IT 1002 – Mathematics for Computing

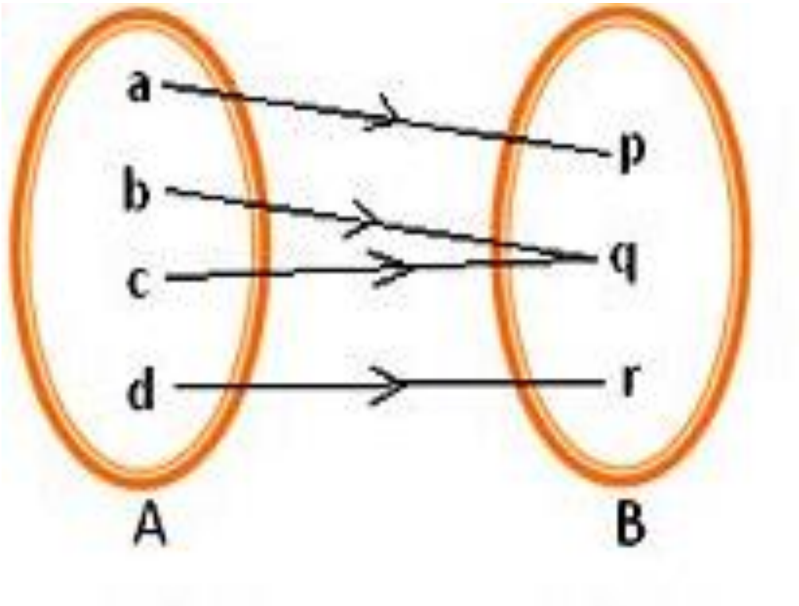
Relations

Representation of relations on finite sets

- There are 3 ways of picturing a relation R from A to B
  - Arrow Diagram
  - Matrix
  - Directed Graph

Arrow Diagram

- Eg: Let A={a,b,c,d} and B={p,q,r}
- R= {(a,p),(b,q),(c,q),(d,r)}



Matrix

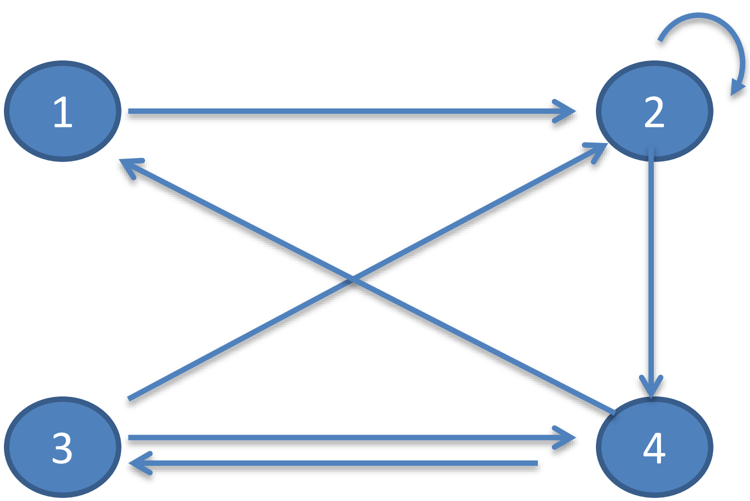
- Eg: Let A={a,b,c,d} and B={p,q,r}
- R= {(a,p),(b,q),(c,q),(d,r)}

$$
\begin{array}{c c}
 & \begin{array}{ccc} p & q & r \end{array} \\
\begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{array}
$$

Directed Graph

- There is another way of picturing a relation R when R is a relation from a finite set A to itself
- Eg: Let A={1,2,3,4}

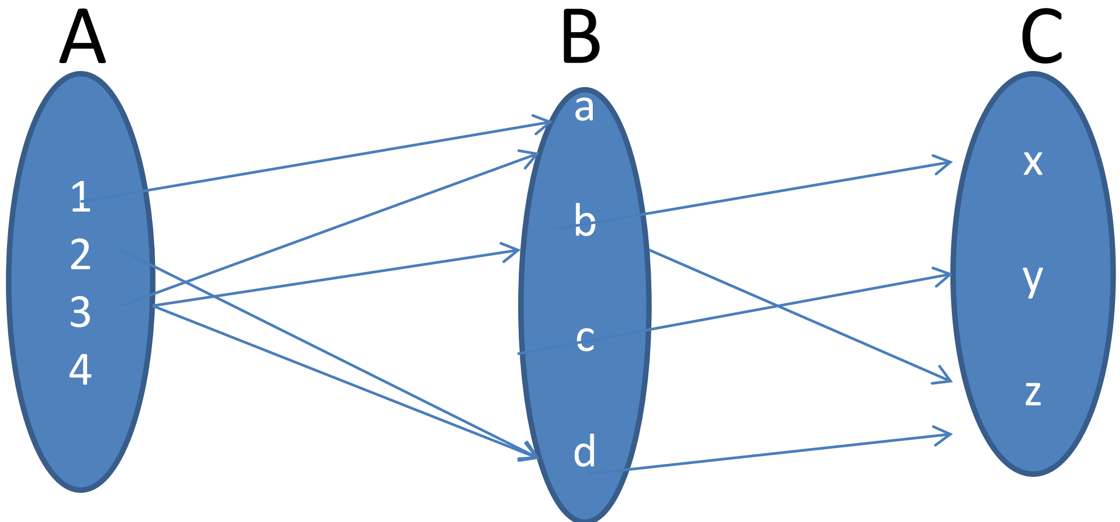
  R={(1,2),(2,2),(2,4),(3,2),(3,4),(4,1),(4,3)}



Composition of relations

- Let A,B,C be sets and let R be a relation from A to B. let S be a relation from B to C. then R and S give rise to a relation from A to C denoted by RoS and defined as follows
- RoS={(a,c): there exists $b \in B$ for which $(a,b) \in R$ and $(b,c) \in S$}
- This relation RoS is called the composition of R and S (Ref:Descrete Mathematics 3rd Ed Schamu's series pg 27)

- Eg: Let A={1,2,3,4} , B={a,b,c,d} and C={x,y,z}
- R={(1,a),(2,d),(3,a),(3,b),(3,d)}
- S={(b,x),(b,z),(c,y),(d,z)}
- RoS={(3,x),(3,z),(2,z)}



- If R is a relation on set A then RoR ,the composition of R with itself is always defined & sometimes denoted as R^2
- Note: Let A,B,C,D be sets. Suppose R is a relation from A to B, S is a relation from B to C, T is a relation from C to D. then

  $(RoS)oT = Ro(SoT)$

Composition of relation and matrices

- There's another way of finding RoS using matrices. Let M_R and M_S denote the matrices of the relations R & S respectively
- $M_{RoS} = M_R.M_S$

- Eg: A = {1,2,3,4}   B = {a,b,c,d} C = {x,y,z}
- R ={(1,a),(2,d),(3,a),(3,b),(3,d)}
- S={(b,x),(b,z),(c,y),(d,z)}

M_R

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

M_s

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$M_R . M_s$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Types of Relations

- Reflexive relation
  - A relation *R* is said to be reflexive over a set *A* if (a,a) $\in$ *R* for every *a* $\in$ *R*.
  - The relation R = {(1,1)(2,2)(3,3)(1,2)(1,3)} is reflexive over the set A = {1,2,3}.
- Symmetric relation
  - A relation *R* is said to be symmetric if (a,b) $\in$ R => (b,a) $\in$ R
  - The relation R = {(1,2)(2,2)(2,1)} is symmetric over the set A = {1,2,3}.

- Anti symmetric relations
  - for any a, and b in A, whenever (a, b) $\in$ R , and (b, a) $\in$ R , a = b must hold
  - The relation R = {(1,3)(2,2)(2,3)} is antisymmetric over the set A = {1,2,3}.
- Transitive relation
  - A relation *R* is said to be transitive if (a,b) $\in$ R, (b,c) $\in$ R => (a,c) $\in$ R
  - The relation R = {(1,3)(2,3)(1,2)(1,1)} is transitive over the set A = {1,2,3}.