



IT 1002 – Mathematics for Computing



Matrices



Inverse of a matrix

- For a square matrix A the inverse is written as A^{-1}
- Not all square matrices have inverses
- A square matrix which has an inverse is called invertible
- A square matrix without inverse is called singular matrix
- $AA^{-1} = A^{-1}A = I$



- Note: $(AB)^{-1} = B^{-1}A^{-1}$
- $(A+B)^{-1} \neq A^{-1}+B^{-1}$



Finding inverse for 2x2 matrix

- If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then
- $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
 $= \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$



Find inverse of a matrix : Adjoint Method

$$\begin{aligned}\bullet \quad A^{-1} &= \frac{1}{\det A} (\text{Adjoint } A) \\ &= \frac{1}{\det A} (\text{Cofactor matrix of } A)^T\end{aligned}$$

1. Create a matrix of minors

for each element of the matrix

- Ignore the values on the current row & column
- Calculate the determinant of the remaining values
- Put those determinants into a matrix



2. Create a matrix of cofactors

According to the following rule add – signs to the elements in the matrix of minors

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

3. Create adjoint matrix

transpose all elements of ‘matrix of cofactors’

4. Multiply by $\frac{1}{\text{determinant}}$

multiply the top row elements by their matching minor determinants



- Find inverse of $A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

- $A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

$$\text{MoM} = \begin{bmatrix} 2 & 2 & 2 \\ -2 & 3 & 3 \\ 0 & -10 & 0 \end{bmatrix}$$

$$\text{MoC} = \begin{bmatrix} 2 & -2 & 2 \\ 2 & 3 & -3 \\ 0 & 10 & 0 \end{bmatrix}$$



- $\text{Adj } A = \begin{bmatrix} 2 & 2 & 0 \\ -2 & 3 & 10 \\ 2 & -3 & 0 \end{bmatrix}$
- $\text{Det } A = (3*2) - (0*2) + (2*2)$
 $= 10$

- $A^{-1} = \frac{1}{10} \begin{bmatrix} 2 & 2 & 0 \\ -2 & 3 & 10 \\ 2 & -3 & 0 \end{bmatrix}$