Derivation of the Choked Flow Equation Using Density

1 Standard Choked Flow Equation

The choked mass flow rate equation for an ideal gas, using the specific gas constant R_s, is given by:

$$\dot{m} = A^* P_0 \sqrt{\frac{\gamma}{R_s T_0} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}}} \tag{1}$$

where:

- $\dot{m}$ = mass flow rate
- A^* = throat area
- P_0 = stagnation pressure
- T_0 = stagnation temperature
- R_s = specific gas constant (R/M, where R is the universal gas constant and M is the molar mass)
- γ = heat capacity ratio

2 Expressing R_s in Terms of Density

From the ideal gas law:

$$P = \rho R_s T \tag{2}$$

At stagnation conditions:

$$P_0 = \rho_0 R_s T_0 \tag{3}$$

Rearranging for R_s:

$$R_s = \frac{P_0}{\rho_0 T_0} \tag{4}$$

3 Substituting R_s into the Choked Flow Equation

Substituting R_s from equation (3) into equation (1):

$$\dot{m} = A^* P_0 \sqrt{\frac{\gamma}{\frac{P_0}{\rho_0 T_0} T_0} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}}} \tag{5}$$

Simplifying:

$$\dot{m} = A^* P_0 \sqrt{\frac{\gamma \rho_0}{P_0} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}}} \tag{6}$$

Since $P_0/P_0 = 1$, we obtain:

$$\dot{m} = A^* \sqrt{\gamma \rho_0 P_0 \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}}} \tag{7}$$

4 Final Form Using Density

The final equation in terms of density is:

$$\dot{m} = A^* \sqrt{\gamma \rho_0 P_0 \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}}} \tag{8}$$

This confirms that the choked flow equation can be rewritten in terms of density using the ideal gas law.