



The University of Jordan

Mechatronics engineering department

Selected Topics in Mechatronics: Switched-mode Converters

Buck Converter Simulation and Control project

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## Part I

### 1.1 The buck converter switching circuit:

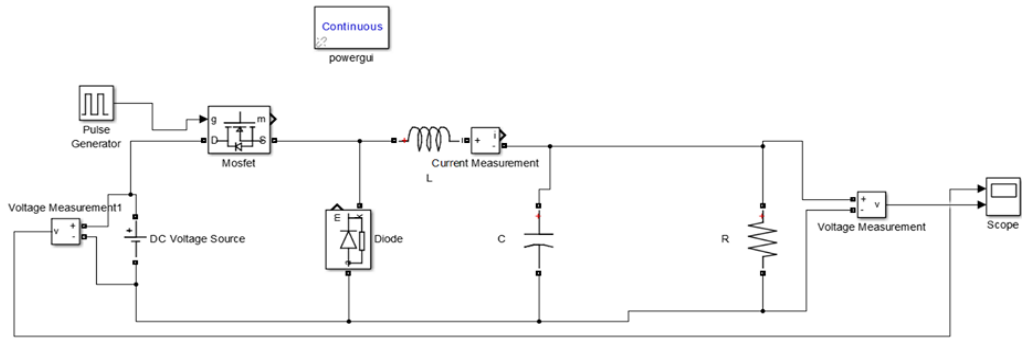


Figure 1: Buck convertor.

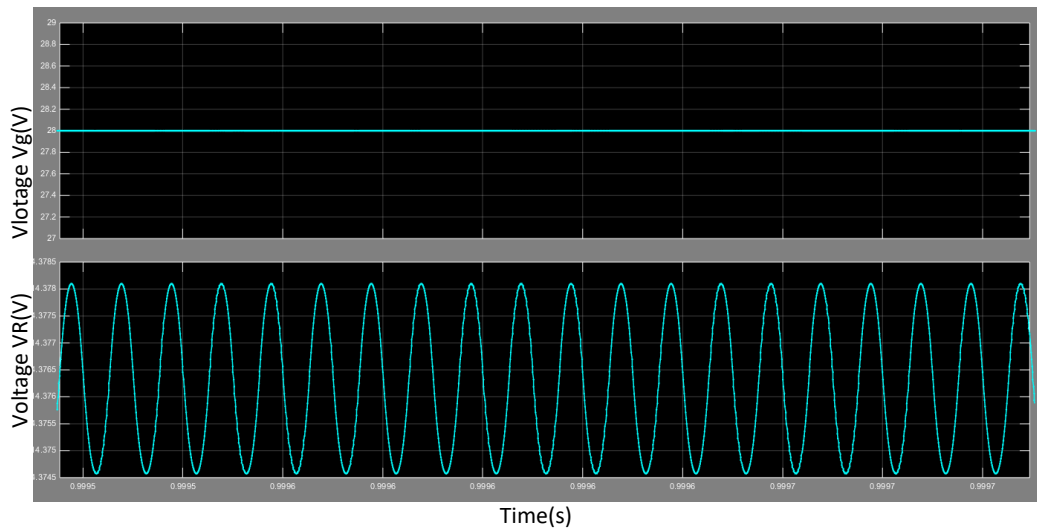


Figure 2: the waveform for the input voltage with the load voltage.

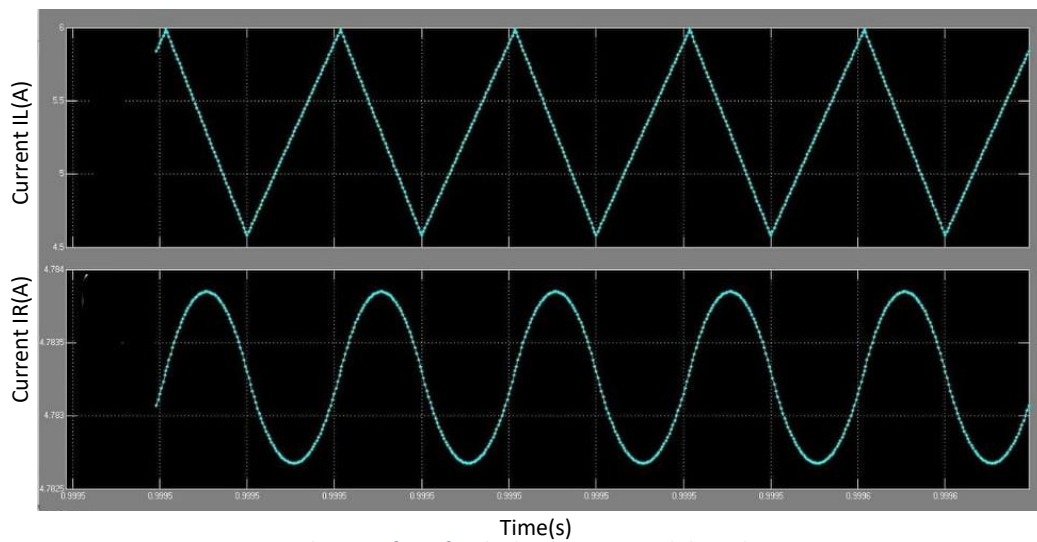


Figure 3: the waveform for the load current and the inductor current

The buck convertor steady-state equivalent circuit:

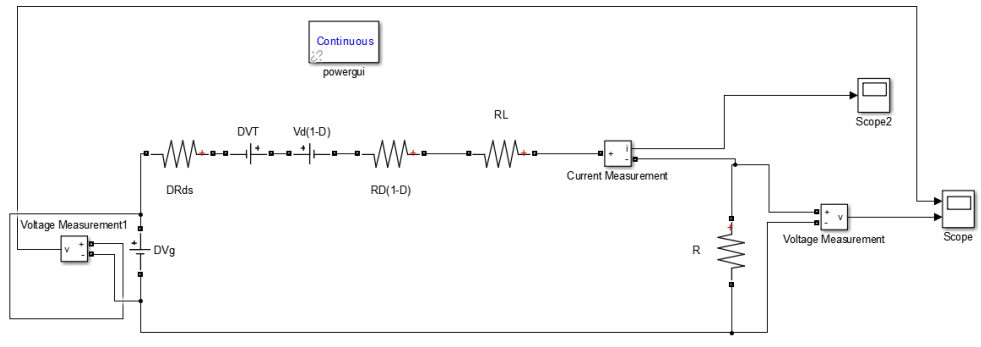


Figure 4: The DC equivalent circuit.

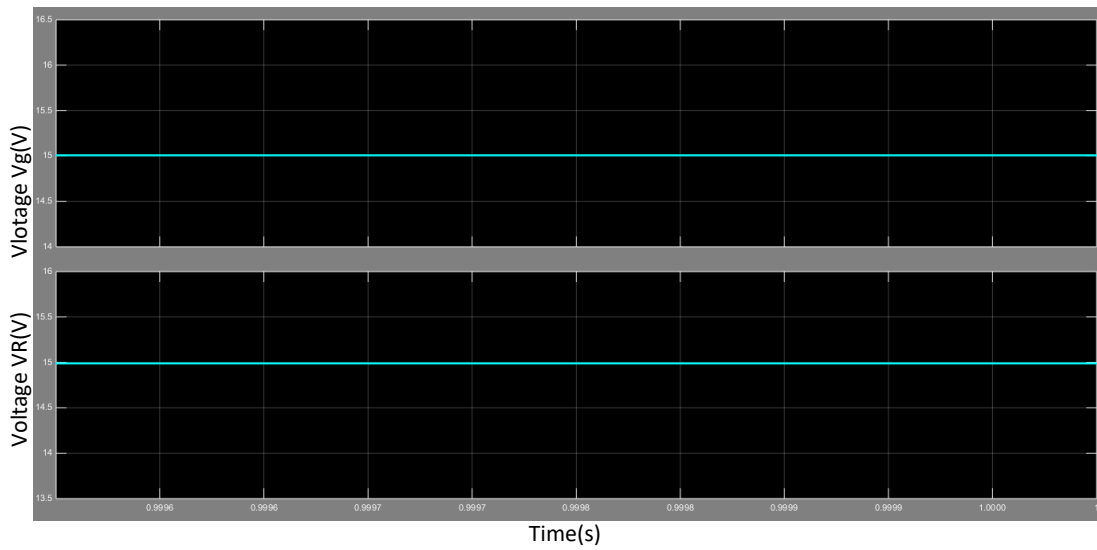


Figure 5: The input voltage and the load voltage of the DC equivalent.

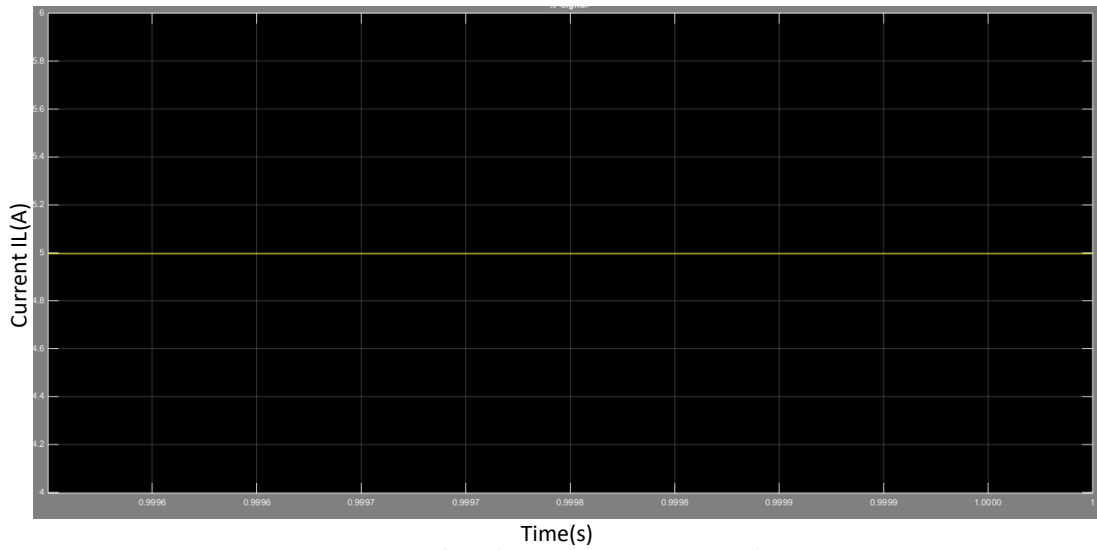


Figure 6: The inductor current in DC equivalent.

## 1.2 a) The first scenario, stepping the voltage up 4V:

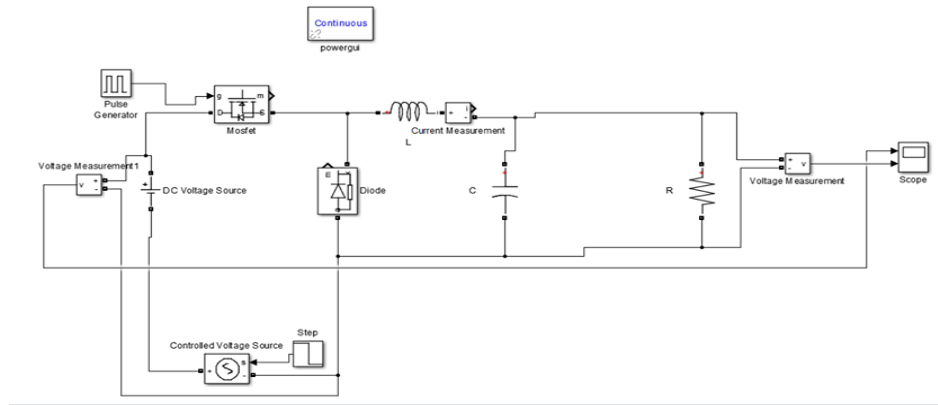


Figure 7: Buck convertor with 4V step up input voltage.

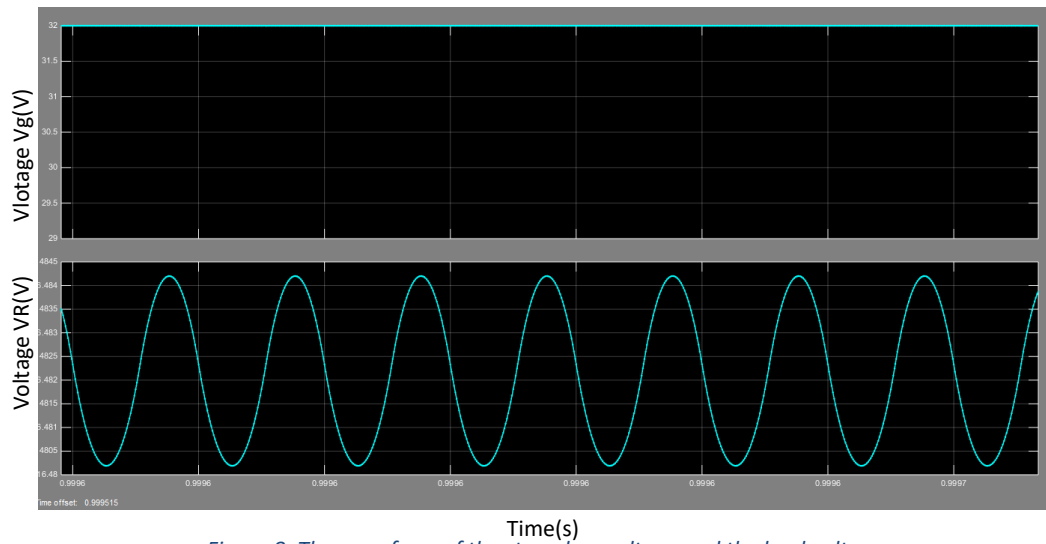


Figure 8: The waveform of the stepped up voltage and the load voltage.

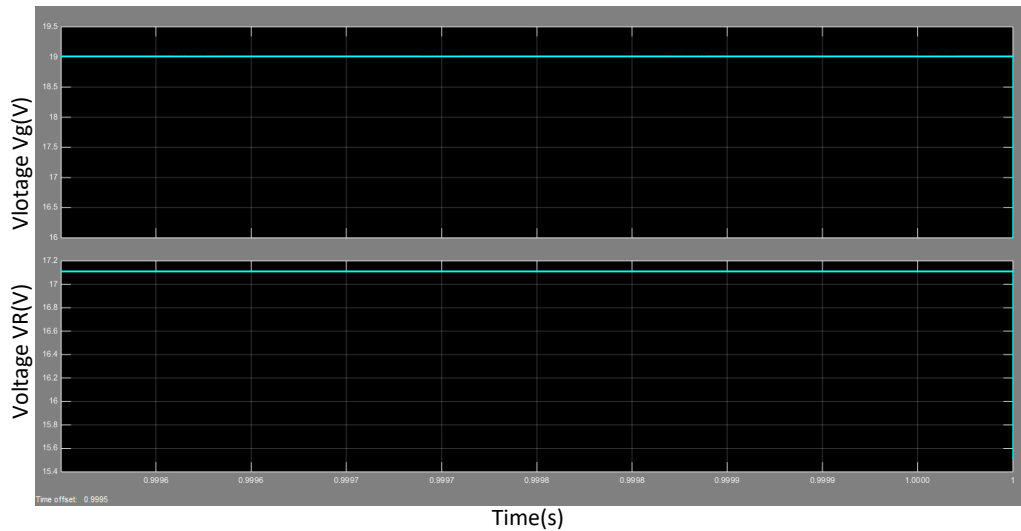


Figure 9: The waveform of the stepped up voltage and the load voltage for the DC equivalent circuit.

b) second scenario, stepping the current down with 0.5A:

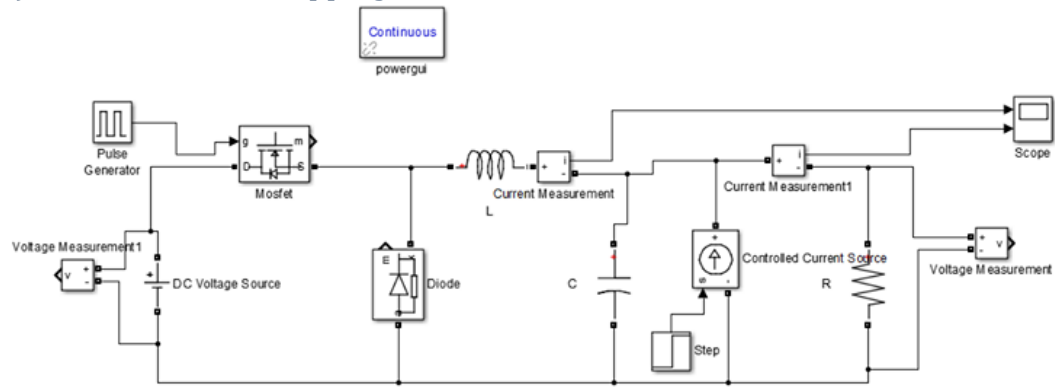


Figure 10: Buck convertor with 0.5A step down load current.

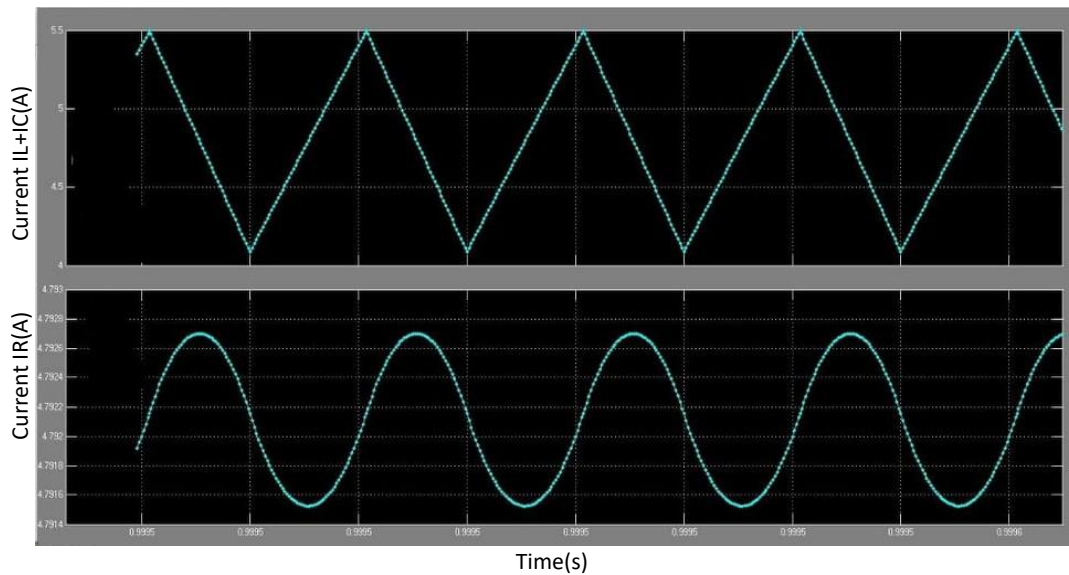


Figure 11: The waveform for the stepped down current and the load current.

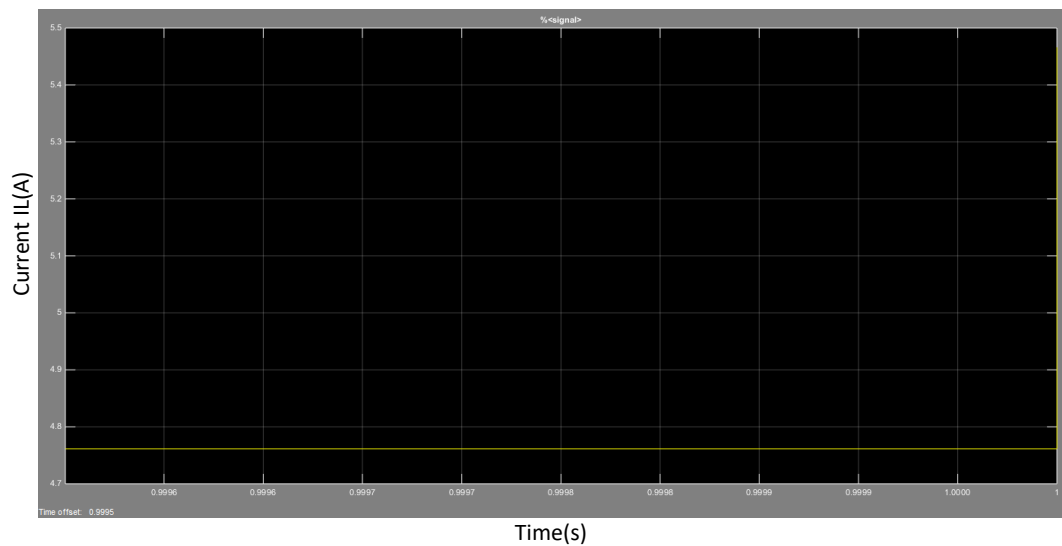


Figure 12: The waveform of the stepped down current in the DC equivalent.

### 1.3 Comparing the ripple current.

- from the waveform in figure (11) is:

$$\Delta i_L = \frac{5.5-4.2}{2} = 0.65A$$

- The small ripple in the inductor current formula:

$$T_s = \frac{1}{f_s}$$

*Equation 1*

$$= \frac{1}{100k} = 1 \times 10^{-5}s$$

$$\Delta i_L = \frac{V_g - V}{2L} D T_s$$

*Equation 2*

$$\Delta i_L = \frac{28-15}{2 \times 50 \times 10^{-6}} \times 0.536 \times 10^{-5} = 0.6968A$$

### 1.4 Notes and Discussion

- Losses were considered in building the steady-state equivalent circuit model
- The generated waveforms of current and voltage agree with the theoretical equations regarding charging and discharging periods and the shape of the waveform.
- The output voltage and current vary significantly between the two circuits i.e. between the buck converter model and its equivalent, non-ideal circuit, which signifies the importance of considering non-ideal components.
- The difference between the current ripple between the theoretical and simulation-derived could be due to imprecise assignment of the current values as observed from the simulation.



## Part II

### 2.1 System analysis Before Compensator

We analyze the system before adding a compensator. Since the desired output voltage is 15 V, and assuming that the output voltage successfully follows the reference voltage, we obtain the feedback gain  $H(s)$  as in equation (4):

$$H = \frac{V_{ref}}{V} = \frac{5.29}{15} = .3526$$

Equation 4

The duty cycle is given by equation (5):

$$D = \frac{V}{V_g} = \frac{15}{28} = .536$$

Equation 5

The control voltage is given by equation (6):

$$V_c = D * V_m = .536 * 4 = 2.144$$

Equation 6

The linearized model of the buck converter, including disturbances is shown in Figure 13:

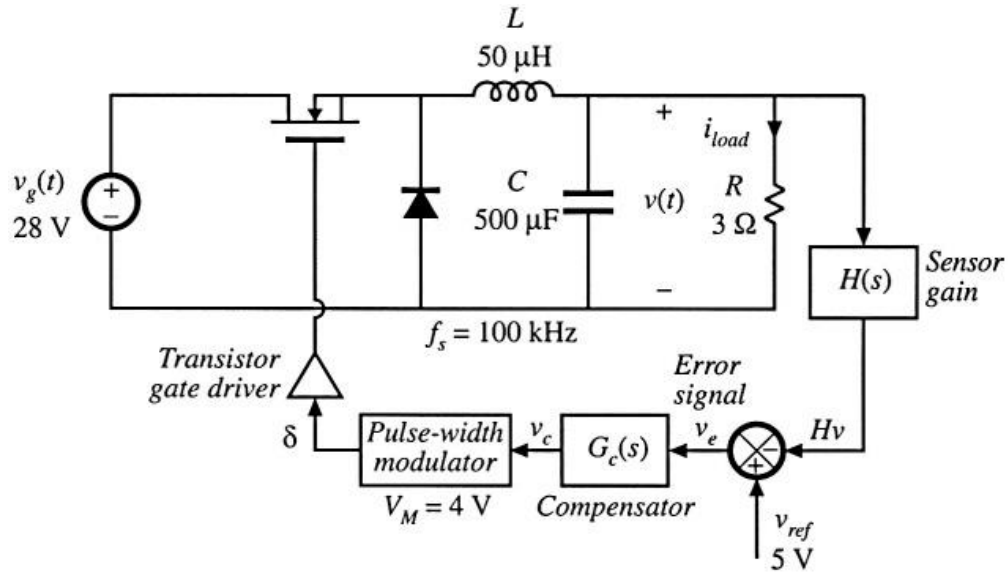


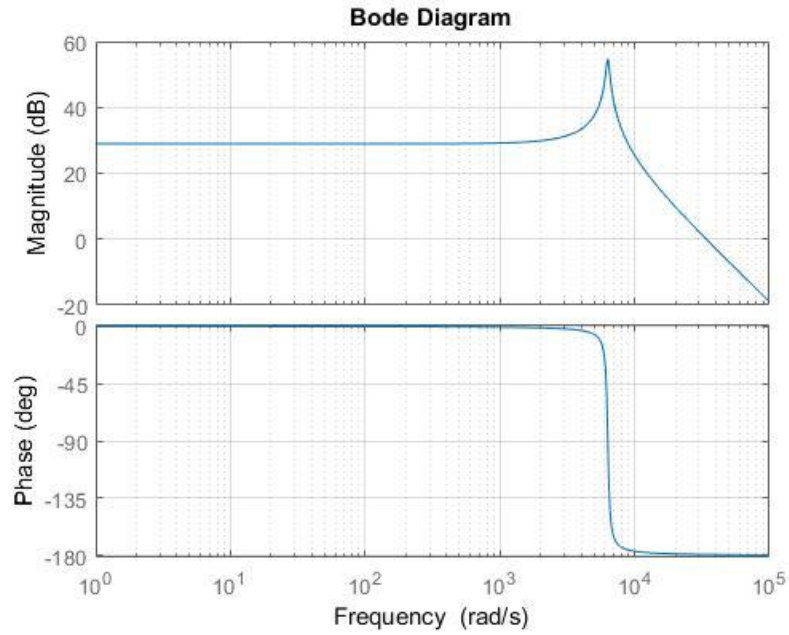
Figure 13. Buck Converter with its control circuit

From Figure 13 we can obtain the open-loop transfer function as in equation (7):

$$G_{vd} = 28 * \frac{1}{1 + \left(\frac{50\mu}{3}\right)s + (50\mu * 500\mu)S^2}$$

*Equation 7. Uncompensated Gvd*

The bode plot of the Uncompensated open-loop transfer function is shown in Figure 14



*Figure 14. Uncompensated Gvd*

The System block Diagram is shown in Figure 15, the loop gain of the system T(s) can be found from the system block diagram and is shown in equation (8).

$$T(s) = 28 * \frac{1}{1 + \left(\frac{50\mu}{3}\right)s + (50\mu * 500\mu)S^2} * \frac{.3526}{4}$$

*Equation 8. Uncompensated Loop Gain*

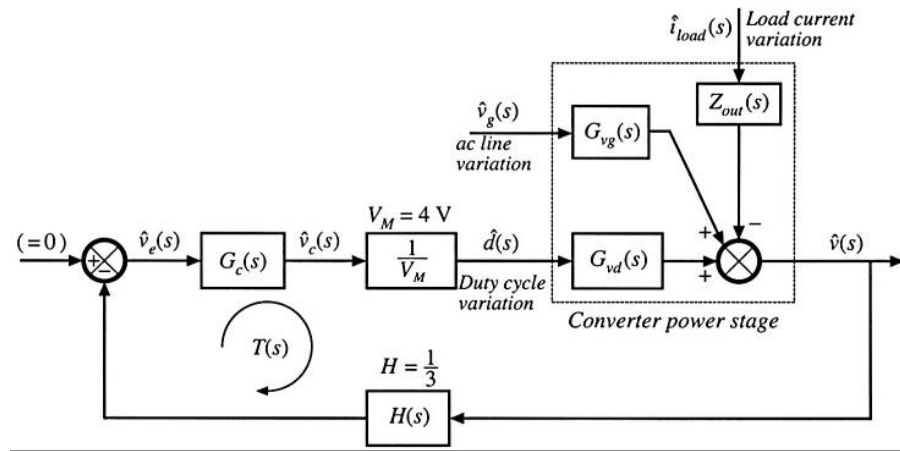


Figure 15. Block Diagram of the Buck Converter Control System

The bode plot of the uncompensated loop gain is shown in Figure 16:

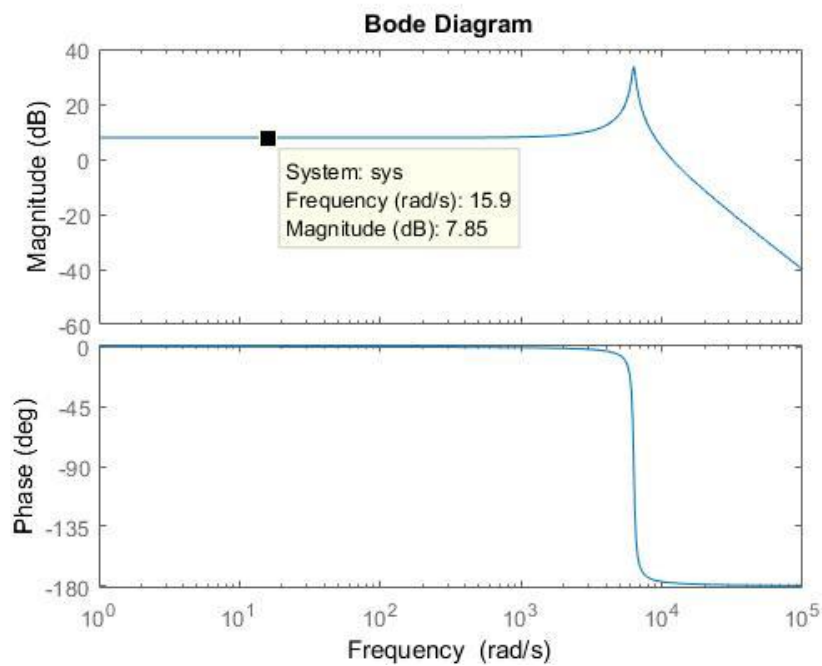


Figure 16. bode plot of the uncompensated loop gain

The uncompensated loop-gain has a crossover frequency of 1.88k Hz and a phase margin of 4.56°.

## 2.2 Lead Compensator Design

A compensator has to be designed such that the cross-over frequency would be 5k Hz and the phase margin to be 59.5°. A lead compensator is used to alter the response, it's designed as shown in equations (9), (10), (11), and (12).

$$f_z = f_c \sqrt{\frac{1 - \sin(\phi)}{1 + \sin(\phi)}}$$

Equation 9. Zero-Frequency

$$f_z = 1.363 \text{ kHz}$$

$$f_p = f_c \sqrt{\frac{1 + \sin(\phi)}{1 - \sin(\phi)}}$$

Equation 10. Pole-Frequency

$$f_p = 18.34 \text{ kHz}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

Equation 11. Corner Frequency

$$f_o = 1 \text{ kHz}$$

$$G_{co} = \left(\frac{f_c}{f_o}\right)^2 \frac{1}{T_o} \sqrt{\frac{f_c}{f_p}}$$

Equation 12. Low-Frequency Gain

$$G_{co} = 0.0058$$

$$G_c(s) = G_{co} \frac{\left(1 + \frac{s}{w_z}\right)}{\left(1 + \frac{s}{w_p}\right)}$$

Equation 3. Lead Compensator TF

$$G_c(s) = \frac{0.0003184 s + 2.727}{8.678 \times 10^{-6} s + 1}$$

The transfer function of the lead converter is calculated from equation (13), a bode plot of the lead compensator is shown in figure 17, and the bode plot of the loop gain after including the lead compensator is shown in figure 18.

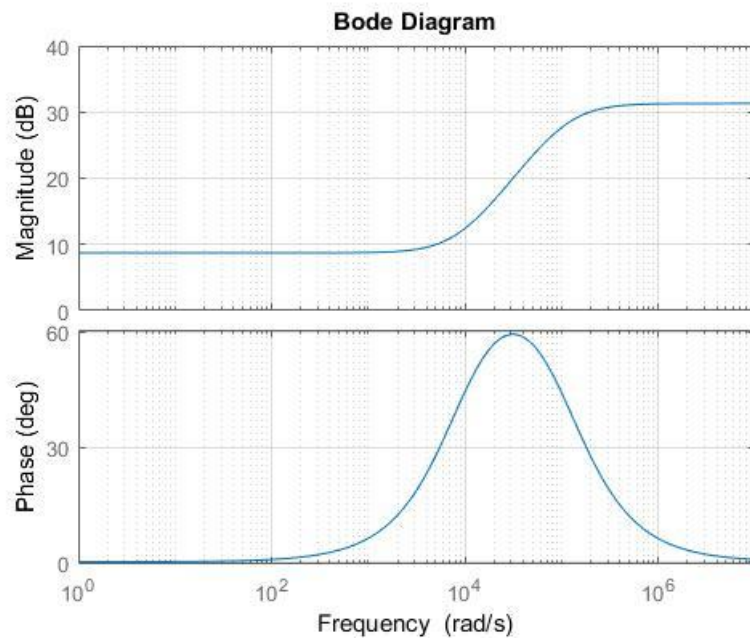


Figure 17. Bode Plot of Lead Compensator

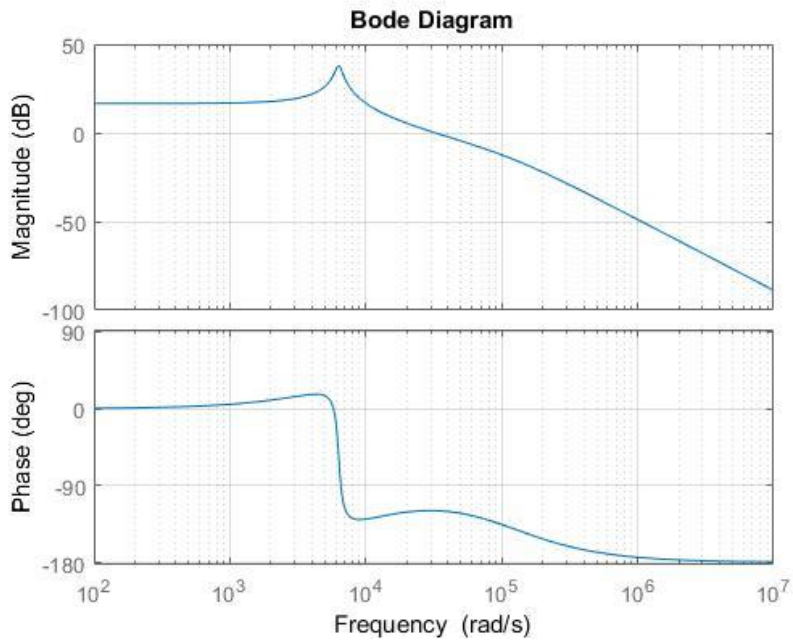


Figure 18. Loop Gain  $T(s)$  with Lead Compensator

- The phase-margin after adding the lead compensator is  $60.7^\circ$
- The crossover frequency after adding the lead compensator is 32.5 krad/s, which equals 5.17 kHz

## 2.3 Lead-Lag Compensator Design

In order to eliminate the effect of disturbances, the loop-gain at low frequencies should be increased, this is done by adding a lag compensator. The transfer function of the lag compensator is shown in equation (14), where  $F_L$  was arbitrarily chosen to be 400 Hz.

$$G_c(s) = \left(1 + \frac{W_L}{s}\right)$$

Equation 4. Lag Compensator TF

Combining both compensators to get the desired values, the lead-lag compensator transfer function is given by:

$$G_c(s) = G_{cm} \frac{\left(1 + \frac{s}{W_z}\right) \left(1 + \frac{W_L}{s}\right)}{\left(1 + \frac{s}{W_p}\right)}$$

Equation 5. Lead-Lag Compensator TF

$$G_c(s) = \frac{0.0003184 s^2 + 3.22 s + 6082}{7.701 \times 10^{-6} s^2 + s}$$

The Lead-Lag compensator bode plot is shown in figure 19.

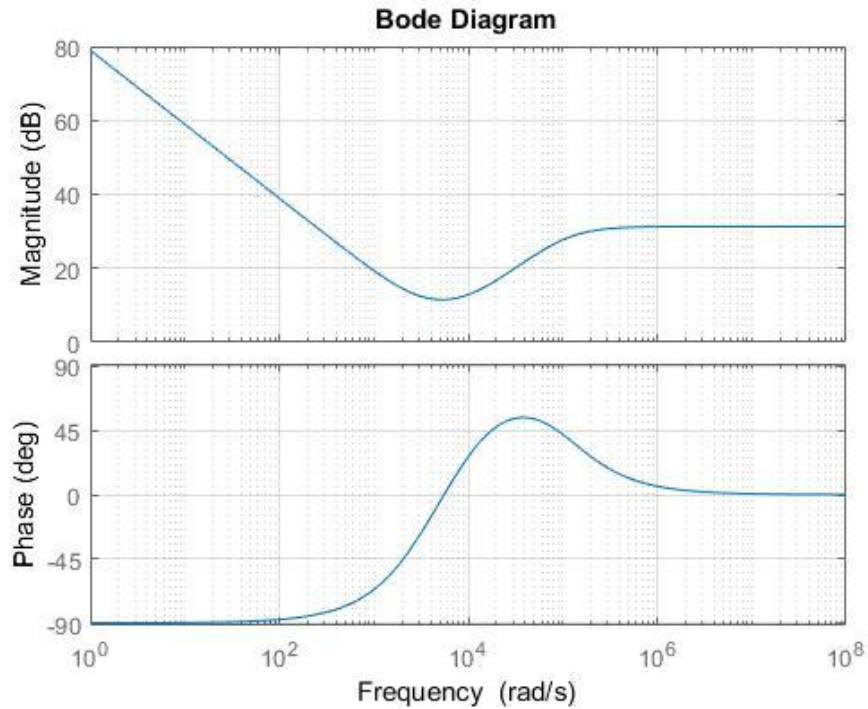


Figure 19. Lead-Lag compensator bode plot

The bode plot of the loop gain  $T(s)$  with lead-lag compensator is shown in figure 20, after slight modifications to  $W_L$  and  $W_Z$  to fulfill the desired requirements.

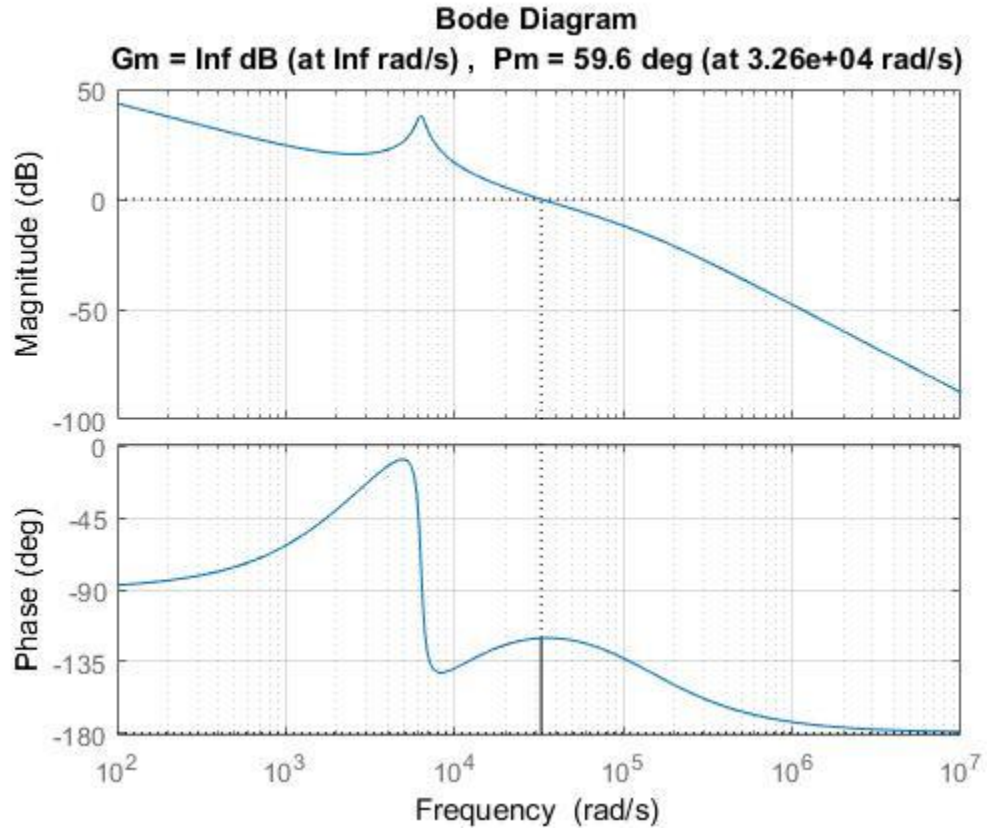


Figure 20. Loop Gain with Lead-Lag Bode Plot

- Achieved Phase-Margin: 59.6
- Achieved Crossover Frequency:  $\frac{32600}{2\pi} = 5.188 \text{ kHz}$

## 2.4 Simulink Models

A block diagram of the buck converter model was built using Simulink as shown in figure 21. Then the output voltage was observed with and without a controller, as shown in figures 22 and 23 respectively.

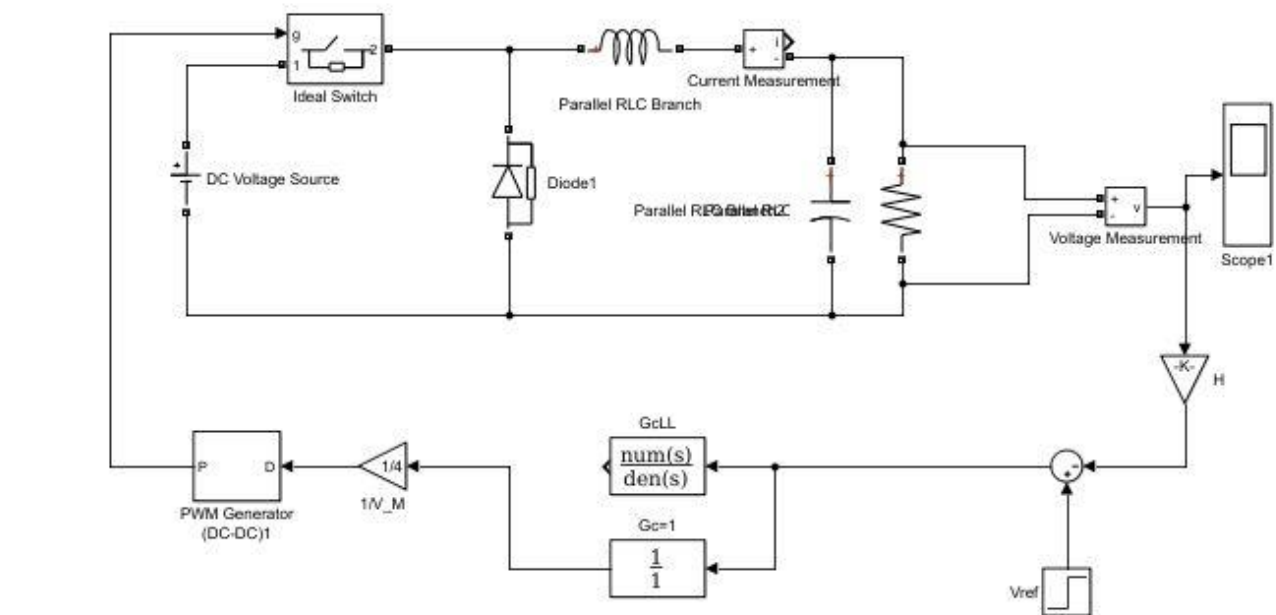


Figure 21. Buck Converter Simulink Model

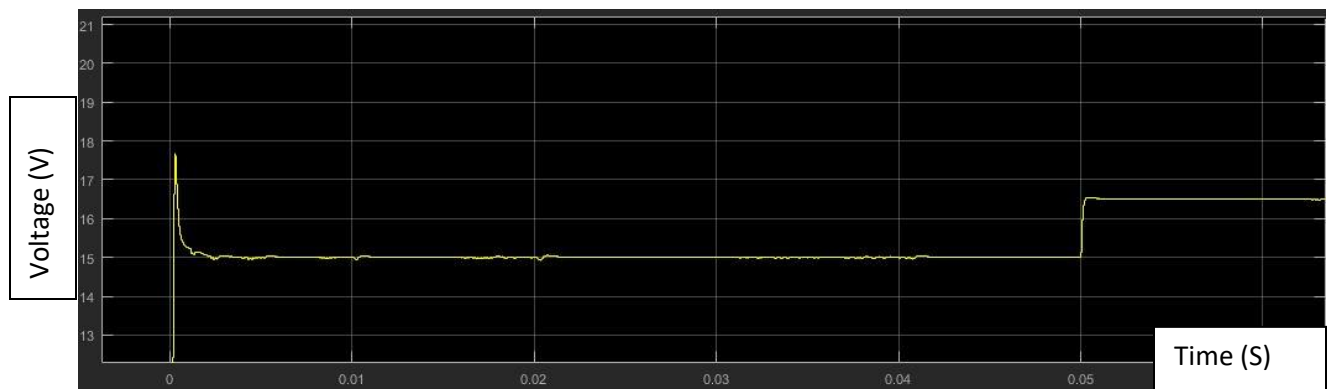


Figure 22. V with Controller

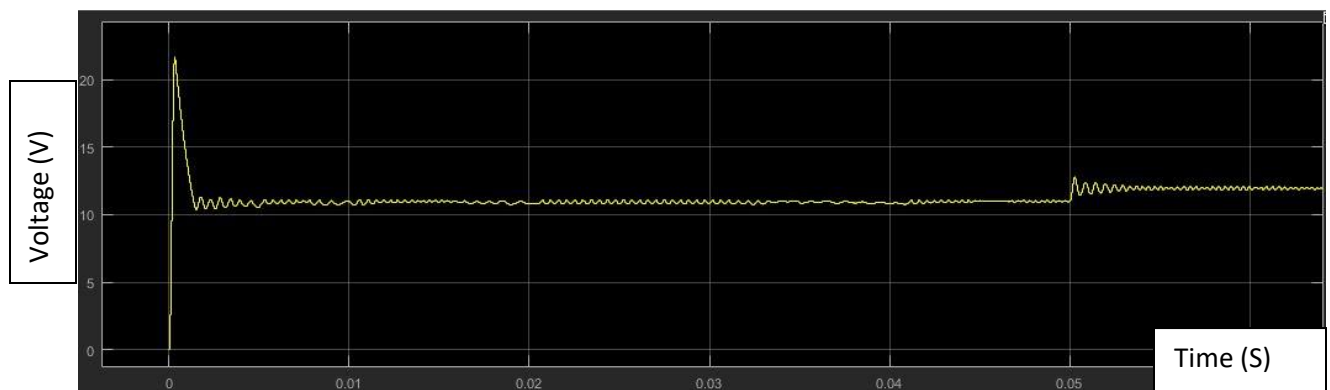


Figure 23. V without Controller



Table (1) illustrates the comparison of the output voltage with and without a controller.

	Without Controller	With Controller
<b>Peak Overshoot</b>	95.45%	16.6%
<b>Steady-State Error</b>	26.7%	0

*Table 1. Output Voltage Response Comparison*

The controller has a notable impact on the performance of the buck converter.

**The End**