Subscripts

a	actual
g	gas
i	ideal, or a particular species in a mixture
max	maximum
opt	optimum nozzle expansion
s	solid
sep	point of separation
t	nozzle throat
x	any direction or section within rocket nozzle
y	any direction or section within rocket nozzle
0	stagnation or impact condition
1	nozzle inlet or combustion chamber
2	nozzle exit
3	atmospheric or ambient

PROBLEMS

1. Certain experimental results indicate that the propellant gases from a liquid oxygen–gasoline reaction have a mean molecular mass of 23.2 kg/kg-mol and a specific heat ratio of 1.22. Compute the specific heats at constant pressure and at constant volume, assuming perfect-gas relations apply.

2. The actual conditions for an optimum expansion nozzle operating at sea level are given below. Calculate v_2, T_2, and C_F. Use $k = 1.30$ and the following parameters: $\dot{m} = 3.7$ kg/sec; $p_1 = 2.1$ MPa; $T_1 = 2585$ K; $\mathfrak{M} = 18.0$ kg/kg-mol

3. A certain nozzle expands a gas under isentropic conditions. Its chamber or nozzle entry velocity equals 90 m/sec, its final velocity 1500 m/sec. What is the change in enthalpy of the gas? What percentage of error is introduced if the initial velocity is neglected?

4. Nitrogen ($k = 1.38$, molecular mass = 28.00 kg/kg-mol) flows at a Mach number of 2.73 and 500 °C. What are its local and acoustic velocities?

5. The following data are given for an optimum rocket propulsion system:

Average molecular mass	24 kg/kg-mol
Chamber pressure	2.533 MPa
External pressure	0.090 MPa
Chamber temperature	2900 K
Throat area	0.00050 m^2
Specific heat ratio	1.30

Determine **(a)** throat velocity; **(b)** specific volume at throat; **(c)** propellant flow and specific impulse; **(d)** thrust; **(e)** Mach number at the throat.

6. Determine the ideal thrust coefficient for Problem 5 by two methods.

7. A certain ideal rocket with a nozzle area ratio of 2.3 and a throat area of 5 in.^{2} delivers gases at $k = 1.30$ and $R = 66$ ft-lbf/lbm-°R at a chamber pressure of 300 psia and a constant chamber temperature of 5300 °R against a back atmospheric pressure of 10 psia. By means of an appropriate valve arrangement, it is possible to throttle the propellant flow to the thrust chamber. Calculate and plot against pressure the following quantities for 300, 200, and 100 psia chamber pressure: **(a)** pressure ratio between chamber and atmosphere; **(b)** effective exhaust velocity for area ratio involved; **(c)** ideal exhaust velocity for optimum and actual area ratio; **(d)** propellant flow; **(e)** thrust; **(f)** specific impulse; **(g)** exit pressure; **(h)** exit temperature.

8. For an ideal rocket with a characteristic velocity $c^* = 1500$ m/sec, a nozzle throat diameter of 20 cm, a thrust coefficient of 1.38, and a mass flow rate of 40 kg/sec, compute the chamber pressure, the thrust, and the specific impulse.

9. For the rocket propulsion unit given in Example 3–2 compute the new exhaust velocity if the nozzle is cut off, decreasing the exit area by 50%. Estimate the losses in kinetic energy and thrust and express them as a percentage of the original kinetic energy and the original thrust.

10. What is the maximum velocity if the nozzle in Example 3–2 was designed to expand into a vacuum? If the expansion area ratio was 2000?

11. Construction of a variable-area conventional axisymmetric nozzle has often been considered to operate a rocket thrust chamber at the optimum expansion ratio at any altitude. Because of the enormous difficulties of such a mechanical device, it has never been successfully realized. However, assuming that such a mechanism could eventually be constructed, what would have to be the variation of the area ratio with altitude (plot up to 50 km) if such a rocket had a chamber pressure of 20 atm? Assume that $k = 1.20$.

12. Design a supersonic nozzle to operate at 10 km altitude with an area ratio of 8.0. For the hot gas take $T_0 = 3000$ K, $R = 378$ J/kg-K, and $k = 1.3$. Determine the exit Mach number, exit velocity, and exit temperature, as well as the chamber pressure. If this chamber pressure is doubled, what happens to the thrust and the exit velocity? Assume no change in gas properties. How close to optimum nozzle expansion is this nozzle?

13. The German World War II A-4 propulsion system had a sea-level thrust of 25,400 kg and a chamber pressure of 1.5 Mpa. If the exit pressure is 0.084 MPa and the exit diameter 740 mm, what would be its thrust at 25,000 m?

14. Derive Eq. 3–34. (*Hint*: Assume that all the mass flow originates at the apex of the cone.) Calculate the nozzle angle correction factor for a conical nozzle whose divergence half angle is 13°.

15. Assuming that the thrust correction factor is 0.985 and the discharge correction factor is 1.050 in Example 3–2, determine **(a)** the actual thrust; **(b)** the actual exhaust velocity; **(c)** the actual specific impulse; **(d)** the velocity correction factor.

16. An ideal rocket has the following characteristics:

Chamber pressure	27.2 atm
Nozzle exit pressure	3 psia
Specific heat ratio	1.20
Average molecular mass	21.0 lbm/lb-mol
Chamber temperature	4200 °F

Determine the critical pressure ratio, the gas velocity at the throat, the expansion area ratio, and the theoretical nozzle exit velocity.

Answers: 0.5645; 3470 ft/sec; 14.8; and 8570 ft/sec.

17. For an ideal rocket with a characteristic velocity c^* of 1220 m/sec, a mass flow rate of 73.0 kg/sec, a thrust coefficient of 1.50, and a nozzle throat area of 0.0248 m^2, compute the effective exhaust velocity, the thrust, the chamber pressure, and the specific impulse.

Answer: 1830 m/sec; 133,560 N; 3.590×10^6 N/m^2; 186.7 sec.

18. Derive Eqs. 3–24 and 3–25.

19. An upper stage of a launch vehicle propulsion unit fails to meet expectations during sea-level testing. This unit consists of a chamber at 4.052 MPa feeding hot propellant to a supersonic nozzle of area ratio $\epsilon = 20$. The local atmospheric pressure at the design condition is 20 kPa. The propellant has a $k = 1.2$ and the throat diameter of the nozzle is 9 cm.

a. Calculate the ideal thrust at the design condition.
b. Calculate the ideal thrust at the sea-level condition.
c. State the most likely source of the observed nonideal behavior.

Answer: **(a)** 44.4 kN, **(b)** 34.1 kN, **(c)** separation in the nozzle.

20. Assuming ideal flow within some given propulsion unit:

a. State all necessary conditions (realistic or not) for

$$c^* = c = v_2$$

b. Do the above conditions result in an *optimum* thrust for a given p_1/p_3?
c. For a launch vehicle designed to operate at some intermediate Earth altitude, sketch (in absolute or relative values) how c^*, c, and v_2 would vary with altitude.

21. A rocket nozzle has been designed with $A_t = 19.2$ in.^{2} and $A_2 = 267$ in.^{2} to *operate optimally* at $p_3 = 4$ psia and produce 18,100 lbf of *ideal thrust* with a chamber pressure of 570 psia. It will use the proven design of a previously built combustion chamber that operates at $T_1 = 6000\,°R$ with $k = 1.25$ and $R = 68.75$ ft-lbf/lbm°R, with a c^*-efficiency of 95%. But test measurements on this thrust system, at the stated pressure conditions, yield a thrust of only 16,300 lbf when the measured flow rate is 2.02 lbm/sec. Find the applicable correction factors ($\zeta_F, \zeta_d, \zeta_{C_F}$) and the actual specific impulse *assuming frozen flow* throughout.

Answers: $\zeta_F = 0.90; \zeta_d = 1.02; \zeta_{C_F} = 0.929; (I_s)_a = 250$ sec.