بسم الله الرحمن الرحيم

Control System Design: Assignment#5
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Bode Plot:

Consider the following 1st order transfer function:

$$G(s) = \frac{k}{1 + \tau.s}$$

Where we have:

$$k = 1,\ \omega_c = 2\pi * 100\ {}^{rad}/_{s},$$

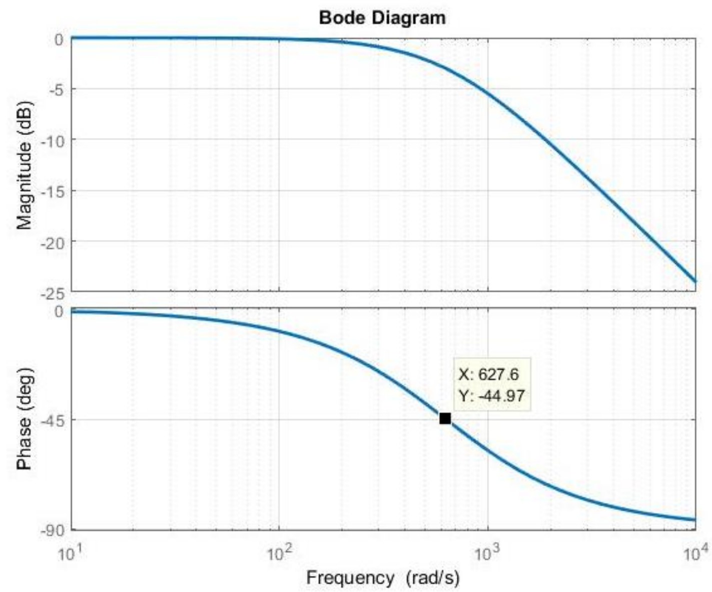
We can know that the time constant is the reverse of the corner frequency:

$$\tau = \frac{1}{\omega_c} = \frac{1}{2\pi.100} \approx 1.59e - 3\ \text{sec}$$

From the Bode plot we find that the maximum value is when frequency goes to zero, at which the amplitude is 0dB and phase is 0° which means no the response is the same as the input in amplitude and it is in phase.

The Bode plot is obtained from:

```
wc = 2*pi*100; % Corner frequency
tao = wc^-1; % Time constant
s = tf('s');
G = 1/(1+tao*s);
bode(G)
```



Now, consider a second order system:

$$G(s) = \frac{1}{s^2+2\zeta\omega_n s+\omega_n^2},$$

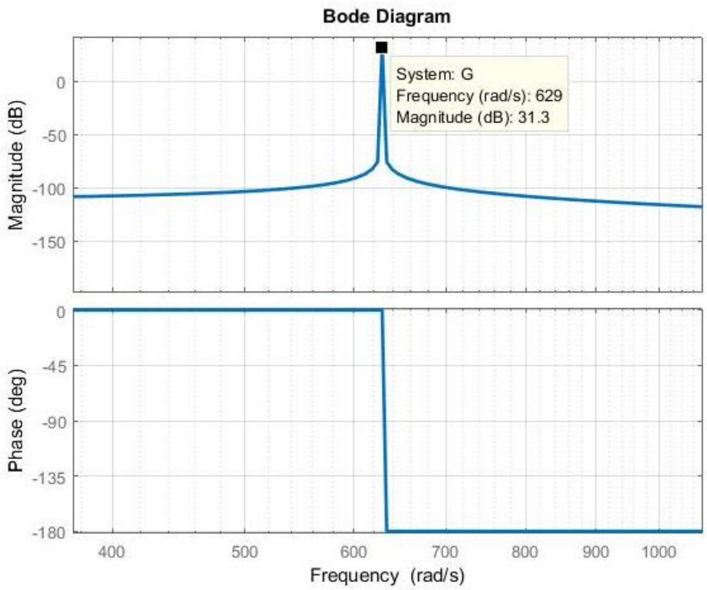
Assume:

$$\omega_n = 2\pi * 100\ {}^{rad}/_{s},$$

$$\zeta = 0.707.$$

To obtain the Bode plot:

```
wn = 2*pi*100; % Natural frequency
zeta = 0.707 % Damping ratio
s = tf('s');
G = 1/(s^2 + 2*wn*zeta + wn^2);
bode(G)
```



The value of time constant is found by:

$\tau = \frac{1}{\sigma}$,

$$\sigma = \zeta * \omega_n = 0.707 * 2\pi * 100$$

$$= 444.2212$$

$$\therefore \tau = 0.0023\ sec.$$

We found that the max $|G(j\omega)|$ is 31.3 dB = 36.8 absolute, at which the response to the input will be amplified to 36.8 time and the phase will lag by 180°. This frequency known as the resonance frequency.

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