IT 1002 – Mathematics for Computing

Matrices

Find inverse of nxn matrix : Gauss Jordan Elimination Method

1. Given an nxn matrix A, we create the augmented matrix (A | I) by appending the nxn identity matrix to the right of A
   – Augmented matrix is a matrix obtained by appending the columns of two given matrices for the purpose of performing the same elementary row operations

- Perform elementary row operations to convert (A|I) into (I|A^{-1})
  - Elementary row operations are
    - Swapping two rows
    - Multiply a row by non zero number
    - Adding a multiple of one row to another row

Steps to convert (A|I) to (I|A^{-1})

- first place A and I adjacent to each other
- Now proceed by changing the columns of A left to right to reduce A to the form

$$\begin{bmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{bmatrix}$$ where * can be any number

This form is called an upper triangular form

- Continue the row operations to reduce the elements above the leading diagonal to zero

- Note: if a whole row becomes 0 then **cannot** find the inverse

- Ex： Find the inverse matrix of $A=\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$

Using Gauss-Jordan Elimination

Sol:

$$[A \;\vdots\; I]=\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ -6 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2-R_1\to R_2}\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ -6 & 2 & 3 & 6 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3+6R_1\to R_3}\left[\begin{array}{ccc:ccc}1 & -1 & 0 & 1 & 0 & 0\\ 0 & 1 & -1 & -1 & 1 & 0\\ 0 & -4 & 3 & 6 & 0 & 1\end{array}\right]$$

$$\xrightarrow{R_3+4R_2\to R_3}\left[\begin{array}{ccc:ccc}1 & -1 & 0 & 1 & 0 & 0\\ 0 & 1 & -1 & -1 & 1 & 0\\ 0 & 0 & -1 & 2 & 4 & 1\end{array}\right]$$

$$\xrightarrow{(-1)R_3\to R_3}\left[\begin{array}{ccc:ccc}1 & -1 & 0 & 1 & 0 & 0\\ 0 & 1 & -1 & -1 & 1 & 0\\ 0 & 0 & 1 & -2 & -4 & -1\end{array}\right]$$

$$\xrightarrow{R_2+R_3\to R_2}\left[\begin{array}{ccc:ccc}1 & -1 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & -3 & -3 & -1\\ 0 & 0 & 1 & -2 & -4 & -1\end{array}\right]$$

$$\xrightarrow{R_2+R_1\to R_1}\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & -1 \\ 0 & 1 & 0 & -3 & -3 & -1 \\ 0 & 0 & 1 & -1 & -4 & -1 \end{array}\right]$$

$$=\left\lfloor I \,\middle|\, A^{-1} \right\rfloor$$

Thus

$$A^{-1}=\begin{bmatrix} -2 & -3 & -1 \\ -3 & -3 & -1 \\ -2 & -4 & -1 \end{bmatrix}$$