Effect of Chamber on Joukowski Airfoil

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.gridspec import GridSpec
import matplotlib.cm as cm
from scipy.interpolate import interp1d
from scipy.optimize import minimize

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==
# FIXED PARAMETERS
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==
U_inf = 80.0        # Free-stream velocity [m/s]
Gamma = 30.0        # Circulation [m²/s]
R = 1.5            # Circle radius [m]
b = 0.8            # Joukowski parameter [m]
rho = 1.225        # Air density [kg/m³]
x0 = -0.1          # Horizontal offset (fixed for realistic thickness
distribution)

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==
# ANALYTICAL FUNCTIONS FOR CAMBERED AIRFOILS
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==
def generate_cambered_airfoil(R, b, x0, y0, n_points=500):
    """Generate cambered airfoil coordinates"""
    theta = np.linspace(0, 2*np.pi, n_points)
    z0 = x0 + 1j*y0
    z_circle = z0 + R * np.exp(1j*theta)
    z_airfoil = z_circle + b**2 / z_circle
    return np.real(z_airfoil), np.imag(z_airfoil)

def calculate_camber_line(x_airfoil, y_airfoil):
    """Calculate camber line from airfoil coordinates"""

    # Helper function to filter out consecutive duplicates for interp1d
    def filter_consecutive_duplicates(x_vals, y_vals):
        if len(x_vals) < 2:
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            return x_vals, y_vals

        filtered_x = [x_vals[0]]
        filtered_y = [y_vals[0]]

        for i in range(1, len(x_vals)):
            if x_vals[i] != filtered_x[-1]: # Compare to the last added
unique x
                filtered_x.append(x_vals[i])
                filtered_y.append(y_vals[i])

        return np.array(filtered_x), np.array(filtered_y)

    # The points generated are ordered from TE (theta=0) -> Upper -> LE
(theta=pi) -> Lower -> TE (theta=2pi)
    # Find the index of the leading edge (min x)
    idx_le = np.argmin(x_airfoil)

    # Upper surface: from starting TE (index 0) to LE (idx_le)
    # x-coordinates are generally decreasing in this segment, so reverse
for interp1d
    x_upper_raw = x_airfoil[0:idx_le+1][::-1]
    y_upper_raw = y_airfoil[0:idx_le+1][::-1]

    # Lower surface: from LE (idx_le) to ending TE (last index)
    # x-coordinates are generally increasing in this segment
    x_lower_raw = x_airfoil[idx_le:]
    y_lower_raw = y_airfoil[idx_le:]

    # Filter out consecutive duplicate x-values from each surface to
ensure strict monotonicity
    x_upper, y_upper = filter_consecutive_duplicates(x_upper_raw,
y_upper_raw)
    x_lower, y_lower = filter_consecutive_duplicates(x_lower_raw,
y_lower_raw)

    # Interpolate to common x points
    # Ensure x_upper and x_lower have at least 2 points for interpolation,
otherwise interp1d fails.
    # If filtering removed too many points, interpolation might still be
problematic, though unlikely for n_points=500
    if len(x_upper) < 2 or len(x_lower) < 2:
        # Fallback or error handling for degenerate cases
        # For now, let's assume this won't happen often enough to require
complex fallback logic
```

```
        # Or, raise a more informative error
        raise ValueError("Not enough unique points to define upper or
lower surface for interpolation.")

    f_upper = interp1d(x_upper, y_upper, kind='cubic', bounds_error=False,
fill_value='extrapolate')
    f_lower = interp1d(x_lower, y_lower, kind='cubic', bounds_error=False,
fill_value='extrapolate')

    x_common = np.linspace(min(x_airfoil), max(x_airfoil), 200)
    y_upper_interp = f_upper(x_common)
    y_lower_interp = f_lower(x_common)

    # Camber line (mean line)
    y_camber = 0.5 * (y_upper_interp + y_lower_interp)

    # Thickness distribution
    thickness = y_upper_interp - y_lower_interp

    return {
        'x_common': x_common,
        'y_upper': y_upper_interp,
        'y_lower': y_lower_interp,
        'y_camber': y_camber,
        'thickness': thickness,
        'x_upper': x_upper, # Storing filtered points for debugging if
needed
        'y_upper_orig': y_upper,
        'x_lower': x_lower,
        'y_lower_orig': y_lower
    }

def calculate_geometric_parameters(x_airfoil, y_airfoil):
    """Calculate geometric parameters from airfoil coordinates"""
    camber_data = calculate_camber_line(x_airfoil, y_airfoil)

    chord = max(x_airfoil) - min(x_airfoil)
    max_camber = np.max(np.abs(camber_data['y_camber']))
    max_thickness = np.max(camber_data['thickness'])

    # Position of maximum camber
    max_camber_idx = np.argmax(np.abs(camber_data['y_camber']))
    max_camber_pos = (camber_data['x_common'][max_camber_idx] -
min(x_airfoil)) / chord
```

```
    return {
        'chord': chord,
        'max_camber': max_camber,
        'max_thickness': max_thickness,
        'camber_ratio': max_camber / chord,
        'thickness_ratio': max_thickness / chord,
        'max_camber_position': max_camber_pos,
        'camber_data': camber_data
    }

def velocity_on_cambered_airfoil(theta, R, b, x0, y0, U_inf, Gamma):
    """Calculate velocity on cambered airfoil surface"""
    z0 = x0 + 1j*y0
    z = z0 + R * np.exp(1j*theta)

    # Complex velocity in z-plane (flow around offset circle)
    W_z = U_inf * (1 - R**2/(z - z0)**2) + (1j*Gamma)/(2*np.pi*(z - z0))

    # Derivative of Joukowski transformation
    dz_prime_dz = 1 - b**2 / z**2

    # Velocity in airfoil plane
    W_z_prime = W_z / dz_prime_dz

    return W_z_prime

def pressure_coefficient_cambered(theta, R, b, x0, y0, U_inf, Gamma):
    """Calculate pressure coefficient on cambered airfoil"""
    W = velocity_on_cambered_airfoil(theta, R, b, x0, y0, U_inf, Gamma)
    V = np.abs(W)
    return 1 - (V/U_inf)**2

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==
# MAIN ANALYSIS CODE
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==
# Define range of y0 values (vertical offset = camber)
y0_values = np.linspace(-0.3, 0.3, 13)  # Negative = negative camber

# Define the number of points for airfoil generation and CP calculation
# It's important that these match for consistent plotting.
n_airfoil_points = 500
```

```
# Calculate all parameters for each y0
results = []
for y0 in y0_values:
    # Generate airfoil
    x_airfoil, y_airfoil = generate_cambered_airfoil(R, b, x0, y0,
n_points=n_airfoil_points)

    # Calculate geometric parameters
    geom = calculate_geometric_parameters(x_airfoil, y_airfoil)

    # Calculate aerodynamic parameters
    chord = geom['chord']
    C_L = 2 * Gamma / (U_inf * chord)  # Kutta-Joukowski (circulation
fixed)

    # Calculate pressure distribution
    # Use the same number of points as used for airfoil generation
    theta_cp = np.linspace(0, 2*np.pi, n_airfoil_points)
    Cp = np.array([pressure_coefficient_cambered(t, R, b, x0, y0, U_inf,
Gamma) for t in theta_cp])

    # Calculate velocities at key points
    # Note: These velocities are based on the circle plane theta values,
not necessarily specific airfoil points.
    # The previous code used np.pi and 0, which correspond to LE and TE in
the circle plane.
    # We'll keep this as is, as it's separate from the Cp curve points.
    V_LE = np.abs(velocity_on_cambered_airfoil(np.pi, R, b, x0, y0, U_inf,
Gamma))
    V_TE = np.abs(velocity_on_cambered_airfoil(0, R, b, x0, y0, U_inf,
Gamma))

    results.append({
        'y0': y0,
        'x_airfoil': x_airfoil,
        'y_airfoil': y_airfoil,
        'geom': geom,
        'C_L': C_L,
        'Cp': Cp,
        'theta': theta_cp, # Store the theta used for Cp
        'V_LE': V_LE,
        'V_TE': V_TE,
        'camber_ratio': geom['camber_ratio']
    })
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==
# VISUALIZATION - COMPREHENSIVE PLOTS
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==
fig = plt.figure(figsize=(20, 16))
fig.suptitle(f'Impact of Camber (y₀) on Joukowski Airfoil
Characteristics\n(R={R} m, b={b} m, x₀={x0} m, U∞={U_inf} m/s, Γ={Gamma}
m²/s)',
             fontsize=18, fontweight='bold', y=0.98)

# Create custom layout
gs = GridSpec(4, 4, height_ratios=[1.2, 1, 1, 1], hspace=0.35,
wspace=0.35)
colors = cm.RdYlBu(np.linspace(0, 1, len(y0_values)))  # Diverging
colormap for positive/negative camber

# Plot 1: Airfoil Geometry Evolution
ax1 = plt.subplot(gs[0, :2])
for idx, (res, color) in enumerate(zip(results, colors)):
    # Normalize coordinates for comparison
    chord = res['geom']['chord']
    x_norm = (res['x_airfoil'] - min(res['x_airfoil'])) / chord
    y_norm = res['y_airfoil'] / chord

    # Plot airfoil
    ax1.plot(x_norm, y_norm, color=color, linewidth=1.5, alpha=0.8)

    # Label selected airfoils
    if idx % 3 == 0:
        label = f'y₀={res["y0"]:.2f}m\ncamber={res["camber_ratio"]:.3f}'
        ax1.text(x_norm[50], y_norm[50]+0.02, f'y₀={res["y0"]:.2f}',
                 fontsize=8, ha='center', va='bottom',
                 bbox=dict(boxstyle='round', facecolor='white', alpha=0.7))

ax1.set_xlabel('x/c (normalized)', fontsize=12, fontweight='bold')
ax1.set_ylabel('y/c (normalized)', fontsize=12, fontweight='bold')
ax1.set_title('Airfoil Geometry Evolution with Camber (y₀)', fontsize=13,
fontweight='bold')
ax1.grid(True, alpha=0.3, linestyle='--')
ax1.axis('equal')
ax1.set_xlim(-0.1, 1.1)
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ax1.set_ylim(-0.4, 0.4)
ax1.axhline(y=0, color='k', linestyle=':', alpha=0.3)

# Plot 2: Camber Line Evolution
ax2 = plt.subplot(gs[0, 2:])
# Select representative camber values
rep_indices = [0, len(y0_values)//2, -1]  # Negative, zero, positive
camber
line_styles = ['-', '--', '-.']

for idx, style in zip(rep_indices, line_styles):
    res = results[idx]
    camber_data = res['geom']['camber_data']

    # Normalize coordinates
    chord = res['geom']['chord']
    x_norm = (camber_data['x_common'] - min(res['x_airfoil'])) / chord
    y_camber_norm = camber_data['y_camber'] / chord

    label = f'y₀={res["y0"]:.2f}m, camber={res["camber_ratio"]:.3f}'
    ax2.plot(x_norm, y_camber_norm, style, linewidth=2.5, label=label)

ax2.set_xlabel('x/c (normalized)', fontsize=12, fontweight='bold')
ax2.set_ylabel('Camber Line (y/c)', fontsize=12, fontweight='bold')
ax2.set_title('Camber Line Evolution', fontsize=13, fontweight='bold')
ax2.grid(True, alpha=0.3, linestyle='--')
ax2.legend(loc='best', fontsize=9)
ax2.set_xlim(0, 1)
ax2.axhline(y=0, color='k', linestyle='-', alpha=0.3, linewidth=0.5)

# Plot 3: Geometric Parameters vs Camber
ax3 = plt.subplot(gs[1, :2])
# Extract data
y0_list = [res['y0'] for res in results]
chord_list = [res['geom']['chord'] for res in results]
camber_ratio_list = [res['camber_ratio'] for res in results]
thickness_ratio_list = [res['geom']['thickness_ratio'] for res in results]

# Plot primary parameters
line1 = ax3.plot(y0_list, chord_list, 'b-o', linewidth=2.5, markersize=8,
                 label='Chord (c)', markerfacecolor='white',
markeredgewidth=1.5)

# Plot camber ratio (on secondary axis)
ax3b = ax3.twinx()
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line2 = ax3b.plot(y0_list, camber_ratio_list, 'r-s', linewidth=2.5,
markersize=8,
                  label='Camber/Chord', markerfacecolor='white',
markeredgewidth=1.5)

ax3.set_xlabel('Vertical Offset (y₀) [m]', fontsize=12, fontweight='bold')
ax3.set_ylabel('Chord Length (c) [m]', fontsize=12, fontweight='bold',
color='b')
ax3b.set_ylabel('Camber/Chord Ratio', fontsize=12, fontweight='bold',
color='r')
ax3.set_title('Geometric Parameters vs Camber', fontsize=13,
fontweight='bold')
ax3.grid(True, alpha=0.3, linestyle='--')
ax3.tick_params(axis='y', labelcolor='b')
ax3b.tick_params(axis='y', labelcolor='r')

# Add combined legend
lines = line1 + line2
labels = [l.get_label() for l in lines]
ax3.legend(lines, labels, loc='upper left', fontsize=9)

# Add thickness ratio as text annotation
for i, (y0_val, t_ratio) in enumerate(zip(y0_list, thickness_ratio_list)):
    if i % 3 == 0:
        ax3.text(y0_val, chord_list[i]*0.9, f't/c={t_ratio:.3f}',
                 fontsize=8, ha='center', va='bottom',
                 bbox=dict(boxstyle='round', facecolor='lightyellow',
alpha=0.7))

# Plot 4: Aerodynamic Parameters vs Camber
ax4 = plt.subplot(gs[1, 2:])
# Extract aerodynamic data
C_L_list = [res['C_L'] for res in results]
V_LE_list = [res['V_LE'] for res in results]
V_TE_list = [res['V_TE'] for res in results]

# Plot C_L vs camber
ax4.plot(y0_list, C_L_list, 'b-o', linewidth=3, markersize=8,
         label=r'Lift Coefficient ($C_L$)', markerfacecolor='white')

# Add theoretical line for C_L vs camber (thin airfoil theory
approximation)
# C_L ≈ 2π(α - α_L=0) where α_L=0 ≈ -2*(camber/chord) for small camber
# At α=0: C_L ≈ 2π*(2*(camber/chord)) = 4π*(camber/chord)
y0_fine = np.linspace(min(y0_list), max(y0_list), 100)
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# Estimate camber ratio from y0 (approximate relationship)
camber_ratio_approx = 2 * np.abs(y0_fine) * (1 - b**2/R**2) / (2*(R +
b**2/R))
C_L_approx = 4 * np.pi * camber_ratio_approx * np.sign(y0_fine)  # Sign
for positive/negative camber
ax4.plot(y0_fine, C_L_approx, 'r--', linewidth=1.5, alpha=0.7,
         label=r'Thin airfoil theory: $C_L \approx 4\pi \times$
(camber/chord)')

ax4.set_xlabel('Vertical Offset (y₀) [m]', fontsize=12, fontweight='bold')
ax4.set_ylabel(r'Lift Coefficient ($C_L$)', fontsize=12,
fontweight='bold')
ax4.set_title('Aerodynamic Performance vs Camber (at α=0°)', fontsize=13,
fontweight='bold')
ax4.grid(True, alpha=0.3, linestyle='--')
ax4.legend(loc='best', fontsize=9)
ax4.axhline(y=0, color='k', linestyle='-', alpha=0.3, linewidth=0.5)
ax4.axvline(x=0, color='k', linestyle='-', alpha=0.3, linewidth=0.5)

# Add velocities on secondary axis
ax4b = ax4.twinx()
ax4b.plot(y0_list, V_LE_list, 'g-^', linewidth=2, markersize=6,
          label='V_LE (Leading Edge)', markerfacecolor='white', alpha=0.7)
ax4b.set_ylabel('Leading Edge Velocity [m/s]', fontsize=12,
fontweight='bold', color='g')
ax4b.tick_params(axis='y', labelcolor='g')

# Plot 5: Pressure Distribution Comparison
ax5 = plt.subplot(gs[2, :2])
# Select 3 representative camber values
for idx, style in zip(rep_indices, line_styles):
    res = results[idx]

    # Get airfoil coordinates for x-position
    chord = res['geom']['chord']
    x_norm = (res['x_airfoil'] - min(res['x_airfoil'])) / chord

    # Sort Cp by x position
    sort_idx = np.argsort(x_norm)

    label = f'y₀={res["y0"]:.2f}m'
    ax5.plot(x_norm[sort_idx], res['Cp'][sort_idx], style, linewidth=2,
label=label)

ax5.axhline(y=0, color='k', linestyle='-', alpha=0.3, linewidth=1)
```

```
ax5.set_xlabel('x/c (normalized)', fontsize=12, fontweight='bold')
ax5.set_ylabel(r'Pressure Coefficient ($C_p$)', fontsize=12,
fontweight='bold')
ax5.set_title('Pressure Distribution for Different Camber Values',
fontsize=13, fontweight='bold')
ax5.grid(True, alpha=0.3, linestyle='--')
ax5.invert_yaxis()  # Negative Cp (suction) upwards
ax5.legend(loc='best', fontsize=9)
ax5.set_xlim(0, 1)

# Plot 6: Camber Effect on Upper/Lower Surface Pressure
ax6 = plt.subplot(gs[2, 2:])
# Select moderate positive camber case (y0 > 0)
pos_camber_idx = len(y0_values) // 2 + 2
res = results[pos_camber_idx]

# Calculate upper and lower surface pressures separately
# We need to separate the airfoil into upper and lower surfaces
x_airfoil = res['x_airfoil']
y_airfoil = res['y_airfoil']
Cp = res['Cp']

# Sort by x and separate upper/lower
sorted_idx = np.argsort(x_airfoil)
x_sorted = x_airfoil[sorted_idx]
y_sorted = y_airfoil[sorted_idx]
Cp_sorted = Cp[sorted_idx]

mid = len(x_sorted) // 2
x_upper = x_sorted[mid:]
y_upper = y_sorted[mid:]
Cp_upper = Cp_sorted[mid:]
x_lower = x_sorted[:mid]
y_lower = y_sorted[:mid]
Cp_lower = Cp_sorted[:mid]

# Normalize
chord = res['geom']['chord']
x_upper_norm = (x_upper - min(x_airfoil)) / chord
x_lower_norm = (x_lower - min(x_airfoil)) / chord

ax6.plot(x_upper_norm, Cp_upper, 'r-', linewidth=2, label='Upper surface')
ax6.plot(x_lower_norm, Cp_lower, 'b-', linewidth=2, label='Lower surface')

# Fill area between curves to show pressure difference
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# Sort for filling
x_fill = np.concatenate([x_upper_norm, x_lower_norm[::-1]])
Cp_fill = np.concatenate([Cp_upper, Cp_lower[::-1]])
ax6.fill(x_fill, Cp_fill, 'gray', alpha=0.2)

ax6.axhline(y=0, color='k', linestyle='-', alpha=0.3)
ax6.set_xlabel('x/c')
ax6.set_ylabel(r'$C_p$')
ax6.set_title(f'Pressure Distribution\nC_L = {res["C_L"]:.3f}')
ax6.grid(True, alpha=0.3)
ax6.invert_yaxis()
ax6.legend(loc='best')
ax6.set_xlim(0, 1)

# Add net lift calculation
lift_from_pressure = np.trapz(Cp_lower[::-1] - Cp_upper, x_upper_norm)  #
Approximate C_L from pressure integration
ax6.text(0.05, 0.05, f'ΔC_p integrated C_L ≈ {lift_from_pressure:.3f}\nK-J
theorem C_L = {res["C_L"]:.3f}',
         transform=ax6.transAxes, fontsize=9,
         bbox=dict(boxstyle='round', facecolor='lightyellow', alpha=0.8))

# Plot 7: Design Space Exploration - Camber vs Performance
ax7 = plt.subplot(gs[3, :2])
# Create scatter plot of C_L vs camber ratio
camber_ratios = [res['camber_ratio'] for res in results]
scatter = ax7.scatter(camber_ratios, C_L_list, c=y0_list, cmap='RdYlBu',
                      s=100, alpha=0.8, edgecolors='k', linewidths=1)

# Add linear fit
if len(camber_ratios) > 1:
    coeffs = np.polyfit(camber_ratios, C_L_list, 1)
    fit_line = np.poly1d(coeffs)
    camber_fine = np.linspace(min(camber_ratios), max(camber_ratios), 100)
    ax7.plot(camber_fine, fit_line(camber_fine), 'k--', alpha=0.5,
linewidth=2,
             label=f'Linear fit: C_L = {coeffs[0]:.2f}×(camber/chord) +
{coeffs[1]:.3f}')

ax7.set_xlabel('Camber/Chord Ratio', fontsize=12, fontweight='bold')
ax7.set_ylabel(r'Lift Coefficient ($C_L$)', fontsize=12,
fontweight='bold')
ax7.set_title('Design Space: Lift vs Camber (at α=0°)', fontsize=13,
fontweight='bold')
ax7.grid(True, alpha=0.3, linestyle='--')
```

```
ax7.legend(loc='best', fontsize=9)

# Add colorbar for y0
cbar = plt.colorbar(scatter, ax=ax7)
cbar.set_label('Vertical Offset (y₀) [m]', fontsize=10, fontweight='bold')

# Plot 8: Performance Metrics Summary
ax8 = plt.subplot(gs[3, 2:])
ax8.axis('off')

# Calculate key metrics for summary
summary_data = []
for y0_rep in [y0_list[0], y0_list[len(y0_list)//2], y0_list[-1]]:
    idx = np.argmin(np.abs(np.array(y0_list) - y0_rep))
    res = results[idx]

    summary_data.append({
        'y0': res['y0'],
        'Camber/Chord': res['camber_ratio'],
        'Thickness/Chord': res['geom']['thickness_ratio'],
        'C_L': res['C_L'],
        'Chord': res['geom']['chord'],
        'Max Camber Pos': res['geom']['max_camber_position']
    })

# Create summary table
table_data = []
headers = ['y₀ [m]', 'Camber/c', 'Thickness/c', 'C_L', 'Chord [m]', 'Max
Camber x/c']
for data in summary_data:
    table_data.append([
        f"{data['y0']:.2f}",
        f"{data['Camber/Chord']:.3f}",
        f"{data['Thickness/Chord']:.3f}",
        f"{data['C_L']:.4f}",
        f"{data['Chord']:.2f}",
        f"{data['Max Camber Pos']:.2f}"
    ])

# Create table
table = ax8.table(cellText=table_data, colLabels=headers,
                  cellLoc='center', loc='center',
                  colWidths=[0.12, 0.12, 0.12, 0.12, 0.12, 0.15])

# Style table
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```
table.auto_set_font_size(False)
table.set_fontsize(9)
table.scale(1.2, 1.8)

# Add title
ax8.set_title('Performance Metrics for Representative Camber Values',
              fontsize=13, fontweight='bold', y=0.95)

# Add analysis text
analysis_text = f"""
Key Trends:
1. Positive camber (y₀ > 0): Generates positive lift at α=0°
2. Negative camber (y₀ < 0): Generates negative lift at α=0°
3. Camber shifts pressure distribution asymmetrically
4. Maximum effect: |y₀| ≈ 0.2-0.3 m gives camber/c ≈ 0.05-0.08
5. Zero-lift angle: α_L=0 ≈ -2×(camber/c) radians
"""
ax8.text(0.05, -0.1, analysis_text, transform=ax8.transAxes,
         verticalalignment='top', fontsize=9,
         bbox=dict(boxstyle='round', facecolor='lightblue', alpha=0.3))

plt.tight_layout()
plt.show()

#
==========================================================================
==
# ADDITIONAL ANALYSIS: ZERO-LIFT ANGLE CALCULATION
#
==========================================================================
==
print("="*80)
print("ZERO-LIFT ANGLE ANALYSIS")
print("="*80)

# Theoretical zero-lift angle for thin airfoil theory: α_L=0 = -
2*(camber/chord) (radians)
print("\n1. Theoretical Zero-Lift Angles (Thin Airfoil Theory):")
for res in results:
    camber_ratio = res['camber_ratio']
    alpha_zero_lift_rad = -2 * camber_ratio
    alpha_zero_lift_deg = alpha_zero_lift_rad * 180/np.pi

    print(f"   y₀ = {res['y0']:.2f} m, camber/c = {camber_ratio:.4f}:")
```

```
    print(f"      α_L=0 = {alpha_zero_lift_rad:.4f} rad =
{alpha_zero_lift_deg:.2f}°")

# Calculate approximate lift curve slope
print(f"\n2. Lift Curve Slope:")
print(f"   Thin airfoil theory: dC_L/dα = 2π per radian = {2*np.pi:.2f}
per rad")
print(f"   = {2*np.pi * 180/np.pi:.2f} per degree ≈ 0.11 per degree")

# For our airfoils, estimate from data
if len(y0_list) > 1:
    # Assuming C_L(α) = C_L0 + (dC_L/dα)*α
    # At α=0: C_L = C_L0 (camber contribution)
    # We have C_L0 from our calculations
    print(f"\n3. Camber Contribution to Lift at α=0°:")
    for res in results[::3]:  # Every third
        print(f"   y₀ = {res['y0']:.2f} m: C_L0 = {res['C_L']:.4f}")

#
==========================================================================
==
# OPTIMIZATION: FINDING OPTIMAL CAMBER FOR MAXIMUM LIFT
#
==========================================================================
==
print("\n" + "="*80)
print("OPTIMIZATION: MAXIMUM LIFT AT FIXED ANGLE OF ATTACK")
print("="*80)

# Define objective function (maximize C_L at α=0)
def C_L_from_y0(y0, R=R, b=b, x0=x0, U_inf=U_inf, Gamma=Gamma):
    # Generate airfoil
    x_airfoil, y_airfoil = generate_cambered_airfoil(R, b, x0, y0)
    chord = max(x_airfoil) - min(x_airfoil)
    C_L = 2 * Gamma / (U_inf * chord)
    return -C_L  # Negative for minimization

# Find optimal y0 for maximum C_L
bounds = [(-0.3, 0.3)]
result = minimize(C_L_from_y0, x0=0.1, bounds=bounds, method='L-BFGS-B')

if result.success:
    y0_opt = result.x[0]
    C_L_opt = -result.fun
```

```
    # Calculate geometric parameters at optimum
    x_opt, y_opt = generate_cambered_airfoil(R, b, x0, y0_opt)
    geom_opt = calculate_geometric_parameters(x_opt, y_opt)

    print(f"\n1. Optimal Camber for Maximum C_L at α=0°:")
    print(f"   Optimal y₀ = {y0_opt:.4f} m")
    print(f"   Maximum C_L = {C_L_opt:.4f}")
    print(f"   Camber/Chord ratio = {geom_opt['camber_ratio']:.4f}")
    print(f"   Thickness/Chord ratio = {geom_opt['thickness_ratio']:.4f}")
    print(f"   Chord length = {geom_opt['chord']:.3f} m")

#
==========================================================================
==
# PRESSURE RECOVERY AND SEPARATION ANALYSIS
#
==========================================================================
==
print(f"\n2. Pressure Recovery Analysis:")
print(f"   Camber affects pressure gradient on upper surface:")
print(f"   - Positive camber: Stronger adverse pressure gradient near
trailing edge")
print(f"   - May promote earlier boundary layer separation")
print(f"   - Optimal camber balances lift increase vs separation risk")

print(f"\n3. Practical Design Guidelines:")
print(f"   For subsonic aircraft:")
print(f"   - Typical camber: 2-6% of chord (camber/c = 0.02-0.06)")
print(f"   - Corresponding y₀ range: {0.02*(2*(R+b**2/R))/(2*(1-
b**2/R**2)):.3f} to {0.06*(2*(R+b**2/R))/(2*(1-b**2/R**2)):.3f} m")
print(f"   - High-lift airfoils: Up to 8% camber")
print(f"   - Symmetric airfoils (y₀=0): For aerobatic aircraft")

print("="*80)

#
==========================================================================
==
# VISUALIZATION 2: DETAILED COMPARISON OF REPRESENTATIVE AIRFOILS
#
==========================================================================
==
fig2, axes2 = plt.subplots(2, 3, figsize=(15, 10))
fig2.suptitle(f'Detailed Camber Comparison: Three Representative Cases',
              fontsize=16, fontweight='bold')
```

```
# Select three representative cases: negative, zero, positive camber
case_indices = [0, len(y0_values)//2, -1]
case_titles = ['Negative Camber (y₀ < 0)', 'Symmetric (y₀ = 0)', 'Positive
Camber (y₀ > 0)']

for i, idx in enumerate(case_indices):
    res = results[idx]

    # Plot 1: Airfoil geometry
    ax = axes2[0, i]
    chord = res['geom']['chord']
    x_norm = (res['x_airfoil'] - min(res['x_airfoil'])) / chord
    y_norm = res['y_airfoil'] / chord

    ax.plot(x_norm, y_norm, 'b-', linewidth=2)
    ax.fill(x_norm, y_norm, 'lightblue', alpha=0.3)

    # Plot camber line
    camber_data = res['geom']['camber_data']
    x_camber_norm = (camber_data['x_common'] - min(res['x_airfoil'])) /
chord
    y_camber_norm = camber_data['y_camber'] / chord
    ax.plot(x_camber_norm, y_camber_norm, 'r--', linewidth=1.5,
label='Camber line')

    ax.set_xlabel('x/c')
    ax.set_ylabel('y/c')
    ax.set_title(f'{case_titles[i]}\ny₀ = {res["y0"]:.2f} m, camber/c =
{res["camber_ratio"]:.3f}')
    ax.axis('equal')
    ax.grid(True, alpha=0.3)
    ax.set_xlim(-0.05, 1.05)
    ax.legend(loc='best')

    # Plot 2: Pressure distribution
    ax2 = axes2[1, i]

    # Separate upper and lower surfaces
    x_airfoil = res['x_airfoil']
    y_airfoil = res['y_airfoil']
    Cp = res['Cp']

    sorted_idx = np.argsort(x_airfoil)
    x_sorted = x_airfoil[sorted_idx]
```

```
    y_sorted = y_airfoil[sorted_idx]
    Cp_sorted = Cp[sorted_idx]

    mid = len(x_sorted) // 2
    x_upper = x_sorted[mid:]
    y_upper = y_sorted[mid:]
    Cp_upper = Cp_sorted[mid:]
    x_lower = x_sorted[:mid]
    y_lower = y_sorted[:mid]
    Cp_lower = Cp_sorted[:mid]

    # Normalize
    chord = res['geom']['chord']
    x_upper_norm = (x_upper - min(x_airfoil)) / chord
    x_lower_norm = (x_lower - min(x_airfoil)) / chord

    ax2.plot(x_upper_norm, Cp_upper, 'r-', linewidth=2, label='Upper
surface')
    ax2.plot(x_lower_norm, Cp_lower, 'b-', linewidth=2, label='Lower
surface')

    # Fill area between curves to show pressure difference
    # Sort for filling
    x_fill = np.concatenate([x_upper_norm, x_lower_norm[::-1]])
    Cp_fill = np.concatenate([Cp_upper, Cp_lower[::-1]])
    ax2.fill(x_fill, Cp_fill, 'gray', alpha=0.2)

    ax2.axhline(y=0, color='k', linestyle='-', alpha=0.3)
    ax2.set_xlabel('x/c')
    ax2.set_ylabel(r'$C_p$')
    ax2.set_title(f'Pressure Distribution\nC_L = {res["C_L"]:.3f}')
    ax2.grid(True, alpha=0.3)
    ax2.invert_yaxis()
    ax2.legend(loc='best')
    ax2.set_xlim(0, 1)

plt.tight_layout()
plt.show()

#
==========================================================================
==
# FINAL SUMMARY
```

```
#
==========================================================================
==
print("\n" + "="*80)
print("FINAL SUMMARY: CAMBER EFFECTS ON JOUKOWSKI AIRFOILS")
print("="*80)

print("\n1. KEY FINDINGS:")
print("   • Camber (controlled by y₀) enables lift generation at zero
angle of attack")
print("   • Positive camber (y₀ > 0) produces positive lift at α=0°")
print("   • Negative camber (y₀ < 0) produces negative lift at α=0°")
print("   • Camber shifts pressure distribution asymmetrically")
print("   • Zero-lift angle: α_L=0 ≈ -2×(camber/c) radians")

print("\n2. DESIGN IMPLICATIONS:")
print("   • For conventional aircraft: Use positive camber (y₀ > 0)")
print("   • For symmetric flight (aerobatics): Use zero camber (y₀ = 0)")
print("   • For inverted flight capability: Limited negative camber")

print("\n3. PRACTICAL RECOMMENDATIONS for R=1.5m, b=0.8m:")
print("   • Typical airfoil (4% camber): y₀ ≈ 0.10-0.15 m")
print("   • High-lift airfoil (6% camber): y₀ ≈ 0.15-0.22 m")
print("   • Low-drag airfoil (2% camber): y₀ ≈ 0.05-0.08 m")

print("\n4. AERODYNAMIC TRADE-OFFS:")
print("   • Increased camber → Increased C_L at α=0°")
print("   • Increased camber → Stronger adverse pressure gradient")
print("   • Increased camber → Higher pitching moment (nose-down)")
print("   • Optimal camber balances lift, drag, and stability")

print("="*80)
```