Relations in Structuralism

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Structuralism

Philosophical structuralism/Mathematical practice issues

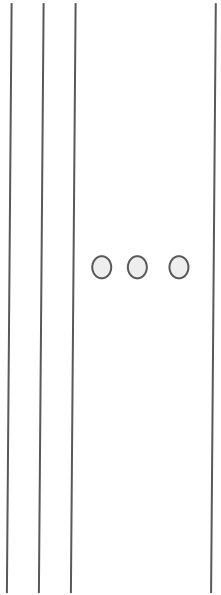
- “...does the view allow for a face-value interpretation of mathematical statements?” (Hellman and Shapiro, 2018)
- Problem of Identity of Indiscernibles
    - Shapiro’s mathematical practice response

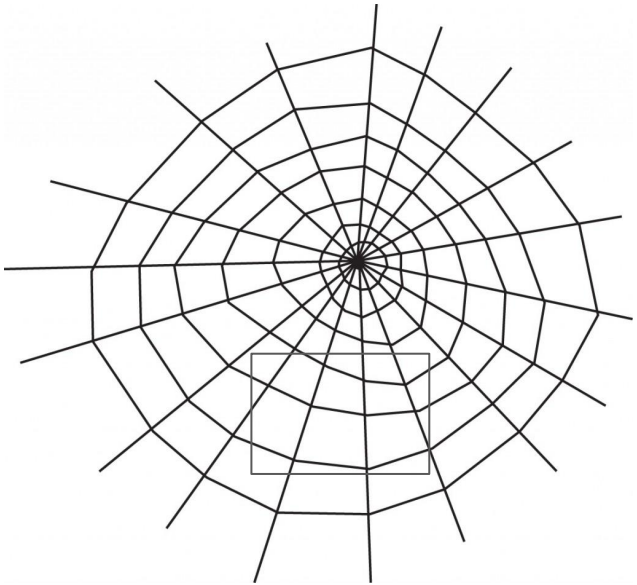
Mathematical Structuralism

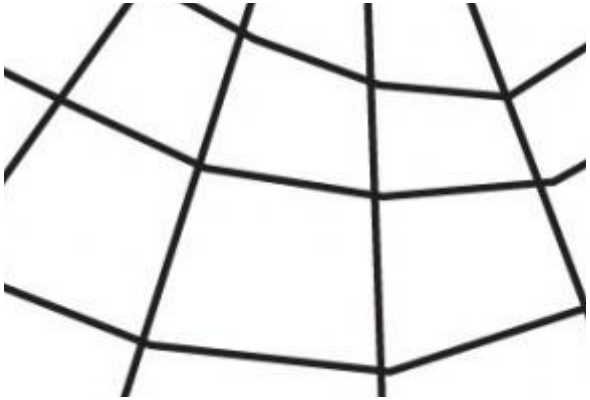
- Set Theory Structuralism
- Category Theory Structuralism
    - Bourbaki Group

Motivation

Take 25 strings







The Metaphysics of Relations

Given relation R and objects a, b
Example a = cat, b = mat, R = ‘is adjacent to’ so that we have ‘cat is adjacent to mat’

Directionalism (or Standard View)

- aRb is a directed relation from a to b, different from the converse bR*a where the direction is in the opposite direction
- Problem: Cannot have symmetry, e.g. R* = R due to different directions.

Positionalism

- Relation R has two empty slots α and β such that $\alpha R \beta$
- Objects a and b need to be slotted into α and β respectively
- Problem: One state of affairs cannot give rise to two formalisms

Donnelly’s Relative Postionalism

- Positions are not construed as objects as Fine does
- They are relative properties
- Problem: Need to take these properties as non reducible primitive

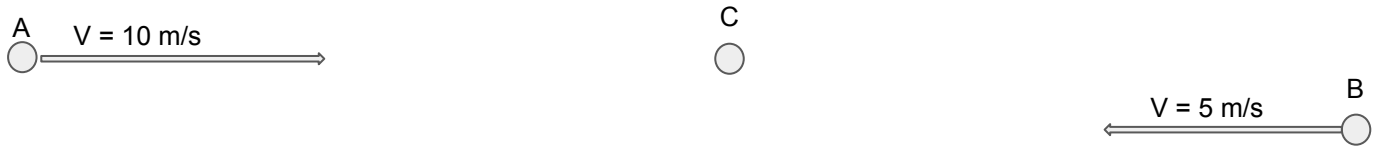
Antipositionalism

- R has no positions
- Symmetry solved
- Two relations relating objects ‘in the same manner’ are identical relations
- “Under these transitions, the concept of relation has become successively simpler and the concept of application successively more complex.” (p.32, Fine 2000)

Frames of Reference: or how I learned to stop worrying and love Relativity

- One state of affairs necessarily gives to rise to two formalisms
- Not an issue as long as they're translatable to each other (covariant)

Example:



- $x = vt + x_0$
- This equation is not contingent on one frame of reference but holds true in all non-inertial frames
- Shows that one state of affairs can give rise to several formalisms

Mechanism of FoR

- Fine's main problem is not problematic in physics formalisations
- Generalised procedure gives us an invariant answer
- Taking 'cat adjacent to mat' example
  - Set a reference (arbitrary)
  - Set a measure (arbitrary)
  - We get invariant results under any FoR
  - Or invariant form: e.g. Object a is 5 cm from reference, Object b is 6 cm from reference ⇒ Object a and b are 1 cm apart
- This is what I contend structuralism to be, a generalisation of relations
  - Objects are defined mathematically, either axiomatically (e.g. number 0 and successor function) or in the process (e.g. function, operator, etc…)
- Directionalism on the observational level due to the nature of experimentation
- Positionalism theoretical

The structural case in mathematics

- The focus becomes relations
  - Dedekind, Hilbert, etc.
- Objects are dealt with as positions, defined axiomatically.
- Have structural properties of positions
  - Korbmacher (2018), Linnebo (2008)
- Proofs have to be traced from axioms

Hilbet-Bourbaki formalism

“The implicit philosophical belief of the working mathematician is today the Hilbert-Bourbaki formalism. Ideally, one works within a closed system: the basic principles are clearly enunciated once for all, including (that is an addition of twentieth century science) the formal rules of logical reasoning clothed in mathematical form.The basic principles include precise definitions of all mathematical objects…” (p.3, Cartier 2000)

The case of physics

- Objects are defined as things that ‘exist’ in the outside world
- Relations are between these objects
- Truth comes from experimentation
- Object abstraction is from those in the physical world
- Mathematical proofs in physics
  - Lack of rigour
    - If it works, it works
  - Based on physical constraints
  - Experimentation is messy
    - “Science often requires methods that ‘eliminate both detail and, in some sense, precision.” (P.414 Urquhart 2008a)
  - “The entities that appear in these calculations, such as infinitesimals or representative variable, are logically anomalous objects.” p.429
    - Dirac Delta

Witten’s topological QFT

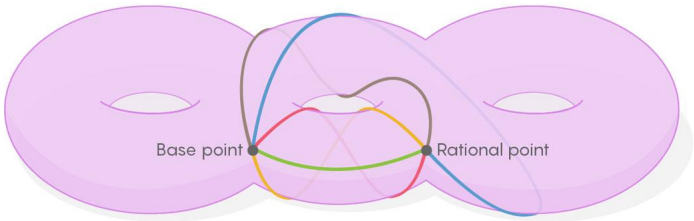
- … understanding these [mathematical] theories as quantum field theories involves constructing theories in which all of the observables are topological invariants… Something that can be computed from a manifold M as a topological space (perhaps with a smooth structure) without a choice of metric is called a "topological invariant" (or a "smooth invariant") by mathematicians. To a physicist, a quantum field theory defined on a manifold M without any a priori choice of a metric on M is said to be generally covariant. Obviously, any quantity computed in a generally covariant quantum field theory will be a topological invariant.

- Were it not for the seeming nuisance that knots must be framed to define the Wilson lines as quantum observables, one would end up proving that the Jones knot invariants were trivial.

The subsequent debate

Example 2 : Kim (Mark's article that he sent)

- To Kim, rational solutions are somehow like the trajectory of light on a torus
- Solution to Diophantine equations
- Look at solution
- Sociological aspect
  - "I was hiding it because for many years I was somewhat embarrassed by the physics connection," he said. "Number theorists are a pretty tough-minded group of people, and influences from physics sometimes make them more skeptical of the mathematics."



THREE-HOLED TORUS: Paths connect the base point with a rational point.

Conclusion

- By focusing on relations, one begins to get a grasp of the mathematical-physical distinction with regards to practice
- Objects are an issue
  - For the mathematician they come from axioms
  - For physicisits, they are abstraction from physical objects
- Relations with(out) objects as generalised procedures
  - For physics: from physical phenomena with no axiomatic trace
  - For mathematicians: needs to be traced back to the axioms (Hilbert-Bourbaki Formalism)

“At a great distance from its empirical source, or after much ‘abstract’ inbreeding, a mathematical subject is in danger of degeneration… Whenever this stage is reached, the only remedy seems to me to be the rejuvenating return to the source: the reinjection of more or less empirical ideas.” (Von Neumann 1947)

Bibliography

Urquhart 2008 a and b

Cartier 2000

Kim 2014

Korbmacher 2018

Linnebo 2008

Dedekind

Hilbert

Fine 2000

Russell 1908

Hellman and Shapiro 2018