Lab 3 Report

Robotics Integration Group Project I

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Group #31 2025-11-26

Abstract

See Resources on github.com/RamessesN/Robotics_MIT.

1 Introduction

2 Procedure

2.1 Individual Work

2.1.1 Transformations in Practice

1. **MESSAGE VS. TF**

- **Assume we have an incoming `geometry_msgs::Quaternion quat_msg` that holds the pose of our robot. We need to save it in an already defined `tf2::Quaternion quat_tf` for further calculations. Write one line of C++ code to accomplish this task.**

```
tf2::fromMsg(quat_msg, quat_tf);
```

More specifically, we can find the official documentation of `fromMsg()` at this page:

Figure 1: tf2 Quaternion doc

- **Assume we have just estimated our robot's newest rotation and it's saved in a variable called `quat_tf` of type `tf2::Quaternion`. Write one line of C++ code to convert it to a `geometry_msgs::Quaternion` type. Use `quat_msg` as the name of the new variable.**

```
geometry_msgs::Quaternion quat_msg = tf2::toMsg(quat_tf);
```

More specifically, we can find the official documentation of `toMsg()` in the same link as `fromMsg()`:

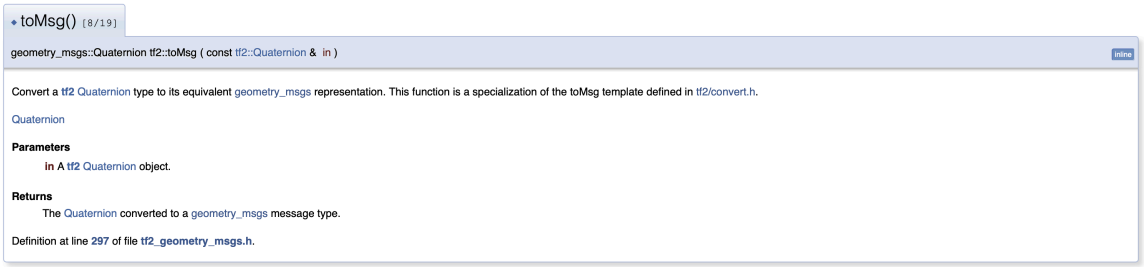


Figure 2: geometry_msgs Quaternion doc

- **If you just want to know the scalar value of a `tf2::Quaternion`, what member function will you use?**

```
double scalar = quat_tf.getW();
```

More specifically, we find the official documentation of `getW()` here:

Figure 3: Quaternion get_w doc

2. CONVERSION

- **Assume you have a `tf2::Quaternion quat_t`. How to extract the yaw component of the rotation with just one function call?**

```
double yaw = tf2::getYaw(quat_t);
```

More specifically, the doc of `getYaw()` is shown at this page:

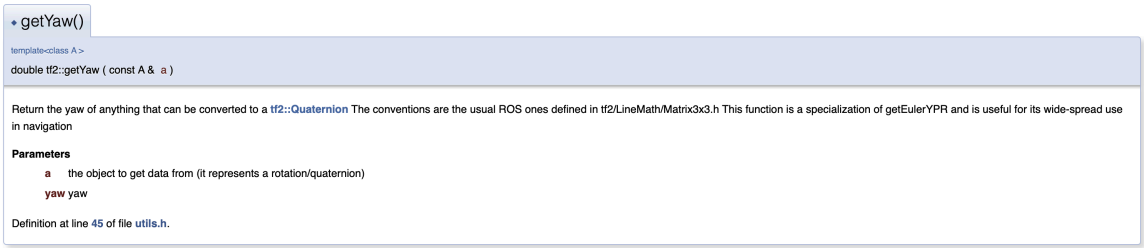


Figure 4: Quaternion get_yaw doc

- **Assume you have a `geometry_msgs::Quaternion quat_msg`. How to you convert it to an Eigen 3-by-3 matrix? Refer to this for possible functions. You probably need two function calls for this.**

```cpp
#include <tf2_eigen/tf2_eigen.h>

Eigen::Quaterniond eigen_quat;

// The first function to call
tf2::fromMsg(quat_msg, eigen_quat);

// The second function to call
Eigen::Matrix3d eigen_mat3 = eigen_quat.toRotationMatrix();
```

More specifically, the doc of `toRotationMatrix()` can be found here:

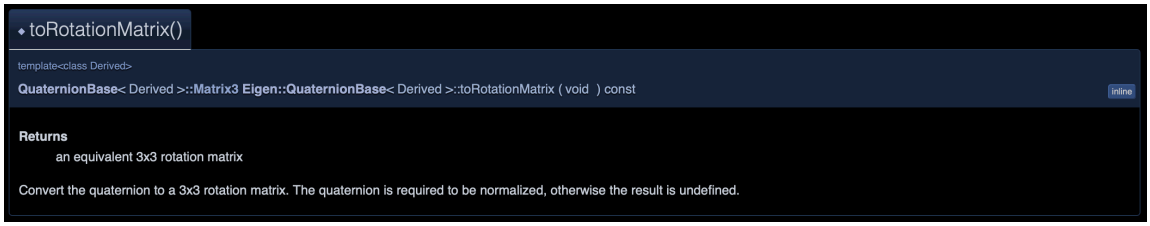
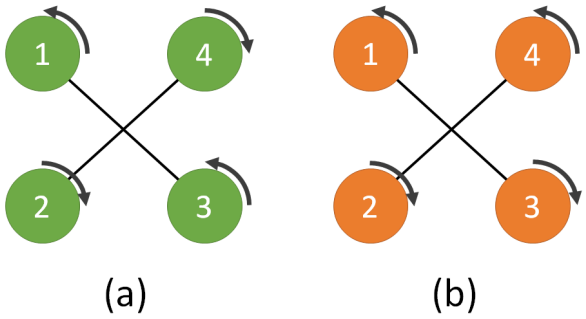


Figure 5: Eigen toRotationMatrix doc

2.1.2 Modelling and control of UAVs

1. **STRUCTURE OF QUADROTORS**



The figure above depicts two quadrotors (a) and (b). Quadrotor (a) is a fully functional UAV, while for Quadrotor (b) someone changed propellers 3 and 4 and reversed their respective rotation directions.

Show mathematically that quadrotor (b) is not able to track a trajectory defined in position $[x, y, z]$ and yaw orientation Ψ.

In order to proof quadroter (b) is not able to track a trajectory, we have to judge whether the rank of the matrix $\boldsymbol{F}$ is full.

The quadrotor has four inputs - the thrust of 4 motors: f_1, f_2, f_3, f_4, and four outputs free degrees (total thrust T and three-axis torque τ_{roll}, τ_{pitch}, and τ_{yaw}). The linear equation is:

$$u = \mathrm{F} f$$

which $u = [T, \tau_{\text{roll}}, \tau_{\text{pitch}}, \tau_{\text{yaw}}]^T$ is the output vector, $f = [f_1, f_2, f_3, f_4]^T$ is the input vector.

	Quadrotor (a)	**Quadrotor (b)**
Thrust T	$[1,1,1,1]$	$[1,1,1,1]$
Roll τ_{roll}	$[-d,-d,d,d]$	$[-d,-d,d,d]$
Pitch τ_{pitch}	$[d,-d,-d,d]$	$[d,-d,-d,d]$
Yaw τ_{yaw}	$[-c,c,-c,c]$	$[-c,c,c,-c]$
Matrix F	$F_a = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -d & -d & d & d \\ d & -d & -d & d \\ -c & c & -c & c \end{bmatrix}$	$F_b = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -d & -d & d & d \\ d & -d & -d & d \\ -c & c & c & -c \end{bmatrix}$
Rank	4 (full)	3 (not full)

Table 1: Comparison of Quadrotors

$\because \text{rank}_b = 3 < 4 \therefore$ the matrix F_b isn't full rank, which means that the output space of the system has only 3 dimensions so that it is not able to track a trajectory defined with $[x, y, z, \Psi]$.

2. CONTROL OF QUADROTORS

Assume that empirical data suggest you can approximate the drag force (in the body frame) of a quadrotor body as:

$$\boldsymbol{F^b = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix} \left(v^b\right)^2}$$

With $\left(v^b\right)^2 = [-v_x^b \cdot |v_x^b|, -v_y^b \cdot |v_y^b|, -v_z^b \cdot |v_z^b|]^T$, and v_x, v_y, v_z being the quadrotor velocities along the axes of the body frame.

With the controller discussed in class (see referenced paper [1]), describe how you could use the information above to improve the tracking performance.

From the referenced paper, we have:

$$\begin{cases} m\dot{v} &= mge_3 - fRe_3 \\ \vec{b}_{3d} &= -\frac{-k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d}{\| -k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d \|} \\ f &= -(-k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d) \cdot Re_3 \end{cases}$$

$\therefore$ Expected resultant force vector: $u_{\text{nominal}} = -k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d$

$\because$ Drag Force: $D_{\text{inertial}} = R \cdot F^b_{\text{drag}} = R \cdot \left(\begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix} (v_b)^2_{\text{signed}} \right)$

$\therefore u_{\text{new}} = u_{\text{nominal}} - D_{\text{inerital}} = -k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d - R \cdot F^b_{\text{drag}}$

$\therefore$ we have $\begin{cases} \vec{b}_{3d} = -\frac{u_{\text{new}}}{\|u_{\text{new}}\|} \\ f = -u_{\text{new}} \cdot Re^3 \end{cases}$.

2.2 Team Work

2.2.1 Trajectory tracking for UAVs

2.2.2 Launching the TESSE simulator with ROS bridge

2.2.3 Implement the controller

2.2.4 Simulator conventions

2.2.5 Geometric controller for the UAV

3 Reflection and Analysis

4 Conclusion

5 Source Code

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