University of Bristol

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Sensors, Signals and Control

Part 1: Identification of Transfer Functions

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1 Introduction and Open Loop Discussion

This report compares the experimental and analytical transfer functions of a 3-degrees-of-freedom Quanser-Control rig. Control system design is important in understanding the behaviour of dynamic systems, to improve the performance. Sensors and actuators are used in the Quanser to measure and vary performance characteristics of the Quanser. In this experiment, an elevation angle change was introduced to the Quanser revealing an oscillating damped behaviour. Measuring this response an empirical transfer function was then created, to closely match the observed behaviour.

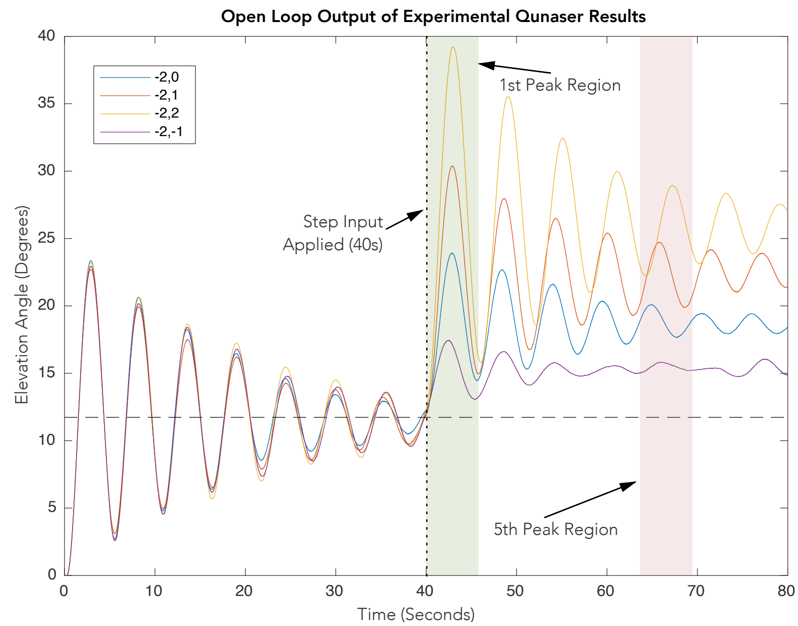

Figure 1: Graph Showing the Open Loop Nature of the Experimental Quanser Response

An Open-Loop system is a where behavioural characteristics can be controlled manually. These changes are not fed back into the system, therefore the output has no effect on the input of the system [1]; meaning self-correction is not possible. In the case of the Quanser-Control Rig, elevation angle was independent of the output and was manually controlled, whereas pitch and travel data was manually fed back to the system. Figure 1 shows the open loop behaviour of the elevation axis, as the elevation angle was varied between 10 and 40 degrees.

2 Method

2.1 Finding Experimentation Results from Quansers

1. Replace the elevator input with a step block, using the parameters: Start Time: 40s, Start Value: -2, End Value: -1 to 2. This was used to automate the step input at a specific time for each elevator input test. A step input time of 40s was chosen to allow the initial response to settle to an acceptable level (see Figure 1).
2. Use range of elevator input values, varying from an initial value of -2 to 2 in steps of 1. Repeat each test case, saving workspace variables.
3. Check the data for anomalies, average valid repeats to obtain the experimental response across the different step inputs, see Figure 1.

2.2 Analyse Transfer Function

Isolate elevation values for corresponding inputs from 40s to 80s; this captures the response after the step input.In order to estimate the transfer function, the following parameters were calculated: Natural Frequency ω_d, Undamped Natural Frequency ω_n and the Damping ratio ζ. Refer to Table 1 for equations used.

2.2.1 Second Order

To calculate the Second Order Transfer-Function, a forced response behaviour was noted. Using Table 1 equations and the x and y values taken from the peaks highlighted in Figure 1, the logarithmic decrement and the period of damped oscillation was approximated. Using these equations values for the Damping Ratio ζ and Natural Frequency ω_n were obtained and substituted into the formal equation shown in equation 1.

$$\frac{y(s)}{u(s)} = \frac{k \cdot \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{1}$$

In order to obtain a single transfer function which represents the system as a whole for varying elevators angle steps, analytical transfer functions were found for each test case. These parameters were then averaged obtaining a single fit transfer function where gain was changed to model different elevator angle step inputs. This captured some of the behaviour of the system as step level varied.

Table 1: Table Showing Key Parameter Calculations [2]

1-DOF Parameters	Definition	Units	Identification Strategy
Λ_i	Logarithmic Decrement	-	$\Lambda_i = \frac{1}{N}\ln(\frac{y_i}{y_{i+N}})$
ζ_i	Damping Ratio	-	$\zeta_i = \frac{\Lambda_i}{2\pi}$
T_D	Period of damped Oscillation	s	$T_D = \frac{x_{i+N} - x_i}{N}$
ω_D	Damped Natural frequency	rads^{-1}	$\omega_D = \frac{2\pi}{T_D}$
ω_n	Undamped Natural Frequency	rads^{-1}	$\omega_n = \frac{\omega_D}{\sqrt{1-\zeta_i^2}}$

To calculate a representative gain scaling factor k for each of the step inputs, max elevation for each step input was taken (from the first peak shown in Figure 1). These values were plotted to find their correlation, shown in figure 2. The gain k value was heuristically adjusted to match the analytical amplitude with the experimental results from -2 to 2, where $k = 0.26$ showed a good fit. This value was used to translate the correlation equation between points into scaling factors.

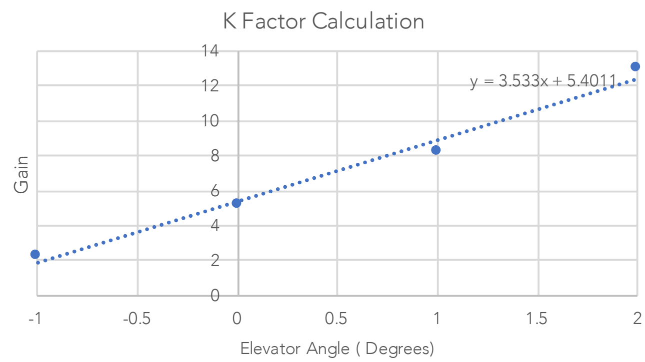

Figure 2: Graph Finding the Gain For Each Step Input

After applying this method, amplitudes for all the steps fitted more closely. Finally damping ratio and undamped natural frequency were tweaked to give the final fit, again by trial and error.

2.2.2 First Order

Standard First Order Response Transfer Function:

$$\frac{y(s)}{u(s)} = \frac{k}{\tau s + 1} \tag{2}$$

Due to the Quanser-Control Rig being a Second Order System, Equation 2 was not applicable for finding the First Order Transfer Function. Instead this was estimated by considering the Second Order Transfer Function case where: $\zeta = 1$ and $s^2 = 0$.

$$\frac{k \cdot \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \rightarrow \frac{k \cdot \omega_n^2}{2\omega_n s + \omega_n^2} \tag{3}$$

3 Results

2nd Order Transfer Function :

$$k \cdot \frac{1.109}{s^2 + 0.1313s + 1.109} \tag{4}$$

Where: $\zeta = 0.0623, \quad \omega_n = 1.0532$

1st Order Transfer Function :

$$k \cdot \frac{1.109}{2.106s + 1.109} \tag{5}$$

Where: $\zeta = 1, \quad \omega_n = 1.0532$

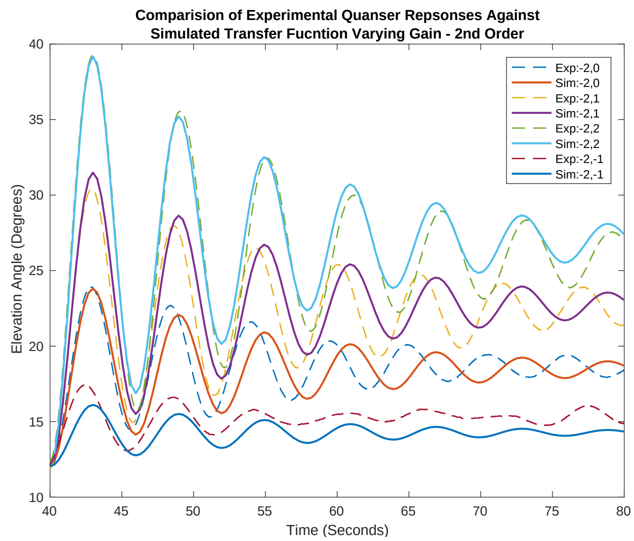

Figure 3: 2nd Order Simualted Transfer Function Comparision

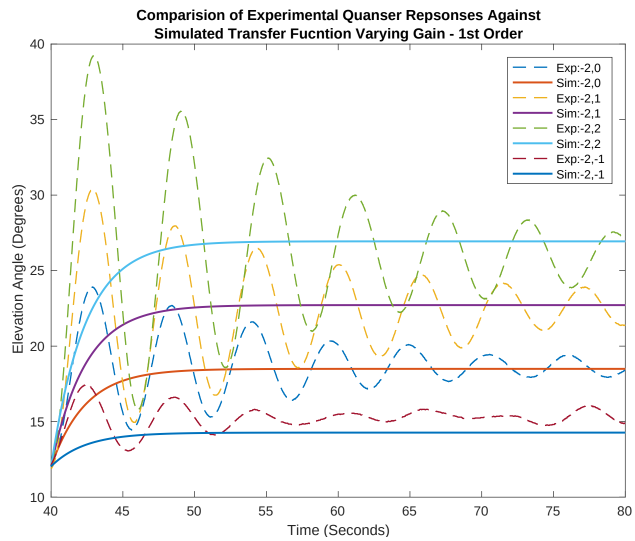

Figure 4: 1st Order Simualted Transfer Function Comparision

Second Order Step Information:

$$t_r = 1.0385 \qquad t_s = 57.3382$$

First Order Step Information:

$$t_r = 4.1719 \qquad t_s = 7.4287$$

4 Observations and Analysis:

4.1 Experimental Elevator Input:

- There is an observable phase and amplitude deviation, particularly after the third peak. The phase shift becomes increasingly out of phase as the initial step input is reduced.
- Peak amplitude prediction becomes worse for lower amplitude cases, however there is a very good match for 0,1 and 2 test cases first oscillations, the k value equation provided a good match. Note this equation was set heuristically to a step input of 2.
- The oscillations around the expected steady level were slightly larger above the steady-state level rather than below. This could be an artefact of the Quanser accumulating elevation drift.
- Steady state values match well for an elevator input of 0 and 1, however are noticeably different for -1 and 2.

4.2 Root-Locus plot (See Figure 5)

- The grid lines represent lines of constant damping and lines of natural frequency
- The Second Order Transfer Function Root-Locus has a pair of points with an imaginary axis component (complex root) - This shows the stable underdamped nature of the system as $0 < \zeta < 1$
- The First Order Transfer Function Root-Locus diagram point exists purely in the real axis component, meaning the function is critically damped as expected and stable.
- Increase in gain for the Second Order shows the roots becoming more positive and negative in the imaginary axis respectively
- Increase in gain for the First Order shows the root becoming more negative along the real axis

5 Discussion

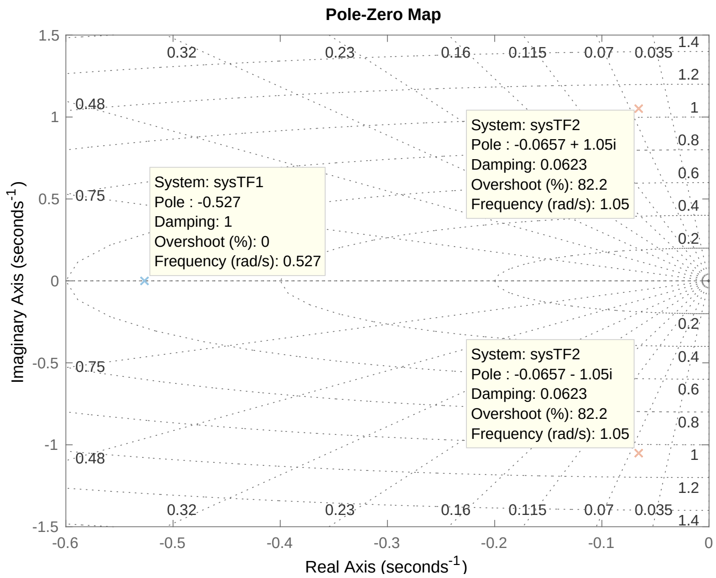

Figure 5: Map of the Poles, sysTF1 = First Order, sysTF2 = Second Order

Observing Figure 3 and 4 many deviation's from MATLAB's analytical transfer function step response were observed. During the Quanser operation, a significant observable error was drift, more noticeable during longer runs; potentially due to accumulating error increasing with time. Whilst the Quansers have error correcting features (through the closed-loop nature of the other parameters), this is only sensitive to a finite degree so could be considered imperfect. An initial elevator input was set to -2 (to get the fans running), and then after 40 seconds, an elevator step was input into the system. As a result the step input may have been applied during mid oscillation past the steady state elevation position; this likely either amplified or decreased the actual step input depending on which stage of oscillation the Quanser was at, at the moment of step input. The 40 second run reduced the steady-state level to 10-20% of the steady state value. Errors may accumulate for a greater run time as the system fails to accurately correct for inconsistencies.

Another source of deviation is inaccuracy in step input time. In theory, the step input is modelled as an immediate action. A step block was introduced in the Simulink model to automate the `elevator input' at 40 seconds, closer observation of the corresponding pitch and travel data plots revealed a slight delay. Accumulated lag in the system and controller and a delayed response time both contributed to latency. Latency causes a phase and amplitude shift in experimental results, as well as introducing error in parameters such as the logarithmic decrement Λ_i and damped frequency ω_d calculations.

From a mechanical perspective, friction in the Quanser hinge support and tension from the power cables resulted in a reduction in expected elevation. This adds to the natural damping of the system, one explaining for the compacted peaks in experimental data (see Figure 3), and could explain some variation in repeats which was larger than would be expected for a fixed experiment.

As with any control system, noise can be introduced by external and internal factors. In the Quanser system, gyro noise and wind resistance are contributing factors. Sensors in any system measure quantities which need to be controlled. In this case, the elevator sensor sampling was storing values at discrete points. Whilst the sampling time was quite small, capturing the general nature of the oscillating damped curves, some critical points may have been missed. An example can be observed at the point of highest amplitude, the inflexion behaviour begins after a region of constant amplitude. Computationally we see a quicker inflexion transition in this region.

References

[1] E. Tutorials. (). Open-loop system and open-loop control systems, [Online]. Available: http://www.electronics-tutorials.ws/systems/open-loop-system.html (visited on 22/02/2017).

[2] D. B. Titurus, ``Vibrations 2 lecture notes - 2 degree of freedom, tuned vibration absorber'', University of Bristol, Tech. Rep., 2016.