Lab 3 Report

Robotics Integration Group Project I

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Abstract

See Resources on github.com/RamessesN/Robotics_MIT.

1 Introduction

2 Procedure

2.1 Individual Work

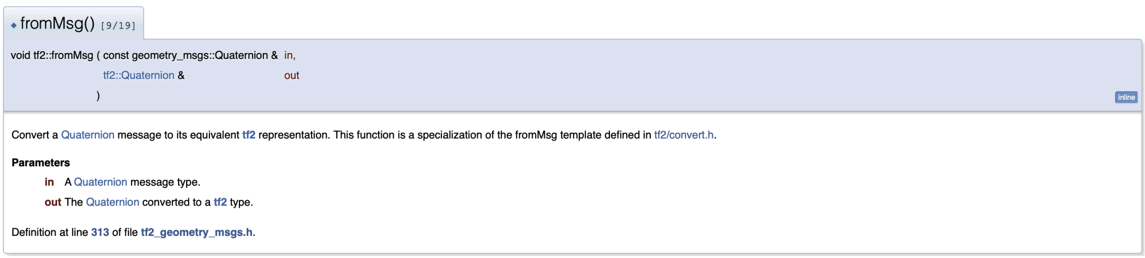
2.1.1 Transformations in Practice

1. **MESSAGE VS. TF**

   - **Assume we have an incoming `geometry_msgs::Quaternion quat_msg` that holds the pose of our robot. We need to save it in an already defined `tf2::Quaternion quat_tf` for further calculations. Write one line of C++ code to accomplish this task.**

   ```
   tf2::fromMsg(quat_msg, quat_tf);
   ```

   More specifically, we can find the official documentation of `fromMsg()` at this page:

   ◆ fromMsg() [9/19]

   void tf2::fromMsg (const geometry_msgs::Quaternion & in,
   tf2::Quaternion & out
)
   inline

   Convert a Quaternion message to its equivalent tf2 representation. This function is a specialization of the fromMsg template defined in tf2/convert.h.

   Parameters
   in A Quaternion message type.
   out The Quaternion converted to a tf2 type.

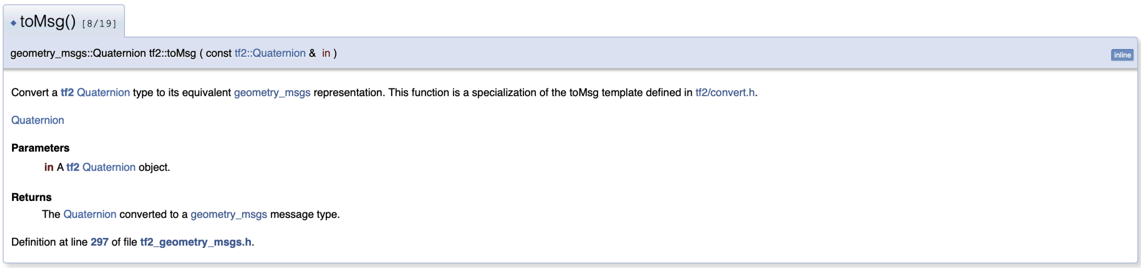
   Definition at line 313 of file tf2_geometry_msgs.h.

   Figure 1: tf2 Quaternion doc

   - **Assume we have just estimated our robot's newest rotation and it's saved in a variable called `quat_tf` of type `tf2::Quaternion`. Write one line of C++ code to convert it to a `geometry_msgs::Quaternion` type. Use `quat_msg` as the name of the new variable.**

   ```
   geometry_msgs::Quaternion quat_msg = tf2::toMsg(quat_tf);
   ```

More specifically, we can find the official documentation of `toMsg()` in the same link as `fromMsg()`:


◆ toMsg() [8/19]

geometry_msgs::Quaternion tf2::toMsg (const tf2::Quaternion & in) inline

Convert a tf2 Quaternion type to its equivalent geometry_msgs representation. This function is a specialization of the toMsg template defined in tf2/convert.h.

Quaternion

Parameters

in A tf2 Quaternion object.

Returns

The Quaternion converted to a geometry_msgs message type.

Definition at line 297 of file tf2_geometry_msgs.h.

Figure 2: geometry_msgs Quaternion doc

- **If you just want to know the scalar value of a `tf2::Quaternion`, what member function will you use?**

```
double scalar = quat_tf.getW();
```

More specifically, we find the official documentation of `getW()` here:


◆ getW()

const TF2SIMD_FORCE_INLINE tf2Scalar& tf2::Quaternion::getW ( ) const inline

Definition at line 348 of file Quaternion.h.

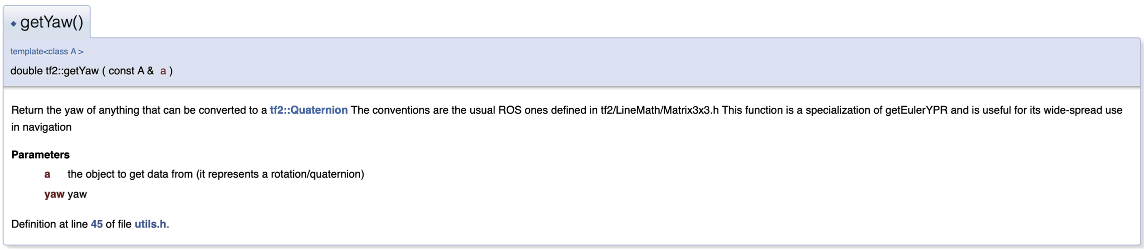
Figure 3: Quaternion get_w doc

2. **CONVERSION**

- **Assume you have a `tf2::Quaternion quat_t`. How to extract the yaw component of the rotation with just one function call?**

```
double yaw = tf2::getYaw(quat_t);
```

More specifically, the doc of `getYaw()` is shown at this page:


◆ getYaw()

template<class A >

double tf2::getYaw (const A & a)

Return the yaw of anything that can be converted to a tf2::Quaternion The conventions are the usual ROS ones defined in tf2/LineMath/Matrix3x3.h This function is a specialization of getEulerYPR and is useful for its wide-spread use in navigation

Parameters

a the object to get data from (it represents a rotation/quaternion)

yaw yaw

Definition at line 45 of file utils.h.

Figure 4: Quaternion get_yaw doc

- **Assume you have a `geometry_msgs::Quaternion quat_msg`. How to you convert it to an Eigen 3-by-3 matrix? Refer to this for possible functions. You probably need two function calls for this.**

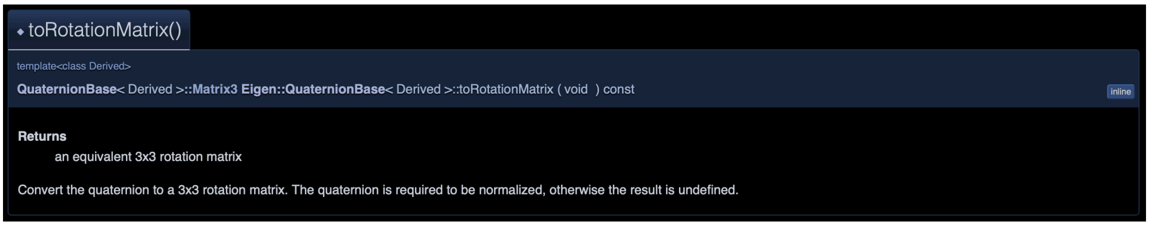
```
#include <tf2_eigen/tf2_eigen.h>

Eigen::Quaterniond eigen_quat;

// The first function to call
tf2::fromMsg(quat_msg, eigen_quat);

// The second function to call
Eigen::Matrix3d eigen_mat3 = eigen_quat.toRotationMatrix();
```

More specifically, the doc of `toRotationMatrix()` can be found here:


◆ toRotationMatrix()

template<class Derived>
QuaternionBase< Derived >::Matrix3 Eigen::QuaternionBase< Derived >::toRotationMatrix (void) const inline

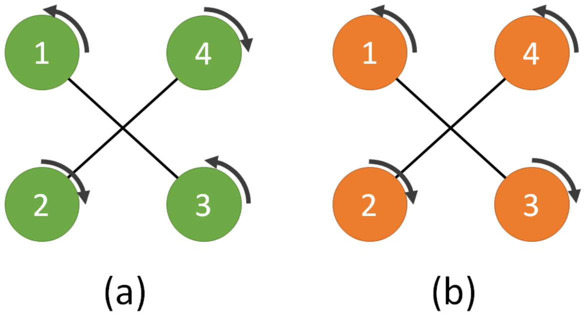
Returns
an equivalent 3x3 rotation matrix

Convert the quaternion to a 3x3 rotation matrix. The quaternion is required to be normalized, otherwise the result is undefined.

Figure 5: Eigen toRotationMatrix doc

2.1.2 Modelling and control of UAVs

1. **STRUCTURE OF QUADROTORS**



The figure above depicts two quadrotors (a) and (b). Quadrotor (a) is a fully functional UAV, while for Quadrotor (b) someone changed propellers 3 and 4 and reversed their respective rotation directions.

Show mathematically that quadrotor (b) is not able to track a trajectory defined in position $[x, y, z]$ and yaw orientation Ψ.

In order to proof quadroter (b) is not able to track a trajectory, we have to judge whether the rank of the matrix $\boldsymbol{F}$ is full.

The quadrotor has four inputs - the thrust of 4 motors: f_1, f_2, f_3, f_4, and four outputs free degrees (total thrust T and three-axis torque τ_{roll}, τ_{pitch}, and τ_{yaw}). The linear equation is:

$$u = \mathrm{F}f \tag{1}$$

which $u = \left[T, \tau_{\text{roll}}, \tau_{\text{pitch}}, \tau_{\text{yaw}}\right]^T$ is the output vector, $f = \left[f_1, f_2, f_3, f_4\right]^T$ is the input vector.

	Quadrotor (a)	**Quadrotor (b)**
Thrust T	$[1,1,1,1]$	$[1,1,1,1]$
Roll τ_{roll}	$[-d,-d,d,d]$	$[-d,-d,d,d]$
Pitch τ_{pitch}	$[d,-d,-d,d]$	$[d,-d,-d,d]$
Yaw τ_{yaw}	$[-c,c,-c,c]$	$[-c,c,c,-c]$
Matrix F	$F_a = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -d & -d & d & d \\ d & -d & -d & d \\ -c & c & -c & c \end{bmatrix}$	$F_b = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -d & -d & d & d \\ d & -d & -d & d \\ -c & c & c & -c \end{bmatrix}$
Rank	4 (full)	3 (not full)

Table 1: Comparison of Quadrotors

$\because \text{rank}_b = 3 < 4 \therefore$ the matrix F_b isn't full rank, which means that the output space of the system has only 3 dimensions so that it is not able to track a trajectory defined with $[x, y, z, \Psi]$.

2. **CONTROL OF QUADROTORS**
Assume that empirical data suggest you can approximate the drag force (in the body frame) of a quadrotor body as:

$$\boldsymbol{F^b} = \begin{bmatrix} \mathbf{0.1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0.2} \end{bmatrix} \left(\boldsymbol{v^b}\right)^2 \tag{2}$$

With $\left(v^b\right)^2 = [-v_x^b \cdot |v_x^b|, -v_y^b \cdot |v_y^b|, -v_z^b \cdot |v_z^b|]^T$, and v_x, v_y, v_z being the quadrotor velocities along the axes of the body frame.

With the controller discussed in class (see referenced paper [1]), describe how you could use the information above to improve the tracking performance.

From the referenced paper, we have:

$$\begin{cases} m\dot{v} &= mge_3 - fRe_3 \\ \vec{b}_{3d} &= -\frac{-k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d}{\| -k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d \|} \\ f &= -(-k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d) \cdot Re_3 \end{cases} \tag{3}$$

$\therefore$ Expected resultant force vector: $u_{\text{nominal}} = -k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d$

$\because$ Drag Force: $D_{\text{inertial}} = R \cdot F^b_{\text{drag}} = R \cdot \left(\begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix} (v_b)^2_{\text{signed}} \right)$

$\therefore u_{\text{new}} = u_{\text{nominal}} - D_{\text{inerital}} = -k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d - R \cdot F^b_{\text{drag}}$

$\therefore$ we have $\begin{cases} \vec{b}_{3d} = -\frac{u_{\text{new}}}{\|u_{\text{new}}\|} \\ f = -u_{\text{new}} \cdot Re^3 \end{cases}$.

2.2 Team Work

2.2.1 Trajectory tracking for UAVs

2.2.2 Launching the TESSE simulator with ROS bridge

2.2.3 Implement the controller

2.2.4 Simulator conventions

2.2.5 Geometric controller for the UAV

3 Reflection and Analysis

4 Conclusion

5 Source Code

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