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$(p \wedge q) \wedge r \iff p \wedge (q \wedge r)$	$(p \vee q) \vee r \iff p \vee (q \vee r)$	Associative laws
$p \wedge (q \vee r) \iff (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \iff (p \vee q) \wedge (p \vee r)$	Distributive laws
$p \wedge p \iff p$	$p \vee p \iff p$	Idempotent laws
$p \wedge \text{True} \iff p$	$p \vee \text{False} \iff p$	Identity laws
$p \wedge \neg p \iff \text{False}$	$p \vee \neg p \iff \text{True}$	Negation laws
$p \wedge \text{False} \iff \text{False}$	$p \vee \text{True} \iff \text{True}$	Domination laws
$p \wedge (p \vee q) \iff p$	$p \vee (p \wedge q) \iff p$	Absorption laws
$\neg(p \wedge q) \iff (\neg p) \vee (\neg q)$	$\neg(p \vee q) \iff (\neg p) \wedge (\neg q)$	De Morgan's laws
	$\neg(\neg p) \iff p$	Involution law
	$(p \implies q) \iff (\neg p) \vee q$	Conditional equivalence
	$(p \implies q) \iff ((\neg q) \implies \neg p)$	Contrapositive

Table 2.1: Logic laws. These laws (excluding the last 2) are the laws of *Boolean algebra*.

Example 2.14. Prove that $(p \wedge q) \implies p$.

We need to show that the proposition is always **True**. We can use either truth tables (try it!), or logic laws: we use the laws to rewrite the proposition in equivalent forms (using \iff by the rules in Table 2.1), until we reach **True**.

$$\begin{aligned}
 & (p \wedge q) \implies p \\
 \iff & (\neg(q \wedge p)) \vee p && \text{(by conditional equivalence)} \\
 \iff & ((\neg q) \vee (\neg p)) \vee p && \text{(by De Morgan)} \\
 \iff & (\neg q) \vee ((\neg p) \vee p) && \text{(by associativity)} \\
 \iff & (\neg q) \vee (p \vee \neg p) && \text{(by commutativity)} \\
 \iff & (\neg q) \vee \text{True} && \text{(by negation laws)} \\
 \iff & \text{True} && \text{(by domination laws)}
 \end{aligned}$$