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Modeling and simulation of vibrations for a bus, with Passive suspension system – a Python approach

ABSTRACT

In this work, a simple bus is modeled as a half-car model of two dynamics, longitudinal and lateral. Vibrations response to a road disturbance is plotted using python and results been discussed.

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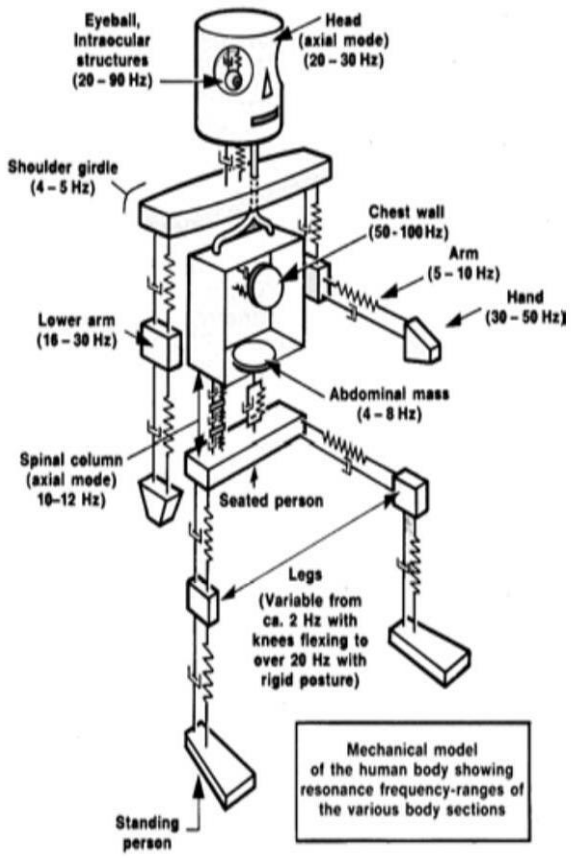
1. Introduction

The traffic participants particularly the users of vehicles of each means of transport are affected by vibrations .The drivers of heavy motor vehicle drivers are affected by higher intensity vibration .As a result the effects of vibrations have certain side effects on comfort and ability to work which are highly prominent in case they affect someone for a long period of time, and they reach a certain level when they become health threatening and decrease safety.

According to the body part that is affected there are whole body vibrations and local vibrations. Whole body vibrations affects the body in different positions and those are transmitted across the whole body (legs, lower back, back). Those vibrations are important in frequency range of $(1 - 80\ Hz)$; because some organs (head, eyes, stomach, spine)of body are located in this frequency range. Local vibrations have an effect on human body on frequency range $(8 - 1000\ Hz)$.



The most important physiological effect of vibration refer to biochemical changes, neuro-vegetative system impairments, cardiovascular illnesses and musculoskeletal disorders.

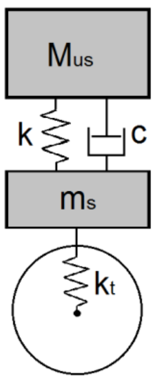
Elimination of vibration is essential in order to improve both the comfort and the safety of the passenger; we shall try to find a good model for a bus with a suitable value to achieve comfort ride, long life, and stability of the bus.

2. Analysis Methodology

In order to perform dynamical analysis, either a quarter-car (Gopala Rao and Narayanan, 2009; Huang and Chen, 2006) or a half-car (Ihsan *et al.*, 2009; Sapiński and Rosoł, 2008) full car model can be used.

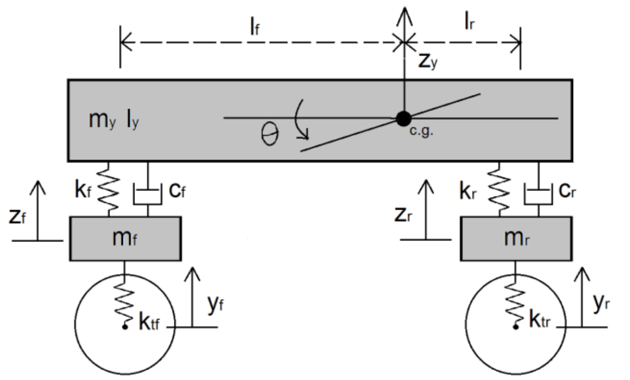
2.1. The quarter-car model

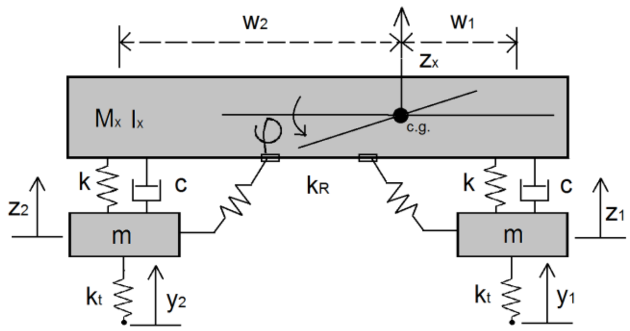
It is the most used in analysis of vibrations. It consists of a non-spring supported mass (a wheel with partial of suspension) and a spring-supported mass (1/4 car body) is a two degrees of freedom model and is usually used for testing of the performance of control algorithms. (see the Figure)



2.2. The half-car model

Consists of a four-degrees of freedom consist of two non-spring supported masses and a spring-supported one (1/2 car body). It additionally includes rotation angle of the body and allows analysis of the response to the excitation applied to both wheels of the vehicle. The half-car model analysis is assume decoupled dynamics between lateral and longitudinal model. (see the Figures)





2.3. The full-car model

The full-car model consists of an eight –degrees of freedom: un-sprung displacement, four sprung displacements, and three angles: yaw, roll and pitch. Due to this number of DOF it the most difficult to analyse and it is also the least used model.

3. The Half-Car Model Dynamics

For design purposes a **half-car** model was used with longitudinal and lateral dissection method.

The dynamics of the model been developed in the following approach:

$$\boldsymbol{M}\ddot{\bar{x}} + \boldsymbol{C}\ddot{\bar{x}} + \boldsymbol{K}\bar{x} = \bar{F}$$

Where:

M: mass matrix

C: damping matrix

K: stiffness matrix

$\bar{x}$: dof vector

$\bar{F}$: force vector.

3.1. Longitudinal Dynamics

Let the dof vector be:

$$\bar{p} = \begin{bmatrix} z_y \\ \theta \\ z_r \\ z_f \end{bmatrix}$$

And let the force vector be:

$$\bar{y} = \begin{bmatrix} 0 \\ 0 \\ \boldsymbol{y_r} \\ \boldsymbol{y_f} \end{bmatrix}$$

∴The dynamics eq. is:

$$\boldsymbol{M}\ddot{\bar{p}} + \boldsymbol{C}\ddot{\bar{p}} + \boldsymbol{K}\bar{p} = \boldsymbol{K_t}\bar{y}$$

And the system matrices are as follows:

$$M = \begin{bmatrix} m_y & 0 & 0 & 0 \\ 0 & I_y & 0 & 0 \\ 0 & 0 & m_r & 0 \\ 0 & 0 & 0 & m_f \end{bmatrix}$$

$$C = \begin{bmatrix} c_f + c_r & c_r l_r - c_f l_f & -c_r & -c_f \\ c_r l_r - c_f l_f & c_f l_f^2 + c_r l_r^2 & -c_r l_r & c_f l_f \\ -c_r & -c_r l_r & c_r & 0 \\ -c_f & c_f l_f & 0 & c_f \end{bmatrix}$$

$$K = \begin{bmatrix} k_f + k_r & k_r l_r - k_f l_f & -k_r & -k_f \\ k_r l_r - k_f l_f & k_f l_f^2 + k_r l_r^2 & -k_r l_r & k_f l_f \\ -k_r & -k_r l_r & k_r + k_t & 0 \\ -k_f & k_f l_f & 0 & k_f + k_t \end{bmatrix}$$

$$K_t = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ k_{tf} & 0 \\ 0 & k_{tr} \end{bmatrix}$$

3.2. Lateral Dynamics

Let the dof vector be:

$$\bar{p} = \begin{bmatrix} z_x \\ \varphi \\ z_1 \\ z_2 \end{bmatrix}$$

And let the force vector be:

$$\bar{x} = \begin{bmatrix} 0 \\ 0 \\ \boldsymbol{x_1} \\ \boldsymbol{x_2} \end{bmatrix}$$

∴The dynamics eq. is:

$$\boldsymbol{M}\ddot{\bar{q}} + \boldsymbol{C}\ddot{\bar{q}} + \boldsymbol{K}\bar{q} = \bar{x}$$

And the system matrices are as follows:

$$M = \begin{bmatrix} m_x & 0 & 0 & 0 \\ 0 & I_x & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m \end{bmatrix}$$

$$C = \begin{bmatrix} 2c & c(w_2 - w_1) & -c & -c \\ c(w_2 - w_1) & c_f w_2^2 + c_r w_1^2 & -cw_1 & cw_2 \\ -c & -cw_1 & c & 0 \\ -c & cw_2 & 0 & c \end{bmatrix}$$

$$K = \begin{bmatrix} 2k & k(w_1 - w_2) & -k & -k \\ k(w_1 - w_2) & k_f l_f^2 + k_r l_r^2 + k_R & -kw_1 & kw_2 \\ -k & -kw_1 & k + k_t & 0 \\ -k & kw_2 & 0 & k + k_t \end{bmatrix}$$

$$K_t = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ k_t & 0 \\ 0 & k_t \end{bmatrix}$$

4. System Parameters

4.1. Mass

After looking into numerous models and designs the gross mass (maximum allowed mass of the whole buss, passengers included) of the buss was predetermined to be 18 Tons.

The mass distribution was made so that 12 Tons of the gross mass were to be centred on the drive axle and the other 6 Tons on the front axle for design reasons and for the sake of other parameter constants such as the spring constant K.

4.2. Damper

Dampers have three types , those types are :

4.2.1. Passive Damper

The passive damper are used to control a vehicles Vertical ,Spinning , and Rolling motion ,it limits the wheel and body's relative motion to give a ride comfort . Dampers are desired to provide good tire-road contact and low vertical aces. , a soft damper achieves this but at the cost of handling performance ,while a stiff one reduces comfortability ,so an optimal damper is balanced according to expected road profiles.

4.2.2. Semi-active Damper

Semi-active systems can only vary the damping co-efficient of shock absorber. They require less energy to function and are less expensive.

4.2.3. Active Damper

An active suspension system controls the relative motion between the wheel and the chassis using an on-board system, it is divided into (pure active, and semi active) systems.

And both use actuators to independently raise and lower the chassis at each wheel.

****P.S** (From the above the 'Passive Damper' is the type being used for the design mainly because of its properties and its non-on-board system nature for better analysis).

The value of a simple dashpot damper is calculated using the following formula:

$$\boldsymbol{C} = \mu\left[\frac{\boldsymbol{3 * \pi L D^3}}{\boldsymbol{4d^3}}\left(\boldsymbol{1}+\frac{\boldsymbol{2d}}{\boldsymbol{D}}\right)\right]$$

The following table shows the different damper fluids and their properties and characteristics:

Fluid	T C	D/d	v m^2/s	L m	P kg/m^3	μ Pa.s	C N.s/m
Fluoro-Carbon gel 868 (Grease)	25	5	0.05	0.5	1160	58	12000
PG-44A	25	5.5	0.192	0.5	840	161.28	43000
Polidimethylsiloxane (PDMS) Oil	25	6	1	0.5	965	965	104000

4.3. Spring

A spring is defined as an elastic body whose function is to distort when loaded and to recover its shape when the load is removed.

We use springs to cushion, absorb or control energy due to either shock or vibrations like in car springs, railway buffers, shock absorbers.

From the many spring types a 'Helical Spring' has been chosen for the design, the helical spring is a wire which has been coiled in the form of a helix and is mainly intended for compressive or tensile loads.

As we mentioned above in the introduction, the human comfort zone is in the natural frequency range between 1-80Hz (6.28-502.65 rad/sec).

-Mass on one suspension system,

$$= 18000/6 = 3000kg$$

-Total bus weight is 18tons x9.81,

$$= 176580\ N$$

One tire load $= 176580/6 = \ 29430N$

We used carbon steel to design the spring with:

τ_{max} = 224 Mpa, G=80Gpa, C=6, and safety factor of 1.5, $\tau_{allowable}$ =224/1.5 =149.33 say 150 Mpa

$$\tau = K\frac{G*d}{8*C^3*n}$$

$$K = \left(\frac{4C-1}{4C-4} + \frac{0.615}{C}\right)\left(1 + \frac{1}{2C}\right)$$
$$= 1.3\text{Mpa}$$

$$\tau = 1.3 * \frac{(8 \text{ x } 29340 \text{ x } 6)}{\pi d^2}$$

-We found that:

d=63mm D=378mm

-Determining (K):

K_{tire}=1.7 x 10^6 N/m

For the comfort zone ω_n=6.28 – 502.65 rad/sec

For $\omega_n = \sqrt[2]{\frac{\text{Keq.}}{m}} = \sqrt[2]{\frac{\text{Keq.}}{3000}} = 6.28$rad/sec

$K_{eq.}$=118.44N/mm

For $\omega_n = \sqrt[2]{\frac{\text{Keq.}}{m}} = \sqrt[2]{\frac{\text{Keq.}}{3000}}$ =502.65 rad/sec

$K_{eq.}$=757985.62N/mm

$$K_{eq} = (K_{sus.} \text{ x } K_{tire}) / (K_{sus.} + K_{tire})$$

For $Keq. = 118.44N/mm$

→ $Ksus. = 127.3\ N/mm$

-We can't have a value for $K_{eq.}$ more than K_{tire}

For comfort zone $Ksus. > \ 127.3\ N/mm$, so, we took the value of $Ksus. = 150N/mm$.

∂=(8WC^3n)/(Gd)

K=W/ ∂

K=(80 x10^3 x63)/(8 x6^3x n)=150N/mm

n=19.44 say 20 coils.

-All upper calculations are for the front tires suspension system.

-As the load on the rear axis is doubled, we put:

M=6000kg W=58860N d=89mm D=534mm

K=300N/mm n=14 coils.

Then we calculated:

d(mm)	K(N/mm)
63	145.5
65	150
70	162

5. Natural Frequencies and Modes

We can calculate the natural frequencies and modes of a multi DOF system by solving the following determinant:

$$|-\omega^2 * \boldsymbol{M} + \boldsymbol{K}| = 0$$

Then we solve for the different spring constants above using python:

5.1. Longitudinal Dynamics

Natural frequencies,

K1 = [4.6403743   7.29827544
40.89423663 55.48674144]
k = [4.66212519 7.37812361
40.89424439 55.56408549]
k2 = [4.70766677 7.56812998
40.89426269 55.74955645]

5.2. Lateral Dynamics

Natural frequencies,

K1 = [3.85218123 4.33591536
57.92877338 57.92941624]
K2 = [3.91268919 4.40556266
58.00788259 58.00856742]
K2 = [4.052938   4.56730762 58.1973906
58.19818072]

6. System Response

We arrange the system dynamics into state space form,

$$\boldsymbol{A}\dot{\bar{x}} + \boldsymbol{B}\bar{x} = \boldsymbol{D}\bar{X}$$

Where:

$$A = \begin{bmatrix} I_4 & \bar{0}_4 \\ \bar{0}_4 & \boldsymbol{M} \end{bmatrix}$$

$$B = \begin{bmatrix} \bar{0}_4 & -I_4 \\ -\boldsymbol{K} & -\boldsymbol{C} \end{bmatrix}$$

$$D = \begin{bmatrix} \bar{0}_4 & \bar{0}_4 \\ \bar{0}_4 & K_t \end{bmatrix}$$

then we solve the system of equations numerically using python.

6.1. Longitudinal Dynamics Response

$$\boldsymbol{A}\dot{\bar{P}} + \boldsymbol{B}\bar{P} = \boldsymbol{D}\bar{Y}$$

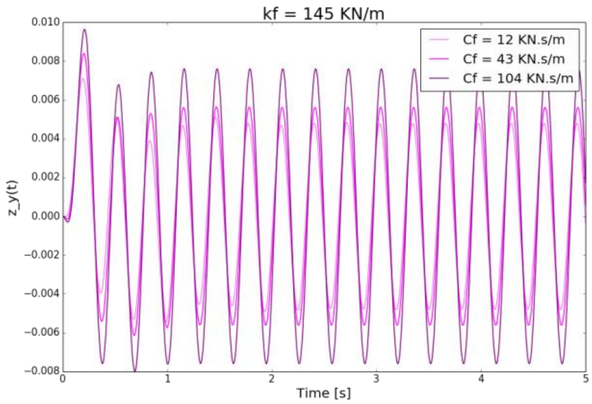
Where:

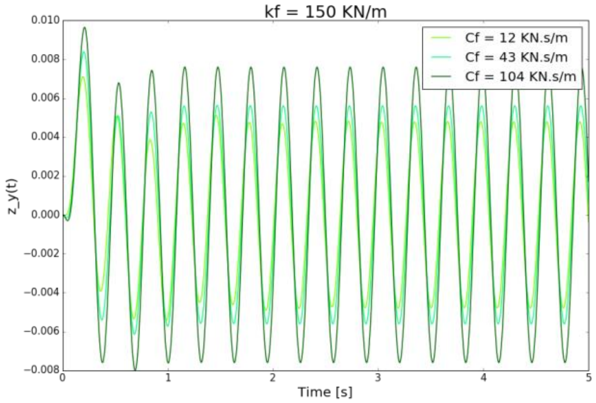
$$\dot{\bar{P}} = \begin{bmatrix} \dot{\bar{p}} \\ \ddot{\bar{p}} \end{bmatrix},\ \bar{P} = \begin{bmatrix} \bar{p} \\ \dot{\bar{p}} \end{bmatrix}, \bar{Y} = \begin{bmatrix} \bar{0}_{4x1} \\ \bar{y} \end{bmatrix}$$

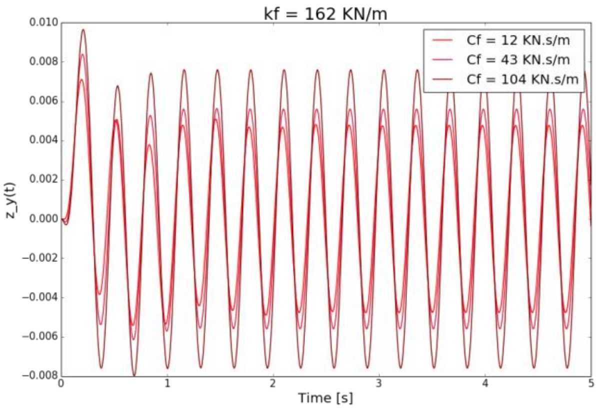
We mainly interested in main mass vertical displacement, so a comparession based on its response is made to find a minimum cost and good performance damping and stiffness components to be used.

We find out the following results:

We started out by testing the damping performance of several damping coefficients while holding one value of stiffness at each.







We find that the amplitude of vibrations is increased as we increase the input frequency, but in general they are all stable.

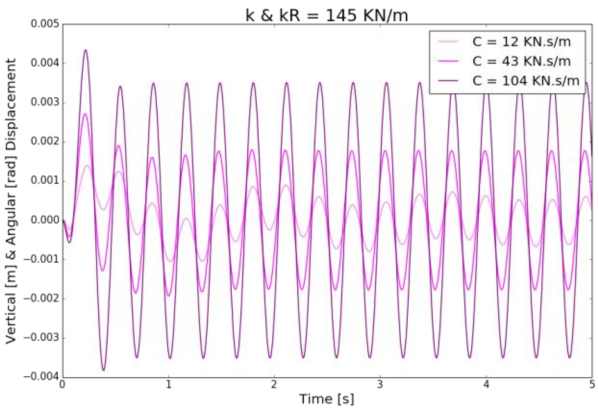
6.2. Lateral Dynamics Response

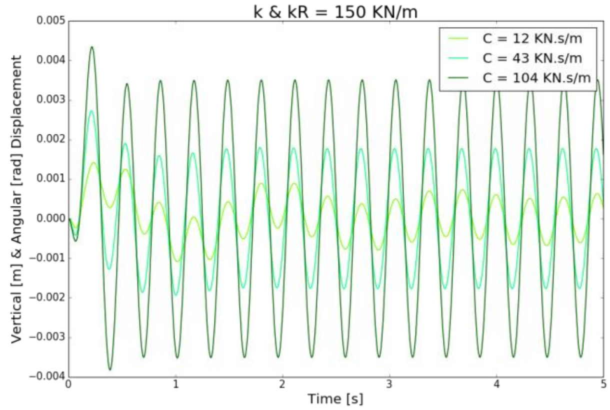
$$\boldsymbol{A}\dot{\bar{Q}} + \boldsymbol{B}\bar{Q} = \boldsymbol{D}\bar{X}$$

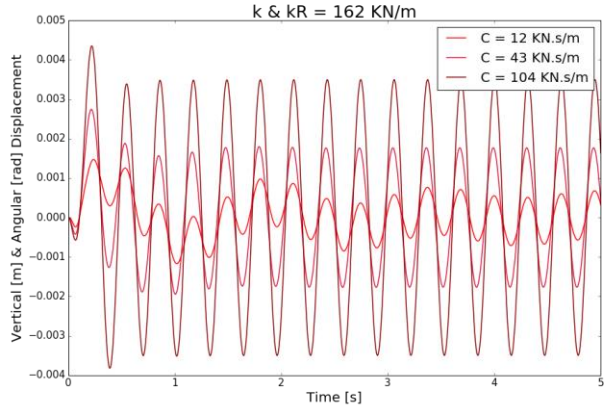
Where:

$$\dot{\bar{Q}} = \begin{bmatrix} \dot{\bar{q}} \\ \ddot{\bar{q}} \end{bmatrix},\ \bar{Q} = \begin{bmatrix} \bar{q} \\ \dot{\bar{q}} \end{bmatrix}, \bar{X} = \begin{bmatrix} \bar{0}_{4x1} \\ \bar{x} \end{bmatrix}$$

We repeat the same analysis as the longitudinal dynamics.



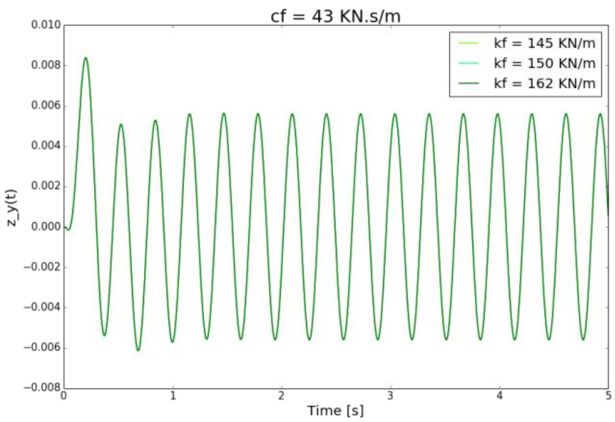


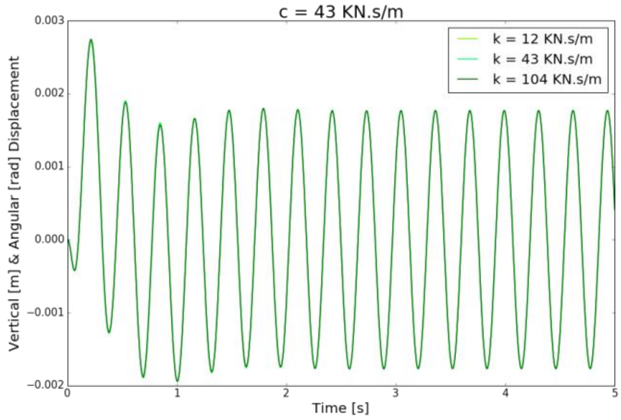


We find as before for the longitudinal that the amplitude of vibrations is increased as we increase the input frequency, but for the small damping the response is unstable vibration.

6.3. Final Results

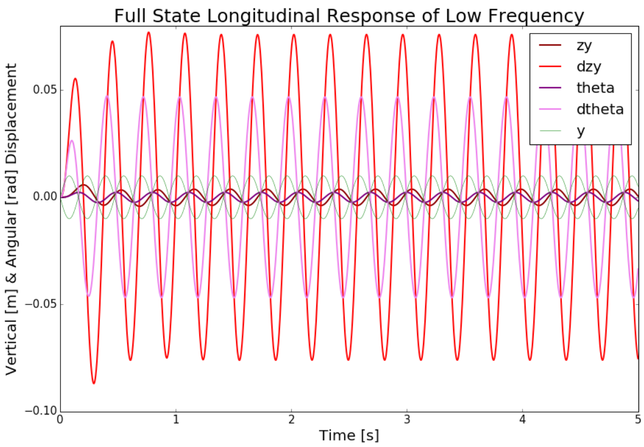
So we choose the second value of the damping coefficient; because it is the minimum best, and plot a response for the system while changing the value of stiffness.

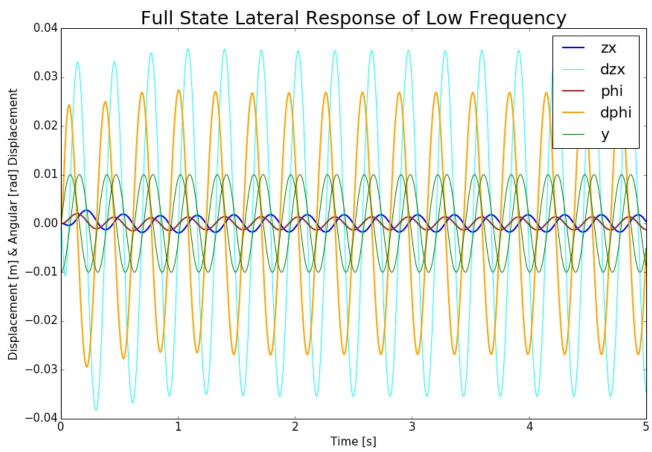




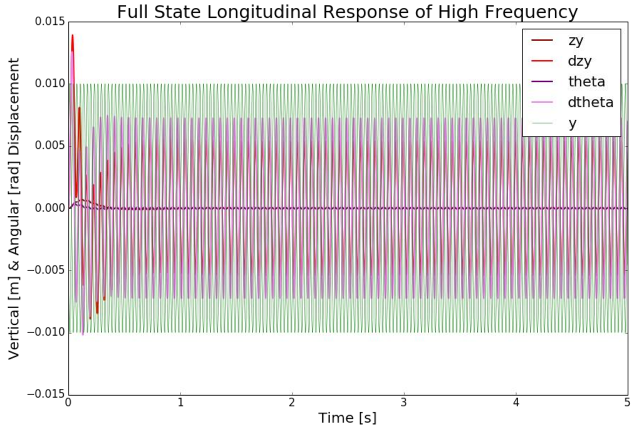
We note that there is no obvious difference in the responses, so we will choose the minimum one.

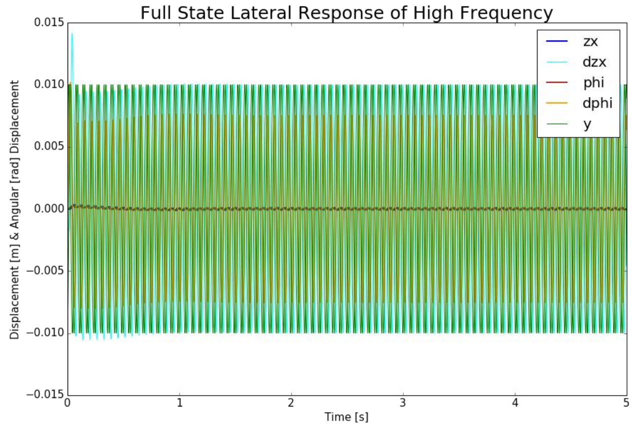
Now we plot the full state response of both Longitudinal & Lateral dynamics at low frequency.





Finally we plot the full state response of both Longitudinal & Lateral dynamics at high frequency road disturbance.





7. Comments and Conclusion

We note that when we increase the damping coefficients the amplitude of the response is decreased with some amount, however, when changing the stiffness the response doesn’t change a much. The final full state response is acceptable with some negligible vibrations.

8. References

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