IT 1002 – Mathematics for Computing

Matrices

Matrix multiplication

- A rxn matrix A can be multiplied by a nxc matrix B and the resulting matrix is a rxc matrix which is denoted by AB
- i.e. the resulting matrix has the same number of rows as the first matrix & same number of columns as the 2nd matrix
- Note: to multiply two matrices number of columns of the 1st matrix must be equal to the number of rows of the 2nd matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{pmatrix}$$

Multiplying a matrix by a vector

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + iz \end{bmatrix}$$

Properties of matrix multiplication

- when a matrix is multiplied by the identity matrix, the product is the same as the original matrix

  MI = IM = M

- Matrix multiplication is not commutative

  AB ≠ BA

- Matrix multiplication is associative

  A(BC)=(AB)C

- Matrix multiplication is associative when it is being multiplied by a scalar

$$(kA)B = k(AB) = A(kB)$$

- Matrix multiplication is distributive

$$A(B+C) = AB + AC$$

- Transposing the product of two matrices is as same as taking the product of their transposes in the reverse order

$$(AB)^T = B^T A^T$$

Multiplying a vector & a matrix

- $[x \quad y \quad z]\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} =$

$$[xm_{11} + ym_{21} + zm_{31} \quad xm_{12} + ym_{22} + zm_{32} \quad xm_{13} + ym_{23} + zm_{33}]$$

- $\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}[x \quad y \quad z]$ = undefined

- $$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} xm_{11} + ym_{12} + zm_{13} \\ xm_{21} + ym_{22} + zm_{23} \\ xm_{31} + ym_{32} + zm_{33} \end{bmatrix}$$

- $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$ = undefined

Determinant of a matrix

- A **determinant** is a real number associated with every square **matrix**.
- Determinant tell us whether or not a matrix can be inverted
- The determinant of A is written as $|A|$ or det A

Calculating determinant for a 2x2 matrix

- If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$= ad-bc$$

Calculating determinant for a 3x3 matrix

- If A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then

$$\begin{array}{ccc} + & - & + \end{array}$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Properties of determinants

- If two rows or columns are equal then det A =0
- det A = det A^T
- If a row(or column) of A consists entirely of 0 then det(A)=0
- det(AB) = det A.det B = det(BA)
- Note: if det A=0 then the matrix don't have an inverse