Rocket Propulsion Problems Solution - Sutton Ch 2

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1 Problem 1: Rocket Propulsion Problem Solution

1.1 Problem Statement

The following data are given for a certain rocket unit: thrust, 8896 N; propellant consumption, 3.867 kg/sec; velocity of vehicle, 400 m/sec; energy content of propellant, 6.911 MJ/kg. Assume 100% combustion efficiency.

Determine:

(a) the effective velocity;

(b) the kinetic jet energy rate per unit flow of propellant;

(c) the internal efficiency;

(d) the propulsive efficiency;

(e) the overall efficiency;

(f) the specific impulse;

(g) the specific propellant consumption.

1.2 Solution

Given:

- $F = 8896\,\mathrm{N}$
- $\dot{m} = 3.867\,\mathrm{kg/sec}$
- $u = 400\,\mathrm{m/sec}$
- $Q_R = 6.911\,\mathrm{MJ/kg}$
- $\eta_{\mathrm{comb}} = 1.0$

1.2.1 (a) Effective Velocity

The effective velocity, c, is given by:

$$c = \frac{F}{\dot{m}} = \frac{8896\,\mathrm{N}}{3.867\,\mathrm{kg/sec}} = 2300\,\mathrm{m/sec}$$

1.2.2 (b) Kinetic Jet Energy Rate per Unit Flow of Propellant

The kinetic jet energy rate per unit flow of propellant, E_k, is:

$$E_k = \frac{c^2}{2} = \frac{(2300\,\mathrm{m/sec})^2}{2} = 2.645 \times 10^6\,\mathrm{m^2/sec^2} = 2.645\,\mathrm{MJ/kg}$$

1.2.3 (c) Internal Efficiency

The internal efficiency, η_i, is:

$$\eta_i = \frac{0.5mc^2}{\dot{m}Q_R\eta_{\mathrm{comb}}} = \frac{0.5 \times 3.867 \times 2300^2}{3.867 \times 6.911 \times 10^6} = 0.383 = 38.3\%$$

1.2.4 (d) Propulsive Efficiency

The propulsive efficiency, η_p, is:

$$\eta_p = \frac{2u/c}{1 + (u/c)^2} = \frac{2 \times 400/2300}{1 + (400/2300)^2} = 0.3376 = 33.76\%$$

1.2.5 (e) Overall Efficiency

The overall efficiency, η, is:

$$\eta = \eta_i\eta_p = 0.383 \times 0.3376 = 0.1293 = 12.93\%$$

Alternatively, it can be calculated as:

$$\eta = \frac{Fu}{\dot{m}Q_R} = \frac{8896 \times 400}{3.867 \times 6.911 \times 10^6} = 0.133 = 13.3\%$$

1.2.6 (f) Specific Impulse

The specific impulse, I_s, is:

$$I_s = \frac{F}{\dot{m}g_0} = \frac{8896}{3.867 \times 9.81} = 234.5\,\mathrm{sec}$$

1.2.7 (g) Specific Propellant Consumption

The specific propellant consumption, SFC, is:

$$\mathrm{SFC} = \frac{1}{I_s} = \frac{1}{234.5} = 0.00426\,\mathrm{sec}^{-1}$$

1.3 Conclusion

The answers are:

(a) $2300\,\mathrm{m/sec}$

(b) $2.645\,\mathrm{MJ/kg}$

(c) 38.3%

(d) 33.76%

(e) 13.3%

(f) $234.5\,\mathrm{sec}$

(g) $0.00426\,\mathrm{sec}^{-1}$

2 Problem 2: Rocket Propulsion Problem 4 Solution

2.1 Problem Statement

For the rocket in Problem 1, calculate the specific power, assuming a propulsion system dry mass of 80 kg and a duration of 3 min.

2.2 Given Data from Problem 1

- Thrust, $F = 8896\,\mathrm{N}$
- Propellant consumption rate, $\dot{m} = 3.867\,\mathrm{kg/sec}$
- Effective velocity, $c = 2300\,\mathrm{m/sec}$

2.3 Additional Data for Problem 2

- Dry mass of propulsion system, $m_f = 80\,\mathrm{kg}$
- Duration, $t = 3\,\mathrm{min} = 180\,\mathrm{sec}$

2.4 Solution

2.4.1 Specific Power Calculation

Specific power is given by:

$$\text{Specific Power} = \frac{P_{\text{jet}}}{m_0}$$

where $P_{\text{jet}} = 0.5Fc$ and m_0 is the total initial mass of the rocket including propellant.

First, calculate the total initial mass m_0:

$$m_0 = \dot{m} \times t + m_f = (3.867\,\text{kg/sec}) \times (180\,\text{sec}) + 80\,\text{kg} = 776.06\,\text{kg}$$

Next, calculate the jet power P_{jet}:

$$P_{\text{jet}} = 0.5 \times F \times c = 0.5 \times 8896\,\text{N} \times 2300\,\text{m/sec} = 10,230,400\,\text{W}$$

Finally, calculate the specific power:

$$\text{Specific Power} = \frac{P_{\text{jet}}}{m_0} = \frac{10,230,400\,\text{W}}{776.06\,\text{kg}} = 13.18\,\text{kW/kg}$$

2.5 Conclusion

The specific power for the rocket is:

$$\boxed{13.18\,\text{kW/kg}}$$

3 Problem 3: Rocket Propulsion Problem 7 Solution

3.1 Problem Statement

For a solid propellant rocket motor with a sea-level thrust of 207,000 lbf, determine:

(a) the (constant) propellant mass flow rate $\dot{m}$ and the specific impulse I_s at sea level;

(b) the altitude for optimum nozzle expansion as well as the thrust and specific impulse at this optimum condition and (c) at vacuum conditions.

Given:

- Initial total mass of the rocket motor is 50,000 lbm and its propellant mass fraction is 0.90.
- The residual propellant (called slivers) combustion stops when the chamber pressure falls below a deflagration limit, which amounts to 3% of the burnt.
- Burn time is 50 seconds.
- Nozzle throat area (A_t) is 164.2 in^2.
- Area ratio (A_e/A_t) is 10.
- Chamber pressure (p_c) is 780 psia and the pressure ratio (p_e/p_c) across the nozzle may be taken as 90.0.

3.2 Solution

3.2.1 (a) Propellant Mass Flow Rate and Specific Impulse at Sea Level

First, calculate the usable propellant mass:

$$m_p = m_0 \kappa_f = 50,000 \times 0.90 = 45,000 \, \text{lbm}$$

Since 3% of the burnt propellant remains as residual:

$$\text{usable part} = 45,000 \times 0.97 = 43,650 \, \text{lbm}$$

Now, calculate the propellant mass flow rate $\dot{m}$:

$$\dot{m} = \frac{m_p}{t} = \frac{43650}{50} = 873.0 \, \text{lbm/sec}$$

Calculate the total impulse I_t:

$$I_t = F \times t = 207,000 \times 50 = 10,350,000 \, \text{lb-sec}$$

Finally, calculate the specific impulse I_s at sea level:

$$I_s = \frac{I_t}{W} = \frac{10,350,000}{43650} = 237.1 \, \text{sec}$$

3.2.2 (b) Altitude for Optimum Nozzle Expansion

From Sutton, 9th Edition, Chapter 3, Fig. 3-4, if $k = 1.25$ and with the given chamber pressure (780 psia) and area ratio (10), at optimum expansion, $p_2 = p_3 = \frac{780}{90} = 8.666 \, \text{psia}$.

The nozzle exit area A_e is:

$$A_e = 10A_t = 10 \times 164.2 = 1642 \, \text{in}^2$$

Using Eq. 2-13 from Sutton to solve for the momentum thrust at optimum expansion:

$$\dot{m}v_e = 207,000 - (8.666 \times 14.696) = 216,900 \, \text{lbf}$$

Calculate the specific impulse at optimum expansion:

$$I_s = \frac{216,900}{873} = 248.45 \, \text{sec}$$

From Appendix 2 in Sutton, the altitude for 8.666 psia is approximately 4,200 meters.

3.2.3 (c) Vacuum Conditions

For vacuum conditions, $p_3 = 0$. Using Eq. 2-14:

$$F = 216,900 + 8.67 \times 1642 = 231,000 \, \text{lbf}$$

Calculate the specific impulse in vacuum:

$$I_s = \frac{231,000}{873} = 264.6 \, \text{sec}$$

3.3 Conclusion

The results are:

(a) $\dot{m} = 873.0\,\mathrm{lbm/sec}$, $I_s = 237.1\,\mathrm{sec}$

(b) Altitude for optimum nozzle expansion: 4,200 meters, Thrust: 216,900 lbf, $I_s = 248.45\,\mathrm{sec}$

(c) Thrust in vacuum: 231,000 lbf, $I_s = 264.6\,\mathrm{sec}$