IT 1002 – Mathematics for Computing

Matrices

Solving a system of linear equations

First, arrange the system in the following form:

$a_1 x + b_1 y + c_1 z = d_1$

$a_2 x + b_2 y + c_2 z = d_2$

$a_3 x + b_3 y + c_3 z = d_3$

where $a_{1, 2, 3}$, $b_{1, 2, 3}$, and $c_{1, 2, 3}$ are the x , y , and z coefficients, respectively, and $d_{1, 2, 3}$ are constants

Next, create a 3×4 matrix in the following manner

$$\left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$$

This is equivalent to writing

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

This matrix will be used to solve systems by Cramer's Rule. We divide it into four separate 3×3 matrices:

$$D = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$D_x = \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix}$$

$$D_y = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix}$$

$$D_z = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

D is the 3×3 coefficient matrix, and D_x, D_y, and D_z are each the result of substituting the constant column for one of the coefficient columns in D.

- Cramer's Rule states that:

$$x=\frac{\det D_x}{\det D}$$

$$y=\frac{\det D_y}{\det D}$$

$$z=\frac{\det D_z}{\det D}$$

- Eg: Solve the following system

$$8x + 10z = 7y + 15$$

$$2x + 3y + 8z = 7$$

$$5y + 9 = 4x + 2z$$

- Rearrange the system:

$8x - 7y + 10z = 15$

$2x + 3y + 8z = 7$

$-4x + 5y - 2z = -9$

$$D = \begin{bmatrix} 8 & -7 & 10 \\ 2 & 3 & 8 \\ -4 & 5 & -2 \end{bmatrix}$$

$$D_x = \begin{bmatrix} 15 & -7 & 10 \\ 7 & 3 & 8 \\ -9 & 5 & -2 \end{bmatrix}$$

$$D_y = \begin{bmatrix} 8 & 15 & 10 \\ 2 & 7 & 8 \\ -4 & -9 & -2 \end{bmatrix}$$

$$D_z = \begin{bmatrix} 8 & -7 & 15 \\ 2 & 3 & 7 \\ -4 & 5 & -9 \end{bmatrix}$$

- Find the determinants:

$det D = (-48 + 224 + 100) - (-120 + 320 + 28) = 276 - 228 = 48$

$det D_x = (-90 + 504 + 350) - (-270 + 600 + 98) = 764 - 428 = 336$

$det D_y = (-112 - 480 - 180) - (-280 - 576 - 60) = -772 - (-916) = 144$

$det D_z = (-216 + 196 + 150) - (-180 + 280 + 126) = 130 - 226 = -96$

- X= 336/48
- Y=144/48
- Z= -96/48

- Thus
- X= 7
- Y=3
- Z= -2