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Control System Design: Assignment#3, **MIMO Systems**
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We find the dynamics of the system using conservation of mass principle, we have two dynamical equation for our system:

$$A_1.\frac{dh_1}{dt} = b_1.u - \frac{h_1 - h_2}{R_1},$$

$$A_2.\frac{dh_2}{dt} = b_2.u + \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_1}.$$

Now we can define our states:

let, $x_1 = h_1$ & $x_1 = h_1$,

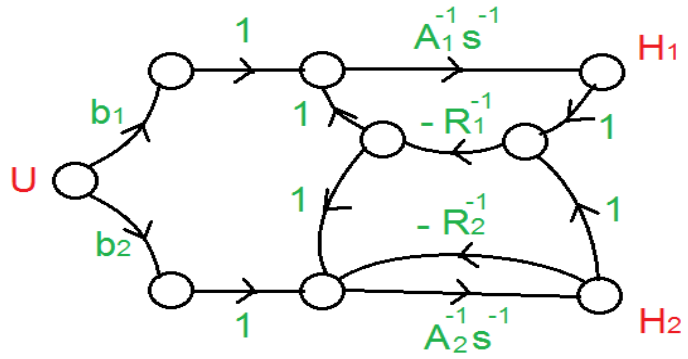
Our state space model:

$\dot{\bar{x}} = A\bar{x} + B\mathrm{u}$, where:

$$A = \begin{bmatrix} \frac{-1}{R_1.A_1} & \frac{1}{R_1.A_1} \\ \frac{1}{R_1.A_2} & \frac{-1}{A_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \end{bmatrix},$$

and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$.

The signal flow graph:

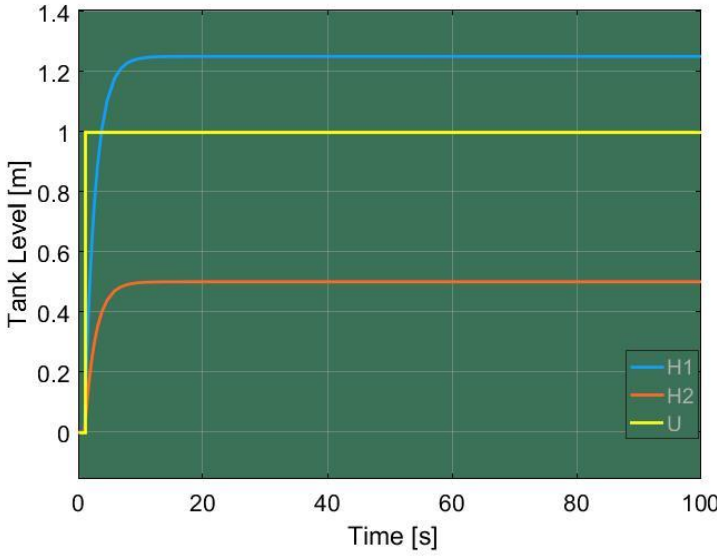


Also we can model our system by either using SIMULINK or LABVIEW, and both of them have been used. (see the attached files)

In MATLAB we defined the following values:

```
% System parameters:
u = 1; %Input [m^3/s]
b1 = 0.75; %Gain#1 []
b2 = 0.25; %Gain#2 []
R1 = 1; %Flow resistance#1 [m/m^3/s]
R2 = 0.5; %Flow resistance#2 [m/m^3/s]
A1 = 1; %Tank#1 capacitance [m^2]
A2 = 1; %Tank#2 capacitance [m^2]
```

Then we obtained the following results, for a unit step input:



We note that the levels of the two tanks are interactive with each other, our input flow is entering both tanks by some factors $b_1 \& b_2$, and both levels are affecting each other through the resistance R_1.

Now we analyze the system to find its eigenvalues and eigenvectors using the following script:

```
% Eig. Val & Vec:
A = [-1/(R1*A1) 1/(R1*A1);
1/(R1*A2) -(1/A2)*(1/R1 +
1/R2)];
B = [b1/A1; b2/A2];
C = eye(2); D = [0; 0];
[T,lam] = eig(A);
```

and we found the following results:

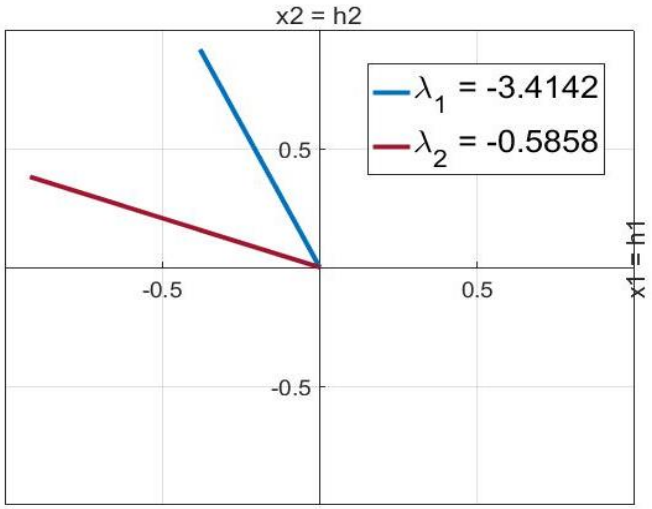
Eigenvalues:

$\lambda_{1,2} = -3.4142, -0.5858,$

Eigenvectors:

$$[\bar{T}_1 \quad \bar{T}_2] = \begin{bmatrix} -0.3827 & -0.9239 \\ 0.9239 & -0.3827 \end{bmatrix}$$

Shown graphically:



We note that the first (fastest) mode is dominant in $h_2$level, and the second (slowest) in $h_1$level.

To check for controllability and observability:

```
if rank(ctrb(A,B))==2
&rank(obsv(A,C))==2
    disp('The system is
controllable and observable!')
end
```

Thus, our system is controllable and observable! ■