Single Stage Rocket Parameters

Initial mass	$m_0 = 100\,\mathrm{kg}$
Propellant mass	$m_p = 20\,\mathrm{kg}$
Average I_{sp}	$I_{sp} = 255\,\mathrm{sec}$
Launch Angle	$\theta_0 = 70^\circ$
Burn time	$t_{burn} = 2\,\mathrm{sec}$
Neglect drag	$C_D = 0$

Find:

(a) Initial and burnout acceleration

(b) Burnout velocity, flight angle, position

(c) Apogee, range, time of flight

Calculations

From

$$(a_x)_x = \frac{F\cos\theta - g}{m_0},$$
$$(a_x)_y = \frac{F\sin\theta - g}{m_0}$$

$$F = \dot{m}_p I_{sp} g = \frac{m_p}{t_{burn}}(I_{sp}g) = \frac{20}{2}(255 \cdot 9.81) = 2510.5\,\mathrm{N}$$

so

$$(a_x)_x = \frac{F\cos\theta - g}{m_0} = \frac{(2510.5)\cos(70^\circ) - 9.81}{100} = 7.75\,\mathrm{m/s}^2,$$
$$(a_x)_y = \frac{F\sin\theta - g}{m_0} = \frac{(2510.5)\sin(70^\circ) - 9.81}{100} = 22.11\,\mathrm{m/s}^2$$
$$a_0 = \sqrt{(a_x)_x^2 + (a_x)_y^2} = 23.66\,\mathrm{m/s}^2$$

From

$$(u_p)_x = -c\ln\left(\frac{m_f}{m_0}\right)\cos\theta_0 = -(255 \cdot 9.81)\ln\left(\frac{80}{100}\right)\cos(70^\circ) = 160.42\,\mathrm{m/s},$$
$$(u_p)_y = c\ln\left(\frac{m_f}{m_0}\right)\sin\theta_0 - gt_p = (255 \cdot 9.81)\ln\left(\frac{80}{100}\right)\sin(70^\circ) - (9.81 \cdot 2) = 507.82\,\mathrm{m/s}$$

$$u_p = \sqrt{(u_p)_x^2 + (u_p)_y^2} = 534.53\,\mathrm{m/s}$$

so

$$\theta_p = \arctan\left(\frac{(u_p)_y}{(u_p)_x}\right) = 71.14^\circ$$

From (gravity loss) $\dot{g}_{t_p}$

$$\dot{g}_{t_p} = -c\ln(M_{t_p}) - u_p = 18.7\,\mathrm{m/s} \implies \dot{g} = \frac{18.7}{2} = 4.145\,\mathrm{m/s}^2$$

power flight:

$$y_f = ct_p\left[1 - \ln\left(\frac{m_f}{m_0}\right)\sin\theta - \frac{1}{2}gt_p = 2510.5(2)\left(1 - \ln\left(\frac{80}{100}\right)\sin(71.14^\circ) - \frac{(9.81\cdot 2)}{2}\right) = 483.11\,\mathrm{m}\right.$$

unpowered:

$$y_z = \frac{(u_p)_y^2}{2g} = \frac{(507.82)^2}{2\cdot 9.81} = 13247.1\,\mathrm{m}$$

so zenith location

$$y_z = 13977.21\,\mathrm{m} \quad \text{(will be zenith location)} = \text{apogee}$$

From

$$g_0 = g\left[\frac{R_e}{R_e + h}\right] = 9.81\left[\frac{6378383}{6378383 + 483.11}\right] = 9.78\,\mathrm{m/s}^2$$

so

$$(a_g)_x = \frac{F\cos\theta - g}{(m_f} = \frac{(2510.5)\cos(71.14^\circ) - 9.78}{80} = 10.1\,\mathrm{m/s}^2,$$
$$(a_g)_y = \frac{F\sin\theta - g}{m_f} = \frac{(2510.5)\sin(71.14^\circ) - 9.78}{80} = 20.01\,\mathrm{m/s}^2$$
$$a_g = \sqrt{(a_g)_x^2 + (a_g)_y^2} = 22.38\,\mathrm{m/s}^2$$

From

$$t_{f_{max}} = \sqrt{\frac{2y_z}{g}} = \sqrt{\frac{2\cdot 13977.21}{9.81}} = 53.97\,\mathrm{sec} + t_{f_{max}} = t_f = 53.97\,\mathrm{sec} \quad \text{(only effect by gravity)}$$

since no drag

$$x_p = ct_p\left[1 - \ln\left(\frac{m_f}{m_0}\right)\cos\theta_0 = 2510.5(2)\left(1 - \ln\left(\frac{80}{100}\right)\cos(70^\circ)\right) = 140.42\,\mathrm{m}\right.$$

so

$$x_f = 140.42(51.17) + 140.07 = 10011.72\,\mathrm{m}$$

From

$$t_{down} = \sqrt{\frac{2y_z}{g}} = \sqrt{\frac{2 \cdot (13977.21)}{9.81}} = 53.97\,\mathrm{sec} \quad \mathrm{TOF} = t_f + t_{down} = 53.97 + 53.97 = 105.94\,\mathrm{sec}$$

so

$$x_{range} = x_f + x_{down} = 10011.72 + 10008.05 = 20019.77\,\mathrm{m}$$

KE input

$$\mathrm{KE}_{\mathrm{input}} = \frac{1}{2}m_0\left[(u_0)_x^2 + (u_0)_y^2\right] = 0.5(100)\left((140.42)^2 + (507.82)^2\right) = 13.09\,\mathrm{MJ}$$