

Vibration Analysis and Control of Building Subjected to Wind and Seismic Waves.

ABSTRACT

In this work, a simple two models of a building is considered. In both models the response due to wind waves and seismic waves is analyzed and reduced to a comfortable level using different techniques. Then a generalized response is been given. The results was shown and comments been given on it.

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1. Introduction

Vibration in the buildings is one of the most import consideration, specially in the active seismic regions, and for tall buildings under wind and tornado forces. The analysis here is to show and give a simplified demonstrations for this phenomenon, showing graphically the movement of the building and giving some explanations.

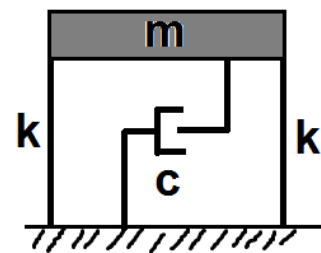
The analysis is taken by the dynamic models of the building, solving their equation using different tools provided from PYTHON. The used code files are attached with this document.

2. Analysis Methodology

In order to perform dynamical analysis, we will study two models separately, first begin with a single degree of freedom model to have a simple idea about the building vibration and control, and then a two degree of freedom model is used and analysed.

2.1. One Story Building, SDOF

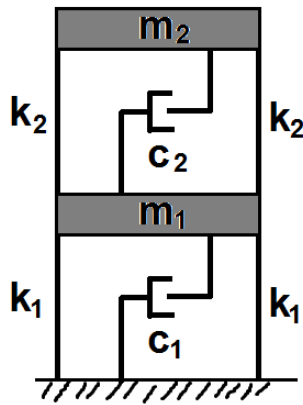
It is the simplest model to analyse. It is a SDOF model consists of a mass representing the whole story. The column can be represented as a cantilever spring with negligible mass compared to the floor mass, also there can be some internal damping due to friction and viscoelastic materials of the wall. (See the Figure)



2.2. Two Stories Building, 2DOF

We will consider another story similar to the previous one on the original model represent a 2DOF system of two stories. Consists of a two mass attached to each other by columns. Consider that the floor mass, columns stiffness and internal

damping as the previous configuration.
(See the Figure)



3. System Parameters

Our system is consists of the main three mechanical dynamical system components: Mass, Spring and Damper.

3.1. Mass

In our mass calculation we ignored the block of columns and took into account the roof mass which is the most influential in the calculation by choosing a reinforced concrete.

3.2. Spring

In our case we will take the columns as springs with the assumption that they will work as a cantilever beam to store the motion in horizontal movement.

3.3. Damper

There are a lot of factors that must be calculated in the process of damping buildings and cannot be theoretically calculated, but must be calculated in practice taking these in consideration. Therefore will use the available standard data:

Currently Used Damping Values (RC Buildings)			
Country	Actions/Stress Levels	Structures	Damping ratios ζ_i (%)
Australia (AS1170.2)	Serviceability Ultimate & Permissible	RC or Prestressed C RC or Prestressed C	0.5 - 1.0 5
Austria	(ÖNORM B4014)		
China	(GB50191-93)	RC Structures RC (TV) Towers Prestressed RC Tower	5 5 3
France	Earthquake	Standard Reinforced Standard Reinforced	1.6 0.65 3 - 4 2
Germany	Wind	(DIN 1055)	
Italy	Wind Earthquake	(EUROCODE 1)	
Japan	Habitability Earthquake		5 1 3
Singapore			2
Sweden	(Swedish Code of Practice)		1.4
United Kingdom	Wind	(ESDU)	
USA	(US Atomic Energy Commission)		

A small amount of damping is been choosen, which is given in term of damping ratio of $\zeta = 5\%$.

Hence, the following approximated data been taken :

- Concrete density, $\rho = 1000 \text{ kg/m}^3$
- Young modulus of concrete,
 $E_c = 0.17e11 \text{ Pa}$
- Young modulus of steel, $E_s = 2.7e11 \text{ Pa}$
- Column modulus,
 $E = 0.2E_s + 0.8E_c = 6.76e10 \text{ Pa}$
- Floor length & width, $L = W = 5 \text{ m}$
- Story height, $H = 5 \text{ m}$
- Floor thickness, $h = 0.5 \text{ m}$
- Floor Volume, $V = L.W.h = 12.5 \text{ m}^3$
- Column width, $w = 0.65 \text{ m}$
- Column inertia, $I = \frac{w^4}{12} = 0.014876 \text{ m}^4$

from above data the dynamical properties of the building is calculate:

- Floor mass,

$$m = m_1 = m_2 = \rho \cdot V = 12500 \text{ kg}$$

- Stiffness of each column

$$k_o = \frac{3EI}{H^3} = 1.6224 \text{ GN/m (Cantilever)}$$

- Equivalent stiffness of each story,

$$k = 4 * k_o = 6.4896 \text{ GN/m (Parallel)}$$

- Critical damping coefficient,

$$c_c = 2 * \sqrt{m \cdot k} \cong 18e6$$

- Story Equivalent damping coefficient,

$$c = 2\zeta \cdot c_c = 1.8 \text{ MN.s/m}$$

4. Input Data

Two different in nature input been will be studied that affect the building. The first type input is the force of the wind flowing around the building, while the second type is the displacement of the earth bellow the building caused by the seismic waves.

4.1. Wind Load

Wind load is a special kind of load on buildings as it is actually capable of creating many types of forces based on the height and the shape of the building in which depending on them different loads will have different degrees of impact on the structure.

The types of the loads can be classified as:

1. Static Loads:

They are responsible for generating elastic bending and twisting on the building.

2. Dynamic Loads:

This type generates fluctuating forces all over the structure, its most common form are oscillations, they mostly affect the taller more slender structures.

For high aspect-ratio structures, the analysis and study of wind loads and their effects is vital for the stability and long term safety of the structure, because these loads can induce oscillation forces and resonance with certain frequencies that if they coincide with the Natural frequency of the structure could lead to catastrophic amounts of damage and in worst case scenarios Total Failure of the structure.

In our analysis we will take an assumption that the wind is blowing in a very fast velocity so that our building can resist any severe condition

The following data of wind parameters been considered:

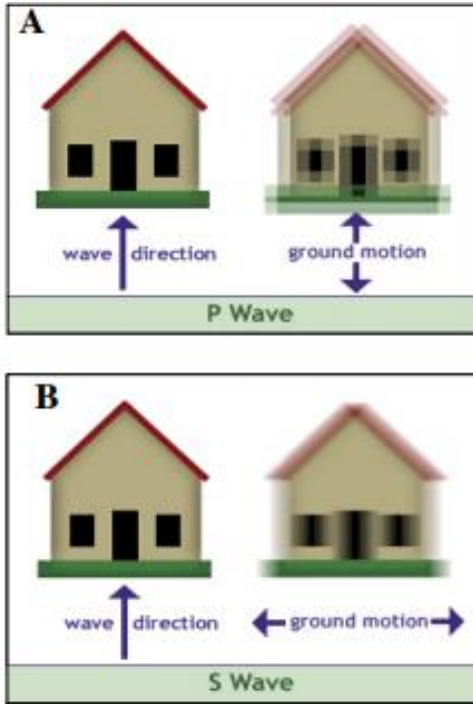
- Air density, $\rho_a = 1.2 \text{ kg/m}^3$
- Air velocity, $v_a = 140 \text{ m/s}$
- Area affected, $A_w = LH = 25 \text{ m}^2$
- Air Force, $F_{wo} = \frac{1}{2} \rho_a v_a^2 A_w$

To simplify our analysis we will choose that the wind is blowing by a sinusoid wave with different range of frequencies:

$$F_w(t) = F_{wo} \cos(\omega_w t)$$

4.2. Seismic Earthquake Waves

Buildings are move due to shake motion either on vertical direction (P-wave or compressional wave (p-waves travel fastest and are generally felt first, they usually cause very little damage)) or on horizontal direction (S-wave or secondary or shear wave (cause the most affection and it's used in most analysis))



In this paper we will take in consideration the S-wave.

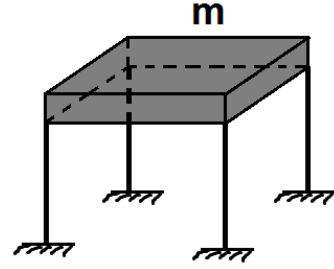
Also to simplify our analysis we will choose that the seismic waves are of a sinusoid nature with different range of frequencies:

$$y(t) = Y_o \sin(\omega_{eq} t)$$

Where the wave amplitude is $Y_o = 5 \text{ m}$.

5. Analysis of One Story Building

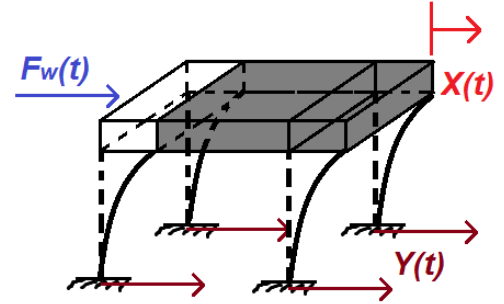
The one story building considered - as mentioned earlier- that it represents a SDOF system. (See the Figure)



The equations of motion for the model is given bellow:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t)$$

This model is subjected to wind forces, and seismic waves, and its responses due to them is studied separately since we are working with linear systems.



5.1. Wind Input Response

The dynamics of the building due to wind input is as follow:

$$m\ddot{x}_w + c\dot{x}_w + kx_w = F_{wo} \cos(\omega_w t)$$

where: the response due to wind is $x_w(t)$.

The steady state solution is as follows:

$$x_w(t) = X_w \cos(\omega_w t - \phi_w)$$

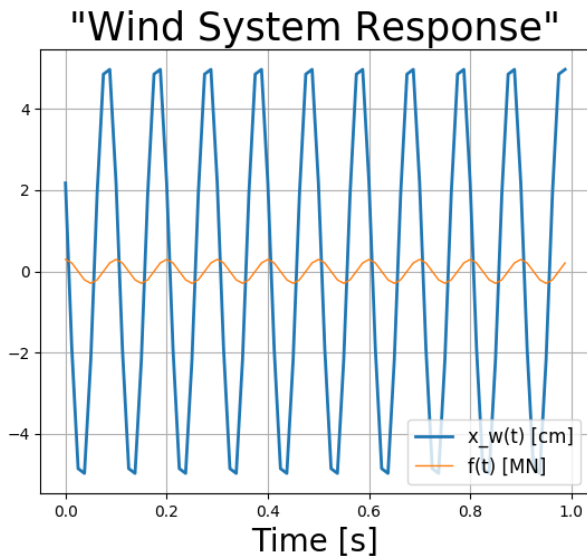
where:

- Amplitude, $X_w = \frac{F_{wo}}{\sqrt{(k - m\omega_w^2)^2 + (c\omega_w)^2}}$
- Phase angle, $\phi_w = \tan^{-1}\left(\frac{c\omega_w}{k - m\omega_w^2}\right)$

Take the wind frequency to be,

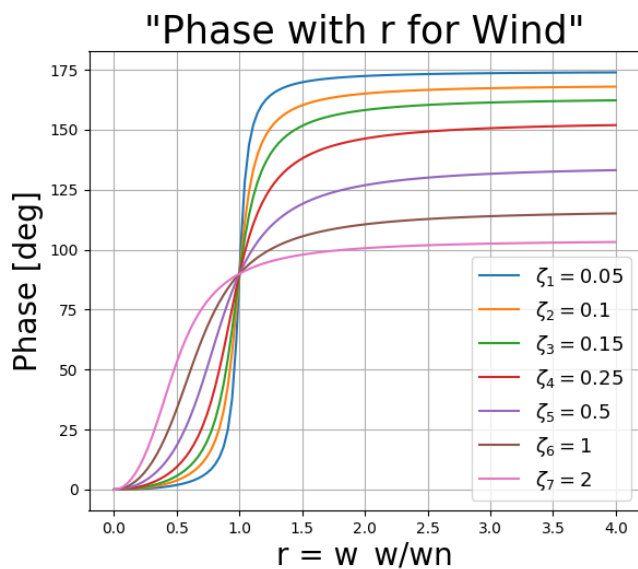
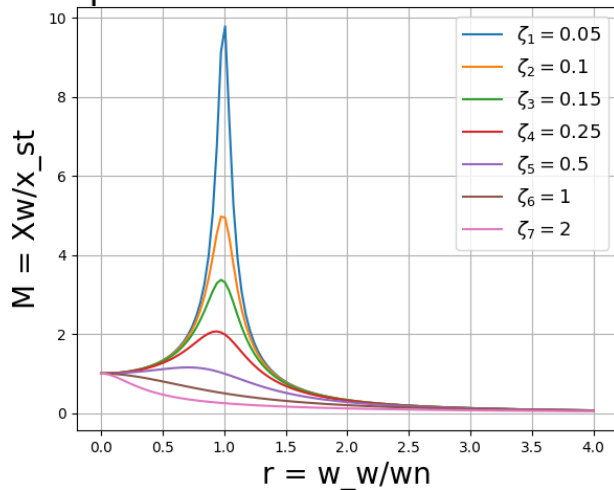
$$f_w = 20 \text{ Hz}$$

So that the following results been obtained;



We show the change in amplification factor as a function of frequency ratio:

"Amplification factor with r for Wind"



The dynamics of the building due to wind input is as follow:

$$m\ddot{x}_{eq} + c\dot{x}_{eq} + kx_{eq} = c\dot{y} + ky$$

The steady state solution is as follows:

$$x_{eq}(t) = X_{eq}\cos(\omega_{eq}t - \phi_{eq})$$

where:

- Amplitude, $X_{eq} = \frac{Y_o\sqrt{K^2 + (c\omega_{eq})^2}}{\sqrt{(k - m\omega_{eq}^2)^2 + (c\omega_{eq})^2}}$

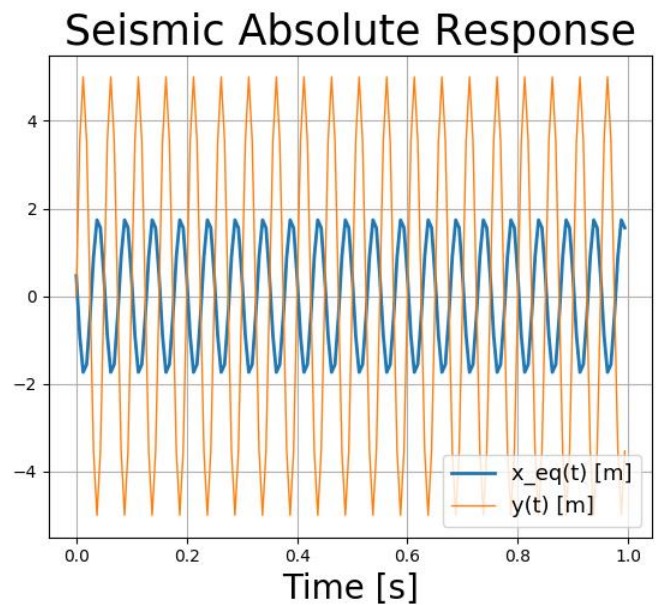
- Phase angle,

$$\phi_{eq} = \tan^{-1}\left(\frac{mc\omega_{eq}^3}{k(k - m\omega_w^2) + (c\omega_{eq})^2}\right)$$

Take the wind frequency to be,

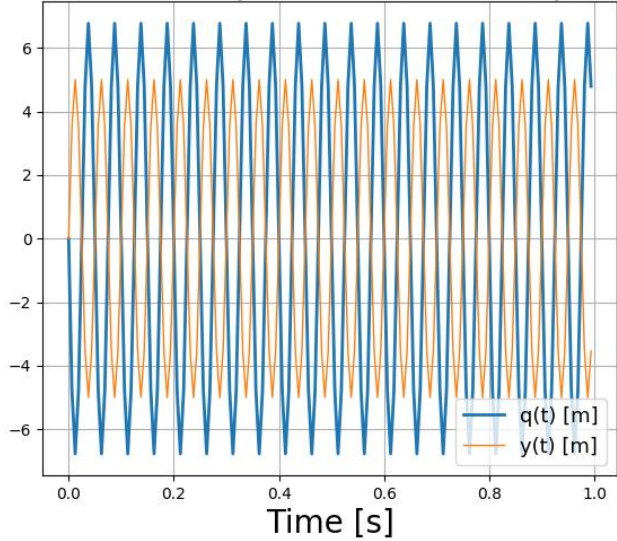
$$f_{eq} = 20 \text{ Hz}$$

So that the following results been obtained for the absolute response and for the relative response:



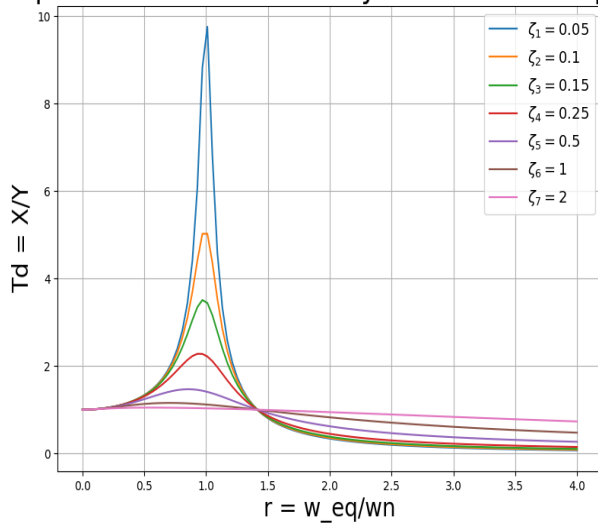
5.2. Seismic Input Response

Relative response for Earthquake

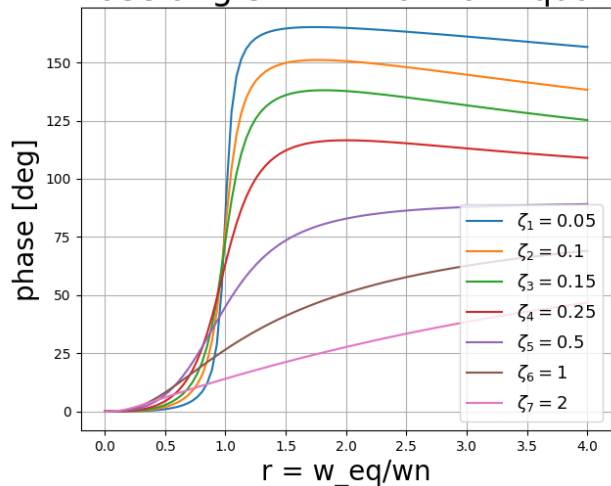


We show the change in displacement transmissibility T_d and its phase angle,

Displacement transmissibility with r for Earthquake

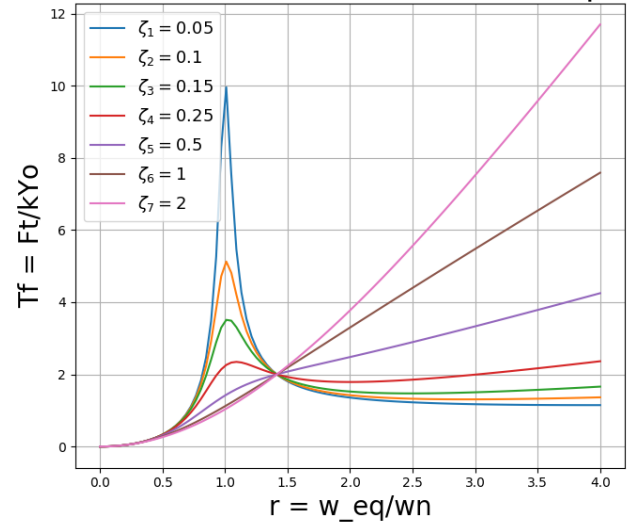


Phase angle with r for Earthquake



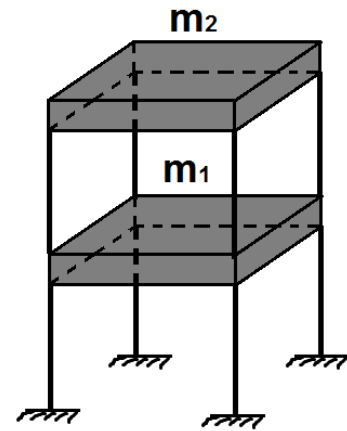
and force transmitted T_f as a function of frequency ratio:

Force transmitted with r for Earthquake



6. Analysis of Two Stories Building

The two stories building considered that it represents a 2DOF system.



The equations of motion for the model is given bellow:

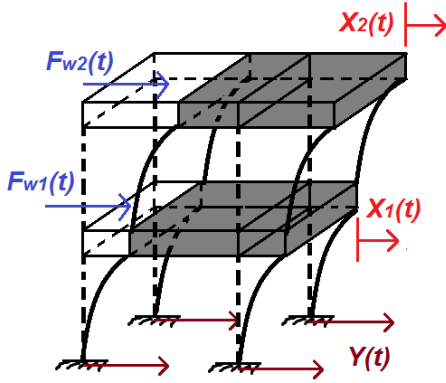
$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix}$$

Since that the parameters are equivalent:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2c & -c \\ -c & c \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}$$

$$+ \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix}$$

This model is subjected to wind forces, and to seismic waves, and its responses due to them is also studied separately.



6.1. Wind Input Response

The dynamics due to wind input is as follow:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_{w1} \\ \ddot{x}_{w2} \end{Bmatrix} + \begin{bmatrix} 2c & -c \\ -c & c \end{bmatrix} \begin{Bmatrix} \dot{x}_{w1} \\ \dot{x}_{w2} \end{Bmatrix} + \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} x_{w1} \\ x_{w2} \end{Bmatrix} = \begin{Bmatrix} F_{w1o} \cos(\omega_w t) \\ F_{w1o} \cos(\omega_w t) \end{Bmatrix}$$

The solution is obtained using impedance matrix:

$$[Z(i\omega_w)] \vec{X}_w = \vec{F}_{w0}$$

Where:

$$\vec{X}_w = \begin{Bmatrix} X_{w1} \\ X_{w2} \end{Bmatrix}, \quad \vec{F}_{w0} = \begin{Bmatrix} F_{w1o} \\ F_{w1o} \end{Bmatrix}$$

$$[Z(i\omega_w)] = \begin{bmatrix} Z_{11}(i\omega_w) & Z_{12}(i\omega_w) \\ Z_{21}(i\omega_w) & Z_{22}(i\omega_w) \end{bmatrix}$$

$$Z_{11}(i\omega_w) = (-\omega_w^2 m + i2c\omega_w + 2k)$$

$$Z_{12}(i\omega_w) = Z_{21}(i\omega_w) = -(ic\omega_w + k)$$

$$Z_{22}(i\omega_w) = (-\omega_w^2 m + ic\omega_w + k)$$

And then using Cramer's law to obtain the solution:

$$\vec{X}_w = [Z(i\omega_w)]^{-1} \vec{F}_{w0}$$

Hence:

$$X_{w1} = \frac{1}{|Z(i\omega_w)|} \begin{vmatrix} F_{w1o} & Z_{12}(i\omega_w) \\ F_{w2o} & Z_{22}(i\omega_w) \end{vmatrix}$$

$$X_{w2} = \frac{1}{|Z(i\omega_w)|} \begin{vmatrix} Z_{11}(i\omega_w) & F_{w1o} \\ Z_{21}(i\omega_w) & F_{w2o} \end{vmatrix}$$

Assume steady state harmonic solution:

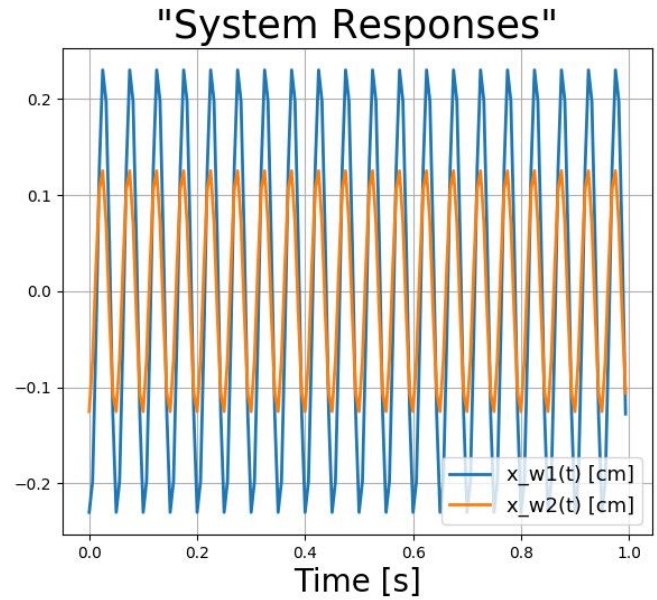
$$x_{wj}(t) = X_{wj}(i\omega_w) \cos(\omega_w t - \phi_{wj})$$

where:

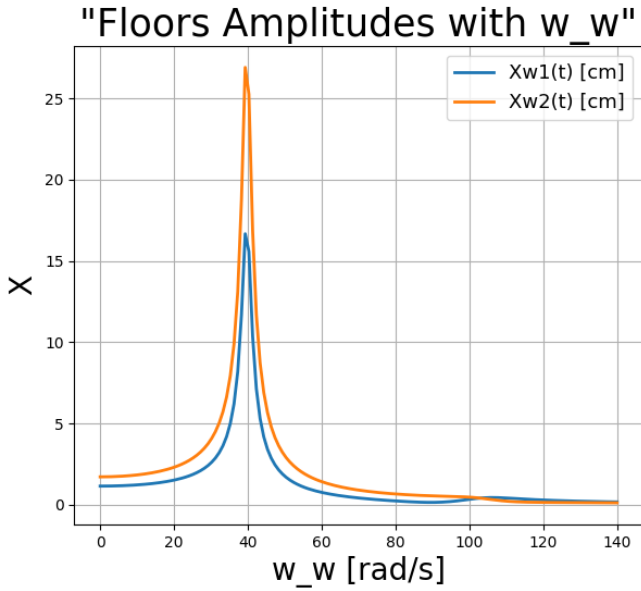
- Amplitude, $X_{wj} = |X_{wj}(i\omega_w)|$

- Phase angle, $\phi_{wj} = \angle X_{wj}(i\omega_w)$

The following results been obtained;



The critical frequencies at which the amplitude is high, are shown bellow:



6.2. Seismic Input Response

The dynamics due to wind input is as follow:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_{eq1} \\ \ddot{x}_{eq2} \end{Bmatrix} + \begin{bmatrix} 2c & -c \\ -c & c \end{bmatrix} \begin{Bmatrix} \dot{x}_{eq1} \\ \dot{x}_{eq2} \end{Bmatrix} + \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} x_{eq1} \\ x_{eq2} \end{Bmatrix} = \begin{Bmatrix} c\dot{y} + ky \\ 0 \end{Bmatrix}$$

The solution is obtained using impedance matrix:

$$[Z(i\omega_{eq})]\vec{X}_{eq} = \vec{F}_{w0}$$

Where:

$$\vec{X}_{eq} = \begin{Bmatrix} X_{eq1} \\ X_{eq2} \end{Bmatrix}, \quad \vec{F}_{w0} = \begin{Bmatrix} F_{eq10} \\ F_{eq10} \end{Bmatrix}$$

$$[Z(i\omega_{eq})] = \begin{bmatrix} Z_{11}(i\omega_{eq}) & Z_{12}(i\omega_{eq}) \\ Z_{21}(i\omega_{eq}) & Z_{22}(i\omega_{eq}) \end{bmatrix}$$

$$Z_{11}(i\omega_{eq}) = (-\omega_{eq}^2 m + i2c\omega_{eq} + 2k)$$

$$Z_{12}(i\omega_{eq}) = Z_{21}(i\omega_{eq}) = -(ic\omega_{eq} + k)$$

$$Z_{22}(i\omega_{eq}) = (-\omega_{eq}^2 m + ic\omega_{eq} + k)$$

And then using Cramer's law to obtain the solution:

$$\vec{X}_{eq} = [Z(i\omega_{eq})]^{-1} \vec{F}_{w0}$$

Hence:

$$X_{eq1} = \frac{1}{|Z(i\omega_{eq})|} \begin{vmatrix} Y(i\omega_{eq}) & Z_{12}(i\omega_{eq}) \\ 0 & Z_{22}(i\omega_{eq}) \end{vmatrix}$$

$$X_{eq2} = \frac{1}{|Z(i\omega_{eq})|} \begin{vmatrix} Z_{11}(i\omega_{eq}) & Y(i\omega_{eq}) \\ Z_{21}(i\omega_{eq}) & 0 \end{vmatrix}$$

Assume steady state harmonic solution:

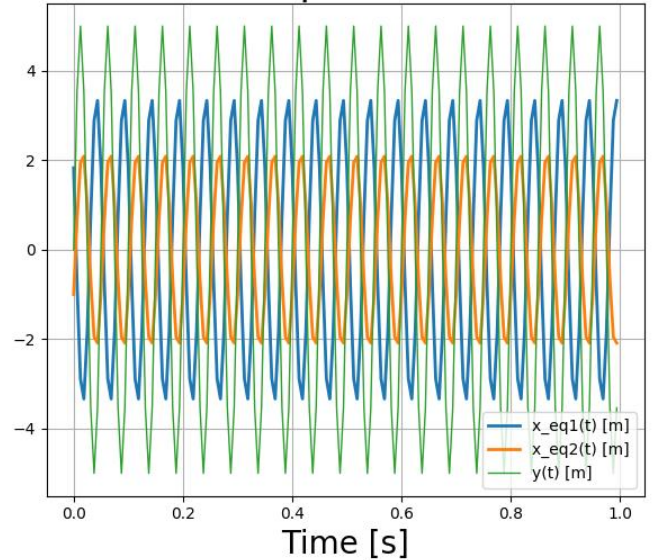
$$x_{eqj}(t) = X_{eqj}(i\omega_{eq}) \cos(\omega_{eq}t - \phi_{eqj})$$

where:

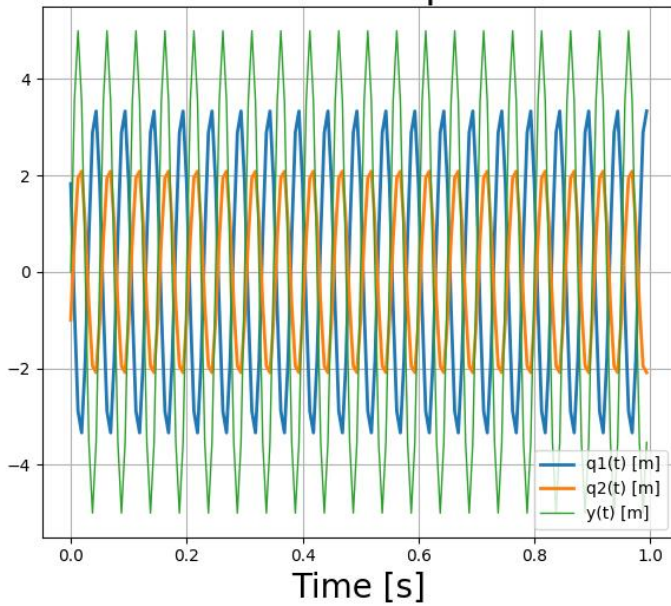
- Input impedance, $Y(i\omega_{eq}) = ic\omega_{eq} + k$
- Amplitude, $X_{eqj} = |X_{eqj}(i\omega_{eq})|$
- Phase angle, $\phi_{eqj} = \angle X_{eqj}(i\omega_{eq})$

The following results been obtained for absolute and relative responses:

"Absolute Response for Seismic"

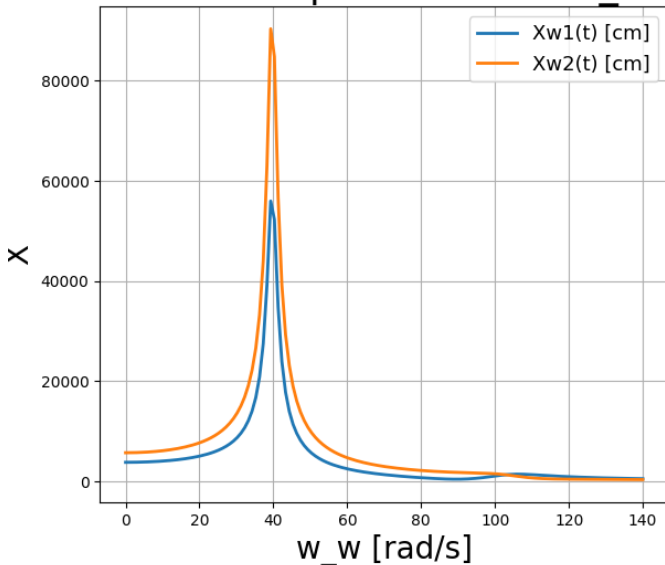


"Relative Response"



The critical frequencies at which the amplitude is high, are shown bellow:

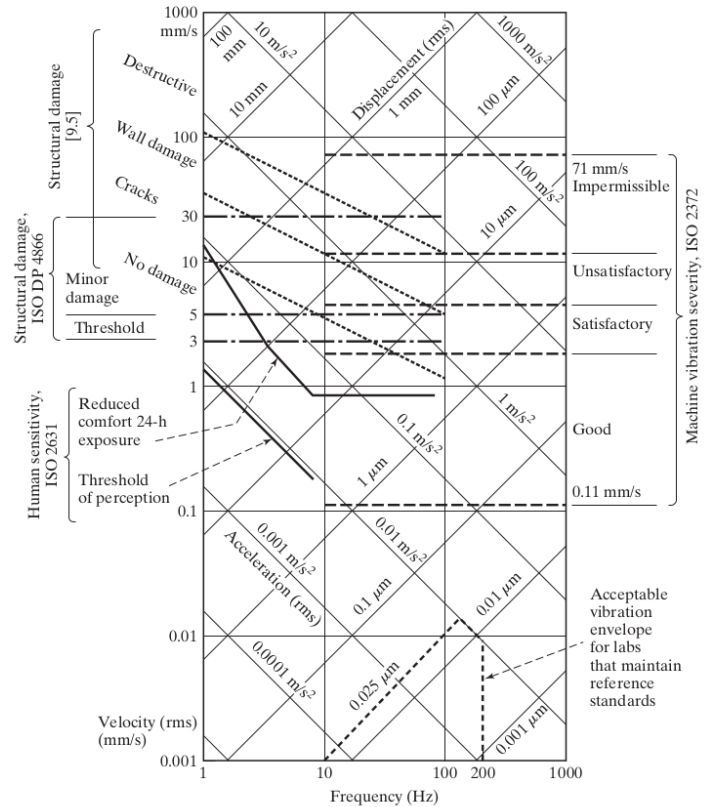
"Floors Amplitudes with w_w "



7. Vibration Control

The vibration of the building have negative effects on the building itself due to fatigue of the building materials, and also on the occupant of the building such as humans, valuable furniture, laboratory facilities, tools, machines,... etc. The vibration effect should be reduced or to be better that it eliminated. The satisfactory

levels of vibration is obtained from the NOMOGRAPH:



The comfort zone for human specially is found to be in $100\&10\mu m$.

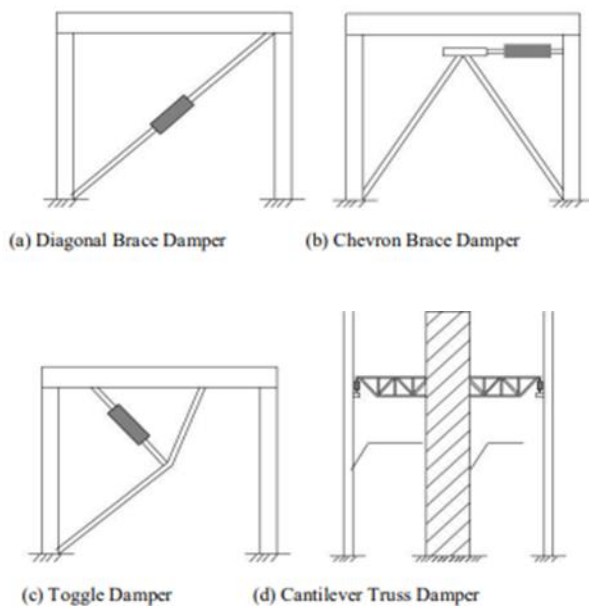
Several techniques can be used to do so, it can be done by isolating the source of vibration, or changing the natural frequencies of the building itself, however this is difficult to obtain due to several constrains. We will study two method of reducing the effect of vibration, by introducing isolation and using absorbers, these two methods are studied separately on each model and their results been showed and also their effects been compared.

7.1. Vibration Isolation

Viscous Damping is one of many different methods for allowing a structure to achieve optimal performance when it is subjected to seismic, wind, blast or other types of transient shock and vibration

disturbances. The addition of fluid viscous dampers to a structure can provide damping as high as 30% of critical, and sometimes even more. This provides a significant decrease in earthquake excitation. The addition of fluid dampers to a structure can reduce horizontal floor accelerations and lateral deformations by 50% and sometimes more.

Viscous Dampers will work and induce damping only if the two points it connects have significant relative movement. Viscous damper can be installed in structures in different configuration to achieve relative movement. (the figure shows some configurations often used in high rise buildings).



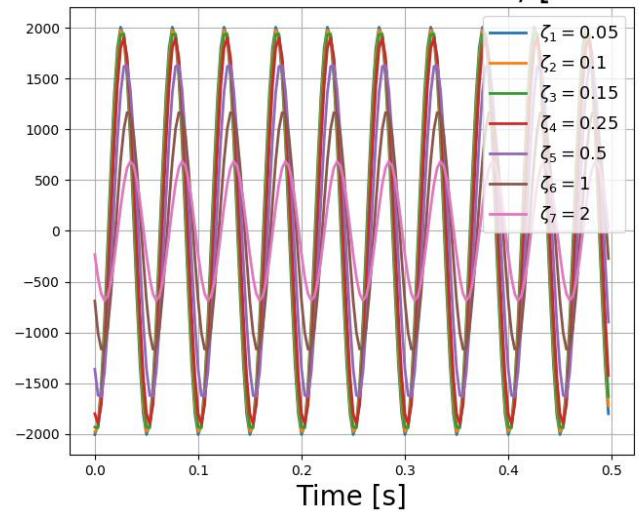
We will consider the damping configuration as manipulation of the damping ratio to simplify the analysis, so we will show the resulted response while increasing the damping ratio from

$\zeta = 5\%: 100\%$.

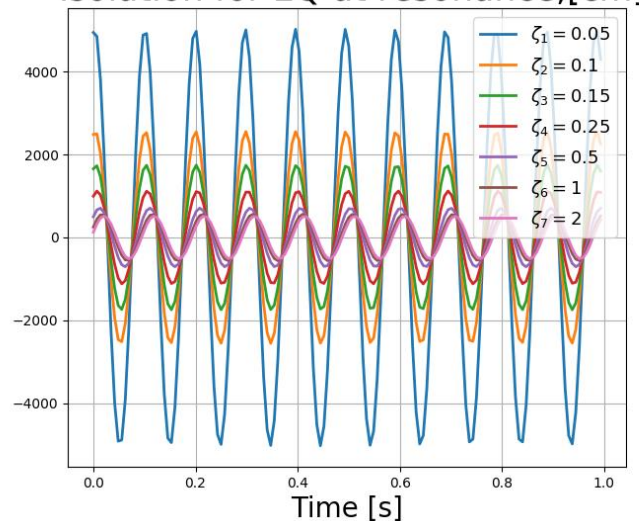
For the SDOF model, the vibration response cause by the wing and

earthquakes as we increase the damping ratio is as follow:

"Isolation for Wind at resonance, [microm]"



"Isolation for EQ at resonance, [cm]"



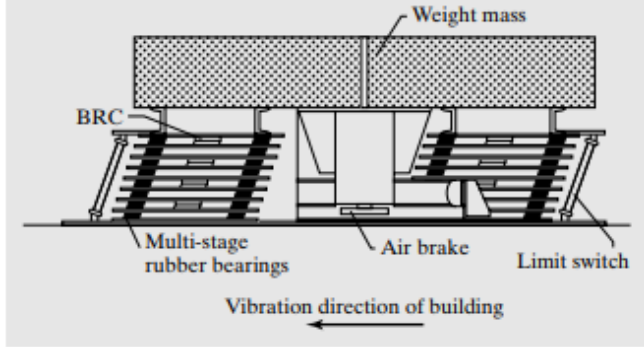
Due to that these results are not satisfactory, we didn't use isolation for the 2DOF systems.

7.2. Vibration Absorber

We will take into consideration the absorption by TMD.

The TMD absorbing system (Short for Tuned Mass Damper) is an energy absorbing device used to reduce the effects of undesirable vibrations of a structure subjected to harmonic excitations. The frequency of the damper is tuned to a

particular structural frequency so that when that frequency is excited, the damper will resonate out of phase with the structural motion. Energy is dissipated by the damper inertia force acting on the structure. (See the Figure)



The TMD concept was first applied by Frahm in 1909 to reduce the rolling motion of ships as well as ship hull vibration.

The dynamic model for SDOF system absorber:

$$\begin{bmatrix} m & 0 \\ 0 & m_a \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{x}_a \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_a \\ -c_a & c_a \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{x}_a \end{Bmatrix} + \begin{bmatrix} k_1 + k_a & -k_a \\ -k_a & k_a \end{bmatrix} \begin{Bmatrix} x \\ x_a \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ 0 \end{Bmatrix}$$

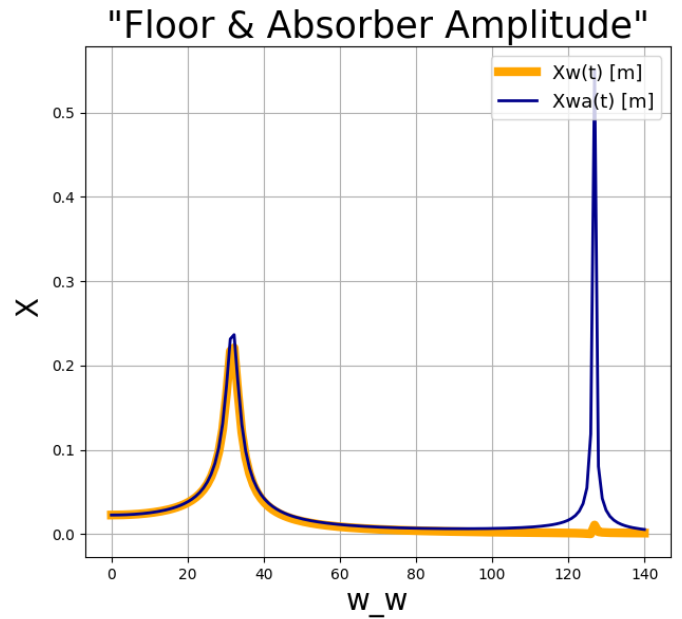
And for 2DOF system:

$$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m_a & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m_b \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_a \\ \ddot{x}_2 \\ \ddot{x}_b \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_a & -c_2 & 0 \\ -c_a & c_a & 0 & 0 \\ -c_2 & 0 & c_1 + c_2 & -c_a \\ 0 & 0 & -c_a & c_a \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_a \\ \dot{x}_2 \\ \dot{x}_b \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 + k_a & -k_a & -k_2 & 0 \\ -k_a & k_a & 0 & 0 \\ -k_2 & 0 & k_2 + k_b & -k_b \\ 0 & 0 & -k_b & k_b \end{bmatrix} \begin{Bmatrix} x_1 \\ x_a \\ x_2 \\ x_b \end{Bmatrix}$$

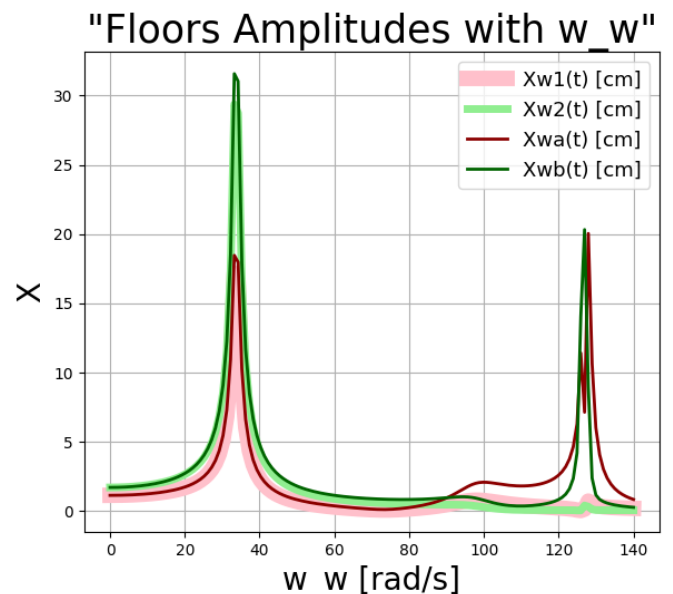
$$= \begin{Bmatrix} F_1(t) \\ 0 \\ F_2(t) \\ 0 \end{Bmatrix}$$

7.2.1. Wind Input Absorber

We can obtain a particular band for our system to operate within depend on the absorber designed frequency:

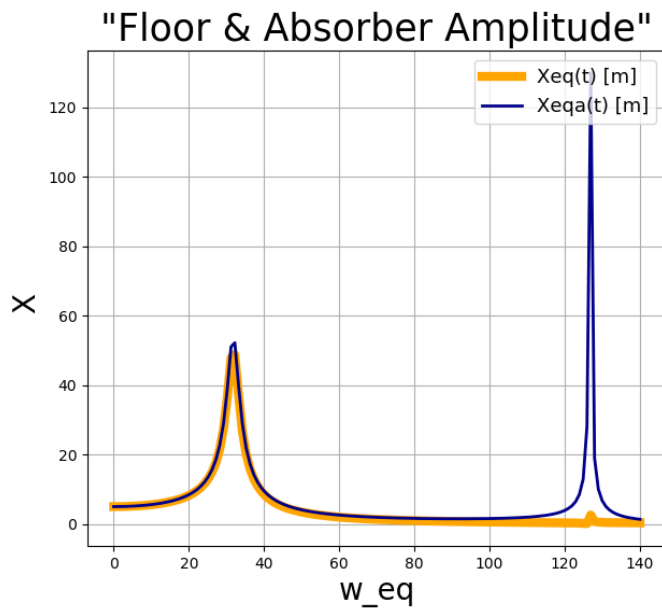


And for a 2DOF system:

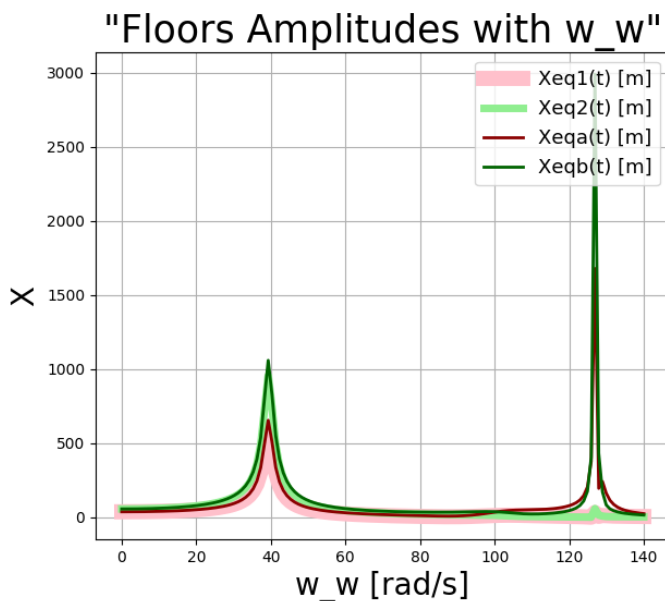


7.2.2. Seismic Input Absorber

Using this technique in SDOF give us the following results for absolute and relative responses:



And for a 2DOF system:

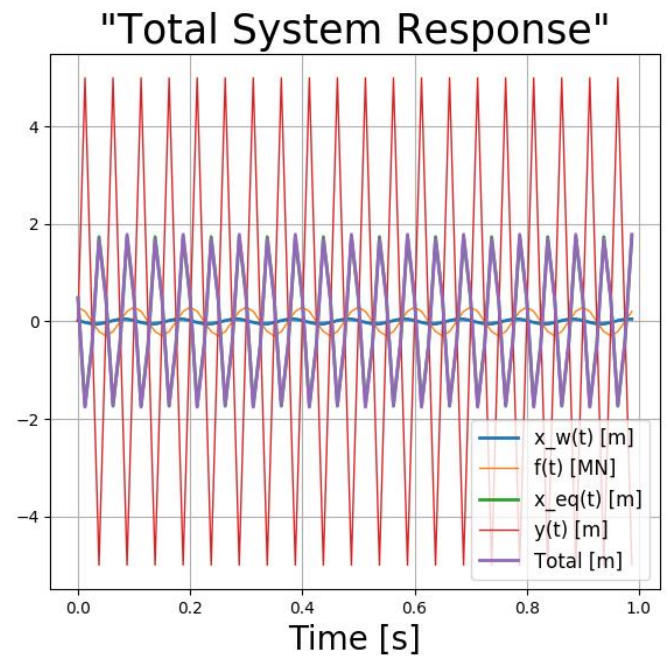


8. Final Results

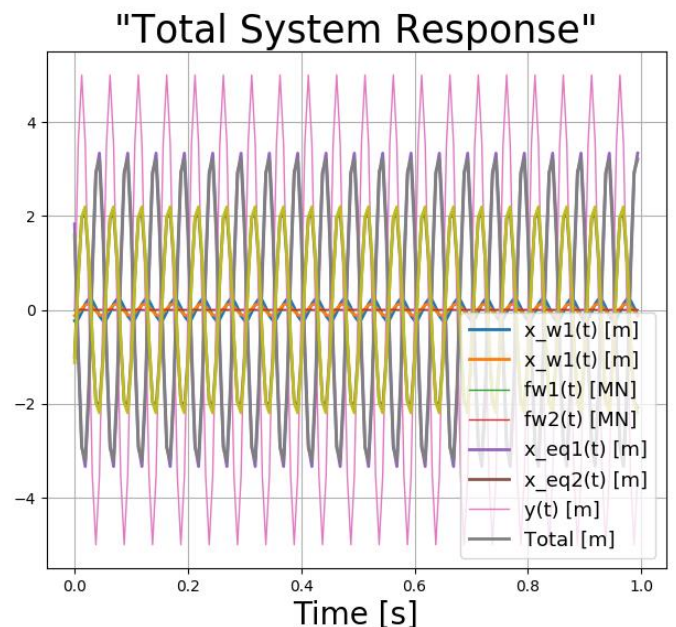
From the previous section we found that the ABSORBER method is better for eliminating the vibration, although we design for high bandwidth of eliminated vibration region by increasing the design frequency.

8.1. Total System Response

Also we will test our absorber to eliminate the total vibration for both wind and earthquakes together without neither isolation or absorption:



And for the 2DOF system:



8.2. System Response Under Saw Seismic Wave

We will consider another form of seismic wave called Saw wave, and it is a

good approximation of the real seismic waves, hence it give more realistic results.

8.2.1. Saw Wave Approximation

This Saw wave can be converted to an approximate harmonic wave using Fourier series, hence we can using it as an input to our model:

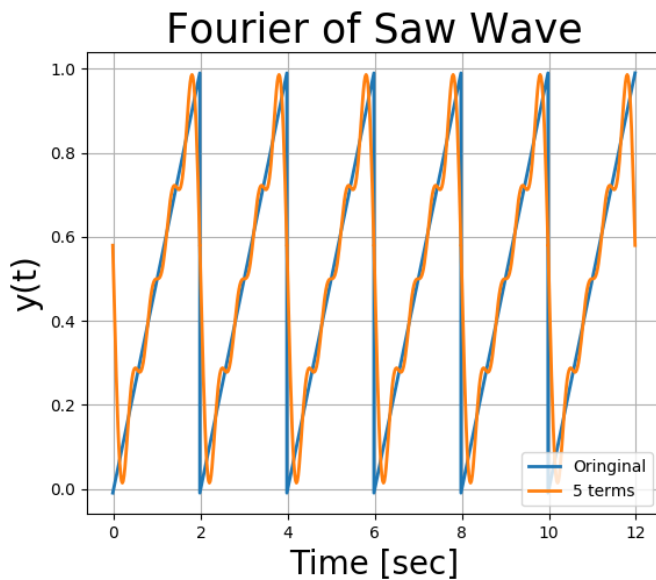
- Saw wave function,

$$y(t) = Y \frac{t}{\tau}; 0 \leq t \leq \tau$$

- Fourier series of saw function,

$$y(t) = \frac{Y}{\pi} \left[\frac{\pi}{2} - \sum_{n=1}^{\infty} \left\{ \frac{\sin(n\omega t)}{n} \right\} \right]$$

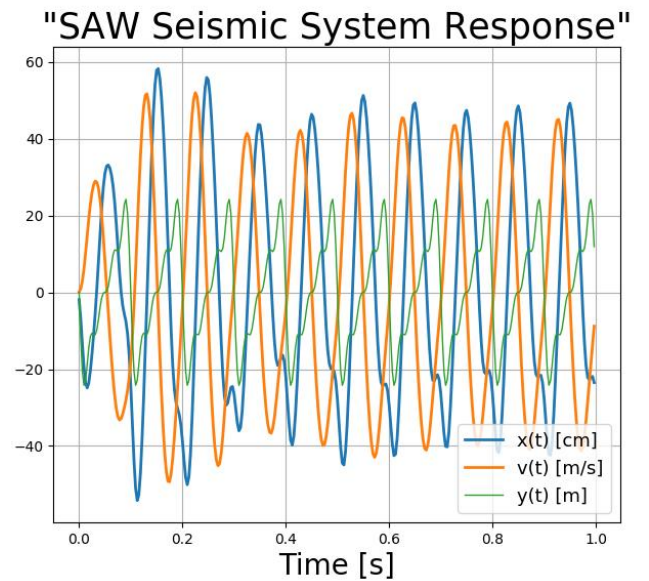
We selected 5 terms of Fourier to approximate the saw wave, hence we obtain the following:



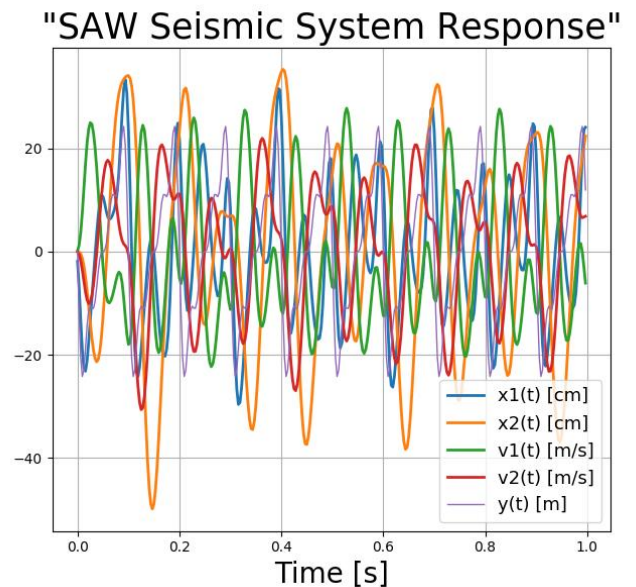
8.2.2. System Response

The analytic response due to the Fourier approximation is tedious, so we will solve it numerically using state response representation and RK4 integration.

Hence the following results been obtained;



And for the 2DOF system:

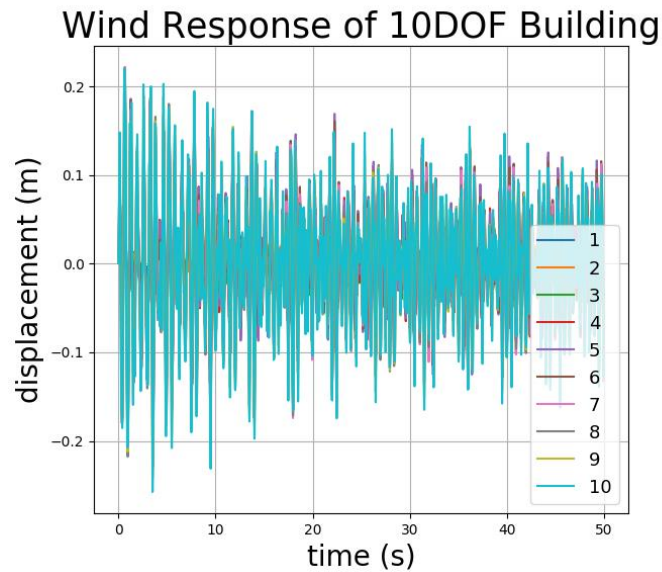


8.3. Generalization

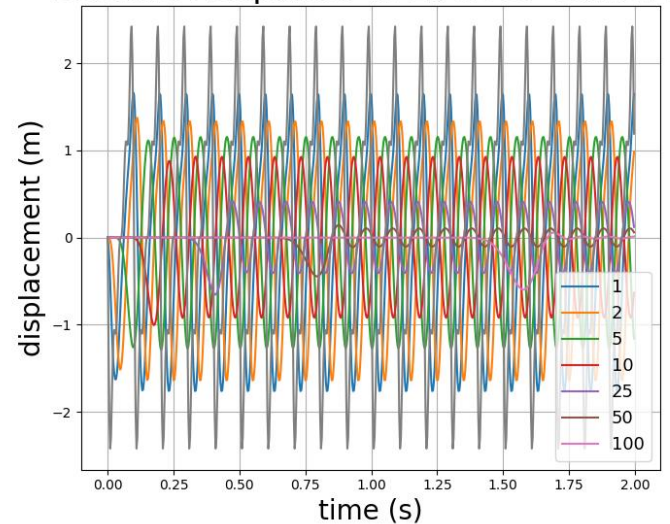
We can solve for the vibration response to any number of stories, but as the order of the system is increased the powerfull of using numerical techniques is shown.

Therefor with the help of state space modelling we can provide a response to n number of degree of freedoms, and we can show results for that to both inputs: wind force and seismic saw waves.

For a 10DOF and 100DOF building:



Seismic Response of 100DOF Building



9. Comments and Conclusion

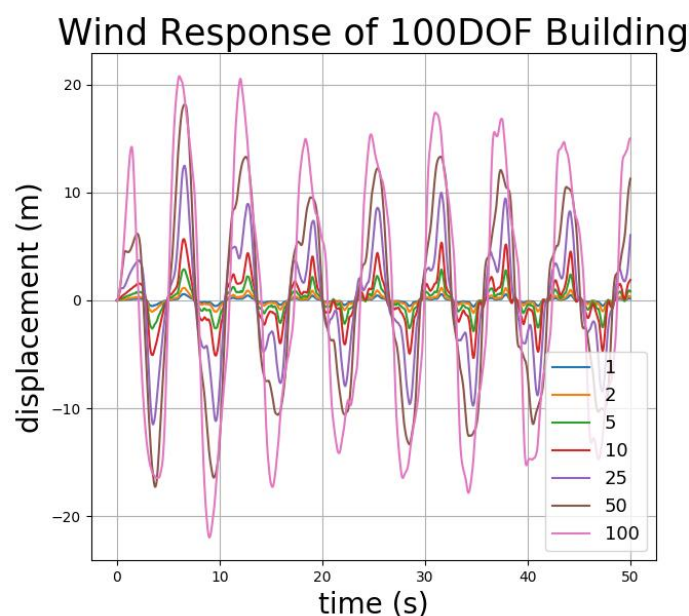
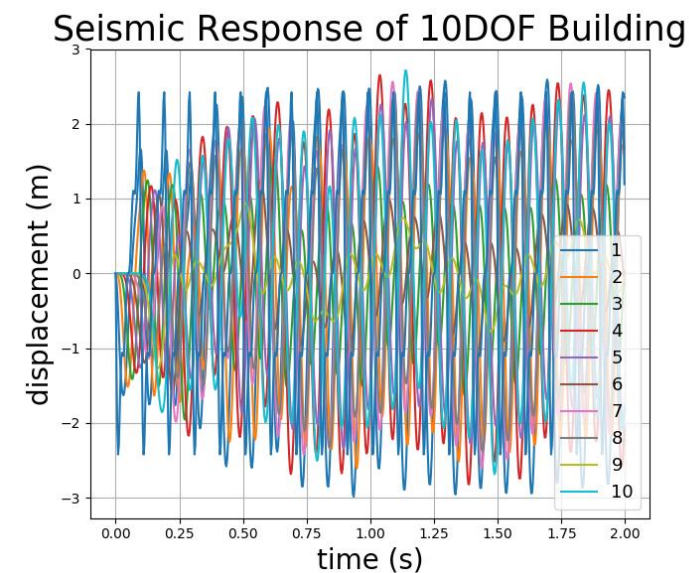
It has been found the using an ABSORBER method gives satisfactory results on total system response and another form of input signal. It has been notice the effect of wind is small compared with earthquake wave, hence the design of absorber can be with respect to the earthquake with guarantee on reducing the wind effect on the building.

We notice that the effect of wind forces is increase as the height of the building is increased but the seismic have less changes is the height is increased.

We have used a lot of approximations in our analysis hence we have obtain inaccurate results, however it is useful that it gave as a good insight in the building vibration and the associated techniques of eliminating the vibrations.

10. References

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