16.485: VNAV - Visual Navigation for Autonomous Vehicles

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整理: 范浩

based on slides by Markus Ryll

Lecture Outline

- 四旋翼的动力学模型（Dynamical model of a Quadrotor）
- 四旋翼无人机的动力模型介绍
- 四旋翼无人机的动力飞控模型介绍

Agile Navigation with Aerial Vehicles

- 一点也不流线型的无人机，要空气动力学有啥用？【差评君】_哔哩哔哩_bilibili
- 四轴无人机的结构及飞行原理_哔哩哔哩_bilibili

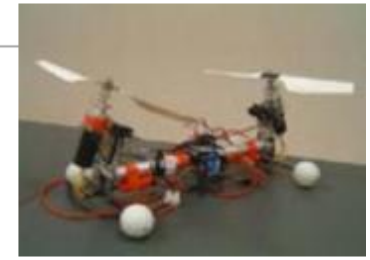
设计理念 Design Concepts

- Single rotor

✓ Good Controllability and maneuverability

- Complex mechanics
- Large rotor

- Tandem rotor



✓ Good Controllability and maneuverability

- Complex mechanics
- Large size

- Coaxial rotor

✓ Compactness

✓ Simple mechanics

- Complex aerodynamics

- Quadrotor

✓ Good maneuverability

✓ Simple mechanics

✓ Big payload

- High energy consumption

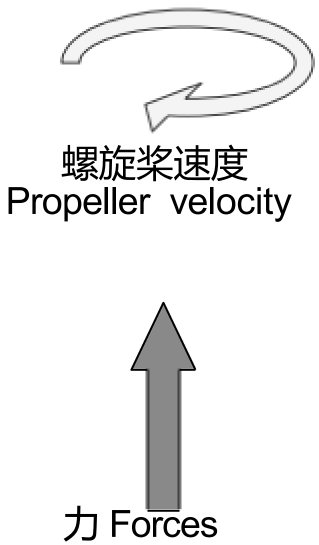
设计理念 Design Concepts

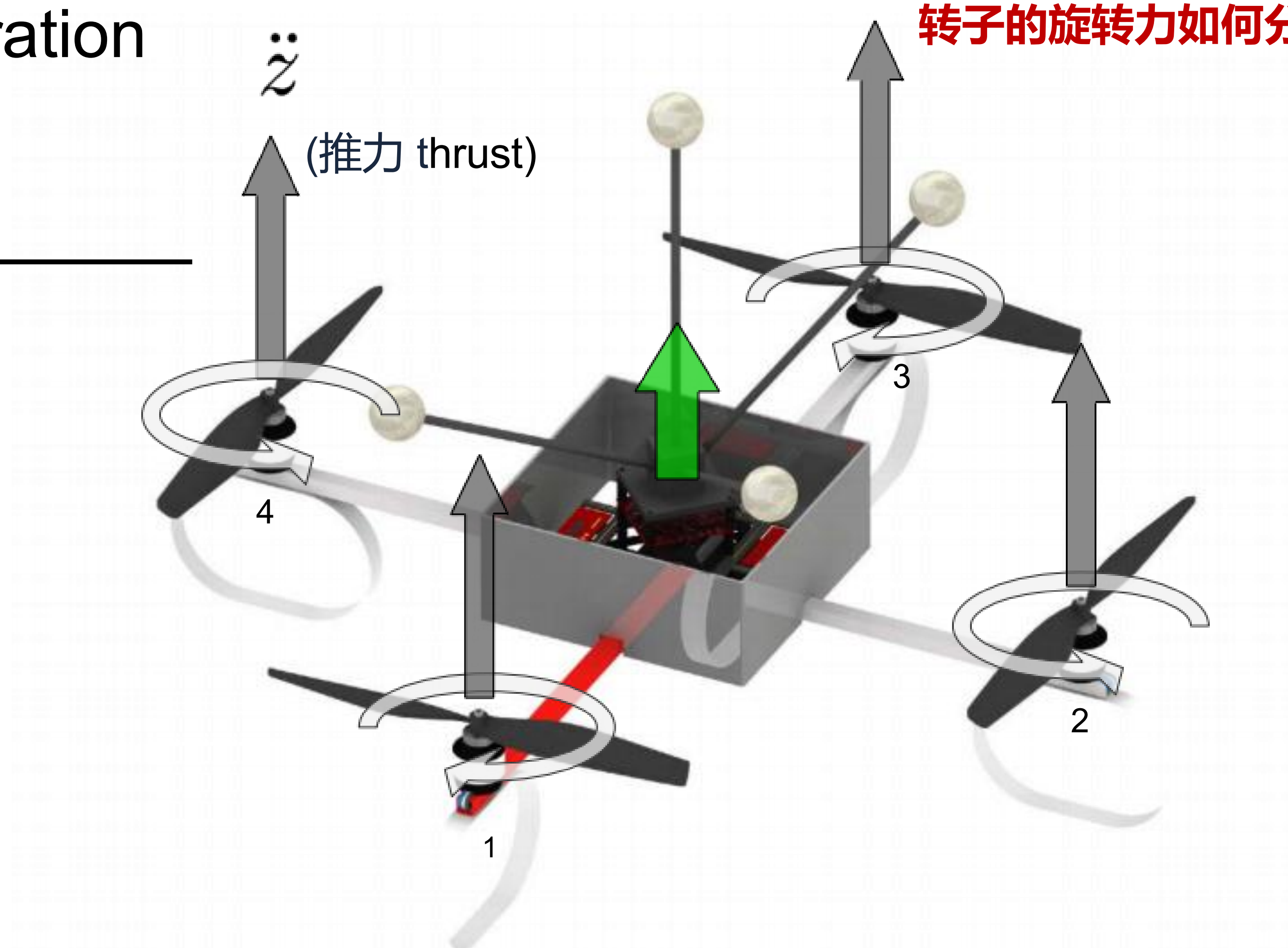
[看了这个视频才知道，无刷电机的结构原理原来这么简单_哔哩哔哩_bilibili]

Vertical acceleration

垂直加速度

转子的旋转力如何分析?

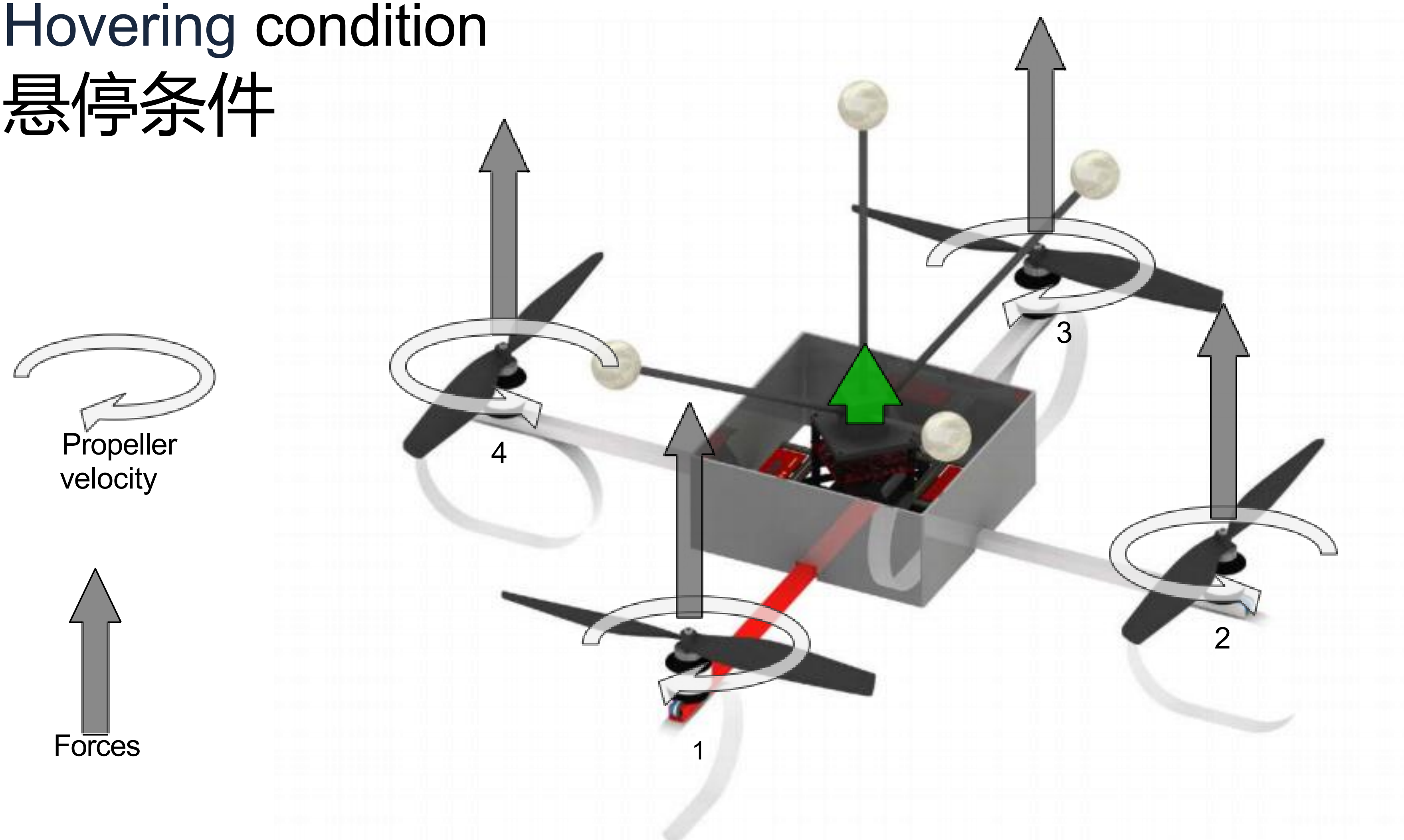




四旋翼的旋转方向，利用对称力矩抵消自旋。

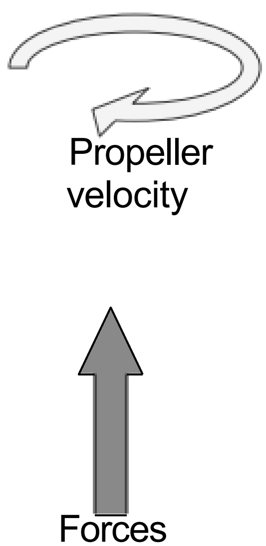
Design Concepts

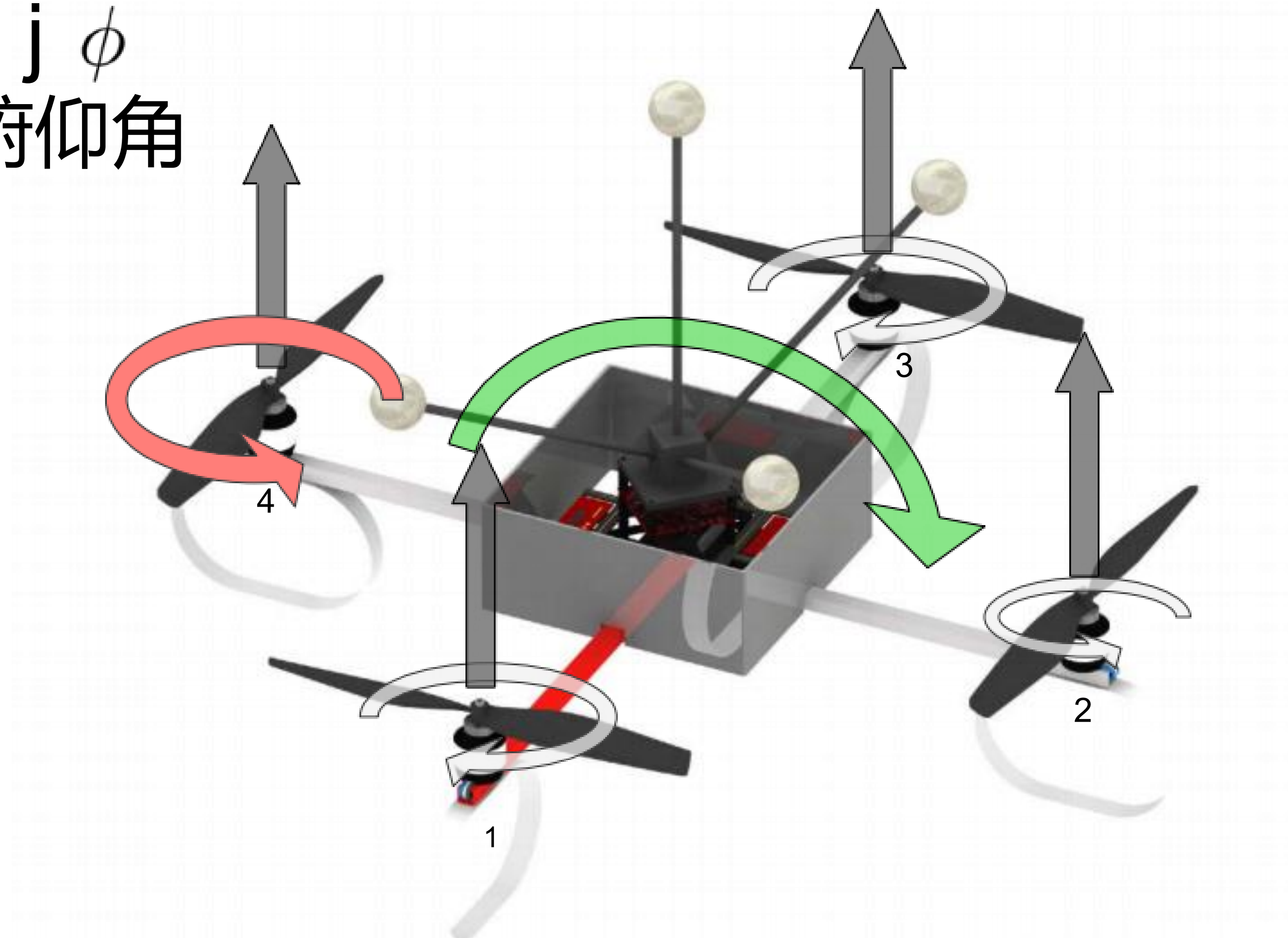
Hovering condition
悬停条件



Design Concepts

Roll/Pitch j $\ddot{\phi}$
翻滚角/俯仰角

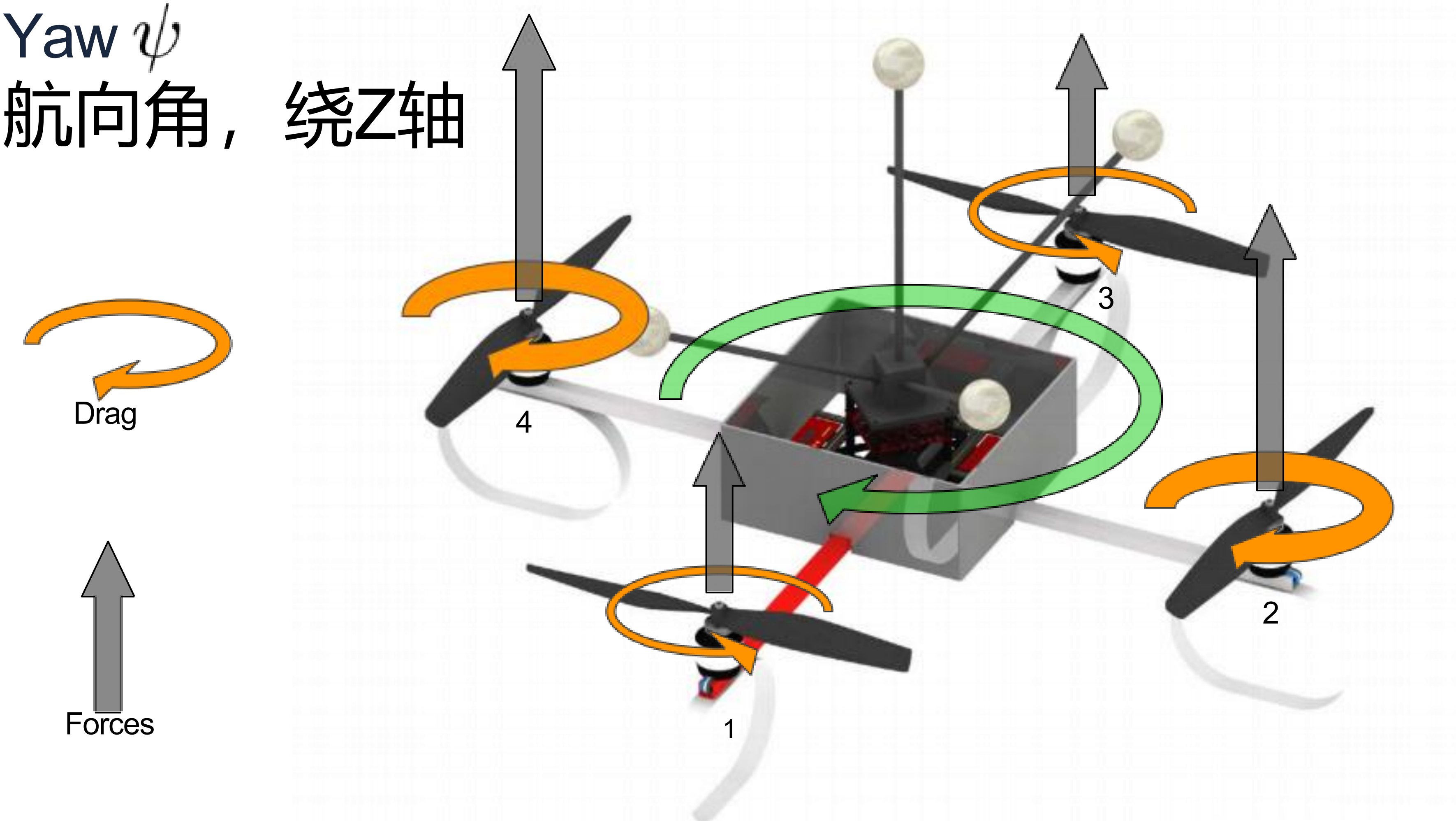




Design Concepts

Yaw $\ddot{\psi}$

航向角，绕Z轴



System view

[无人机的工作原理_哔哩哔哩_bilibili](#)

Source: http://ardupilot.org/copter/_images/ Pixhawk-Inforgraphic2.jpg

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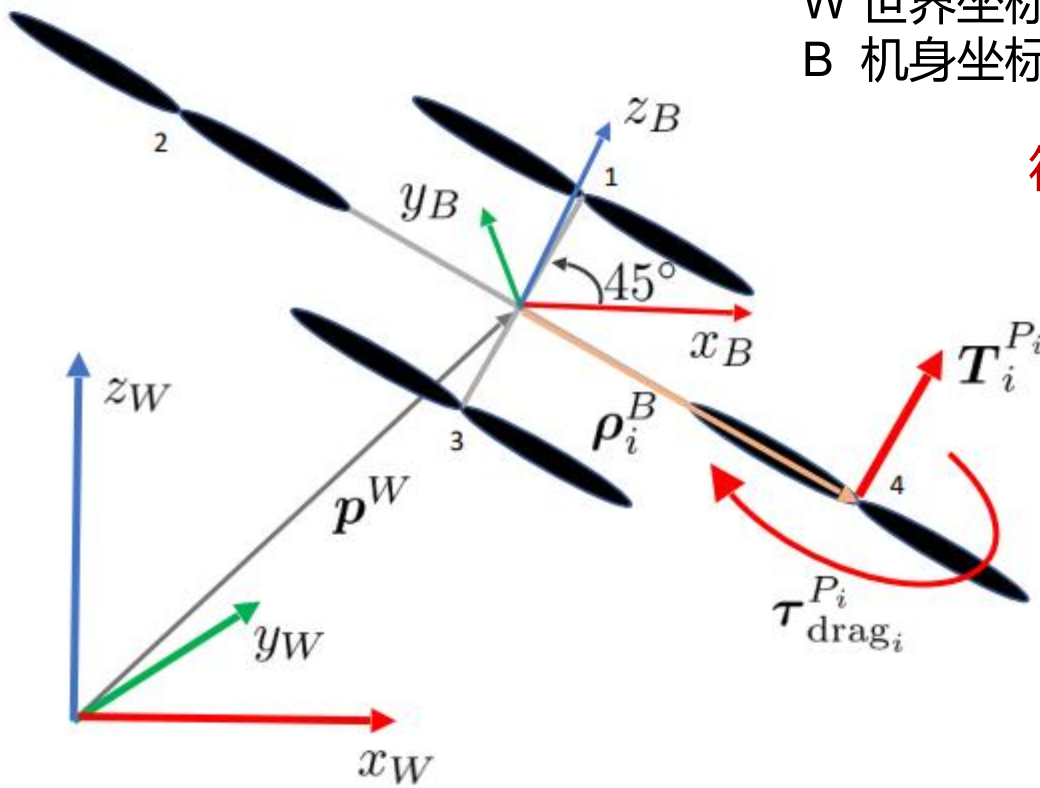
四旋翼无人机模型

W 世界坐标系 the world frame
B 机身坐标系 the body frame

符合相机拍摄视角



Pi 旋翼坐标系

Thrust 推力
Tor 力矩，扭矩
$\boldsymbol{\rho}_i^{\mathrm{B}}$ being the position (vector) of the propeller i in the body frame.

四旋翼无人机模型

- 刚体的旋转与平移.

$$\begin{bmatrix} \boldsymbol{f}^{\mathrm{B}} \\ \boldsymbol{\tau}^{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} m\mathbf{I}_3 & \mathbf{0} \\ \mathbf{0} & \mathcal{J} \end{bmatrix} \begin{bmatrix} \boldsymbol{a}^{\mathrm{B}} \\ \boldsymbol{\alpha}^{\mathrm{B}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\omega}^{\mathrm{B}} \times \mathcal{J}\boldsymbol{\omega}^{\mathrm{B}} \end{bmatrix}$$

如何理解?

推力	$\boldsymbol{f}^{\mathrm{B}} \in \mathbb{R}^3$	applied total forces expressed in the body frame
力矩	$\boldsymbol{\tau}^{\mathrm{B}} \in \mathbb{R}^3$	applied total torques expressed in the body frame
	$m \in \mathbb{R}_+$	body mass 机身质量
	$\mathcal{J} \in \mathbb{R}^{3\times 3}$	moment of inertia about the center of mass 关于质心的转动惯量
	$\boldsymbol{I}_n$	$n \times n$ identity matrix
加速度	$\boldsymbol{a}^{\mathrm{B}} \in \mathbb{R}^3$	translational acceleration of the center of mass expressed in the body frame
角加速度	$\boldsymbol{\alpha}^{\mathrm{B}} \in \mathbb{R}^3$	angular acceleration of the body expressed in the body frame
	$\boldsymbol{\omega}^{\mathrm{B}} \in \mathbb{R}^3$	angular velocity of the body expressed in the body frame

角速度

四旋翼无人机模型

- 在世界坐标系“w”下的推力：

$$\begin{bmatrix} \boldsymbol{f}^{\mathrm{W}} \\ \boldsymbol{\tau}^{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_{\mathrm{B}}^{\mathrm{W}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_3 \end{bmatrix} \begin{bmatrix} m\boldsymbol{I}_3 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\mathcal{J}} \end{bmatrix} \begin{bmatrix} \boldsymbol{a}^{\mathrm{B}} \\ \boldsymbol{\alpha}^{\mathrm{B}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\omega}^{\mathrm{B}} \times \boldsymbol{\mathcal{J}} \boldsymbol{\omega}^{\mathrm{B}} \end{bmatrix}$$

- 考虑的第一个外力是重力，其他力和扭矩是由于空气动力学效应造成的. 空气动力学的影响

$$\begin{bmatrix} \boldsymbol{f}^{\mathrm{W}} \\ \boldsymbol{\tau}^{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_{\mathrm{B}}^{\mathrm{W}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_3 \end{bmatrix} \begin{bmatrix} m\boldsymbol{I}_3 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\mathcal{J}} \end{bmatrix} \begin{bmatrix} \boldsymbol{a}^{\mathrm{B}} \\ \boldsymbol{\alpha}^{\mathrm{B}} \end{bmatrix} + \begin{bmatrix} \boxed{-m\boldsymbol{g}^{\mathrm{W}}} \\ \boldsymbol{\omega}^{\mathrm{B}} \times \boldsymbol{\mathcal{J}} \boldsymbol{\omega}^{\mathrm{B}} \end{bmatrix}$$

推力 thrust force

- 在第一个近似中，单个旋翼 i 的推力，以旋翼 i 的参考系（又名螺旋桨架 Pi）表示，可以计算为

$$T_i^{\mathrm{P}i} = c_f w_i |w_i| e_3$$

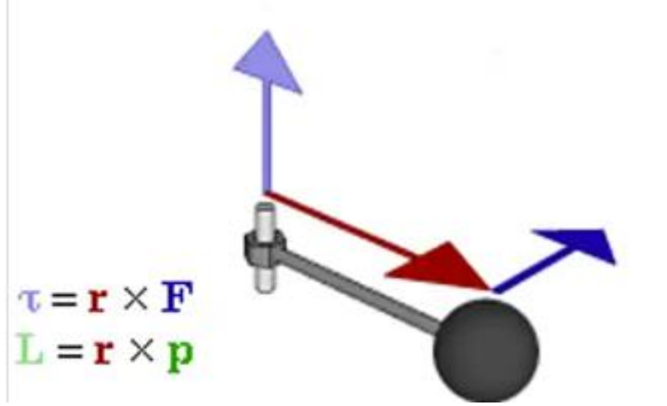
c_f是常量，e3=[0,0,1]T。

- $Wi|Wi|$是转子旋转速度的有符号平方.

$$f_{thrust}^{\mathrm{B}} = \sum_{i=1}^{4} R_{\mathrm{P}_i}^{\mathrm{B}} T_i^{\mathrm{P}i};$$

$R_{\mathrm{P}_i}^{\mathrm{B}}$ being the rotation matrix from propeller- to body-frame ($R_{\mathrm{P}_i}^{\mathrm{B}} = \mathbf{I}_3$ for standard quadrotors)

力矩 torque



- 力矩包含2个部分:

转子拖力 + 桨叶升力分力--推力

$$\boldsymbol{\tau}^{\mathrm{B}} = \boldsymbol{\tau}^{\mathrm{B}}_{drag} + \boldsymbol{\tau}^{\mathrm{B}}_{thrust}$$

转子拖力 $$\boldsymbol{\tau}^{\mathrm{B}}_{drag} = \sum_{i=1}^{4} \boldsymbol{R}^{\mathrm{B}}_{\mathrm{P}_i} \boldsymbol{\tau}^{\mathrm{P}_i}_{drag_i}$$

$$\boxed{\boldsymbol{\tau}^{\mathrm{P}_i}_{drag_i} = (-1)^{(i+1)} c_d w_i |w_i| \boldsymbol{e}_3}$$

桨叶升力分力

$$\boldsymbol{\tau}^{\mathrm{B}}_{thrust} = \sum_{i=1}^{4} (\boldsymbol{\rho}^{B}_{i} \times \boldsymbol{R}^{\mathrm{B}}_{\mathrm{P}_i} \boldsymbol{T}^{\mathrm{P}_i}_{i})$$

$\boldsymbol{\rho}^{\mathrm{B}}_{i}$ being the position (vector) of the propeller i in the body frame.

- 力矩是一个矢量，定义是距离与力的叉乘。力矩的量纲是*ML2T-2*，在国际单位制中的单位是 kg·m2·s-2 。
- 力矩会改变物体的角动量，即物体的转动性质，或者使物体发生扭曲。处于平衡状态的物体不仅受力为零，所受力矩也为零。

四旋翼模型 Quadrotor model

- 模型中的力和力矩 the forces and torques

$$\begin{bmatrix} m\boldsymbol{a}^{\mathrm{W}} \\ \mathcal{J}\boldsymbol{\alpha}^{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} m\boldsymbol{g}^{\mathrm{W}} \\ -\boldsymbol{\omega}^{\mathrm{B}} \times \mathcal{J}\boldsymbol{\omega}^{\mathrm{B}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{R}_{\mathrm{B}}^{\mathrm{W}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_3 \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_{thrust}^{\mathrm{B}} \\ \boldsymbol{\tau}_{drag}^{\mathrm{B}} + \boldsymbol{\tau}_{thrust}^{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} -mg\boldsymbol{e}_3 \\ -\boldsymbol{\omega}^{\mathrm{B}} \times \mathcal{J}\boldsymbol{\omega}^{\mathrm{B}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{R}_{\mathrm{B}}^{\mathrm{W}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_3 \end{bmatrix} \boldsymbol{F}\boldsymbol{w}$$

$$\begin{bmatrix} f_x^{\mathrm{B}} \\ f_y^{\mathrm{B}} \\ f_z^{\mathrm{B}} \\ \tau_x^{\mathrm{B}} \\ \tau_y^{\mathrm{B}} \\ \tau_z^{\mathrm{B}} \end{bmatrix} = \boldsymbol{F}\boldsymbol{w} = \begin{bmatrix} c_f\boldsymbol{e}_3 & c_f\boldsymbol{e}_3 & c_f\boldsymbol{e}_3 & c_f\boldsymbol{e}_3 \\ c_d\boldsymbol{e}_3 + c_f\boldsymbol{\rho}_1^{\mathrm{B}} \times \boldsymbol{e}_3 & -c_d\boldsymbol{e}_3 + c_f\boldsymbol{\rho}_2^{\mathrm{B}} \times \boldsymbol{e}_3 & c_d\boldsymbol{e}_3 + c_f\boldsymbol{\rho}_3^{\mathrm{B}} \times \boldsymbol{e}_3 & -c_d\boldsymbol{e}_3 + c_f\boldsymbol{\rho}_4^{\mathrm{B}} \times \boldsymbol{e}_3 \end{bmatrix} \boldsymbol{w}$$

最终想得到的转速向量

W=[w1,w2,w3,w4]T

- 最后，我们观察到（线性和角）速度和加速度由以下方式相关:

$$\dot{\boldsymbol{p}}^{\mathrm{W}} = \boldsymbol{v}^{\mathrm{W}} \qquad \dot{\boldsymbol{v}}^{\mathrm{W}} = \boldsymbol{a}^{\mathrm{W}}$$

$$\dot{\boldsymbol{R}}_{\mathrm{B}}^{\mathrm{W}} = \boldsymbol{R}_{\mathrm{B}}^{\mathrm{W}}[\boldsymbol{\omega}^{\mathrm{B}}]^{\wedge} \qquad \dot{\boldsymbol{\omega}}^{\mathrm{B}} = \boldsymbol{\alpha}^{\mathrm{B}}$$

四旋翼模型 Quadrotor model

- 基于一阶微分方程的四旋翼动力学模型:

$$
\begin{bmatrix} m\dot{\boldsymbol{v}}^{\mathrm{W}} \\ \boldsymbol{\mathcal{J}}\dot{\boldsymbol{\omega}}^{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} -mg\boldsymbol{e}_3 \\ -\boldsymbol{\omega}^{\mathrm{B}} \times \boldsymbol{\mathcal{J}}\boldsymbol{\omega}^{\mathrm{B}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{R}_{\mathrm{B}}^{\mathrm{W}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_3 \end{bmatrix} \boldsymbol{F}\boldsymbol{w}
$$

$$
\dot{\boldsymbol{p}}^{\mathrm{W}} = \boldsymbol{v}^{\mathrm{W}}
$$

$$
\dot{\boldsymbol{R}}_{\mathrm{B}}^{\mathrm{W}} = \boldsymbol{R}_{\mathrm{B}}^{\mathrm{W}}[\boldsymbol{\omega}^{\mathrm{B}}]^{\wedge}
$$

the *state* of the quadrotor is $\boldsymbol{p}^{\mathrm{W}}, \boldsymbol{v}^{\mathrm{W}}, \boldsymbol{R}_{\mathrm{B}}^{\mathrm{W}}, \boldsymbol{\omega}^{\mathrm{B}}$ and the *control actions*

the propeller velocities, included in the vector w.

差平整度特性

- 在本节中，我们表明以螺旋桨w的四个角速度为输入的四旋翼动力学是微分平坦的.
- 状态和输入可以写成四个精心挑选的平面输出及其导数的代数函数。我们选择的平坦输出由下式给出

$$\boldsymbol{\sigma} = [\sigma_1\ \sigma_2\ \sigma_3\ \sigma_4]^\mathsf{T} = [x,\ y,\ z,\ \psi]^\mathsf{T},$$

where $\boldsymbol{p}^{\mathrm{w}} = [x,\ y,\ z]^\mathsf{T}$ are the coordinates of the center of mass of the quadrotor in the world coordinate system and ψ is the yaw angle. We will define a trajectory, $\boldsymbol{\sigma}(t)$, as a smooth curve in the space of flat outputs:

$$\boldsymbol{\sigma}(t) : [t_o, t_m] \to \mathbb{R}^3 \times SO(2). \tag{6.26}$$

➢ 四旋翼动力模型推导_哔哩哔哩_bilibili

多旋翼无人机偏航运动与反扭力的深度解析

Lecture Outline

- Dynamical model of a Quadrotor
- 四旋翼无人机的动力模型介绍
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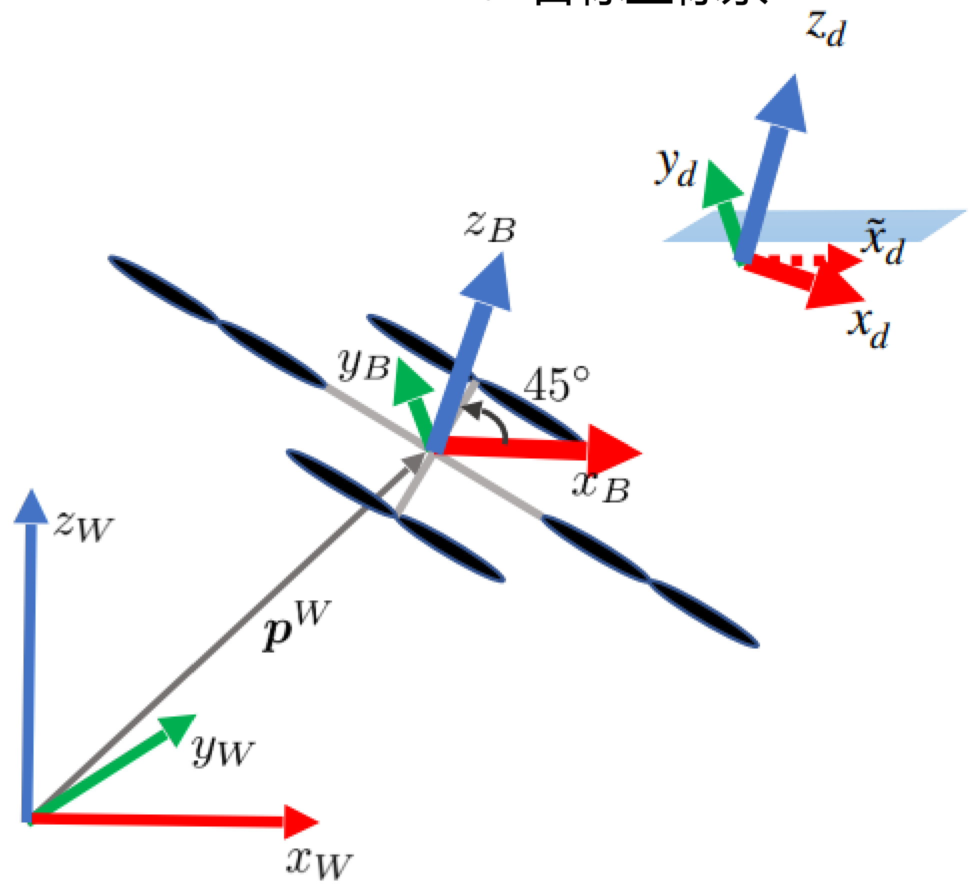
四旋翼控制器

W 世界坐标系 the world frame
B 机身坐标系 the body frame
d 目标坐标系

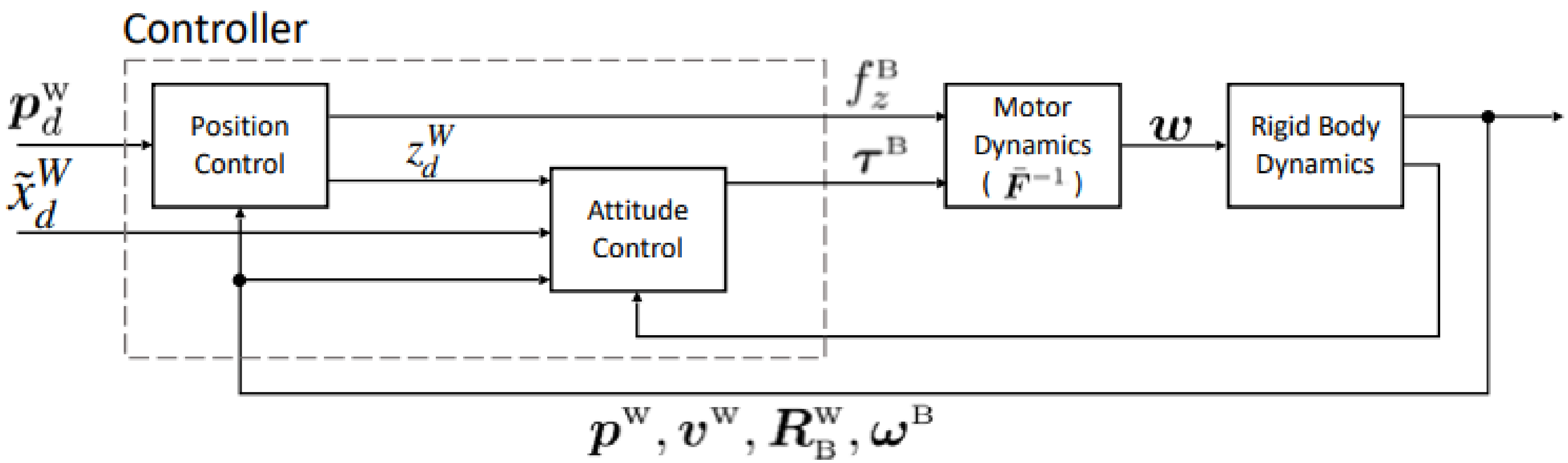
$$R_B^W = [x_B^W \ y_B^W \ z_B^W]$$

$$R_d^W = [x_d^W \ y_d^W \ z_d^W]$$



四旋翼控制器

- 用于位置和姿态控制的几何控制回路。



位置，速度，旋转，角速度

几何控制器1

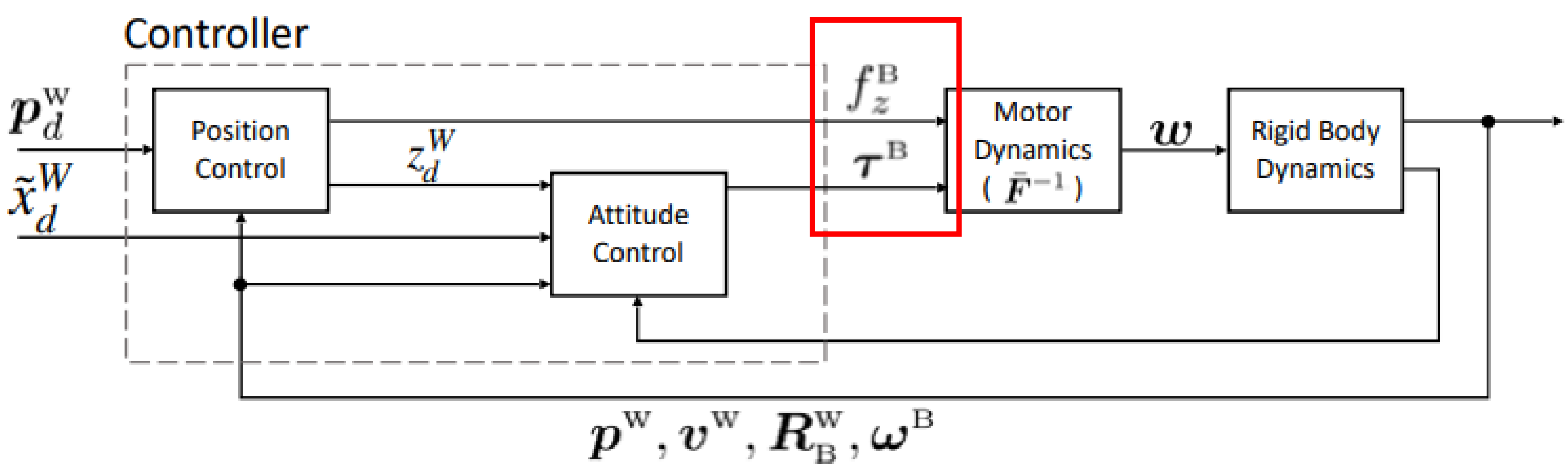
- 四旋翼有 4 个控制输入（4 个转子的速度），但有 6 个自由度（3 个用于平移，3 个用于旋转）。由于我们的输入比控制少，因此我们将无法独立控制所有自由度。
- 直观地说，如果四旋翼具有俯仰角或横滚角，其动力学将迫使其移动，因此四旋翼无法同时保持所需的位置以及所需的横滚和俯仰。横滚和俯仰时，机身必然移动！
- 因此，在本次讲座中，我们将只讨论如何控制 4 个自由度。

跟踪误差和控制设计

- 在第 6 讲中，我们已经看到四旋翼动力学满足:

$$\begin{bmatrix} m\ddot{\mathbf{p}}^{\mathrm{W}} \\ \mathcal{J}\dot{\boldsymbol{\omega}}^{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} -mg\mathbf{e}_3 \\ -\boldsymbol{\omega}^{\mathrm{B}} \times \mathcal{J}\boldsymbol{\omega}^{\mathrm{B}} \end{bmatrix} + \begin{bmatrix} \mathbf{z}_{\mathrm{B}}^{\mathrm{W}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_3 \end{bmatrix} \begin{bmatrix} f_z^{\mathrm{B}} \\ \boldsymbol{\tau}^{\mathrm{B}} \end{bmatrix}$$

- 由于 F 是常数，我们总是可以从 w 到 [f ; τ] 来回移动，
- 因此，在下文中我们将直接将 [f ; τ] 视为我们的控制输入.



跟踪错误

- 人们可能会将几何控制器视为 PD（比例导数）控制器，但具有尊重旋转的李群结构的误差指标。错误定义如下:

$$\begin{aligned}
\boldsymbol{e}_p &= \boldsymbol{p}^{\mathrm{w}} - \boldsymbol{p}_d^{\mathrm{w}} && \text{(position error)} \\
\boldsymbol{e}_v &= \boldsymbol{v}^{\mathrm{w}} - \boldsymbol{v}_d^{\mathrm{w}} && \text{(linear velocity error)} \\
\boldsymbol{e}_R &= \frac{1}{2}\left[(\boldsymbol{R}_d^{\mathrm{w}})^{\mathsf{T}}\boldsymbol{R}_{\mathrm{B}}^{\mathrm{w}} - (\boldsymbol{R}_{\mathrm{B}}^{\mathrm{w}})^{\mathsf{T}}\boldsymbol{R}_d^{\mathrm{w}}\right]^{\vee} && \text{(rotation error)} \\
\boldsymbol{e}_\omega &= \boldsymbol{\omega}^{\mathrm{B}} - (\boldsymbol{R}_{\mathrm{B}}^{\mathrm{w}})^{\mathsf{T}}\boldsymbol{R}_d^{\mathrm{w}}\boldsymbol{\omega}_d && \text{(angular velocity error)}
\end{aligned}$$

控制器

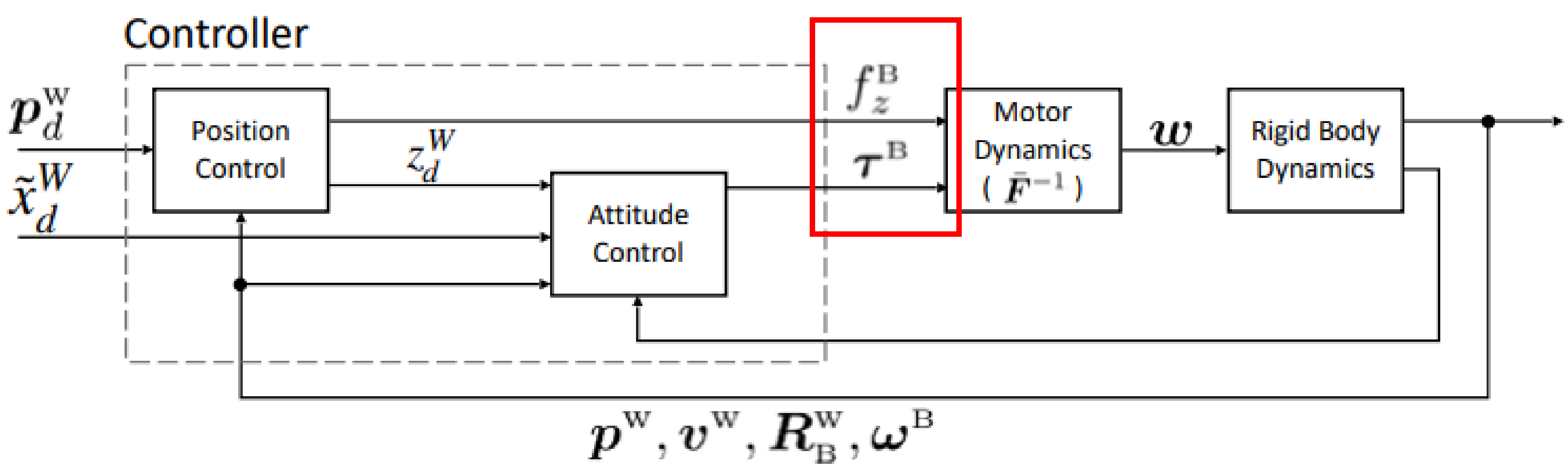
- 我们现在可以介绍几何控制器.

$$f_z^{\mathrm{B}} = (-k_p \boldsymbol{e}_p - k_v \boldsymbol{e}_v + mg\boldsymbol{e}_3 + m\ddot{\boldsymbol{p}}_d^{\mathrm{W}}) \cdot \boldsymbol{R}_{\mathrm{B}}^{\mathrm{W}} \boldsymbol{e}_3,$$

$$\boldsymbol{\tau}^{\mathrm{B}} = -k_R \boldsymbol{e}_R - k_\omega \boldsymbol{e}_\omega + \boldsymbol{\omega}^{\mathrm{B}} \times \boldsymbol{\mathcal{J}} \boldsymbol{\omega}^{\mathrm{B}} - \boldsymbol{\mathcal{J}}([\boldsymbol{\omega}^{\mathrm{B}}]_\times (\boldsymbol{R}_{\mathrm{B}}^{\mathrm{W}})^\top \boldsymbol{R}_d^{\mathrm{W}} \boldsymbol{\omega}_d - (\boldsymbol{R}_{\mathrm{B}}^{\mathrm{W}})^\top \boldsymbol{R}_d^{\mathrm{W}} \dot{\boldsymbol{\omega}}_d),$$

where the desired attitude $\boldsymbol{R}_d^{\mathrm{W}}$ is computed as $\boldsymbol{R}_d^{\mathrm{W}} = [\frac{(\boldsymbol{z}_d^{\mathrm{W}} \times \tilde{\boldsymbol{x}}_d^{\mathrm{W}}) \times \boldsymbol{z}_d^{\mathrm{W}}}{\|(\boldsymbol{z}_d^{\mathrm{W}} \times \tilde{\boldsymbol{x}}_d^{\mathrm{W}}) \times \boldsymbol{z}_d^{\mathrm{W}}\|}, \frac{\boldsymbol{z}_d^{\mathrm{W}} \times \tilde{\boldsymbol{x}}_d^{\mathrm{W}}}{\|\boldsymbol{z}_d^{\mathrm{W}} \times \tilde{\boldsymbol{x}}_d^{\mathrm{W}}\|}, \boldsymbol{z}_d^{\mathrm{W}}] \in \mathrm{SO}(3)$, and:

$$\boldsymbol{z}_d^{\mathrm{W}} = \boldsymbol{R}_d^{\mathrm{W}} \boldsymbol{e}_3 = \frac{-k_p \boldsymbol{e}_p - k_v \boldsymbol{e}_v + mg\boldsymbol{e}_3 + m\ddot{\boldsymbol{p}}_d^{\mathrm{W}}}{\|-k_p \boldsymbol{e}_p - k_v \boldsymbol{e}_v + mg\boldsymbol{e}_3 + m\ddot{\boldsymbol{p}}_d^{\mathrm{W}}\|},$$



近悬停控制器

- 课程介绍了两个标准和众所周知的四旋翼控制器的原理。
- 首先，我们讨论了一个几乎全局稳定的几何控制器，它提供了一种最先进的方法来执行四旋翼机的激进机动。
- 第二个控制器（Near-Hovering Controller）是轻量级级联比例微分（PD）控制器，它在悬停附近工作得很好。

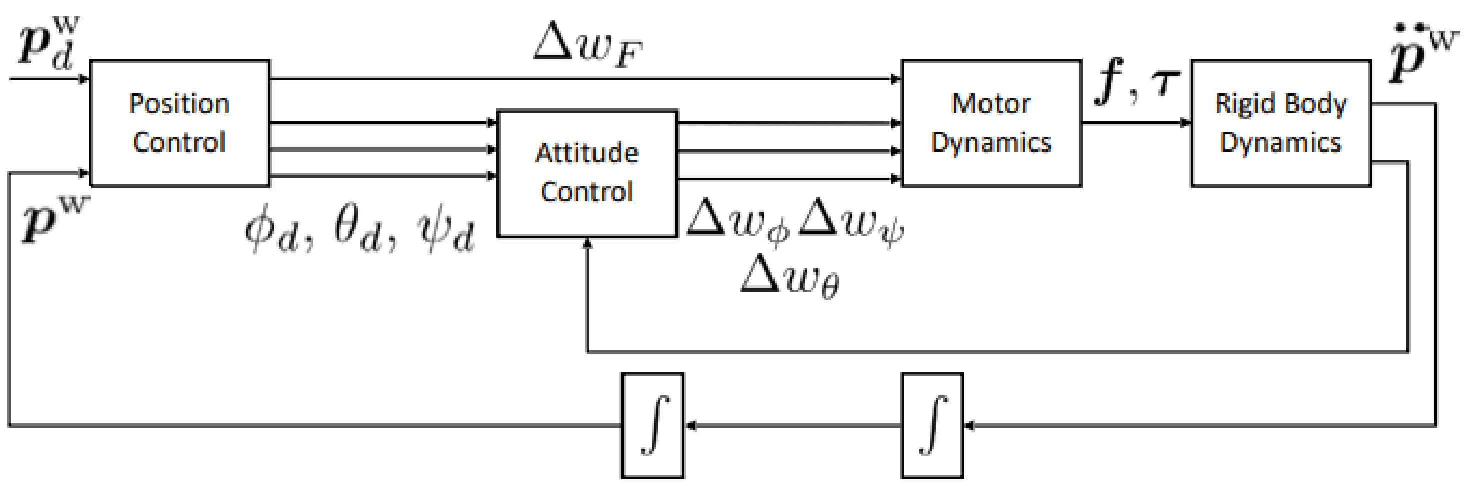


Figure 7.3: The nested control loops for position and attitude control.

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