IT 1002 – Mathematics for Computing

Functions

Definition

- If each element of set A is assigned to a unique element of a set B then the collection of assignments is called a function from A into B.
- It is denoted by f: A -> B
- It is read as “f is a function from A in to B” or “f maps A into B”

- Domain of the function
  - The set A is called the domain of the function
- Target set/ Co-domain
  - The set B is called the target set
- Range or Image of A
  - If f(a)=b we say that b is the image of a
  - The set of all such image values is called the range
  - It is denoted by Im(f) or Ran(f)

Function requirements

- No element of the domain must be left unmapped
- No element of the domain may map to more than one element of the co domain

Inverse function

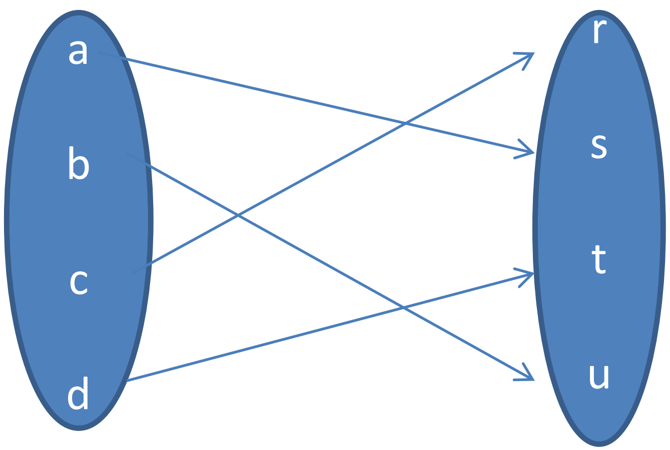
- Let f be a one to one correspondence from set A to set B.
- The inverse function of f is denoted by f^{-1} and $f^{-1}(b) = a$ when $f(a)=b$

Identity function

- Consider any set A. then there’s a function from A in to A which assigns each element in to itself
- It is called the identity function and it is denoted by I_A
- I_A : A-> A

Graph of ‘f’

- Every function f: A-> B gives rise to a relation from A -> B called the graph of f
- Graph of f = {(a,b) : a$\in$A , b = f(a)}
- Eg: A = {a,b,c,d} and B={r,s,t,u}



- Graph of f ={(a,s),(b,u),(c,r),(d,t)}

Composition function

- Consider the functions f: A->B and g:B->C, then we define a new function ($g_o f$): A->C by

  $$(g_o f)(a)=g[f(a)]$$

- here the target set 'B' is the domain of g

$$\begin{tikzcd}
A \arrow[r] \arrow[rr, bend right=60, "g_o f"'] & B \arrow[r] & C \arrow[ll, bend right=60, "f_o g"']
\end{tikzcd}$$

- Eg:Let $f:R\to R$ and $g:R\to R$ defined by $f(x)=x^2$ and $g(x)=(x+3)$. Find $g \circ f(x)$ and $f \circ g(x)$
- $g \circ f(x) = g(f(x))$

  $=g(x^2)$

  $= x^2 + 3$
- $f \circ g(x) = f(g(x))$

  $=f(x+3)$

  $=(x+3)^2$

Types of functions

- One to one function(injective function)
  - A function for which every element of the range corresponds to exactly one element of the domain
  - A function f is injective iff whenever $f(x)=f(y)$;$x=y$
  - If distinct elements in the domain have distinct images

- On to function(surjective function)
  - A function f is surjective if for every y in B there is at least one x in A such that f(x)=y
  - If each element of B is the image of some element of A

- Bijective function
  - A function f is bijective if for every y in B there is exactly one x in A such that $f(x)=y$
  - If the function is bijective then it should be injective & surjective