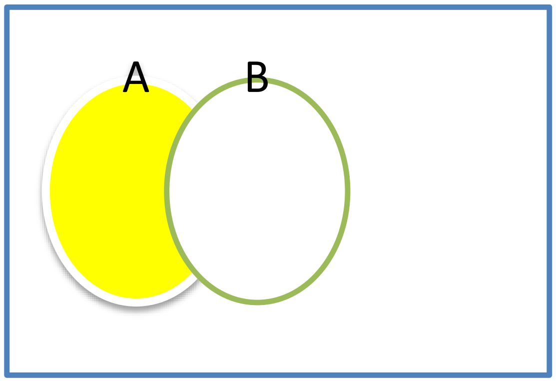
$$n\,(A \cup B) = n(A) + n(B)$$



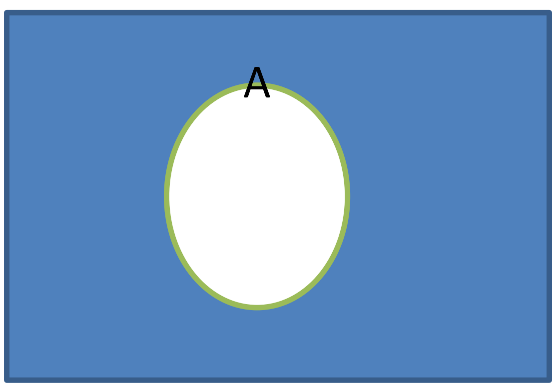
$$n\,(A \cup B) = n(A) + n(B) - n\,(A \cap B)$$



$$n\,(A \cup B \cup C) = n(A) + n(B) + n(C) - n\,(A \cap B) - n\,(A \cap C) - n\,(C \cap B) + n\,(A \cap B \cap C)$$



$$n(A/B) = n(A) - n(A \cap B)$$



$$n(A^c) = n(U) - n(A)$$

Duality

- If we interchange $\cup$ and $\cap$ and also U and ø in any statement about sets, then the new statement is called the dual of the original one.
- For example, each of De Morgan's Laws is the dual of the other.
- The first complement law, $A \cup A' = \mathbf{U}$, is the dual of the second: $A \cap A' = \emptyset$.

Inclusion Exclusion Principle

- Inclusion Exclusion Principle determine the number of elements in a union of sets when some of the sets overlap
- The Inclusion Exclusion Principle for Two or Three Sets
  - $n(P \cup Q) = n(P) + n(Q) - n(P\ )$
  - $n(P \cup Q \cup R) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(P \cap R) - n(Q \cap R) + n(P \cap Q \cap R)$