Physics 30

Learn Everyware

we envision

Momentum and Impulse

Unit A

Red Deer Catholic Regional Schools
Pembina Hills Regional School Division
Black Gold Regional Schools
Peace Wapiti School Division No. 76
Calgary Board of Education
Edmonton Public Schools
Alberta Education

Physics 30

Learn EveryWare

Momentum and Impulse

Unit A

Physics 30
Unit A: Momentum and Impulse
Student Module Booklet
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

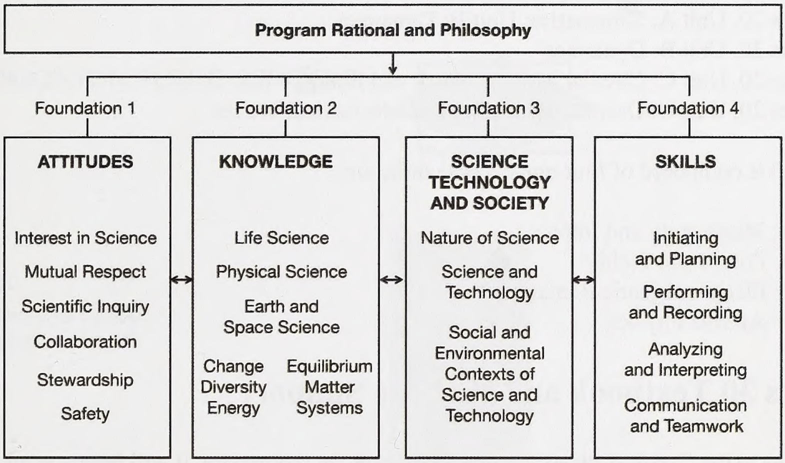
- Alberta Education, **http://www.education.gov.ab.ca**
- Learning Resources Centre, **http://www.lrc.education.gov.ab.ca**
- Tools4Teachers, **http://www.tools4teachers.ca**

Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Physics 30

Welcome to Physics 30

In Physics 30 you will learn more than facts. You will be encouraged to develop positive attitudes and to acquire and use knowledge and skills in responsible ways. Your studies will lead you to achievements in each of the following four areas prescribed by the Alberta Program of Studies.



Foundation 1

Attitudes—*Students will be encouraged to* develop attitudes that support the responsible acquisition and application of scientific and technological knowledge to the mutual benefit of self, society, and the environment.

Foundation 2

Knowledge—*Students will* construct knowledge and understandings of concepts in life science, physical science, and Earth and space science and apply these understandings to interpret, integrate, and extend their knowledge.

Foundation 3

Science, Technology, and Society (STS)—*Students will* develop an understanding of the nature of science and technology, the relationships between science and technology, and the social and environmental contexts of science and technology.

Foundation 4

Skills—*Students will* develop the skills required for scientific and technological inquiry, for solving problems, for communicating scientific ideas and results, for working collaboratively, and for making informed decisions.

This course builds upon the scientific concepts from

Science 8, Unit C: Light and Optical Systems
Science 9, Unit D: Electrical Principles and Technologies
Science 9, Unit E: Space Exploration
Physics 20, Unit A: Kinematics; Unit B: Dynamics
Physcis 20, Unit B: Dynamics
Physics 20, Unit C: Circular Motion, Work and Energy
Physics 20, Unit D: Oscillatory Motion and Mechanical Waves

Physics 30 is composed of four units. These units are

Unit A: Momentum and Impulse
Unit B: Forces and Fields
Unit C: Electromagnetic Radiation
Unit D: Atomic Physics

Physics 30 Textbook and Website Support

You will be using *Pearson Physics* as your textbook for this course. It will help you add depth to your understanding of the topics you study. You will find additional support at the textbook's online website, **http://www.physicssource.ca**. (You or your school may have to pay a fee to access this website.) Here, you can use unit pre-quizzes, web links, chapter highlights, study tips, research tools, and other opportunities for further learning.

Physics 30 Partners

Alberta Education

Black Gold Regional Schools

Calgary Board of Education

Peace Wapiti School Division No.76

Edmonton Public Schools

Pembina Hills Regional School Division No.7

Red Deer Catholic Regional Schools

Rocky View School Division

Learning in an Online Environment

This course is delivered to you in an online environment. You can look forward to using resources, such as interactive multimedia, and the Internet for various activities. You will also have access to computer simulations, computer multimedia, computer graphics, and electronic information to support your learning.

LearnAlberta.ca

LearnAlberta.ca is a protected digital learning environment for Albertans. This Alberta Education portal, found at **http://www.learnalberta.ca/**, is a place where you can support your learning by accessing resources for projects, homework, help, review, or study.

For example, LearnAlberta.ca contains a large Online Reference Centre that includes multimedia encyclopedias, journals, newspapers, transcripts, images, maps, and more. The National Geographic site contains many current video clips that have been indexed for Alberta Programs of Study. The content is organized by grade level, subject, and curriculum objective. Use the search engine to quickly find key concepts. Check this site often as new interactive multimedia segments are being added all the time.

If you find a password is required, contact your teacher or school to get one. No fee is required.

Alternative Learning Environments and Distributed Learning

Distributed Learning is a model through which learning is distributed in a variety of delivery formats and mediums—print, digital (online), and traditional delivery methods—allowing teachers, students, and content to be located in different, non-centralized locations. Physics 30 students will be completing this course in a variety of learning environments, including traditional classrooms, online/virtual schools, home education, outreach programs, and alternative programs.

Instructional Design

Explanation

The learning model used in Physics 30 is designed to be engaging and to have you participate in inquiry and problem solving. You will actively interpret and critically reflect on your learning process. Learning begins within a community setting at the centre of a larger process of teaching and learning. You will be encouraged to share your knowledge and experiences by interaction, feedback, debate, and negotiation.

Components

This course uses the following structure and instructional design to connect you to the relevant curriculum and scientific concepts in Physics 30. These components are used consistently throughout the course and will help you in seeing the context and overall content of the program.

Big Picture

Big Picture provides a brief introduction to the module while connecting to your prior learning and personal knowledge. It refers to the essential questions of the module and invites you to reflect on the "big picture" within your own context.

Discuss

Discuss provides opportunities for you to interact with your peers and teacher. Discussion topics and collaborative activities should be independent of delivery mode, given the variety of technology access and delivery methods in schools.

Explore

Explore encourages you to investigate new concepts through preparation and presentation (Read), multimedia interactions (Watch and Listen), hands-on simulations (Labs), and explorative activities (Try This). Components within this section often do not follow a specific order. For example, you can do Watch and Listen after Try This or do Labs before Read.

Get Focused

Get Focused encourages you to focus on the task at hand and the outcomes to achieve. It includes a list of knowledge outcomes, STS outcomes, and/or skills outcomes. It prepares you for the upcoming lesson by providing a lesson rubric, a list of assessment items, and a list of required equipment and materials.

Going Beyond

Going Beyond gives you the choice of challenging and enriching your knowledge beyond the lesson.

Lab

Labs include hands-on activities with available equipment/materials and/or multimedia simulations of a lab.

Lesson

Each lesson consists of the main learning content from which you explore, reflect, and connect. The length of each lesson is defined by content that covers at least one measurable outcome.

Module

Each module consists of content developed around a general or major outcome. Modules are comprised of lessons and include the introductory sections Big Picture, Challenge, In This Module, and Module Summary and Assessment.

Read

The Read component uses textual material to convey concepts to you. This material may appear directly within this component. Alternatively, it may be presented indirectly through another resource. For example, you may be sent to your textbook or provided with a link to a website.

Reflect and Connect

Reflect and Connect provides you with opportunities to check your understanding of concepts introduced in the lesson (Self-Check) and to make connections to prior learning and personal knowledge (Reflect on the Big Picture). It also provides you with opportunities to interact with your peers and your teacher through communication and collaboration (Discuss).

Reflect on the Big Picture

Reflect on the Big Picture, part of Reflect and Connect, provides connections to the Big Picture introduced at the beginning of the module. It connects and adds to the initial essential question(s) and situates the concepts of the lesson within the Big Picture context.

Summary

There are course, unit, module, and lesson summaries. All lesson summaries build toward the unit and course summaries and make connections to the Big Picture introduced at the module level. Each summary provides you with information about what you have accomplished.

Self-Check

Self-Check provides you with opportunities to check your understanding of new concepts learned in the lessons and to make connections to prior learning. These may be in auto-marked form or may require teacher feedback.

Try This

Try This includes opportunities to practise and apply learned concepts outside of a lab environment. These can be simulations, questions, webquests, or other activities that provide you with a space to explore different ways of applying new concepts.

Unit

The units of study are identified in the Program of Studies. Units are defined by subject matter and are not limited by quantity of content or time of study. Each unit is comprised of modules (usually one for each general outcome), includes a general introduction and a visual representation of content structure (e.g., concept organizer/site map) as well as a list of general outcomes to be addressed, and includes a unit summary and assessment.

Watch and Listen

Watch and Listen includes both passive and interactive multimedia content (podcasts, videos, interactive Flash activities, etc.).

Visual Cues (Icons)

You will see icons throughout the course. These icons are clues regarding the type of activity you are about to begin.

Each unit in the course has a different colour theme, and the icons will change colour to match. For example, here are the four different colours of the Big Picture icon that appear in Units A to D, respectively.


Unit A


Unit B


Unit C


Unit D

The icons and their meanings are given.


Big Picture


Assessment


Summary


Discuss


Get Focused


Read


Try This


Lab


Self-Check


Reflect and Connect


Explore


Worked Examples


Reflect on the Big Picture


Going Beyond


Watch and Listen

Special Learning Activities: Labs and Simulations

In Physics 30 you will use a large number of computer simulations to help you master concepts. You will often use the simulations to carry out activities that give you similar experiences to those in a lab. In some ways, simulations provide you with an enhanced lab experience because additional tools to analyze what is going on are available in the simulations.

The majority of the simulations can also be found at LearnAlberta.ca if your usual connection isn't working.

You will be able to change parameters using sliders, choose which items show on the screen using buttons, access information about the simulation and how it works from menus at the top of the screen, and carry out and replay the actions.

Physics 30 was developed with simulations as a major component and requires you to use them regularly.

Reference (Data Tables)

In this course you will need numerical and scientific data for reference. Your textbook, *Pearson Physics*, has data tables in the appendix. For a list of standard equations, you should consult the Data Booklet for physics from Alberta Education. The Data Booklet also contains numerical and scientific data.

You will be allowed to use a copy of the Data Booklet when writing the Physics 30 Diploma Examination. That is a good reason for becoming familiar with the Data Booklet. The Data Booklet can be downloaded from **http://www.education.alberta.ca/admin/testing/diplomaexams.aspx**.

Glossary

A physics glossary—an alphabetical list of physics terms and their meanings—is provided in this course. Your textbook, *Pearson Physics*, also has a glossary for you to refer to.

Lesson Answers (for students)

For many items, answers are provided through a "Check your work" link. Some items, especially assessment items, are to be marked by your teacher. For these items, submit your work to your teacher.

Using the Physics 30 Course Folder

The Physics 30 course folder serves as the organized collection of your work in Physics 30. It exhibits to others your efforts, achievements, self-reflection, and progress throughout the course. When you want to show your friends or family what you've been learning, your work is all there.

You will be expected to put all of your work into the course folder. If you are unsure of the process, your teacher will walk you through it. Throughout the course you will be asked to add things to the course folder.

In addition to being able to show others what you have done, the course folder lets you see your progress. It lets you see how your knowledge, skills, and understandings are growing. It also lets you review and annotate work you have already completed. You may find your course folder useful in preparing for tests and quizzes.

Periodically, you will be asked to share items from your course folder with your teacher. This is not always for grading, as often your teacher may use these items to learn more about you and your interests or as a way of tailoring other work assigned to you.

Physics 30

LearnEveryWare

Momentum and Impulse

Module 1

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Physics 30 Learn EveryWare

Unit A Momentum and Impulse

Unit A Introduction

In Unit A you will investigate the conservation of momentum when objects interact. This will include the concept of impulse and looking at the transfer of kinetic energy in collisions. This unit builds on your work from Physics 20: vector addition skills; the principle of uniformly accelerated motion (unbalanced forces); and the principle of conservation of energy, which is an overall theme for Physics 20 and 30. This unit will prepare you for further study of mechanics in subsequent units and for post secondary studies in physics.

Module 1 will define momentum and examine how it was derived from Newton's laws of motion that you studied in Physics 20. Also, you will investigate the relationship between momentum and impulse, and how the two can be used to make vehicular safety devices more effective. This will lead you to the module assessment to analyze a crash test and the forces acting on crash test dummies.

Module 2 will expand upon momentum by proving the law of conservation of momentum and how it can be applied in one and two dimensions. You will conduct this proof by using vector addition and computer simulations. In keeping with the conservation laws you will use your knowledge of the law of conservation of energy from Physics 20 to examine the energy transfer in collisions by examining whether collisions are elastic or inelastic. This will lead into the module assessment to analyze a traffic accident.

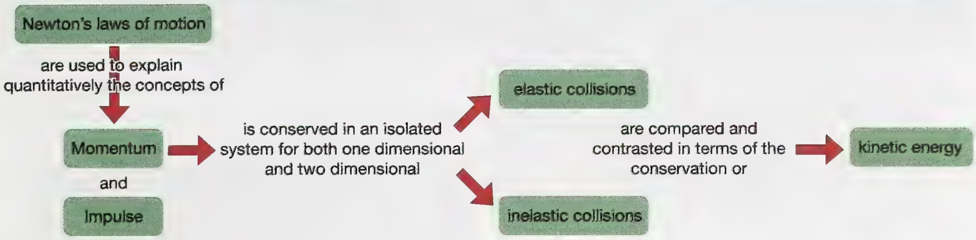
Unit A is based on the physics principle of the conservation of momentum. Although it initially sounds very simple, there are many details and aspects that can cause problems. To support your understanding you will answer questions and do activities that examine common misconceptions and help expand your understanding and ability to apply the principle of conservation of momentum.

Specifically, at the end of the unit, you will be able to

- describe the characteristics of an object that affect its momentum
- describe the relationship between momentum and impulse
- demonstrate that momentum is conserved in elastic and inelastic collisions for both one dimensional and two dimensional interactions
- demonstrate that kinetic energy is not conserved for inelastic collisions and is conserved for elastic collisions
- analyze kinetic energy to compare and discriminate between collisions in isolated and non-isolated systems

The principle of conservation of momentum will be used again in Unit 4: Atomic Physics for the discovery of several subatomic particles.

Unit A Concept Organizer



Module Descriptions

Module 1: Momentum and Impulse

In this module you will work with the relationship between Newton's laws of motion and momentum. You will apply these concepts in the context of the design and functioning of modern vehicle safety devices. Specifically, you will be asked to apply your knowledge to answer the following questions:

- What do we really mean when we say "speed kills"?
- How is it possible to reduce injury and death when a vehicle occupant experiences a large change in momentum during a collision?

Module 2: The Conservation of Momentum in Isolated Systems

In this module you will learn about the conservation of momentum in a closed system. You will analyze elastic and inelastic collisions in both one and two dimensions. You will learn to discriminate between elastic and inelastic collisions using the conservation principles of momentum and kinetic energy. Specifically, you will be asked to apply your knowledge to answer the following questions:

- What happens to mass, velocity, and momentum in a linear collision between two objects?
- What happens to mass, velocity, and momentum in a non-linear collision between two objects?
- Is kinetic energy conserved in a collision between two objects?

Using the answers to these questions and component vector analysis, you will prepare a crash analysis for an inelastic two-dimensional collision.

Physics 30 Learn EveryWare

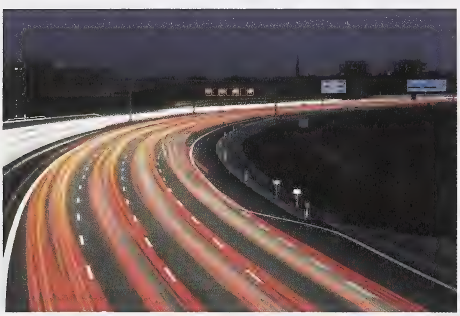
Unit A Momentum and Impulse

Module 1—Momentum and Impulse

Module Introduction

In this module you will work with the relationship between Newton's laws of motion and momentum. You will apply these concepts in the context of the design and function of modern vehicle safety devices.

Steve Allen/Brand X Pictures/Getty Images



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Big Picture

According to the August 2007 Canadian Vehicle Survey conducted by Statistics Canada, there were an estimated 2 533 947 vehicles in Alberta. This number includes an estimated 113 729 vehicles that had masses in the range of 4.5-tonne delivery trucks to 14.9-tonne tractor semitrailers.

Considering the mass of all of the vehicles, and the velocity at which those vehicles travel, there is plenty of momentum on Alberta's roads. Given the large number of vehicles, there is little doubt that accidents involving large forces and changes in momentum will be experienced by the human occupants of colliding vehicles.

Consider the crash test dummy. Smashed, crushed, bent, and battered, the dummy serves as a simulated human being that does not experience the real pain and cost of a vehicle collision.

Crash test dummies are the lone occupants of a vehicle travelling at high velocity, with a large amount of momentum, only to change their momentum dramatically when the vehicle makes contact with another vehicle or barrier. In the collision, forces act to change the momentum of the vehicle and occupant, testing the effectiveness of safety systems, such as air bags and crumple zones, in the vehicle.

But before a vehicle races down the track to certain destruction during collision testing, investigators need to understand how the concepts of momentum and impulse can be applied to understand the function and design requirements of vehicle safety devices.

As you are working in Module 1, keep the following questions in mind:

- What do we really mean when we say "speed kills"?

- How is it possible to reduce injury and death when a vehicle occupant experiences a large change in momentum during a collision?

In this lesson you will be working with vectors as you did in Physics 20. During the lesson, before you have any vector questions, there is an opportunity for you to review vector addition.

Module Assessment

Each lesson has a teacher-marked assignment based on work completed in the lesson. In addition, you will be graded on your contributions to the Discuss section of each lesson.

You will also be asked to complete Self-Check or Try This questions, which you should place in your Physics 30 course folder. These are not formally assessed but are a valuable way to practise the concepts and skills of the lesson. These activities can provide you with reflective feedback on your understanding of the lesson work.

You will be marked for your lesson work on the following items:

- Module 1: Lesson 1 Assignment
- Module 1: Lesson 2 Assignment
- Module 1 Project

In the context of each lesson, you will explain why "speed kills" and how modern vehicle safety devices work to reduce the impulse experienced by vehicle occupants involved in a collision.

After you have completed both lessons, you will do a module assessment that consists of three questions similar to Diploma Exam written-response questions. These questions will require you to apply your knowledge and skills to a practical application of momentum in a crash test scenario. As this is your first module assessment, make sure that you read the marking rubrics and the self-evaluation questions to be sure that you have answered the questions completely and understand what will be evaluated.

If at any point you are having any concerns or difficulties, contact your teacher for specific feedback.

In This Module

Lesson 1—Momentum and Newton's Second Law

In this lesson you will define the vector quantity momentum and its relationship to Newton's laws and address what we really mean when we say "speed kills." To do this, you will explore the following questions:

- What is momentum?
- Why is momentum defined as a vector?
- How is momentum related to Newton's laws?

Lesson 2—Impulse: Changing Momentum

In this lesson you will explore how it is possible to reduce injury and death when a vehicle occupant experiences a large change in momentum during a collision through the function of vehicle safety features. To better understand this, you will explore the concept of momentum and its relationship to Newton's second law. In this lesson you will explore the working principles of safety devices in the context of the following questions:

- What is impulse?
- How is impulse related to a change in momentum?
- How is impulse related to Newton's laws?

Module 1—Momentum and Impulse

Lesson 1—Momentum and Newton's Second Law

Get Focused

The motorcycle has been popular for a long time. Driving a motorcycle has been compared to riding a horse because the rider is completely exposed to the elements. It's also like flying a small aircraft because it's "a full-body experience." It requires a feel for the road—leaning into turns and constantly making adjustments for changing road conditions.

© David P. Smith/shutterstock

There is, however, very little protection for the rider if a crash occurs. Consider the typical motorcycle fatality in Canada. It seems to have a common profile—single vehicle, summer weekend evening, rural road with a posted speed limit over 90 km/h. Motorcycle fatalities in Alberta more than doubled between 2000 (14 fatalities) and 2006 (32). Sadly, the annual number of motorcycle driver deaths is on the increase. Statistics indicate that one contributing factor is the increasing number of very powerful, high-performance motorcycles. Compare the two bikes pictured here.

© John Neff/shutterstock (speeder on highway)
© ann trilling/shutterstock (woman on bike)

The death rate for people riding high-performance motorcycles is about three times that of people riding lower-powered machines. Why do you think that is? Could the phrase "speed kills" apply here?

Part of the answer to this question involves exploring the concept of momentum and how it is related to the vector quantity velocity and the scalar quantity mass.

In this lesson you will answer the following essential questions:

- What is momentum?
- Why is momentum defined as a vector?
- How is momentum related to Newton's laws?

Module 1: Lesson 1 Assignment

Your teacher-marked Module 1: Lesson 1 Assignment requires you to submit responses to the following:

- Assignment—A 1, A 2, A 3, A 4
- Discuss—D 1, or D 2, or D 3
- Reflect on the Big Picture

The other questions in this lesson are not marked by the teacher; however, you should still answer these questions. The Self-Check and Try This questions are placed in this lesson to help you review important information and build key concepts that may be applied in future lessons.

After a discussion with your teacher, you must decide what to do with the questions that are not part of your assignment. For example, you may decide to submit to your teacher the responses to Try This questions that are not marked. You should record the answers to all questions in this lesson and place those answers in your course folder.

Explore

What's the most important piece of equipment for reducing the severity of injuries caused by a motorcycle collision? Most experts agree that it's a well-designed helmet. Approved helmets are designed to withstand the very large forces that can occur at the point of impact. As you learned in previous courses, an unbalanced force always acts when a mass undergoes a change in velocity. A change in velocity could mean a change in the magnitude of the velocity, resulting in the object speeding up or slowing down. A change in velocity could also mean a change in direction. You can learn more about factors that affect the directions of two objects after a collision in the next activity.

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Read

To find out more about collisions, read page 446 of your textbook.

Try This

Follow the instructions and complete "9–1 QuickLab: Predicting Angles After Collisions" on page 447 of your textbook. As you make observations and collect data, use the questions at the end of the activity to help guide your work.

TR 1. Answer question 1 at the end of the QuickLab.

TR 2. Answer question 3 at the end of the QuickLab.

Momentum

You can use the word **momentum** in everyday language to describe various events—sporting events, political campaigns, or economic trends. However, regardless of what is being described, the word momentum always implies sustained movement. For example, the expression "Team X has gained momentum" means that the team is "on the move" and will be difficult to stop—the greater the momentum, the harder it is to stop the team.

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momentum: is the product of the mass and velocity of an object

Expressed as an equation it is $\vec{p} = m\vec{v}$.

In physics, the word momentum has a similar meaning but it is related to the quantity of motion possessed by an object.

Read

You can find out about the exact meaning of the word *momentum* in physics by reading pages 448 and 449 of your textbook.

Self-Check

SC 1. Produce a short summary that includes the key ideas about momentum.

Check your work with the answers in the Appendix.

Problem Solving with Momentum

When solving problems, it is useful to follow the GRASP model for problem solving. The GRASP model is on page 867 in your textbook.

Since momentum is a vector, it is very important to clearly communicate direction. Note how a sign convention is used to handle direction in each of the following examples.

Example Problem 1

Determine the momentum of a vehicle with a mass of 2100 kg moving at a velocity of 22 m/s, east.

Given

$$m = 2100\text{ kg}$$

$$\begin{aligned}\vec{v} &= 22\ \frac{\text{m}}{\text{s}}[\text{E}]\\ &= +22\ \frac{\text{m}}{\text{s}}\end{aligned}$$

Required

The momentum of the vehicle, $\vec{p}$.

Analysis and Solution

Let east be the positive direction.

$$\begin{aligned}\vec{p} &= m\vec{v}\\ &= (2100\text{ kg})\left(+22\ \frac{\text{m}}{\text{s}}\right)\\ &= +4.6\times10^{4}\ \frac{\text{kg}\bullet\text{m}}{\text{s}}\end{aligned}$$

Paraphrase

The momentum of the vehicle is 4.6×10^4 kg·m/s [E].

Example Problem 2

In many countries motorized scooters are a very popular form of transportation. The total momentum of two people on a scooter is 7.20 × 103 kg·m/s [S]. If the combined mass is 250 kg, determine the velocity of the people on the scooter.

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Given

$$\vec{p} = 7.20 \times 10^3 \frac{\text{kg}\bullet\text{m}}{\text{s}} [\text{S}]$$
$$= -7.20 \times 10^3 \frac{\text{kg}\bullet\text{m}}{\text{s}}$$
$$m = 250 \text{ kg}$$

Required

The velocity of the vehicle, $\vec{v}$.

Analysis and Solution

Let south be the negative direction.

$$\vec{p} = m\vec{v}$$
$$\vec{v} = \frac{\vec{p}}{m}$$
$$= \frac{-7.20 \times 10^3 \text{ kg}\bullet\frac{\text{m}}{\text{s}}}{250 \text{ kg}}$$
$$= -28.8 \ \frac{\text{m}}{\text{s}}$$

Paraphrase

The velocity of the motorcycle and riders is 28.8 m/s [S].

Example Problem 3

An airplane has a momentum of 8.3×10^7 kg·m/s [N]. If the airplane is flying at a velocity of 990 km/h [N], determine its mass.

Given

$$\begin{aligned} p &= 8.3\times10^7 \text{ kg}\bullet\text{m/s [N]} \\ &= +8.3\times10^7 \text{ kg}\bullet\text{m/s} \\ v &= 990 \text{ km/h [N]} \\ &= +990\frac{\cancel{\text{km}}}{\cancel{\text{h}}}\times\frac{1000 \text{ m}}{1 \cancel{\text{km}}}\times\frac{1 \cancel{\text{h}}}{3600 \text{ s}} \\ &= +275 \text{ m/s} \end{aligned}$$

Required

The mass of the airplane, *m*.

Analysis and Solution

Let north be the positive direction. This solution involves dividing momentum by velocity. Recall that multiplying or dividing one vector by another is beyond the scope of high school physics. Therefore, the vector notation is dropped.

$$\begin{aligned} p &= mv \\ m &= \frac{p}{v} \\ &= \frac{+8.3\times10^7 \text{ kg}\bullet \cancel{\text{m/s}}}{+275 \cancel{\text{m/s}}} \\ &= 3.0\times10^5 \text{ kg} \end{aligned}$$

Paraphrase

The mass of the airplane is 3.0×10^5 kg.

Scalars and Vectors

As you saw in the example problems, the vector nature of momentum plays a significant role in problem solving. If you'd like a review of vectors and the use of vector notation, take a few minutes to complete "Vector Review," which follows. If you feel confident in your understanding of vectors and the use of vector notation, continue on with the lesson.

Review of Vectors

Have you ever arrived 15 minutes late for a movie? Even though you eventually pick up the plot and manage to enjoy yourself, you know that it is still not quite the same experience that you could have enjoyed had you been there from the beginning. If the movie has an intriguing plot, the effects of missing the beginning can be worse. You may never really feel like you are able to catch the whole meaning of the film.

Module 1 is much like the beginning of a movie with a complex plot. If you miss the main ideas here, you may feel out of touch for the rest of the course. Although the ideas are meant to be a review of Physics 20, you will have to pay close attention to the details of how to communicate vector solutions properly. It is important to learn how to communicate your solutions in a way that will help you be successful in the whole course, including the diploma exam. So, as you review vectors, be sure to check your answers closely with the solutions provided.

Read

Begin your review of vectors by reading or skimming the introduction and the section titled "Vector Methods in One Dimension" on pages 70 to 75 of your textbook. Answer the following questions to help highlight the key points.

Self-Check

SC 2.

a. What is a resultant?

b. How is a resultant drawn?

SC 3. Does order matter when you add vectors together?

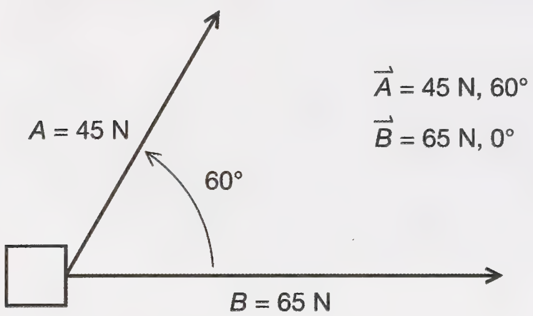
Check your work with the answers in the Appendix.

Read

To review how vectors can be added graphically in two dimensions, read from the bottom of page 76 to page 81 of your textbook.

Self-Check

SC 4. The angle between two vectors does not always have to be a right angle. For example, the two vectors could be arranged as shown in the following diagram.



Use graphical methods to determine the resultant of these two vectors.

Check your work with the answers in the Appendix.

Up to this point, the approach that you have been using has been mainly graphical—solutions could be obtained by drawing the vectors with a ruler and a protractor. However, when a more precise answer is required an alternative mathematical approach is used.

Read

You can review more precise methods of dealing with vectors by reading pages 83 to 89 of your textbook. Pay close attention to the diagrams and to the use of notations in "Example 2.5" on page 88.

Proper Use of Vector Notation in Physics 30

The following guidelines explain the proper use of vector notation in Physics 30:

- You should be able to add and subtract vector quantities from each other. You should also be able to multiply and divide vector quantities by constants or by scalar quantities.

- You are not expected to multiply and divide vector quantities by other vector quantities. In this course it is customary to drop the vector notation when multiplying or dividing vectors. An equation that states the multiplication or division of vectors implies a kind of mathematical treatment that is beyond the scope of this course.

Self-Check

SC 5. Solve question 11 on page 90 of your textbook.

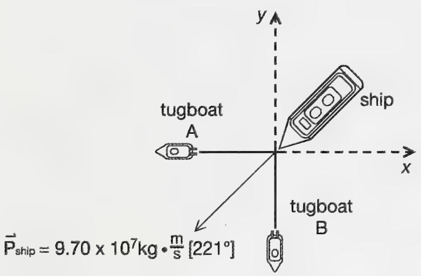
Check your work with the answers in the Appendix.

Self-Check

SC 6. What is a scalar quantity? Give two examples.

SC 7. What is a vector quantity? Give two examples.

SC 8. The following diagram shows a ship being towed out of a harbour by two tugboats.



The momentum of the ship is 9.70×10^7 kg•m/s [221°]. What is the component of the momentum provided by tugboat A? What is the component of the momentum provided by tugboat B?

SC 9. Calculate the momentum of a 70.0-kg person travelling west at 5.0 m/s.

SC 10. A motorcycle has a velocity of 95 km/h [N] and a momentum of 7250 kg•m/s [N]. Determine the mass of the motorcycle.

Check your work with the answers in the Appendix.

Module 1: Lesson 1 Assignment

Go to the Module 1 Assignment Booklet, and complete questions A 1, A 2, A 3, and A 4 in the Lesson 1 Assignment.

Momentum and Newton's Second Law

Recall that Newton's second law states that if an external, non-zero, net force acts on an object, it will accelerate. Acceleration means a change in velocity, which means a change in momentum! The external unbalanced force, therefore, causes a change in momentum. In other words, if there's an $\vec{F}_{\text{net}}$, there's a $\Delta\vec{p}$. Therefore, Newton's second law and momentum are related to one another.

Read

Find out more about the connection between momentum and Newton's second law of motion by reading pages 450 to 452 in your textbook.

Self-Check

SC 11. Show the derivation of Newton's second law in terms of the change in momentum.

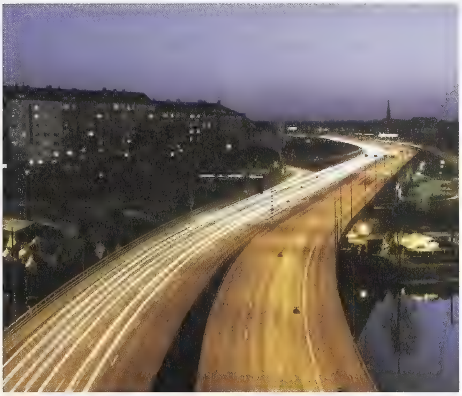
Check your work with the answers in the Appendix.

$$\vec{F}_{\text{net}} = \left(\frac{\Delta\vec{p}}{\Delta t}\right)$$

In this form, Newton's second law tells us that the external, non-zero, net force acting on an object will be equal to the rate of change of the momentum of that object with respect to time. For example, in order to change the momentum of a large mass, such as a passenger car (even if it is moving at a low velocity), in a short time, a very large net force will be required. The same is true for a smaller mass, such as a motorcycle if it travels at high velocity. In this form of Newton's second law, the mass, the time interval and the velocity of the object (momentum) are considered.

Reflect and Connect

A moving vehicle has a momentum defined by the product of its mass and velocity. Changing either of these two quantities will cause a corresponding change in the momentum. Both the mass and average velocity of modern vehicles is much larger than those of the past. This means that the amount of momentum on today's busy freeways is increasing. And according to Newton's second law, a large net force is required to make a large change in momentum quickly. In effect, this means that the modern car crash that involves a larger vehicle and/or one travelling at a high velocity will generate larger net forces since the time of the collision hasn't changed.



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Can you think of what implications this has for both vehicle design and safety? Larger vehicles and/or those that travel at a high rate of speed require larger engines and brake systems and stronger steering systems. All of these design features are required to change large amounts of momentum in short periods of time.

Discuss

In the discussion forum, use the concepts of momentum and Newton's second law to explain why the following statements are true.

D 1. A large freight train moving down the tracks takes a long time to stop.

D 2. A motorcycle can have the same momentum as a large truck.

D 3. In general, a motorcycle can accelerate at a greater rate than a large truck.

You may want to review the Discussion Scoring Guide in the Appendix for the assessment criteria in posting and responding to discussions.

Once you have submitted your explanations, read at least two other postings. Consider how well these responses used the concepts of momentum and Newton's second law. Reflect on your responses and the two others you have read. Choose one of the responses, revise your response and submit both the original and the revised response to your teacher.

Module 1: Lesson 1 Assignment

Copy your revised response to question D 1, D 2, or D 3 into the Lesson 1 Assignment in the Module 1 Assignment Booklet.

Reflect on the Big Picture

In real life, a police officer may warn that "speed kills." In video games, high-speed crashes are common. Suppose a particular scene in a game requires the following elements:

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- A high-performance motorcycle must crash into a barrier and be stopped.
- A small scooter must crash into another section of the same barrier and be stopped.
- The forces exerted as each vehicle collides with the barrier change the shape of that section of the barrier in addition to stopping the vehicle.
- The time required to stop each vehicle must be realistic.

Create a paragraph, illustration, animation, or audio recording to explain how you could use the concept of momentum and net force to make each simulated crash realistic. In general, how could you use momentum and Newton's laws of motion to predict the amount that each section of the barrier changes shape? Refer to the Holistic Scoring Guide in the Appendix for information about how this assignment will be marked.

Module 1: Lesson 1 Assignment

Submit the answer to Reflect on the Big Picture to your teacher in the Lesson 1 Assignment in the Module 1 Assignment Booklet.

Module 1: Lesson 1 Assignment

Make sure you have completed all of the questions for the Lesson 1 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 1 assignments have been completed.

Lesson Summary

In this lesson you explored the following questions:

- What is momentum?
- Why is momentum defined as a vector?
- How is momentum related to Newton's laws?

Momentum is the product of an object's mass and velocity.

Since momentum is the product of a vector (velocity) and a scalar (mass), it is a vector quantity. The direction of the momentum vector is in the same direction and is relative to the same reference frame as the velocity of the object.

Newton's second law tells us that the external, non-zero, net force acting on an object will be equal to the rate of change of the momentum of that object with respect to time. This has particular relevance to situations involving large changes in momentum, such as those of vehicle collisions. In the next lesson you will learn how effective vehicle safety devices can be designed based on the same principles.

Lesson Glossary

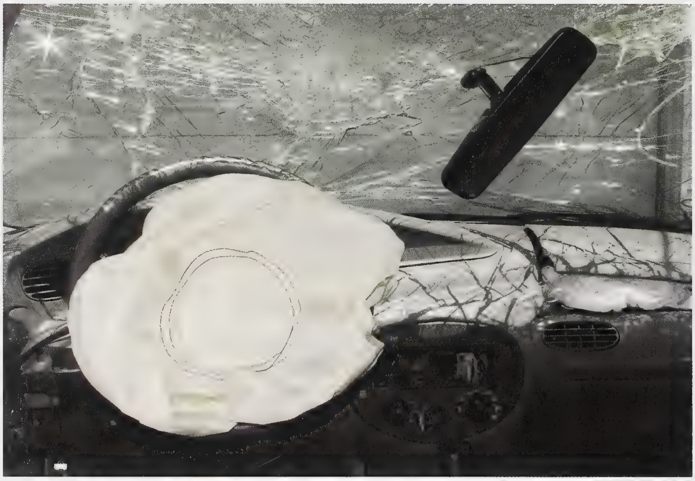
momentum: is the product of the mass and velocity of an object

Expressed as an equation it is $\vec{p} = m\vec{v}$.

Module 1—Momentum and Impulse

Lesson 2—Impulse: Changing Momentum

Get Focused



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In Lesson 1 you learned that large, heavy vehicles or those that travel at high rates of speed have considerable momentum. In the event of a sudden stop, such vehicles will experience a large change in momentum. In the instant prior to the collision, the occupant of a vehicle has the same velocity as the vehicle. For example, the velocity could have been more than 100 km/h. Very shortly after the collision, the vehicle and the occupant would have had a very different velocity—probably zero. Both the vehicle and occupant experienced a huge change in momentum during this collision. How is it possible to reduce injury and death when an occupant in a vehicle experiences a large change in momentum during a collision?

Recall what you learned in Lesson 1. A force is required to change the momentum of the occupant. How can you limit the damage caused by this force? Seat belts and airbags both can play a major role in limiting damage to the occupant. How are Newton's laws of motion applied to the designs of these devices?

To understand this correctly, we need to explore the concept of impulse and how it relates to a change in momentum and Newton's laws of motion.

In this lesson you will explore the following essential questions:

- What is impulse?
- How is impulse related to a change in momentum?
- How is impulse used in the design of safety devices that protect the occupants of vehicles during a crash?

Module 1: Lesson 2 Assignment

Your teacher-marked Module 1: Lesson 2 Assignment requires you to submit responses to the following:

- Assignment—A 1, A 2, A 3, and A 4
- Discuss—D 2

You must decide what to do with the questions that are not marked by your teacher.

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

Explore

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People who ride motorcycles do not have the protection of airbags if they are involved in a crash. So other safety features are used to reduce injuries in this case.

Helmets are an essential component of the safety systems that protect people who ride motorcycles. The role of the cushioning in the lining of the helmet is explored in the next activity.

Try This

Complete "9-2 QuickLab: Softening the Hit" on page 455 of your textbook. If you do not have access to putty-type material, you can try one of these recipes for playdough.

Playdough Recipes

Following are three recipes for playdough. The cooked recipe (#1) usually provides the best results.

Playdough Recipe #1

Materials

- ½ cup salt
- 1 cup flour
- 1 tablespoon cream of tartar
- 1 tablespoon oil
- 1 cup water
- food colouring

Procedure

Combine the ingredients in a saucepan. Heat gently, stirring constantly. When the dough has a stiff consistency, take it off the heat and allow it to cool. When cool, take it out of the pot and knead several times. Store the playdough in an airtight container or a self-sealing plastic bag.

Note: Due to the high salt content of this playdough, it is potentially toxic to domestic animals such as dogs and cats if they eat a large amount of it.

Playdough Recipe #2

Materials

- 2½ cups flour
- 1 cup salt
- 3 tablespoons vegetable oil
- 2 cups boiling water
- 2 packages unsweetened Kool-Aid for colour

Procedure

Mix dry ingredients. Add oil and water. Wearing gloves, knead for 10 minutes. Store it in a self-sealing plastic bag in the refrigerator.

Note: Your hands may smell of the Kool-Aid flavour even after wearing gloves.

Playdough Recipe #3

Materials

- ¼ cup salt
- 1 cup flour
- ¼ cup water

Procedure

Mix the flour and salt in a bowl, and then add the water. Knead and squeeze the dough to make a clay consistency. You may need to add more water until you reach the desired consistency.

Note: This playdough does not last as long as the cooked recipe.

Self-Check

SC 1. Answer questions 1, 2, and 3 of the QuickLab on page 455 of your textbook.

Hint: In your answer to question 3, start with this equation, $\Delta d = \left(\frac{v_f + v_i}{2}\right)\Delta t$.

Check your work with the answers in the Appendix.

The QuickLab demonstrated that if cushioning is present as an object loses its momentum and comes to a stop, then the forces required to stop the object can be reduced. In the case of the putty ball, the evidence of the magnitude of the stopping force is the amount of deformation of the ball.


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Since a motorcyclist rarely travels over cushioned surfaces, the motorcyclist must carry his or her own cushioning—it's in the lining of the helmet. This cushioning helps to reduce the negative impact of forces acting on the rider's head during a crash.

Read

To learn more about the connections between the type of surface and the severity of injuries, read pages 454 to 456 in your textbook. Pay close attention to the relationship between stopping time and force on page 456.

Self-Check

SC 2. Explain why maximizing the stopping time has the effect of minimizing the stopping force.

Check your work with the answers in the Appendix.

Impulse

© Courtesy of ADAC.de

Whether you stop a crash test dummy in a vehicle with an airbag or stop a putty ball with a cushioned surface, the combination of force acting over a time interval plays the essential role. That is why the product of the force and the time interval during which the force acts is given a special name—**impulse**.

impulse: the product of the net force applied to an object and the time interval during an interaction

The equation is as follows:

$$\text{impulse} = \vec{F}_{\text{net}}\,\Delta t$$

Impulse is a different kind of quantity because it does not have its own symbol. Instead, impulse is either represented by the word *impulse* or the variables $\vec{F}\Delta t$. This helps remind you that many combinations of a force acting over a time interval can provide a specific amount of impulse.

DID YOU KNOW?

The unit for impulse is N·s, which is the same as the unit for momentum, kg·m/s, since N = kg·m/s^2.

$$\begin{aligned} \text{N}\bullet\text{s} &= \left(\frac{\text{kg}\bullet\text{m}}{\text{s}^{\cancel{2}}}\right)(\cancel{\text{s}}) \\ &= \frac{\text{kg}\bullet\text{m}}{\text{s}} \end{aligned}$$

Read

You can learn more about impulse and how it can be calculated by reading pages 457, 458, and the top half of page 459 in your textbook. Pay particular attention to the calculations shown in "Example 9.3."

SC 3. Use Newton's second law to identify the two different ways of calculating impulse.

SC 4. Complete the following chart to summarize the symbols and units for each of the quantities that you have learned so far in this module.

Quantity	Symbol	SI Unit
impulse		
net force		
time interval		
momentum		
mass		
velocity		

SC 5. Do "Practice Problems" 1 and 2 on page 458 of your textbook.

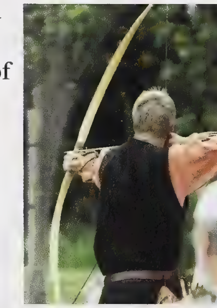
Check your work with the answers in the Appendix.

Module 1: Lesson 2 Assignment

Go to the Module 1 Assignment Booklet, and complete questions A 1 and A 2 in the Lesson 2 Assignment.

Calculating Impulse from Graphs

When the archer in this photograph releases the arrow, the force exerted by the bow on the arrow will not be constant. The following graph shows the magnitude of the net force on the arrow during the first part of its release. How do you think you could determine the magnitude of the impulse in a case like this?



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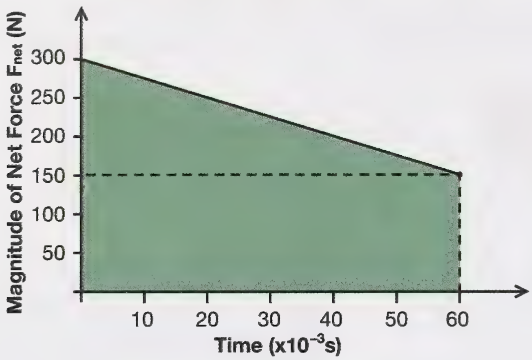
Magnitude of Net Force vs Time



Read

You can find out more about how to determine the magnitude of the impulse from a force-time graph by reading from the middle of page 459 to the bottom of page 460.

Self-Check

SC 6. Return to the graph describing the net force on the arrow vs. time shown earlier in this lesson. Use the information on that graph to determine the magnitude of the impulse acting on the arrow.

Check your work with the answers in the Appendix.

Read

You can use the area under a graph to determine the magnitude of impulse. However, in some circumstances, the area under the graph is not easy to determine. Read pages 461 and 462 of your textbook to learn more about applications and limitations. Pay close attention to "Example Problem 9.4."

Try This

TR 1. Solve practice problem 1 on page 462 of your textbook.

TR 2. How can you get a negative area on a graph and what does it mean in terms of impulse?

Module 1: Lesson 2 Assignment

Go to the Module 1 Assignment Booklet, and complete question A 3 in the Lesson 2 Assignment.

Applying Impulse to the Design of Safety Devices

Roadside barriers are designed to reduce injury during collisions.

The technologies of safety features, such as roadside barriers, have been incorporated into the construction of highways and other major roadways. The intent of roadside barriers is to prevent vehicles from swerving off the highway. However, if this is their only purpose, the most appropriate design would be the sturdiest—the concrete barrier. Why, then, are other designs used? What are the links between the laws of motion and the technology of roadside barriers?

Read

The answers to these questions can be found on pages 463 to 465 of your textbook. As you read these pages, note how the combination of the net force and the time interval are considered in the design devices that protect people from injury.

Self-Check

SC 7.

a. Identify some of the safety devices incorporated into the design of automobiles that protect occupants during a crash.

b. Concisely explain the key physics principle behind the use of these devices.

Check your work with the answers in the Appendix.

Two Ways to Apply an Impulse to an Object

Many of the safety devices that are incorporated into the design of vehicles use the idea that the necessary impulse to change the momentum of an object can be applied with smaller net force acting over a larger time interval or a larger net force acting over a smaller time interval.

impulse = $\vec{F}\Delta t$	
impulse = $\vec{\boldsymbol{F}}\Delta t$	impulse = $\vec{F}\Delta \boldsymbol{t}$
impulse = $\vec{\boldsymbol{F}}\Delta t$	impulse = $\vec{F}\Delta \boldsymbol{t}$
smaller force and larger time	larger force and smaller time

This idea can now be applied to the design of roadside barriers.

Module 1: Lesson 2 Assignment

Go to the Module 1 Assignment Booklet, and complete question A 4 in the Lesson 2 Assignment.

Impulse Can Improve Sports Performance

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This tennis player uses her racquet to change the momentum of the tennis ball. When she took her first tennis lessons, her coach explained that when hitting the ball with the racquet, it was important to continue to swing the racquet after contacting the ball. This technique is often referred to as using proper "follow through." It is thought to be more effective than a shorter swing at giving the ball a high velocity.

Read

You can find out more about the role of impulse in improving sports performance by reading page 466 of your textbook.

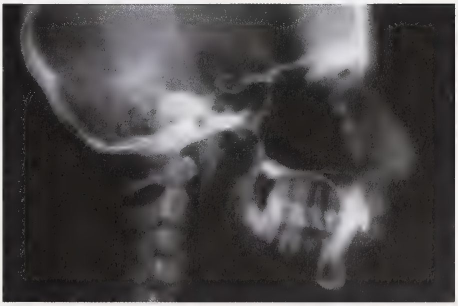
Self-Check

SC 8. Concisely explain why proper follow through increases the change in momentum of a tennis ball being struck by a tennis racquet.

Check your work with the answers in the Appendix.

Protecting Your Head

If you drop an expensive MP3 player onto a cement sidewalk, you might be disappointed if it no longer works, but at least you can have it repaired or replaced. There are many more on store shelves and the newer models often come with improved features. If you hit your head on a cement sidewalk, the long-term consequences could be much more serious. Although medical science has learned a lot about the workings of the brain, only so much damage can be repaired, and, unlike your MP3 player, an off-the-shelf replacement for your brain is out of the question.



© Jim Wehje/Photodisc/Getty Images
The human skull is a thin layer of bone encasing the brain.

People who are exposed to hazards that could lead to head injuries and possible brain damage take protective measures while at work or at play. In short, they wear helmets.

(left to right): Adam Crowley/Photodisc/Getty Images; Eyewire/Getty Images; Photodisc/Getty Images; Eyewire/Getty Images

Here are a few examples of how people protect their heads at work and at play.

Which activities do you participate in that require a helmet? Some possibilities include riding all-terrain vehicles (ATVs), playing hockey, downhill skiing, snowboarding, and inline skating. The activity that is probably on most lists is cycling. Since most people learn to ride a bicycle at a young age and can continue that activity into adulthood, the use of bicycle helmets has received public attention across Canada, including legislation for mandatory use.

Many people have strong opinions regarding the mandatory use of bicycle helmets. At the heart of the issue is this question: "What evidence is available to support the idea that the use of bicycle helmets significantly reduces head injuries among cyclists?" You will have an opportunity to answer this question in the next activity.

Discuss

In this activity you will use the Internet as a research tool.

What evidence is available to support the idea that the use of bicycle helmets significantly reduces head injuries among cyclists? Describe the overall trends in the data you collected.

Post your findings to the discussion area, and compare your findings with those of other students. How do their findings differ from yours? Are the arguments made to support these views consistent with the information you researched? Did other students find additional information unknown to you? Has your opinion of bicycle helmets changed since you started? Explain.

You may want to review the Discussion Scoring Guide in the Appendix for the assessment criteria in posting and responding to discussions.

Module 1: Lesson 2 Assignment

Copy your revised responses to the question posed in Discuss into the Lesson 2 Assignment in the Module 1 Assignment Booklet.

Reflect on the Big Picture

Now that you understand impulse and how the product of net force and time will impact a situation, you understand how a helmet can prevent injuries when people are participating in sports and recreational activities. Using the concept of impulse, explain how a helmet prevents injuries when somebody participates in the sport or recreational activity of your choice. Save your explanation in your course folder.

Module 1: Lesson 2 Assignment

Ensure that you have finished all of the questions for both the Lesson 1 and Lesson 2 Assignments. When you are finished, submit the Module 1 Assignment Booklet to your teacher to be marked.

Lesson Summary

In this lesson you explored the following essential questions:

- What is impulse?
- How is impulse related to a change in momentum?
- How is impulse used in the design of safety devices that protect the occupants of vehicles during a crash?

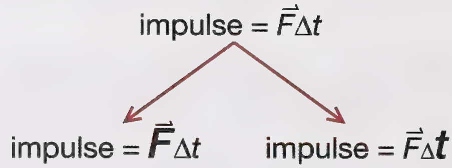
Impulse is the product of the net force applied to an object and the time interval during an interaction. Newton's second law can be used to demonstrate that the impulse applied to an object is equivalent to the change in the momentum of that object.

$$\vec{F}_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t}$$

$$\vec{F}_{\text{net}} \Delta t = \Delta \vec{p}$$

$$\text{impulse} = \vec{F}_{\text{net}} \Delta t, \text{ or impulse} = \Delta \vec{p}$$

In the design of many safety devices, the force providing the impulse to an object or a person can be minimized if the time interval through which the force acts is maximized.



Lesson Glossary

impulse: the product of the net force applied to an object and the time interval during an interaction

The equation is as follows:

$$\text{impulse} = \vec{F}_{\text{net}}\,\Delta t$$

Module 1—Momentum and Impulse

Module Summary

Module 1 explored the concepts of momentum and impulse as they relate to the function and design requirements of vehicle safety devices. In particular, we examined the phrase "speed kills" and how it is possible to reduce injury and death when a vehicle occupant experiences a large change in momentum during a collision.

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Within this context the following questions were examined:

- What is momentum?

- Why is momentum defined as a vector?

Momentum is the product of a vector (velocity) and a scalar (mass), so it is a vector quantity. The direction of the momentum vector is the same as the direction of the velocity vector.

- How is momentum related to Newton's laws?

momentum: the product of the mass and velocity of an object

It is expressed as an equation:

$$\vec{p} = m\vec{v}$$

Quantity	Symbol	SI Unit
momentum	$\vec{p}$	kg·m/s
mass	m	kg
velocity	$\vec{v}$	m/s

Newton's second law explains that the external, non-zero, net force acting on an object will be equal to the rate of change of the momentum of that object. This has particular relevance to situations involving large changes in momentum, such as those of vehicle collisions that occur at a high rate of speed.

Module 1 also explored the working principles of safety devices in the context of the following questions:

- What is impulse?
- How is impulse related to a change in momentum?

impulse: the change in momentum, $\Delta\vec{p}$, and the product of net force and time

It is expressed in equation form:

$$\begin{aligned}\Delta\vec{p} &= \vec{F}_{\text{net}}\Delta t\\ &= m\Delta\vec{v}\end{aligned}$$

Quantity	Symbol	SI Unit
impulse	$\Delta\vec{p}$	N·s or kg·m/s
net force	$\vec{F}_{\text{net}}$	N
time	t	S
momentum	$\vec{p}$	kg·m/s
mass	m	kg
velocity	$\vec{v}$	m/s

Since force and momentum are vector quantities, impulse is a vector quantity. The direction of the impulse vector is the same as the direction of the net force vector.

- How is impulse related to Newton's laws?

By applying Newton's second law, impulse can be described in terms of a change in velocity, caused by a non-zero (unbalanced) net force.

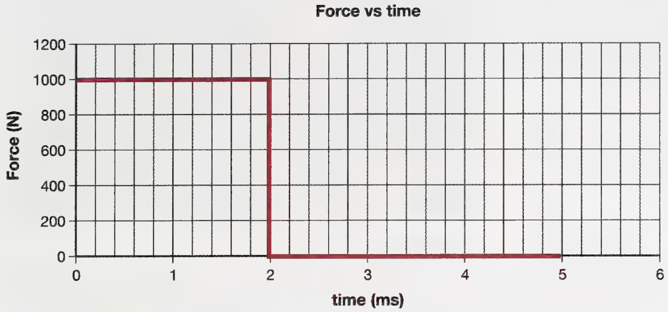
$$\Delta\vec{p} = \vec{F}_{\text{net}}\Delta t$$

$$\Delta\vec{p} = \left(m\vec{a}\right)\Delta t$$

$$\Delta\vec{p} = m\left(\frac{\Delta\vec{v}}{\Delta t}\right)\Delta t$$

$$\Delta\vec{p} = m\Delta\vec{v}$$

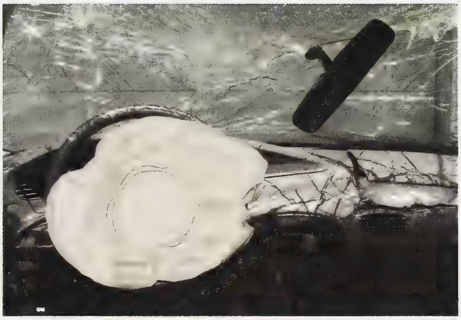
Impulse may also be represented graphically in the form of a force-time graph. In such cases, the product of force and time, or the impulse, is represented by the area of the graph.



We also learned that the force associated with a given impulse can be reduced by applying it over a greater time period, a principle related to the design of vehicle safety devices such as air bags and crumple zones.

Module Assessment

Car safety has changed dramatically since Karl Benz invented the first gasoline-powered vehicle in 1885. The first cars had no safety devices. Drivers had to wear goggles to protect themselves from hitting birds and insects while driving. Windshields were made of plate glass that would shatter into pieces if hit. Today we take it for granted that all cars have seat belts, safety glass, and airbags. There are even cars with multiple airbags, night vision systems, back up cameras and/or sonar and cars, such as the Volvo XC60, that will stop themselves to prevent collisions. Car designers, scientists, and engineers are even working on cars that will drive themselves in the future.



Most of the improvements in safety are the result of crash test information that comes from crash test dummies. Today's crash test dummies are the culmination of many decades of testing. From human volunteers to animal cadavers to modern dummies, the technology to simulate dangerous crashes has improved dramatically. Modern dummies come in multiple sizes to simulate humans from small babies up to large adults; they have joints and move like their human counterparts but are full of sensors to measure the accelerations and forces that they experience during a crash to determine how dangerous the collision would be for real people.

In a crash test, two test dummies with a mass of 77 kg each are placed in the test vehicle. The driver dummy has a seatbelt while the passenger dummy does not have a seatbelt. The vehicle is placed on a track and run into a wall at 56 km/h. Upon impact, everything comes to a halt.

© trialart-info/shutterstock

- Calculate the impulse for each dummy during the collision. (Analytic 5 marks)
- The seatbelt brings the driver dummy safely to a halt whereas the passenger stops violently by hitting the dashboard and windshield. Draw a force vs time graph to demonstrate an example set of forces working on each dummy and show that the graphed impulse for each dummy matches the previous calculations. (Graphing 5 marks)
- Describe why the graph for each dummy is different and how those differences show that the driver dummy stops safely and the passenger dummy does not stop safely. (Holistic 5 marks)

This module assessment is similar to the types of questions asked on a Diploma Exam. It is designed to examine your knowledge and application of several skills and concepts from the first module. Before submitting your solution, read the Analytic Question Scoring Guide and Graphing Question Scoring Guide in the Appendix. These are the same rubrics as used on the Diploma Exam.

If you have any questions about the marking guides or the question itself contact your teacher for guidance.

Module 1 Glossary

Consult the glossary in the textbook for other definitions that you may need to complete your work.

impulse: the product of the net force applied to an object and the time interval during an interaction

The equation is as follows:

$$\text{impulse} = \vec{F}_{\text{net}}\,\Delta t$$

momentum: is the product of the mass and velocity of an object

Expressed as an equation it is $\vec{\rho} = m\vec{v}$.

Appendix

Analytic Question Scoring Guide

(5 marks each)

Check before you submit your work:

Did you state the relevant physics principles as stated on your physics data sheet?

Did you state the equation from the equation sheet?

Did you show the manipulated form of the equation?

Did you show the substitution and units?

Did you calculate the correct final answer and paraphrase the answer with the correct significant digits and appropriate units?

Scoring Guide for Analytic Questions

Physics Principles

Score	Description
4	Both relevant physics principles are stated and both are clearly related to the response. Physics principles for questions involving linear vector addition require explicit communication of the vector nature; e.g., a situational diagram or a free-body diagram (FBD) for forces and a vector addition diagram.
3	Both relevant physics principles are stated, but only one is clearly related to the response.
2	Both relevant physics principles are stated but neither is clearly related to the response. **or** One relevant physics principle is stated and is clearly related to the response.
1	One relevant physics principle is stated.
0	No relevant physics principle is stated.

Substitutions

Score	Description
1	All substitutions are shown. Significant digits are not required in intermediate steps. A response with at most one implicit unit conversion may receive this score. An incomplete or incorrect response may receive this score if all the values substituted are appropriate; i.e., length measurements into length variables, energy measurements into energy variables.
0	Too many substitutions are missing. **or** The response contains one invalid substitution; i.e., electric field strength for energy or speed for electric potential difference.

Formulas

Score	Description
3	All relevant formulas required for the complete solution are present and have been written as they appear on the equations sheet or in the information given with the question.
2	Most relevant formulas are stated. **or** Derived formulas are used as starting points.
1	One relevant formula from the formula sheet is stated.
0	No relevant formula is stated.

Final Answer

Score	Description
2	The value of the answer to the complete problem is stated and calculated consistently with the solution presented. The final answer is stated with the appropriate number of significant digits and with appropriate units. A response in which an inappropriate substitution has been made may receive this score if the incorrect units are consistently carried forward.
1	The value of the final answer is stated, calculated consistently with the solution presented. Units or significant digits are incorrect. **or** The response is incomplete, but an intermediate value is stated and calculated consistently with the solution presented with appropriate units (significant digits not required).

0	The answer stated is unrelated to the solution shown. **or** No answer is given.

Discussion Scoring Guide

Principles involved: momentum				
Criteria	**Level 1 (Below Standard)**	**Level 2 (Approaching Standard)**	**Level 3 (Standard)**	**Level 4 (Above Standard)**
Knowledge				
Demonstrates understanding of the situation, physics principles and technology, and their connections.	Demonstrates a vague and sometimes incorrect understanding of the physics principles involved. Obvious irrelevant or missing information.	Demonstrates a basic understanding of the physics principles involved. May exhibit minor mistakes or vague information or application to the situation.	Demonstrates a good understanding of the physics principles involved and applies them properly to the given situation. All necessary information is given.	Demonstrates a superior understanding of the physics principles involved and their application to the situation. All applications are considered in detail.
Reflection				
The post shows reflection on one's own and other students' work. Contributes to the group discussion.	Does not make an effort to participate. Seems indifferent to discussion.	Occasionally makes meaningful reflections on the group's efforts or discussions. Marginal effort is shown to become involved with the group or discussion.	Frequently makes meaningful reflections on the group's efforts and presents relevant viewpoints for consideration by the group. Interacts freely with group members.	Regularly attempts to motivate the group discussion and delve deeper into concepts. Interacts freely and encourages all group members.

Content and presentation of discussion summary				
The information is logically arranged in a clear and concise manner.	The information is poorly organized with many concepts implied. Irrelevant or rambling sentences make reading difficult.	The information is somewhat organized with implied concepts. Excessive words or awkward sentences are used, which hinder reading.	The information is well-organized and logically arranged. All concepts are explicitly explained. There are a few awkward but understandable sentences.	The information is well-organized and very easy to understand. Well-worded sentences make reading pleasurable.

Graphing Question Scoring Guide

(5 marks)
Check before you submit your work:

Did you put a title on the graph?
Did you label each axis with an appropriate title including units?
Are the axis scales appropriate to the size of the graph?
Is the equation shown?
Did you calculate the area and paraphrase the answer with the correct significant digits and appropriate units?

Scoring Guides for Graphing Skill-Based Questions—Mathematical Treatment

Score	Description
5	• All formulas are present. • All substitutions are given and are consistent with the graphed data. • The relationship between the slope, area, or intercept, and the appropriate physics is explicitly communicated. • The final answer is stated with appropriate significant digits and with appropriate units. Unit analysis is explicitly provided, if required. **Note:** one minor error may be present.*
4	• The response contains implicit treatment.** **or** • The response contains explicit treatment with up to three minor errors or one major error.***

3	• The response is incomplete but contains some valid progress toward answering the question; i.e., coordinates of relevant points are read correctly, including powers of 10 and units, and a valid substitution is shown.
2	• The coordinates of one relevant point are read. • The reason for requiring a point is addressed or implied.
1	• A valid start is present.
0	• Nothing appropriate to the mathematical treatment required is present.

*Minor errors include:

- Misreading a data value while interpolating or extrapolating up to one-half grid off.
- Stating the final answer with incorrect (but not disrespectful) units.
- Stating the final answer with incorrect (but not disrespectful) significant digits.
- Missing one of several different formulas.

**Implicit treatment means:

- Substituting appropriate values into a formula from the data sheets without stating the formula.
- Starting with memorized, derived formulas not given on the equations sheet.
- Substituting the value from one calculation into a second formula without communicating that the physics quantity in the two formulas is the same.

***Major errors include:

- Using off-line points (most often, this is calculating the slope using data points that are not on a linear line of best fit).
- Using a single data point ratio as the slope.
- Missing powers of 10 in interpolating or extrapolating.

Holistic Scoring Guide

<table>
<tr><th>Score</th><th>Description</th></tr>
<tr><td>5</td><td>The nature of a response that will receive a score of 5 has the following characteristics:
The response addresses, with appropriate knowledge, all the major concepts in the question (all bullets must be attempted).
The student applies major physics principles in the response (appropriate physics principles are stated).
The relationships between ideas contained in the response are explicit* (physics principles are clearly linked to the application).
The reader has no difficulty following the strategy or solution presented by the student.
Statements made in the response are supported explicitly.*
Note: the response may contain minor errors or have minor omissions.</td></tr>
<tr><td>4</td><td>The nature of a response that will receive a score of 4 has the following characteristics:
The response addresses, with appropriate knowledge, all the major concepts in the question (all bullets must be attempted).
The student applies major physics principles in the response (appropriate physics principles are stated).
The relationships between the ideas contained in the response are implied**(physics principles are stated but not properly linked to the application).
The reader has some difficulty following the strategy or solution presented by the student.
Statements made in the response are supported implicitly.**
Note: the response is mostly complete and mostly correct, although it may contain errors or have omissions, and contains some application of physics principles.</td></tr>
<tr><td>3</td><td>The nature of a response that will receive a score of 3 has the following characteristics:
The response addresses, with some appropriate knowledge, all the major concepts in the question (all bullets must be attempted).
The student does not apply major physics principles in the response (all appropriate physics principles are not stated).
There are no relationships between the ideas contained in the response (physics principles are stated but not applied).
The reader may have difficulty following the strategy or solution presented by the student.</td></tr>
<tr><td>2</td><td>The nature of a response that will receive a score of 2 has the following characteristic:
The response addresses, with some appropriate knowledge, two of the major concepts in the question (only two bullets are attempted).</td></tr>
</table>

1	The nature of a response that will receive a score of 1 has the following characteristic: • The response addresses, with some appropriate knowledge, one of the major concepts in the question (only one bullet is attempted).
0	• The student provides a solution that is invalid for the question.

*Explicit means the response is clearly stated; the marker does not have to interpret.

**Implicit (implied) means the response is not clearly stated; the marker must interpret.

For example:

Explicit: An electron has a negative charge while a proton has a positive charge.

The answer is clear with no possible misinterpretation.

Implicit: An electron has a negative charge while a proton does not.

The answer is not clear because the marker does not know if a proton is neutral or positively charged. There is more than one possible way to interpret the answer.

Lesson 1

Self-Check Answers

SC 1.

Momentum is the product of the mass and velocity of an object.

Expressed as an equation it is $\vec{p} = m\vec{v}$

Quantity	Symbol	SI Unit
momentum	$\vec{p}$	kg·m/s
mass	m	kg
velocity	$\vec{v}$	m/s

Since momentum is the product of a vector (velocity) and a scalar (mass), it is a vector quantity. The direction of the momentum vector is in the same direction, as the velocity of the object.

SC 2.

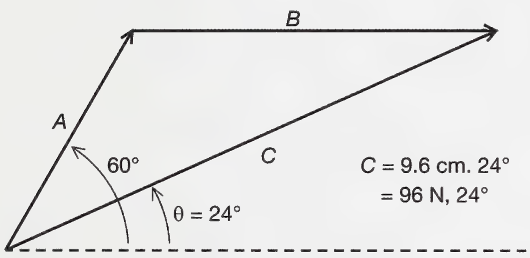
a. A resultant is a single vector that represents the sum of two or more other vectors.

b. The resultant is shown graphically by an arrow that connects the tail of the first vector with the head of the last vector.

SC 3. No, the order of adding vectors has no effect on the resultant.

SC 4.

Scale: Let 1 cm = 10 N



SC 5.

Given

$\Delta\vec{d}_1 = 45.0\text{ m}[310°]$

$\Delta\vec{d}_2 = 35.0\text{ m}[135°]$

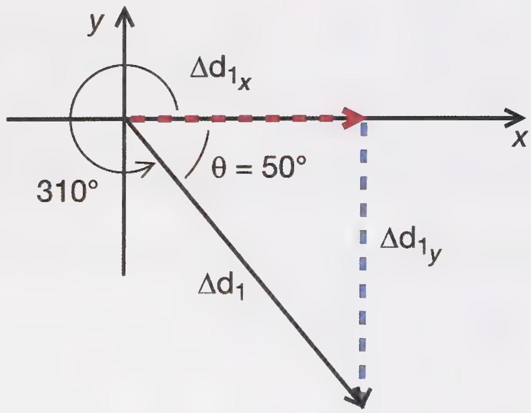
Required

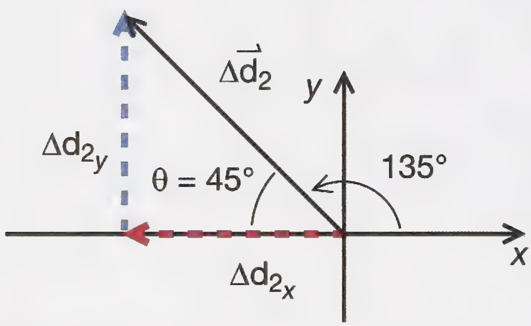
The resultant displacement, $\Delta\vec{d}_R$

Analysis and Solution

Step 1: These vectors are not collinear, so find the x and y components of each one:

$\Delta\vec{d}_1 = 45.0\text{ m}[310°]$ $\quad\quad$ $\Delta\vec{d}_2 = 35.0\text{ m}[135°]$





Note: Vector notation is not used for the components because they are scalars.

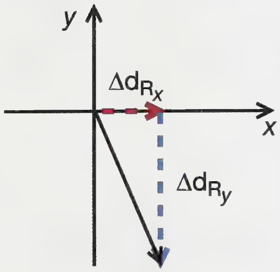
Displacement	x-component	y-component
$\Delta\vec{d}_1 = 45.0\text{ m}[310°]$	$\Delta d_{1x} = \Delta\vec{d}_1\cos\theta$ $= (45.0\text{ m})\cos(50°)$ $= 28.93\text{ m}$	$\Delta d_{1y} = \Delta\vec{d}_1\sin\theta$ $= -(45.0\text{ m})\sin(50°)$ $= -34.47\text{ m}$
$\Delta\vec{d}_2 = 35.0\text{ m}[135°]$	$\Delta d_{2x} = \Delta\vec{d}_2\cos\theta$ $= -(35.0\text{ m})\cos(45°)$ $= -24.75\text{ m}$	$\Delta d_{2y} = \Delta\vec{d}_2\sin\theta$ $= (35.0\text{ m})\sin(45°)$ $= 24.75\text{ m}$

Note: A negative sign was added to Δd_{2x} because it points in the negative x direction.
A negative sign was added to Δd_{1y} because it points in the negative y direction.

Step 2: Determine the magnitude of $\Delta\vec{d}_{\mathrm{R}}$:

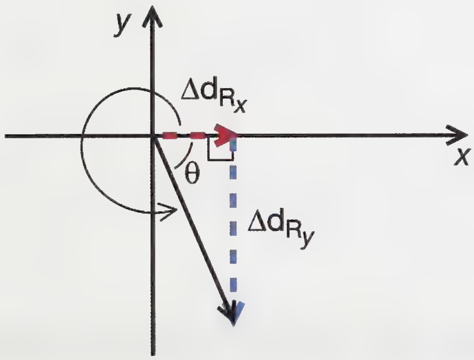
$$\begin{aligned}\Delta d_{\mathrm{Rx}} &= \Delta d_{1x} + \Delta d_{2x}\\ &= 28.93\ \mathrm{m} + -24.75\ \mathrm{m}\\ &= 4.18\ \mathrm{m}\end{aligned}$$

$$\begin{aligned}\Delta d_{\mathrm{Ry}} &= \Delta d_{1\mathrm{y}} + \Delta d_{2\mathrm{y}}\\ &= -34.47\ \mathrm{m} + 24.75\ \mathrm{m}\\ &= -9.72\ \mathrm{m}\end{aligned}$$



$$\begin{aligned}C^2 &= A^2 + B^2\\ \left(\Delta\vec{d}_{\mathrm{R}}\right)^2 &= \left(\Delta d_{\mathrm{Rx}}\right)^2 + \left(\Delta d_{\mathrm{Ry}}\right)^2\\ \Delta\vec{d}_{\mathrm{R}} &= \sqrt{\left(\Delta d_{\mathrm{Rx}}\right)^2 + \left(\Delta d_{\mathrm{Ry}}\right)^2}\\ &= \sqrt{(4.18\ \mathrm{m})^2 + (-9.72\ \mathrm{m})^2}\\ &= 10.6\ \mathrm{m}\end{aligned}$$

Step 3: Determine the direction of $\Delta\vec{d}_{\mathrm{R}}$:



$$\begin{aligned}\tan\theta &= \frac{\Delta d_{\mathrm{Ry}}}{\Delta d_{\mathrm{Rx}}}\\ \theta &= \tan^{-1}\left(\frac{\Delta d_{\mathrm{Ry}}}{\Delta d_{\mathrm{Rx}}}\right)\\ &= \tan^{-1}\left(\frac{9.72\ \cancel{\mathrm{m}}}{4.18\ \cancel{\mathrm{m}}}\right)\\ &= 66.7^\circ\end{aligned}$$

The direction is measured counterclockwise from the x-axis. Therefore the direction is $360^\circ - 66.7^\circ = 293^\circ$.

Paraphrase

The final displacement of the skateboarder is 10.6 m [293°]

SC 6. A scalar quantity is a measurement that has magnitude only. Examples include time and mass.

SC 7. A vector quantity has both magnitude and direction. Examples of vectors include velocity, force and acceleration.

SC 8.

Given

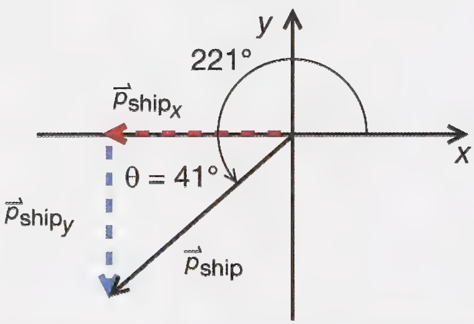
$\vec{p}_{ship} = 9.70 \times 10^7 \text{ kg}\bullet\frac{\text{m}}{\text{s}}[221°]$

Required

The component of the momentum provided by tugboat A, $\vec{p}_{\text{ship x}}$.
The component of the momentum provided by tugboat B, $\vec{p}_{\text{ship y}}$.

Analysis and solution

The momentum of the ship is provided by two tugboats. Tugboat A provides the component in the negative x direction. Tugboat B provides the component in the negative y direction. The component can be determined by using trigonometry.



Momentum	X component	Y component
$\vec{p}_{ship}$	$p_{ship\ x} = p_{ship}\cos\theta$ $= -\left(9.70\times10^7 \text{ kg}\bullet\frac{\text{m}}{\text{s}}\right)\cos(41°)$ $= -7.32\times10^7 \text{ kg}\bullet\frac{\text{m}}{\text{s}}$	$p_{ship\ y} = p_{ship}\sin\theta$ $= -\left(9.70\times10^7 \text{ kg}\bullet\frac{\text{m}}{\text{s}}\right)\sin(41°)$ $= -6.36\times10^7 \text{ kg}\bullet\frac{\text{m}}{\text{s}}$

Paraphrase

The component of the momentum provided by tugboat A is $7.32\times10^7\ \text{kg}\bullet\frac{\text{m}}{\text{s}}$, in the negative x direction.

The component of the momentum provided by tugboat B is $6.36\times10^7\ \text{kg}\bullet\frac{\text{m}}{\text{s}}$, in the negative y direction.

SC 9.

Given

$$m = 70.0\ \text{kg}$$

$$\vec{v} = 5.0\ \frac{\text{m}}{\text{s}}[W]$$
$$= -5.0\ \frac{\text{m}}{\text{s}}$$

Required

The momentum of the person, $\vec{p}$.

Analysis and Solution

Let west be the negative direction.

$$\vec{p} = m\vec{v}$$
$$= (70.0\ \text{kg})\left(-5.0\ \frac{\text{m}}{\text{s}}\right)$$
$$= -3.5\times10^2\ \text{kg}\bullet\frac{\text{m}}{\text{s}}$$

Paraphrase

The momentum of the person is $3.5\times10^2\ \text{kg}\bullet\frac{\text{m}}{\text{s}}$ west.

SC 10.

Given

$$\begin{aligned}\vec{v} &= 95.0\ \frac{\text{km}}{\text{h}}[\text{N}]\\ &= +95\ \frac{\text{km}}{\text{h}}\\ &= +95\ \frac{\text{km}}{\text{h}} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{h}}{3600\ \text{s}}\\ &= 26.3\bar{8}\ \frac{\text{m}}{\text{s}}\end{aligned}$$

$$\begin{aligned}\vec{p} &= 7250\ \text{kg}\bullet\frac{\text{m}}{\text{s}}[\text{N}]\\ &= +7250\ \text{kg}\bullet\frac{\text{m}}{\text{s}}\end{aligned}$$

Required

The mass of the motorcycle, m.

Analysis and Solution

Let north be the positive direction.

This solution involves dividing momentum by velocity. Dividing one vector by another is beyond the scope of high school physics so the vector notation is dropped.

$$\vec{p} = m\vec{v}$$

$$\begin{aligned}m &= \frac{p}{v}\\ &= \frac{+7250\ \text{kg}\bullet\frac{\text{m}}{\text{s}}}{+26.3\bar{8}\ \frac{\text{m}}{\text{s}}}\\ &= 2.7\times10^2\ \text{kg}\end{aligned}$$

Paraphrase

The mass of the motorcycle is 2.7×10^2 kg.

SC 11.

$$\vec{F} = m\vec{a} \text{ and } \vec{a} = \left(\frac{\vec{v}_{\mathrm{f}} - \vec{v}_{\mathrm{i}}}{\Delta t}\right)$$

therefore,

$$\vec{F}_{\text{net}} = m\left(\frac{\vec{v}_{\mathrm{f}} - \vec{v}_{\mathrm{i}}}{\Delta t}\right)$$

$$\vec{F}_{\text{net}} = \left(\frac{m\vec{v}_{\mathrm{f}} - m\vec{v}_{\mathrm{i}}}{\Delta t}\right)$$

$$\vec{F}_{\text{net}} = \left(\frac{\vec{p}_{\mathrm{f}} - \vec{p}_{\mathrm{i}}}{\Delta t}\right)$$

$$\vec{F}_{\text{net}} = \left(\frac{\Delta\vec{p}}{\Delta t}\right)$$

Lesson 2

Self-Check Answers

SC 1.

1. The putty ball that landed on the hardest surface was the most deformed. The putty ball that landed on the surface that was the most cushioned was the least deformed. The putty ball that landed on the third surface—having intermediate cushioning—experienced an intermediate amount of deformation.

2. The putty ball that landed on the hardest surface was the most deformed. The putty ball that landed on the most-cushioned surface was the least deformed.

3. The instant prior to striking the surface, the putty ball had an initial velocity. Once contact was made with the surface, the putty ball began to decelerate and it eventually came to a stop, making $v_{\mathrm{f}} = 0$. The time to bring the ball to a stop is Δt, and the distance required to stop the ball is Δd. The cushioning material is compressed this distance (Δd) while stopping.

   Look at the following analysis of the equation. It shows that the greater the compression of the material, the greater the stopping distance. This shows that the time interval required to bring the ball to a stop is greater.

$$\Delta d = \left(\frac{v_f + v_i}{2}\right)\Delta t$$

$$\Delta t = \Delta d\left(\frac{2}{v_f + v_i}\right), \text{ remember the final velocity is 0}$$

$$\Delta t = \frac{2}{v_i}\Delta d, \text{ the stopping time is directly proportional to the stopping distance}$$

SC 2.

In the previous lesson you learned that Newton's second law can be expressed in terms of the change in momentum.

$$\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$$

If this equation is rearranged, it can be shown that the change in momentum is equal to the net force multiplied by the time interval.

$$\vec{F}_{net}\,\Delta t = \Delta\vec{p}$$
$$\vec{F}_{net}\,\Delta t = m\Delta\vec{v}$$

If identical eggs are dropped from the same height onto a surface, the change in momentum will be identical for both eggs. However, if the stopping time interval for one egg is maximized, then the force that egg experiences can be minimized.

SC 3.

The following equations show how Newton's second law reveals two ways of calculating impulse.

$$\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$$

$$\vec{F}_{net}\,\Delta t = \Delta\vec{p}$$
$$\vec{F}_{net}\Delta t = m\Delta\vec{v}$$

$$\text{impulse} = \vec{F}_{net}\,\Delta t$$

or

$$\text{impulse} = \Delta\vec{p}$$

or

$$\text{impulse} = m\Delta\vec{v}$$

SC 4.

Quantity	Symbol	SI Unit
impulse	$\vec{F}_{\text{net}}\,\Delta t$	N•s or kg•m/s
net force	$\vec{F}_{\text{net}}$	N
time interval	Δt	s
momentum	$\vec{p}$	kg•m/s
mass	m	kg
velocity	$\vec{v}$	m/s

Note that impulse, net force, momentum, and velocity are all vector quantities, so the symbol for each of these quantities includes an arrow.

SC 5.

Solution to Problem 1

Given

$\Delta t = 3.64 \text{ s}$

$\vec{F}_{\text{net}} = 200 \text{ N [S]}$

$m = 1100 \text{ kg}$

Required

a. the impulse provided to the car, $\vec{F}_{\text{net}}\Delta t$

b. the change in velocity of the car, $\Delta\vec{v}$

Analysis and Solution

Let south be the positive direction.

a. $$\begin{aligned}\vec{F}_{\text{net}}\Delta t &= (+200\ \text{N})(3.64\ \text{s})\\ &= +728\ \text{N}\bullet\text{s}\\ &= 728\ \text{N}\bullet\text{s}\ [\text{S}]\end{aligned}$$

b. $$\begin{aligned}\vec{F}_{\text{net}}\Delta t &= m\Delta\vec{v}\\ \Delta\vec{v} &= \frac{\vec{F}_{\text{net}}\Delta t}{m}\\ &= \frac{(+200\ \text{N})(3.64\ \text{s})}{1100\ \text{kg}}\\ &= +0.664\ \text{m/s}\\ &= +0.662\ \text{m/s}\ [\text{S}]\end{aligned}$$

Paraphrase

The impulse provided to the car is 728 N•s [S].

The velocity of the car changes by 0.662 m/s [S].

Solution to Problem 2

Given

$m = 400\ \text{kg}$

$\Delta t = 4.20\ \text{s}$

$$\begin{aligned}\vec{v}_{\text{i}} &= 0.200\ \frac{\text{m}}{\text{s}}\ [\text{backward}]\\ &= -0.200\ \frac{\text{m}}{\text{s}}\end{aligned}$$

$$\begin{aligned}\vec{v}_{\text{f}} &= 1.80\ \frac{\text{m}}{\text{s}}[\text{forward}]\\ &= +1.80\ \frac{\text{m}}{\text{s}}\end{aligned}$$

Required

The average net force acting on the dog team.

Analysis and Solution

Let forward be the positive direction.

$$\begin{aligned}
\vec{F}_{\text{net ave}}\Delta t &= m\Delta\vec{v} \\
\vec{F}_{\text{net ave}} &= \frac{m\Delta\vec{v}}{\Delta t} \\
&= \frac{m\left(\vec{v}_{\text{f}} - \vec{v}_{\text{i}}\right)}{\Delta t} \\
&= \frac{400\ \text{kg}\left(\left(+1.80\ \text{m/s}\right) - \left(-0.200\ \text{m/s}\right)\right)}{4.20\ \text{s}} \\
&= +190\ \text{N} \\
&= 190\ \text{N}\ [\text{forward}]
\end{aligned}$$

Paraphrase

The net force acting on the dog team is 190 N [forward].

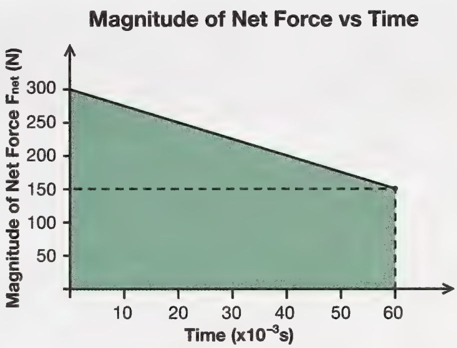
SC 6.

Given

$\vec{F}_{\text{net i}} = 300\ \text{N}$

$\vec{F}_{\text{net f}} = 150\ \text{N}$

$\Delta t = 6.0 \times 10^{-2}\ \text{s}$



Required

The magnitude of the impulse, $\vec{F}_{\text{net}}\Delta t = \Delta\vec{p}$.

Analysis and Solution

The magnitude of the impulse is equal to the area under a net force versus time graph.

$$\begin{aligned}
\text{magnitude of impulse} &= \frac{1}{2}(a+b)h \\
&= \frac{1}{2}(150\ \text{N} + 300\ \text{N})\left(6.0\times10^{-2}\ \text{s}\right) \\
&= 14\ \text{N}\bullet\text{s}
\end{aligned}$$

Paraphrase

The magnitude of the impulse is 14 N•s.

SC 7.

a. Some of the safety devices that are incorporated into the design of automobiles include a padded dashboard, crumple zones built into the body of the vehicle, collapsible steering column, and airbags.

b. Each safety device is designed to change the momentum of an object by applying an equivalent amount of impulse. This is described by the following rearranged version of Newton's second law: $\vec{F}_{\text{net}}\Delta f = \Delta\vec{p}$.

The safety devices mentioned do this by increasing the time interval required to change the momentum so that the average net force required to stop the motion is minimized.

SC 8. To change the momentum of an object like a tennis ball, an athlete must supply an impulse to the ball. Since impulse is the product of the net force applied to the ball and the time interval during the interaction, the athlete attempts to maximize both the net force and the interaction time. The time interval is maximized by ensuring that the ball and the racquet are in contact for as long a time interval as possible. This is what the idea of proper follow through is all about.

Principles involved: momentum				
Criteria	**Level 1 (Below Standard)**	**Level 2 (Approaching Standard)**	**Level 3 (Standard)**	**Level 4 (Above Standard)**
Knowledge				
Demonstrates understanding of the situation, physics principles and technology, and their connections.	Demonstrates a vague and sometimes incorrect understanding of the physics principles involved. Obvious irrelevant or missing information.	Demonstrates a basic understanding of the physics principles involved. May exhibit minor mistakes or vague information or application to the situation.	Demonstrates a good understanding of the physics principles involved and applies them properly to the given situation. All necessary information is given.	Demonstrates a superior understanding of the physics principles involved and their application to the situation. All applications are considered in detail.

Reflection				
The post shows reflection on one's own and other students' work. Contributes to the group discussion.	Does not make an effort to participate. Seems indifferent to discussion.	Occasionally makes meaningful reflections on the group's efforts or discussions. Marginal effort is shown to become involved with the group or discussion.	Frequently makes meaningful reflections on the group's efforts and presents relevant viewpoints for consideration by the group. Interacts freely with group members.	Regularly attempts to motivate the group discussion and delve deeper into concepts. Interacts freely and encourages all group members.
Content and presentation of discussion summary				
The information is logically arranged in a clear and concise manner.	The information is poorly organized with many concepts implied. Irrelevant or rambling sentences make reading difficult.	The information is somewhat organized with implied concepts. Excessive words or awkward sentences are used, which hinder reading.	The information is well-organized and logically arranged. All concepts are explicitly explained. There are a few awkward but understandable sentences.	The information is well-organized and very easy to understand. Well-worded sentences make reading pleasurable.

Physics 30

Learn EveryWare

The Conservation of Momentum in Isolated Systems

Module 2

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Module 2—The Conservation of Momentum in Isolated Systems

Module Introduction

In this module, you will learn how to apply conservation principles related to momentum and energy in the analysis of one- and two-dimensional collisions.


Digital Vision/Getty Images

The essential questions that you will be considering in this module are as follows:

- How is the conservation of momentum principle applied to linear collisions?
- Is it possible to describe mathematically the difference between elastic and inelastic collisions using kinetic energy?
- How are the conservation of momentum principle and vector component analysis applied to solve non-linear collisions?

Big Picture

In Module 1 you explored the need for better-designed vehicle safety devices to keep our roads safe. If you have ever been in a car accident or know someone who has, you know that collisions can happen suddenly and are over very quickly. In some cases, experts are called in to decide who may be at fault and to establish responsibility for the associated costs of an accident. In this process, scientific principles such as the conservation of momentum are applied to develop an approximate reconstruction of the events just prior to a collision. This reconstruction is important for establishing why and how a collision occurred.

S. Meltzer/Photolink/Getty Images

Consider what happens after a vehicle collision:

- Two vehicles collide at an intersection. The vehicle mass is scattered about and various forces act, producing heat, sound, and deformation of metal, plastic, and glass. The tires skid across the pavement and chunks of debris are thrown upwards into the sky. Hot metal fragments cool down as the wreckage waits to be cleared.

- An ambulance arrives at the scene to treat the injured. The police take numerous photographs of the scene, measure skid marks, and note the temperature and road conditions. Investigators estimate vehicle masses and note debris patterns.

- Investigations determine relevant events at the time of the collision and establish fault.

The investigation into the events just prior to the collision is based on the conservation of momentum, a principle that perfectly describes collisions that occur in isolated systems. An isolated system is one in which no mass enters or leaves the system, and no net external force acts on it. You will learn more about this in a later lesson, but such a system is not readily found on our planet (curling stones on a totally frictionless sheet of ice would be a good approximation). And although we know cars don't collide like curling stones, we could analyze car collisions in a similar way, assuming the system to be isolated just at the moment of impact. This assumption allows us to apply scientific principles to produce a reasonable approximation of events for a vehicle crash. In this module we will learn how to use the conservation of momentum to analyze and reconstruct collisions in both one and two dimensions.

Case File

On October 1, Kathy Johnson (not her real name) was driving home from work in her red car. As she approached the University Drive intersection, Bill Peterman (not his real name), driving his white pickup truck, attempted to cross the same intersection on his way to class. Unfortunately, they collided in the centre of the intersection.

As seen in this photo, both vehicles became joined and moved off to the side of the intersection before coming to rest. When the police arrived and began taking statements, Bill claimed that Kathy was well back from the intersection when he attempted to cross and that her high rate of speed was to blame for the accident. At the same time, Kathy insisted she was travelling at the posted speed limit, 50 km/h, and that Bill entered the intersection without proper care and attention.

Both Kathy and Bill received medical attention for non-life-threatening injuries. The property damage to both vehicles was substantial. The case is now before the courts to decide who will pay for the damages. In addition, the police have charged Kathy with driving in excess of the posted speed limit.

In her defence, Kathy has hired a consultant to perform a crash analysis hoping that it will confirm her speed prior to the collision. The intent of this analysis is to prove, beyond a reasonable doubt, that she was actually travelling at the speed limit just prior to the accident. How is this possible? How can conservation principles related to momentum and energy be applied to analyze and understand collisions?

You will refer to this case study as part of your Module 2 Project throughout the lessons of Module 2.

Essential Questions

As you are working in Module 2, keep the following questions in mind:

- How is the conservation of momentum principle applied to linear collisions?
- Is it possible to describe mathematically the difference between elastic and inelastic collisions using kinetic energy?
- How are the conservation of momentum principle and vector component analysis applied to solve non-linear collisions?

Module Assessment

Each lesson has a teacher-marked assignment, based on work completed in the lesson. In addition, you will be graded on your contributions to the Discuss section of each lesson.

You will also be asked to complete Self-Check or Try This questions, which you should place in your Physics 30 course folder. These are not formally assessed but are a valuable way to practise the concepts and skills of the lesson. These activities can provide you with reflective feedback on your understanding of the lesson work.

You will be marked for your lesson work on the following items:

- Module 2: Lesson 1 Assignment
- Module 2: Lesson 2 Assignment
- Module 2: Lesson 3 Assignment
- Module 2 Project

In each lesson you will prepare a crash analysis for your Module 2 Project of the collision between Kathy and Bill described in the Big Picture section. The analysis should be complete at the end of Lesson 3. It must show a complete solution for the mathematical determination of Kathy's speed just prior to the collision. If you have any questions contact your teacher.

In This Module

Lesson 1—Linear Collisions and the Conservation of Momentum

In this lesson you will explore the conservation of momentum in linear collisions. You will explore linear collisions in the context of the following questions:

- Can the complicated details of a linear collision be eliminated during the analysis of an interaction?
- What is the law of conservation of momentum? How can this law be applied?

Lesson 2—Kinetic Energy and Collisions

In this lesson you will explore the difference between elastic and inelastic collisions in terms of energy changes and the conservation of momentum. You will explore elastic and inelastic collisions in the context of the following questions:

- How can the kinetic energy of the objects in a system before and after a collision be used to categorize the type of collision?
- What is the fundamental difference between an elastic and an inelastic collision?
- In a collision between two automobiles, is it better for the occupants if the collision is elastic or inelastic?

Lesson 3—Analyzing Non-Linear Collisions

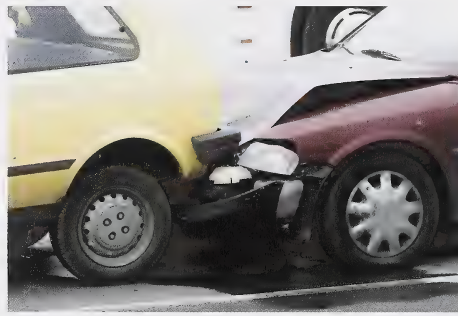
In this lesson you will apply the conservation principle and component analysis to analyze non-linear collisions. You will explore collisions of this nature in the context of the following questions:

- Is momentum conserved in a non-linear collision?
- How is component analysis applied to solve non-linear collision problems?

Module 2—The Conservation of Momentum in Isolated Systems

Lesson 1—Linear Collisions and the Conservation of Momentum

Get Focused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It is obvious that this photo shows a rear-end collision. The vehicles collided along a straight line—a linear collision.

During the collision, the **momentum** of each of the cars involved changed, and forces acted over time to provide an impulse to each car. These interactions are complicated; the forces not only changed the momentum of each vehicle, the forces also changed the shape of each vehicle. It is worthwhile to analyze interactions like this not only to determine who was responsible from a legal point of view, but also to improve the design of vehicles to reduce injuries to the occupants.

momentum: the product of the mass and velocity of an object

In this lesson you will explore linear collisions in the context of the following questions:

- Can the complicated details of a linear collision be eliminated during the analysis of an interaction?
- What is the law of conservation of momentum, and how can it be used to answer the previous question?
- Under what circumstances can this law be applied?

Module 2: Lesson 1 Assignments

Your teacher-marked Module 2: Lesson 1 Assignment requires you to submit a response to the following:

- Lab—LAB 1, LAB 2, and LAB 3
- Assignment—A 1, A 2, and A 3
- Discuss—D 5

Although the other questions in this lesson will not be marked by the teacher, you should still answer these questions. The Self-Check and Try This questions are placed in this lesson to help you review important information and build key concepts that may be applied in future lessons.

After a discussion with your teacher, you must decide what to do with the questions that are not part of your assignment. For example, you may decide to submit to your teacher the responses to Try This questions that are not marked. You should record the answers to all the questions in this lesson and place those answers in your course folder.

You will also begin your Module 2 Project. You will start working on a Crash Analysis, which you will save to your Physics 30 course folder and submit to your teacher for marks when you have finished Module 2.

Explore

Read

Read "Collisions in One Dimension" on pages 468 to 470 of your physics textbook for an introduction to collisions of systems of objects.

Understanding Isolated and Non-isolated Systems

Lawrence M. Sawyer / Sports and Recreation 2/Getty Images

A system is a collection of two or more objects. For example, a collection of curling stones on a sheet of ice represents a system. If the number of stones on the ice does not change, the system is closed. If there were no external forces acting on the stones, such as the players pushing them or the small amount of friction on the ice, the system would also be isolated.

There are other kinds of systems as well, some of which you may be familiar with.

In the *open system*, mass and energy may enter or leave the system, and external forces may influence the system. In the *closed system*, no mass enters or leaves the system but energy can. Also, external forces may influence the closed system. No net external force acts on the *isolated system*, and no mass or energy enters or leaves the system.

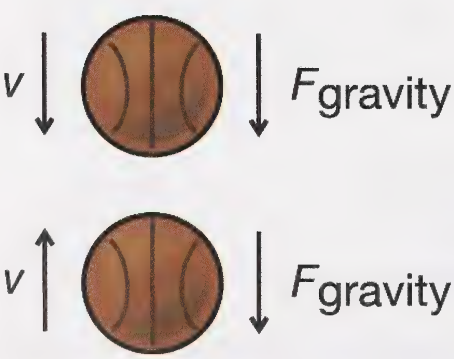
DID YOU KNOW?

A truly isolated system is impossible to create. Isolating a system in terms of mass is fairly straightforward. However, isolating the system with respect to energy and external forces, especially gravity, is far more difficult. For questions in Physics 30 it is assumed that the system is isolated from net external forces in order to simplify questions.

In this lesson when we examine the momentum of a system, it is important that the system under investigation is an isolated system. Assumptions are often made that the force of friction acting in a system is negligible, thereby providing a useful model for approximating real world situations, such as vehicle collisions.

It is easy to tell if a system is closed: no mass enters or exits. However, it is a bit more difficult to determine if a system is isolated—you must be aware of any external forces that may be acting on the system. Here are some examples:

Example 1

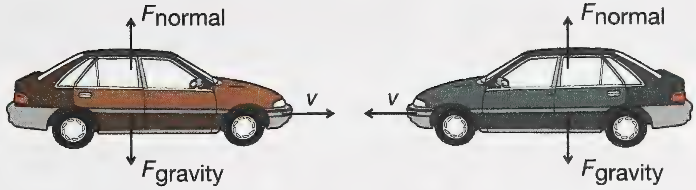


Two basketballs (the system) are moving vertically in the air. One of the balls is falling down, while the other has just been thrown upward. The two balls collide as they move.

The system is not isolated. The external force acting on both balls is the force of gravity.

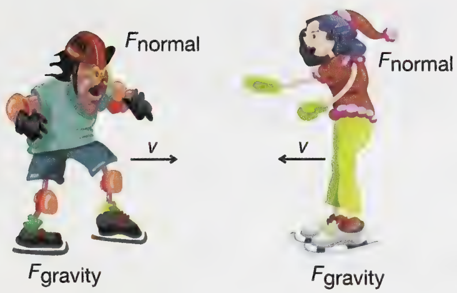
Example 2

Two cars (the system) are driving toward one another on smooth pavement and collide head on.



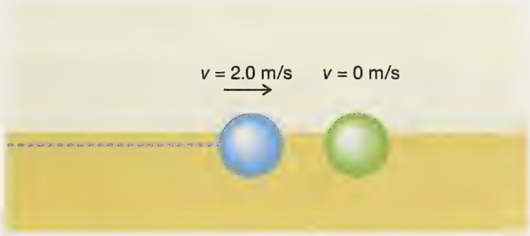
If we assume that the force of friction is negligible, this system can be treated as isolated. Interestingly, in the vertical plane, the normal force of the road balances the force of gravity, so there is no net external force in the vertical plane.

Example 3



Two skaters (the system) are coasting along frictionless ice and are headed straight for each other. They end up crashing into each other.

If we assume that the force of friction is negligible, this system can be treated as isolated. Again, there is no net external force acting in the vertical plane.



Module 1: Lesson 1 explained that any moving object has velocity and momentum. But what happens if two or more objects collide? What happens to the velocity and the momentum of each object? Let's explore this question using a simulation of a rear-end collision.

Lesson 1 Lab: Elastic Collisions Lab

Problem

Is momentum conserved in a linear collision?

Procedure

- Perform five different collisions using the "Collision 1D simulator" on the Physics 30 Multimedia DVD and complete the data tables in Observations.
- For clarity on the simulator, toggle "Show CM" and "Show CM Frame" to the off position (☐ Show CM ☐ Show CM Frame).
- To generate a new collision, click the "New" button (☐) and change the initial velocity of the blue ball by adjusting the velocity slider (Initial Velocity 8.0 m/s) to any value.
- After each collision, view the collision information by clicking the "Data" button (X|Y) and fill in the appropriate table.
- The following equations will be required to complete the tables:

  $\Delta\vec{v} = \vec{v}_{\text{final}} - \vec{v}_{\text{initial}}$ and $\Delta\vec{p} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}}$

Observations

Collision 1 (Sample Data)							
Object	mass (kg)	$\vec{v}_{\text{initial}}$ (m/s)	$\vec{v}_{\text{final}}$ (m/s)	$\Delta\vec{v}$ (m/s)	$\vec{p}_{\text{initial}}$ (kg·m/s)	$\vec{p}_{\text{final}}$ (kg·m/s)	$\Delta\vec{p}$ (kg·m/s)
blue	4.0	+8.0	-0.86	-8.86	+32	-3.44	-35.44
green	9.0	0	+3.94	+3.94	0	+35.46	+35.46
Collision 2							
Object	mass (kg)	$\vec{v}_{\text{initial}}$ (m/s)	$\vec{v}_{\text{final}}$ (m/s)	$\Delta\vec{v}$ (m/s)	$\vec{p}_{\text{initial}}$ (kg·m/s)	$\vec{p}_{\text{final}}$ (kg·m/s)	$\Delta\vec{p}$ (kg·m/s)
blue							
green							

Collision 3							
Object	mass (kg)	$\vec{v}_{\text{initial}}$ (m/s)	$\vec{v}_{\text{final}}$ (m/s)	$\Delta\vec{v}$ (m/s)	$\vec{p}_{\text{initial}}$ (kg·m/s)	$\vec{p}_{\text{final}}$ (kg·m/s)	$\Delta\vec{p}$ (kg·m/s)
blue							
green							
Collision 4							
Object	mass (kg)	$\vec{v}_{\text{initial}}$ (m/s)	$\vec{v}_{\text{final}}$ (m/s)	$\Delta\vec{v}$ (m/s)	$\vec{p}_{\text{initial}}$ (kg·m/s)	$\vec{p}_{\text{final}}$ (kg·m/s)	$\Delta\vec{p}$ (kg·m/s)
blue							
green							
Collision 5							
Object	mass (kg)	$\vec{v}_{\text{initial}}$ (m/s)	$\vec{v}_{\text{final}}$ (m/s)	$\Delta\vec{v}$ (m/s)	$\vec{p}_{\text{initial}}$ (kg·m/s)	$\vec{p}_{\text{final}}$ (kg·m/s)	$\Delta\vec{p}$ (kg·m/s)
blue							
green							

Module 2: Lesson 1 Assignment

Remember to submit the answers to LAB 1, LAB 2, and LAB 3 to your teacher as part of your Module 2: Lesson 1 Assignment.

Analysis

LAB 1.

a. Is there a relationship between the changes in *velocity* of the blue mass and the green mass? If yes, describe the relationship.

b. Is there a relationship between the changes in *momentum* of the blue mass and the green mass? If yes, describe the relationship.

You should see an interesting connection between the changes in momentum of each mass: the changes in momentum should be equal, but opposite. For example, if the blue mass has an increase in momentum, then the green mass has a decrease in momentum by the same amount. This connection leads us to an important concept about momentum and what happens to the total momentum of a system in a collision.

LAB 2. If the change in momentum of one object is exactly equal but opposite to the change in momentum of another object, what does that indicate about the total momentum of the system?

LAB 3. If the total momentum of a system before a collision is equal to the total momentum of a system after a collision, then momentum is conserved. For each of the five collisions, verify that the total *initial* momentum is equal to the total *final* momentum by completing the following table.

Notice the following shorthand way of talking about the sum of the momentums of the blue and green masses. The capital Greek letter sigma (Σ) is often used to denote sums.

$$\sum \vec{p}_{\text{initial}} = \vec{p}_{\text{initial green ball}} + \vec{p}_{\text{initial blue ball}}$$

$$\sum \vec{p}_{\text{final}} = \vec{p}_{\text{final green ball}} + \vec{p}_{\text{final blue ball}}$$

Collision	Total Initial Momentum $\sum \vec{p}_{\text{initial}}$ (kg·m/s)	Total Final Momentum $\sum \vec{p}_{\text{final}}$ (kg·m/s)	Is Momentum Conserved? $\sum \vec{p}_{\text{initial}} = \sum \vec{p}_{\text{final}}$
1			
2			
3			
4			
5			

Based on the lab activity, you should have found that the total momentum is conserved in a collision within an isolated system.

The Law of Conservation of Momentum

For any isolated system, the total momentum does not change. In a collision, momentum is conserved—the total momentum before the collision is equal to the total momentum after the collision. Expressed as an equation, the law of conservation of momentum is

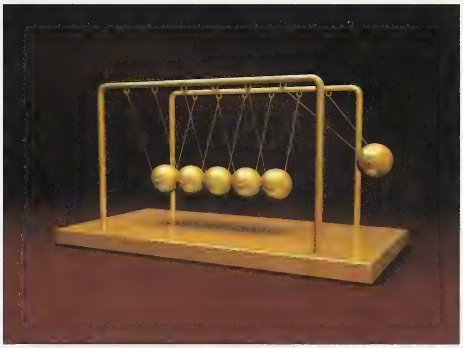
$$\sum \vec{p}_{\text{initial}} = \sum \vec{p}_{\text{final}}$$

sum of initial momentum = sum of final momentum

The conservation principle can also be expressed algebraically as follows:

$$\begin{aligned}
\sum \vec{p}_{\text{i}} &= \sum \vec{p}_{\text{f}} \\
\vec{p}_{1\text{i}} + \vec{p}_{2\text{i}} &= \vec{p}_{2\text{f}} + \vec{p}_{1\text{f}} \\
\vec{p}_{1\text{i}} - \vec{p}_{1\text{f}} &= \vec{p}_{2\text{f}} - \vec{p}_{2\text{i}} \\
\vec{p}_{1\text{f}} - \vec{p}_{1\text{i}} &= -\left(\vec{p}_{2\text{f}} - \vec{p}_{2\text{i}}\right) \\
\Delta \vec{p}_1 &= -\Delta \vec{p}_2
\end{aligned}$$

The law of conservation of momentum governs all physical interactions. It is considered to be one of the fundamental laws of physics. The law has been used to investigate and analyze all types of interactions. In addition to vehicle accidents, it has been used in the study of subatomic particles, such as electrons and protons, all the way up to the astronomical study of planets, stars, and galaxies.



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This law controls the universe by keeping the total quantity of motion in the universe fixed. That is, if one body slows down and comes to rest, another body must speed up and start moving. Newton's cradle of swinging masses is a good example of this, as you can see in the picture on the right.

Watch and Listen

View "The Conservation of Momentum video" on the Physics 30 Multimedia DVD which describes how Rene Descartes' knowledge of straight lines led him to develop the law of conservation of momentum.

Analyzing Collisions

In the lab section you discovered that the total momentum of a system is conserved, as long as there are no external forces acting on a system. The conservation of momentum will be used to analyze various collisions. When solving problems, it is useful to follow the GRASP model for problem solving.

The GRASP model is on page 867 in your textbook.

Watch and Listen

Verify your answers, where possible, by using the "Collision 1D simulator" on the Physics 30 Multimedia DVD.

SC 1. A 5.0-kg object is travelling to the right at 10.0 m/s. It collides with a 7.0-kg object that initially is at rest. After the collision, the 5.0-kg object continues to move to the right, but at 1.17 m/s. In what direction and with what speed is the other object moving?

Check your work with the answer in the Appendix.

Try This

Watch and Listen

Verify your answers, where possible, by using the "Collision 1D simulator" on the Physics 30 Multimedia DVD.

TR 1. When you shoot a gun, it "kicks back," or recoils.

a. Using conservation of momentum, explain why a gun recoils when you shoot it.

b. If a 30-g bullet is shot with a velocity of +300 m/s from a 7.5-kg gun, what is the recoil velocity of the gun?

Module 2: Lesson 1 Assignment

Remember to submit the answers to A 1 and A 2 to your teacher as part of your Module 2: Lesson 1 Assignment.

A 1. Object A, with a mass of 2.0 kg, is travelling to the right at 15.0 m/s. Object B, with a mass of 12.5 kg, is also travelling at 15.0 m/s, but to the left. If the two objects stick together upon impact, what is the final velocity of the system?

A 2. A 250-g firecracker explodes into two pieces. The first piece has a mass of 97 g and flies off to the right at 16 m/s. What is the velocity of the second piece?

Self-Check

SC 2. An object, initially travelling to the right, collides with another object that is at rest. After the collision, the objects stick together and move to the left. Is this collision possible? Explain why or why not.

Check your work with the answer in the Appendix.

Module 2: Lesson 1 Assignment

Remember to submit the answers to A 3 to your teacher as part of your Module 2: Lesson 1 Assignment.

A 3. Are the following collisions possible? Explain why or why not.

a. Object A, initially travelling to the right, collides with object B, which is at rest. After the collision, object A moves back to the left and object B remains at rest.

b. Object A, initially travelling to the right, collides with object B, which is at rest. Object B has more mass than object A. After the collision, object A moves back to the left and object B moves to the right.

c. An object is travelling to the right and collides with another object that is at rest. After the collision, both objects travel to the right, but at different speeds.

Read

Read "Momentum is Conserved in One-dimensional Collisions" on pages 473 to 475 of your physics textbook.

Newton's second and third laws can be used to derive the equation for impulse. In fact Newton's original writings defined force in terms of momentum and time, not the $\vec{F} = m\vec{a}$ that we used in Physics 20. However you haven't done the calculus that is required to understand Newton's original definition of force.

Watch and Listen

Watch "Momentum and Newton's Law" on the Physics 30 Multimedia DVD.

Try This

TR 1. Answer questions 1, 2, 5, 6, and 7 of "9.3 Check and Reflect" on page 486 of your physics textbook.

Reflect and Connect

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In traffic, when one car fails to stop or slow down for another car, a rear-end collision can occur. When this happens, some of the momentum of the moving car is transferred at the moment of impact to the car it strikes. The conservation of momentum can be used to describe how this happens. But it is not that simple; vehicles on a roadway are not an example of an isolated system. Frictional forces interact during and after the collision to bring all the vehicles to a stop. Therefore, the analysis of the momentum would be most useful for determining speeds immediately before and after the point of impact, and no more.

Module 2: Lesson 1 Assignment

Remember to submit the answers to D 5 to your teacher as part of your Module 2: Lesson 1 Assignment.

Discuss

Now that you know about the law of conservation of momentum, you can apply what you have learned to a number of simplified forensics engineering problems. Post your answers to these problems to the discussion area to see how your answers compare to those of other students.

A collision occurred when a 1500-kg car collided with a 15 000-kg truck initially at rest. After the collision, the vehicles locked together and moved with a combined velocity of 1.0 m/s, east.

D 1. What type of collision is described by this situation?

D 2. Use the law of conservation of momentum to solve for the velocity of the car, in metres per second, before the collision occurred.

D 3. Convert your answer from D 2 into kilometers per hour. The posted speed limit in this area was 25 km/h because it was designated for loading passengers and freight. Was the car speeding?

D 4. In answering this question you have made some assumptions. Knowing this, would those assumptions be grounds for reasonable doubt if there were a court case against the car's driver due to damages caused by the collision? Post your answer to the discussion forum and read the other students' postings.

D 5. Read the other students' comments and then rewrite your own answer to D 4 and include a description of what you learned from the other students' responses.

The Discussion Scoring Guide can be found in the Appendix.

Reflect on the Big Picture

Recall the collision of Kathy and Bill from the Big Picture section. Based on what you learned in this lesson, start to analyze this collision.

The Module 2 Project can be found at the back of the Assignment Booklet. You will be required to add to it at the end of each lesson in Module 2 until it is complete and submit it to your teacher as your Module 2 Project for marks.

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Answer the following question in your crash analysis document:

The mass of the truck is 3200 kg, and the mass of the car is 2800 kg. Based on the length of the skid marks at the scene and the mass of the vehicles, police estimate that the combined mass was moving at 7.0 m/s just after impact. From this point, the two vehicles slid and came to rest at the corner of the intersection.

- Determine the magnitude of the momentum just after the collision. (Analytic)
- Is this the same as the magnitude of the momentum just before the collision? Explain why or why not. (Holistic)

The Scoring Guide for Analytic Questions and the Scoring Guide for Holistic Questions can be found in the Appendix.

Remember to store your new crash analysis document in your Physics 30 course folder.

Module 2: Lesson 1 Assignment

Remember to submit the Module 2: Lesson 1 Assignment to your teacher.

You should also have started working on your Crash Analysis document as part of your Module 2 Project.

Lesson Summary

At the start of this lesson, you were asked the following essential questions:

- Can the complicated details of a linear collision be eliminated during the analysis of an interaction?
- What is the law of conservation of momentum, and how can it be used to answer the previous question?
- Under what circumstances can this law be applied?

When collisions are analyzed in terms of momentum, the complicated conditions that exist during the fraction of a second that the collision occurs do not need to be considered. This approach involves identifying all the objects involved in the system and accounting for the momentum values of each object both before and after the collision.

If there are no external net forces acting on the system, then the system is said to be isolated and the law of conservation of momentum can be used to analyze the collision. This law states that the momentum of an isolated system remains constant:

$$\vec{p}_{\text{sys}_\text{i}} = \vec{p}_{\text{sys}_\text{f}}$$

The law of conservation of momentum is a powerful tool for analyzing collisions. It can be applied to any system that is isolated. In forensic engineering, the systems involving automobiles that crash are not friction-free. Strictly speaking, these systems are not isolated. Nevertheless, if the interaction forces involved in the collision are larger than the frictional forces, then the law of conservation of momentum can still be used to provide reasonably accurate descriptions of the conditions that occurred immediately before and after the crash.

Lesson Glossary

momentum: the product of the mass and velocity of an object

Module 2—The Conservation of Momentum in Isolated Systems

Lesson 2—Kinetic Energy and Collisions

Get Focused

left ©Frances Twitty/iStockphoto
right © Arthur Kwiatkowski/iStockphoto

What happens when a moving object strikes a stationary object? The answer to this question depends very much on the characteristics of the system of objects that are colliding.

Consider the case of the cue ball striking the purple ball on the pool table in the photograph on the right. If the collision occurs along a straight line, and spinning is kept to a minimum, then the cue ball will likely stop and the purple ball will move with very nearly the same velocity that the cue ball had prior to impact.

Would a similar chain of events occur when the blue car rear-ends the grey car in the other photo? In other words, would the grey car leave the point of impact moving with very nearly the same velocity of the blue car prior to impact? How does the system of the two cars differ from the system of the two balls on the pool table? What is the significance of the fact that the two cars undergo a permanent change in shape but the pool balls do not? The answers to these questions require you to consider the changes in kinetic energy that occur when a collision occurs within a system of objects.

In this lesson you will explore elastic and inelastic collisions within the context of the following questions:

- How can the kinetic energy of the objects in a system before and after a collision be used to categorize the type of collision?
- What is the fundamental difference between an elastic and an inelastic collision?
- In a collision between two automobiles, is it better for the occupants if the collision is elastic or inelastic?

Module 2: Lesson 2 Assignments

Your teacher-marked Module 2: Lesson 2 Assignment requires you to submit a response to the following:

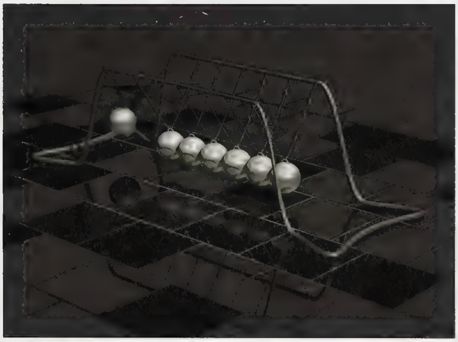
- Lab—LAB 1 and LAB 2
- Assignment—A 1, A 2, and A 3

You must decide what to do with the questions that are not marked by the teacher.

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

You will also continue to add to your Module 2 Project that you started in Lesson 1 and saved in your course folder. The finished project will be submitted to your teacher for marks once you have completed Module 2.

Explore



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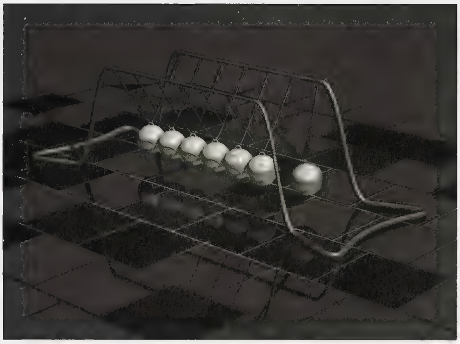

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Watch and Listen

Have you ever played with a device like this? It's called Newton's cradle. Spend a few moments playing with the "Animation of Newton's cradle" on the Physics 30 Multimedia DVD.

As you are playing, see if you can answer these questions:

- What happens if you change the number of balls put into motion?

- Do you ever get a different number of balls leaving the pack than enters it?
- How does the velocity of the entering balls compare to the velocity of the leaving balls?
- Notice the height at which the balls start their swing toward the pack. How does that initial height compare to the final height the balls reach when they swing away from the pack?

During your play with the animation, you surely saw that momentum was conserved. One ball enters and one ball leaves, and the velocities are basically the same. The same was true for two balls or even three. Momentum would have been conserved if one ball entered and two balls left with half the velocity of the original ball. Did that ever happen? (Knowing how the applet works makes the answer to that question an emphatic no!)

The answer to the last of the questions gives you a clue to what is happening. For example, you may have used just one starting ball. Since the starting height of the ball and the ending height of the ball seem to be the same, the energy seems to have been conserved. Just before impact, the energy of the moving ball can be calculated. You'll remember the equation, $E_k = \frac{1}{2}mv^2$. Just after the collision, the new moving ball will have the same kinetic energy. (These concepts would still be true if you had used more than one starting ball.) Kinetic energy appears to be conserved in Newton's cradle, but is kinetic energy always conserved in collisions?

DID YOU KNOW?

The units for energy are joules (J), named after James Prescott Joule. The joule is not a base SI unit but is made of other SI units.

$$1\text{J} = 1\text{N}\bullet\text{m} = 1\text{kg}\bullet\frac{\text{m}}{\text{s}^2}\bullet\text{m} = 1\frac{\text{kg}\times\text{m}^2}{\text{s}^2}$$

Is Kinetic Energy Conserved in a Collision?

Some of the energy in a closed, isolated system is in the form of kinetic energy. This is particularly true for two objects that collide, since one or both must be moving in order for this to occur.

Self-Check

SC 1. Two bumper cars, each with a mass of 50 kg and travelling at 0.75 m/s, are headed straight toward each other.

a. What is the momentum of each car? Do the cars have the same momentum? (Remember: Momentum is a vector quantity.)

b. What is the total momentum of the system?

c. What is the kinetic energy of each car? Do the cars have the same kinetic energy? (Remember: Kinetic energy is a scalar quantity.)

d. What is the total kinetic energy of the system?

Check your work with the answer in the Appendix.

Collisions and Kinetic Energy

In an isolated system, if the total energy is conserved, does this imply that the kinetic energy is conserved? Recall that kinetic energy is the energy of motion $\left(E_{\mathrm{k}} = \frac{1}{2}mv^2\right)$. Any object that is moving not only has momentum but also has kinetic energy. In previous lessons, you discovered that the total momentum of an isolated system is conserved during a collision. But what about kinetic energy—is it also conserved?

Lesson 2 Lab: Kinetic Energy and Collisions

In this lab, a simulation will be used to collect velocity and mass data from five different collisions. The data will be used to test the conservation of kinetic energy and momentum in each collision.

Problem

Are kinetic energy and momentum always conserved in an isolated system that involves the collision of two objects?

Watch and Listen

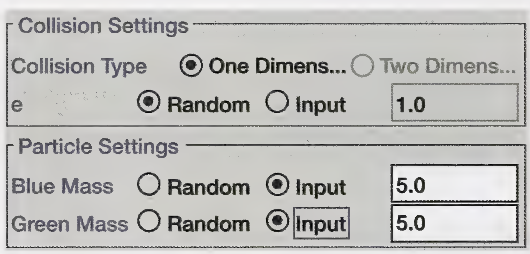
Procedure

- Use the "Collision 1D simulator" on the Physics 30 Multimedia DVD to investigate five different collisions between the same objects and complete the table in LAB 1. The data collected will be used to verify that momentum is conserved and to determine if kinetic energy is also conserved.

- On the simulation, toggle "Show CM" and "Show CM Frame" to the off position (☐ Show CM ☐ Show CM Frame).

- Click the "Options" buttons ( ), and enter the settings shown here.

    - set e to "Random"
    - set Blue Mass to "Input" with a mass of 5.0 kg
    - set Green Mass to "Input" with a mass of 5.0 kg



- To generate a new collision (without changing the value of e or object mass), click the "New" button ( ) and change the initial velocity of the blue ball by adjusting the velocity slider (

) to any value.

- After each collision, view the collision information by clicking the "Data" button ( ) and fill in the appropriate table.

Module 2: Lesson 2 Assignment

Remember to submit the answers to LAB 1 and LAB 2 to your teacher as part of your Module 2: Lesson 2 Assignment.

Observations and Analysis

LAB 1. In the chart below, record the initial velocity and final velocity for each object and the e value for the collision. Calculate the initial and final momentum and energy for each object using the mass and velocity values.

Do not worry about what e represents; it will be discussed in detail in the next section. The value of e will be listed with the other information on the simulation and must be recorded for each collision.

Collision 1 e = _____							
Object	mass (kg)	$\vec{v}_{initial}$ (m/s)	$\vec{v}_{final}$ (m/s)	$\vec{p}_{initial}$ (kg·m/s)	$\vec{p}_{final}$ (kg·m/s)	E_k initial (J)	E_k final (J)
blue	5.0						
green	5.0						
total	-----	-----	-----				

Collision 2 e = _____							
Object	mass (kg)	$\vec{v}_{initial}$ (m/s)	$\vec{v}_{final}$ (m/s)	$\vec{p}_{initial}$ (kg·m/s)	$\vec{p}_{final}$ (kg·m/s)	E_k initial (J)	E_k final (J)
blue	5.0						
green	5.0						
total	-----	-----	-----				

Collision 3 e = _____							
Object	mass (kg)	$\vec{v}_{initial}$ (m/s)	$\vec{v}_{final}$ (m/s)	$\vec{p}_{initial}$ (kg·m/s)	$\vec{p}_{final}$ (kg·m/s)	E_k initial (J)	E_k final (J)
blue	5.0						
green	5.0						
total	-----	-----	-----				

Collision 4 e = _____							
Object	mass (kg)	$v_{initial}$ (m/s)	v_{final} (m/s)	$p_{initial}$ (kg·m/s)	p_{final} (kg·m/s)	E_k initial (J)	E_k final (J)
blue	5.0						
green	5.0						
total	-----	-----	-----				

Collision 5 e = _____							
Object	mass (kg)	$v_{initial}$ (m/s)	v_{final} (m/s)	$p_{initial}$ (kg·m/s)	p_{final} (kg·m/s)	E_k initial (J)	E_k final (J)
blue	5.0						
green	5.0						
total	-----	-----	-----				

Conclusion

LAB 2. Answer the following questions using the data collected in the collision tables:

a. Is momentum conserved in each collision?
b. Is kinetic energy conserved in each collision?

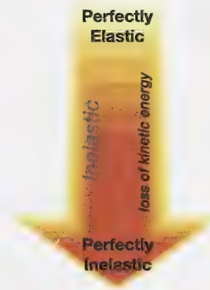
The Difference Between Elastic and Inelastic Collisions

In a collision, momentum is always conserved, but kinetic energy is not. Sometimes, the kinetic energy is nearly conserved. In other collisions, it is hardly conserved at all. What is going on?

During a collision, energy changes form. As objects interact and collide, they change shape and are distorted. When this occurs, the kinetic energy of the colliding bodies is converted into potential energy, or dissipated as sound or heat. The extent to which the initial kinetic energy is converted to final kinetic energy during the collision determines the *elasticity* of the collision.

There is a spectrum of elasticity: collisions can range from being perfectly elastic to perfectly inelastic. As collisions become more and more inelastic, more and more kinetic energy is lost.

Perfectly Elastic Collisions



In a perfectly elastic collision, the total kinetic energy of the system is conserved. Perfectly elastic collisions generally occur only at the subatomic level.

Inelastic Collisions

In an inelastic collision, some kinetic energy is lost, generally as sound or thermal energy. This is a broad range and most collisions fall within this class.

Perfectly Inelastic Collisions

In a perfectly inelastic collision (also called completely inelastic), the colliding objects stick together upon impact. There is the greatest loss of kinetic energy in this type of collision.

Self-Check

SC 2. Imagine throwing a perfectly spherical bouncy, elastic ball against the wall.

a. Describe the shape of the ball before, during, and after the collision.

   before: _______ during: __________ after: __________

b. Describe what happens to the kinetic energy of the ball during the interaction. Is the kinetic energy of the ball conserved? If not, what happened to it?

SC 3. Imagine throwing a blob of playdough against the wall.

a. Describe the shape of the blob before, during, and after the collision.

   before: _________during: _____________ after: ____________

b. Describe what happens to the kinetic energy of the blob during the interaction. Is the kinetic energy of the blob conserved? If not, what happened to it?

Check your work with the answer in the Appendix.

The Coefficient of Restitution Defines Elasticity

Is there a range of elasticity in the collisions that were performed in the lab? Recall that in each collision, a value known as e was recorded at the top of each table. Now we will examine what the e value represents. Use the lab data tables to complete the Module 2: Lesson 2 Assignment.

Module 2: Lesson 2 Assignment

Remember to submit the answers to A 1 and A 2 to your teacher as part of your Module 2: Lesson 2 Assignment.

A 1. Complete the data table below by doing the following:

a. Record the total $E_{\text{k initial}}$, total $E_{\text{k final}}$, and e for each collision from the lab data tables.

b. Calculate the percentage of loss in kinetic energy for each collision using:

$$\% \text{ loss} = \frac{\sum E_{\text{k initial}} - \sum E_{\text{k final}}}{\sum E_{\text{k initial}}} \times 100\%$$

Collision	e	Total E_k Initial (J)	Total E_k Final (J)	Loss in E_k (%)
1				
2				
3				
4				
5				

A 2. Using the same data table above, compare the percentage of total kinetic energy lost during each collision to the value of e; e is called the elasticity of the collision. What is the relationship between the percentage of loss in kinetic energy and e?

Read

Read "Inelastic Collisions" on page 483 of your physics textbook. Continue by studying "Example 9.10" on pages 484 and 485 of your textbook.

Self-Check

SC 4. Answer "Practice Problem 1" on page 484 of your textbook.

Check your work with the answer in the Appendix.

Read

Study “Example 9.11” on page 485 of your textbook.

Self-Check

SC 5. Answer “Practice Problem 1” on page 485 of your textbook.

Check your work with the answer in the Appendix.

Try This

TR 1. Answer “Practice Problem 2” on page 484 of your textbook.
TR 2. Answer “Practice Problem 2” on page 485 of your textbook.

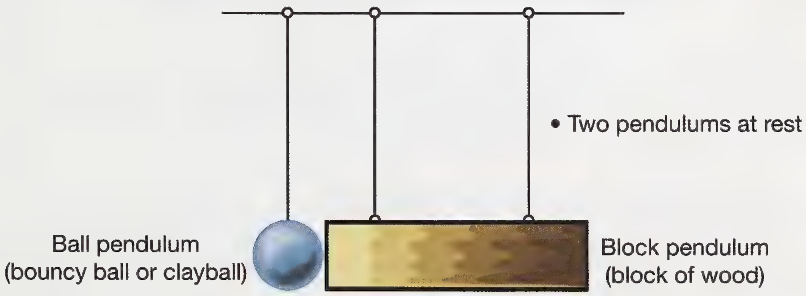
Module 2: Lesson 2 Assignment

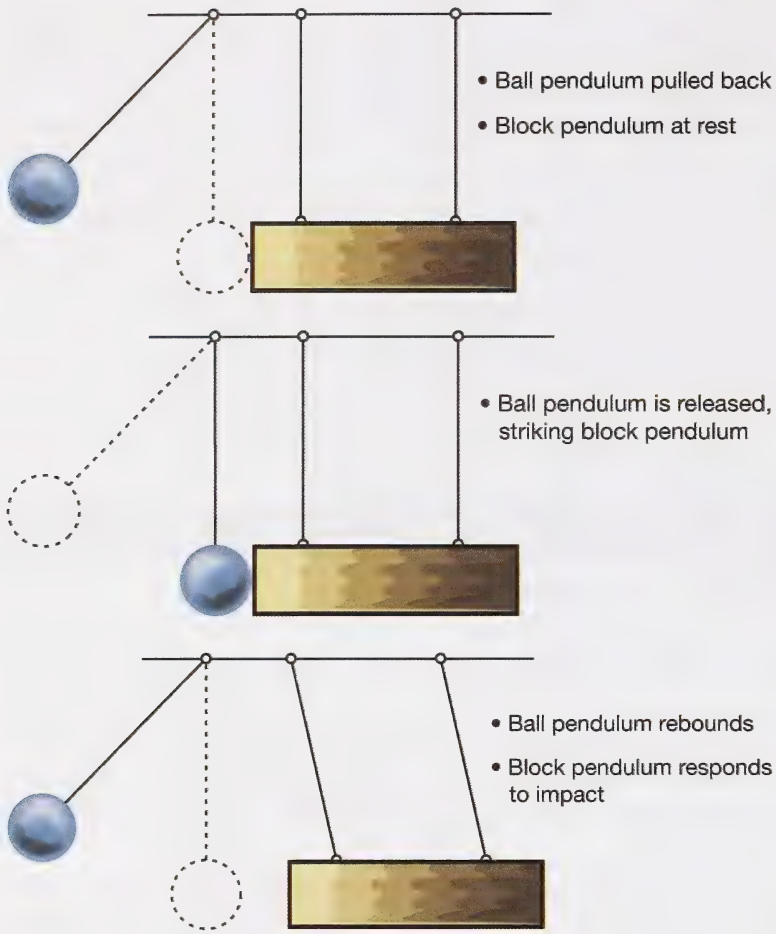
Remember to submit the answers to A 3 to your teacher as part of your Module 2: Lesson 2 Assignment.

A 3. Design an experiment that would let you answer the following problem:

Is the collision between a ball pendulum and a block pendulum elastic, inelastic, or completely inelastic?

The following four diagrams give you a starting point for your experiment.





From "Examples 9.10" and "9.11" of your textbook, you will be familiar with using gravitational potential energy to find the kinetic energy of a ball pendulum. You can deal with the kinetic energy of the block pendulum in a similar fashion.

You need to present the following in your experimental design:

- the measurements that need to be taken
- the calculations that would have to be made
- the criteria you would use to answer the problem

Reflect and Connect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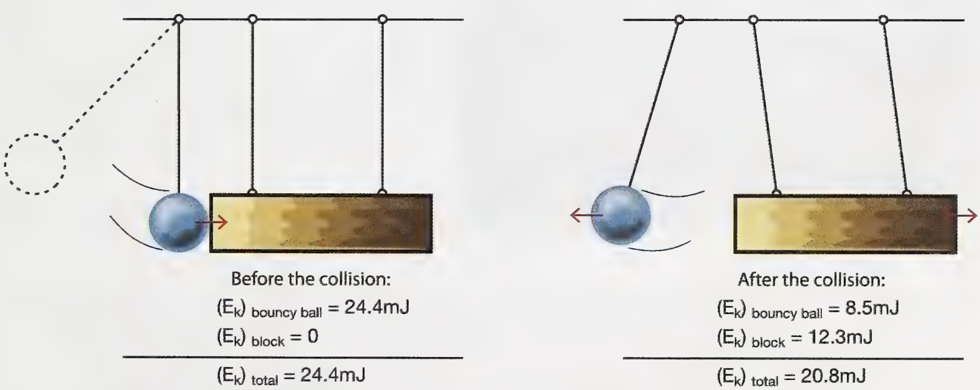
In the Get Focused section of this lesson, you were asked this question: in a collision between two automobiles, is it better for the occupants if the collision is elastic or inelastic?

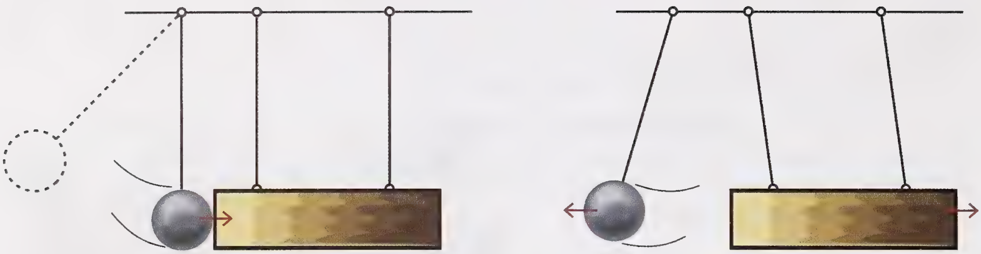
Now that you know the differences between elastic and inelastic collisions, you should be ready to complete the necessary analysis to answer this question. A good place to begin is the collision between the ball pendulum and the block pendulum for which you designed an experiment in A 3.

In one version of this experiment, a bouncy ball attached to a long string collides with a wooden block that is also suspended by a long string. In other words, a bouncy-ball pendulum collides with a wooden-block pendulum. Before the collision, the bouncy ball has all the kinetic energy because the wooden block is at rest. After the collision, both objects have some kinetic energy.

The following diagram summarizes typical results:



In another version of this experiment, the bouncy ball is replaced with a ball of clay. Before the collision, the ball of clay has all the kinetic energy because the block of wood is at rest. After the collision, both objects have some kinetic energy. The following diagram summarizes typical results:



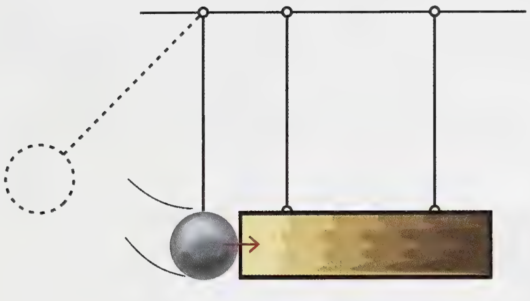
Before the collision:
$(E_k)_{\text{clay ball}} = 24.4\text{mJ}$
$(E_k)_{\text{block}} = 0$
$(E_k)_{\text{total}} = 24.4\text{mJ}$

After the collision:
$(E_k)_{\text{clay ball}} = 0.5\text{mJ}$
$(E_k)_{\text{block}} = 4.3\text{mJ}$
$(E_k)_{\text{total}} = 4.8\text{mJ}$

The results for the pendulum experiment illustrate how the properties of the materials impact the transfer of kinetic energy in a collision. Some of the initial kinetic energy was used to do work to change the shape of the clay ball, so there was less kinetic energy available for the objects after the collision.

An automobile collision is more like the collision between the clay ball and the block than the bouncy ball and the block. Some of the initial kinetic energy is used to do work to change the shape of the crumple zones of the vehicle. The amount of kinetic energy transferred from one vehicle to the other depends upon the amount of work that is done to change the shape of the vehicles. Although this energy transfer has nothing to do with impulse, the resulting impulse that acts on each vehicle in a collision is strongly influenced by *how* the transfer of kinetic energy occurs between the two vehicles. You can see why by returning to typical data from the experiments of the two pendulums.

In the case of the bouncy ball and the wooden block, typical data for the collision is shown in the following diagram:

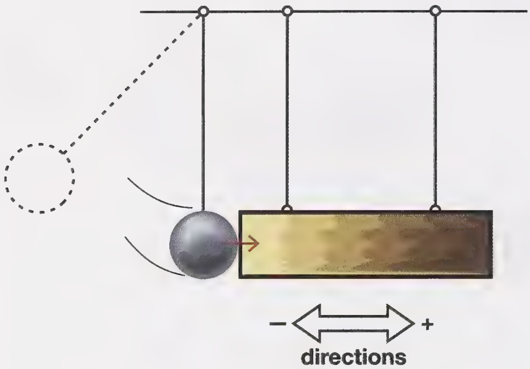


bouncy ball		wooden block
$m_{sb} = 0.052 \text{ kg}$		$m_{wb} = 0.384 \text{ kg}$
$\vec{v}_{sb_i} = +\ 0.96 \ \frac{\text{m}}{\text{s}}$	← Before the collision →	$\vec{v}_{wb_i} = 0$
$\vec{v}_{sb_f} = -0.59 \ \frac{\text{m}}{\text{s}}$	← After the collision →	$\vec{v}_{wb_f} = +\ 0.21\frac{\text{m}}{\text{s}}$

Determine the impulse that acted on the bouncy ball:	Determine the impulse that acted on the wooden block:
$\vec{F}\Delta t = m_{sb}\vec{v}_{sb}$ $= m_{sb}\left(\vec{v}_{sb_f} - \vec{v}_{sb_i}\right)$ $= (0.052 \text{ kg})\left(\left(-0.59\frac{\text{m}}{\text{s}}\right) - \left(0.96\frac{\text{m}}{\text{s}}\right)\right)$ $= (0.052 \text{ kg})\left(-1.55\frac{\text{m}}{\text{s}}\right)$ $= -0.081 \text{ kg}\bullet\frac{\text{m}}{\text{s}}$	$\vec{F}\Delta t = m_{wb}\vec{v}_{wb}$ $= m_{wb}\left(\vec{v}_{wb_f} - \vec{v}_{wb_i}\right)$ $= (0.384 \text{ kg})\left(0.21\frac{\text{m}}{\text{s}} - 0\frac{\text{m}}{\text{s}}\right)$ $= +0.081 \text{ kg}\bullet\frac{\text{m}}{\text{s}}$

As you can see, the impulse for each has the same magnitude but opposite direction.

When a clay ball collides with a wooden block, the velocity values are significantly reduced due to the lower values of kinetic energy. The following diagram shows typical results:



clay ball		**wooden block**
$m_{cb} = 0.052$ kg		$m_{wb} = 0.384$ kg
$\vec{v}_{cb_i} = +\ 0.96\ \frac{\text{m}}{\text{s}}$	← Before the collision →	$\vec{v}_{wb_i} = 0$
$\vec{v}_{cb_f} = -0.14\ \frac{\text{m}}{\text{s}}$	← After the collision →	$\vec{v}_{wb_f} = +\ 0.15\ \frac{\text{m}}{\text{s}}$

Determine the impulse that acted on the clay ball:

$$\begin{aligned}
\vec{F}\Delta t &= m_{\text{CB}}\Delta\vec{v}_{\text{CB}} \\
&= m_{\text{CB}}\left(\vec{v}_{\text{CB}_\text{f}} - \vec{v}_{\text{CB}_\text{i}}\right) \\
&= (0.052\ \text{kg})\big((-0.14\ \text{m/s}) - (0.96\ \text{m/s})\big) \\
&= (0.052\ \text{kg})(-1.1\ \text{m/s}) \\
&= -0.057\ \text{kg}\bullet\text{m/s}
\end{aligned}$$

Determine the impulse that acted on the wooden block:

$$\begin{aligned}
\vec{F}\Delta t &= m_{\text{WB}}\Delta\vec{v}_{\text{WB}} \\
&= m_{\text{WB}}\left(\vec{v}_{\text{WB}_\text{f}} - \vec{v}_{\text{WB}_\text{i}}\right) \\
&= (0.384\ \text{kg})\big((0.15\ \text{m/s}) - (0\ \text{m/s})\big) \\
&= (0.384\ \text{kg})(0.15\ \text{m/s}) \\
&= 0.058\ \text{kg}\bullet\text{m/s}
\end{aligned}$$

Self-Check

SC 6. Explain why it would be better to be in a vehicle that is in an inelastic collision with another object rather than a vehicle in an elastic collision with another object.

Check your work with the answer in the Appendix.

Reflect on the Big Picture

Recall the collision of Kathy and Bill in the Big Picture section. Based on the information in this lesson, you can add to your Module 2 Project.

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Using the principle of conservation of momentum the police determined that Bill's truck was travelling at 2.3 m/s before the collision, and Kathy's car was travelling at 15 m/s before the collision.

Add the answers to the following questions to the document titled "Crash Analysis" that you started in Lesson 1:

- What is the kinetic energy of the vehicles before and after the collision? (Analytic)
- Is kinetic energy conserved? If kinetic energy is not conserved, what does this tell us about the type of collision? (Holistic)

The Scoring Guide for Analytic Questions and the Scoring Guide for Holistic Questions can be found in the Appendix.

Remember to re-save your updated document in your Physics 30 course folder. You will submit the Module 2 Project to your teacher for marks when you have completed the module.

Module 2: Lesson 2 Assignment

Remember to submit the Module 2: Lesson 2 Assignment to your teacher.

Lesson Summary

At the start of this lesson, you were asked the following essential questions:

- How can the kinetic energy of the objects in a system before and after a collision be used to categorize the type of collision?
- What is the fundamental difference between an elastic and an inelastic collision?
- In a collision between two automobiles, is it better for the occupants if the collision is elastic or inelastic?

The kinetic energy values of all the objects in a system can be used to categorize a collision as elastic, inelastic, or perfectly (completely) inelastic. In the case of elastic collisions, the total kinetic energy of the system before the collision equals the total of the kinetic energy of the system after the collision. In other words, kinetic energy is conserved. These collisions are rare with regard to the collision of everyday objects because some of the kinetic energy is usually converted to sound, thermal energy, or deformation. This is the fundamental difference between an elastic collision and an inelastic collision.

In an inelastic collision, the total kinetic energy of the system after the collision is less than the total kinetic energy of the system before the collision. In other words, kinetic energy is not conserved. In some cases, the kinetic energy is completely converted to other forms of energy after the collision. In these cases, the collision is described as being perfectly, or completely, inelastic.

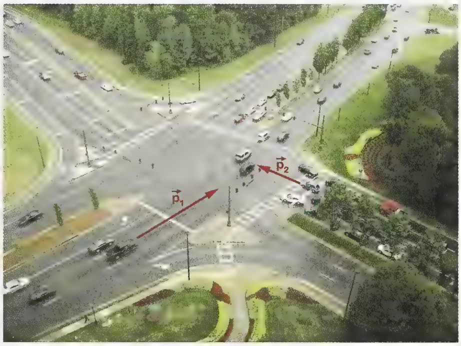
In collisions between two automobiles, it is better for the occupants if the collision is inelastic rather than elastic. Crumple zones of vehicles are designed to transform some of the initial kinetic energy into work. This work changes the shapes of crushable parts of the vehicle. Since the vehicles have less kinetic energy after the collision, there tends to be less reversal of direction. The change in velocity of each vehicle is less, so the change in momentum for each vehicle is less. Thus, the impulse acting on each vehicle is reduced. Since inelastic collisions between vehicles involve the crumpling of crushable parts, the time interval for the interaction is increased, which reduces the forces acting on each vehicle.

Module 2—The Conservation of Momentum in Isolated Systems

Lesson 3—Analyzing Non-Linear Collisions

Get Focused

Intersections are busy places. So busy that, despite traffic lights, signs, markings, and warnings, vehicles still collide. In the previous lesson the rear-end collision was explored in the context of a linear collision. These types of collisions are common at intersections where vehicles moving in the same direction have to stop periodically. But what happens when vehicles travelling at right angles to one another collide, as in the Module 2 project with Kathy and Bill? What happens to mass, velocity, and momentum in a non-linear interaction, such as a T-bone collision?



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In this lesson you will apply the law of conservation of momentum and component analysis to analyze non-linear collisions, such as that of Bill and Kathy in the Big Picture.

You will explore collisions of this nature in the context of the following questions:

- Is momentum conserved in a non-linear collision?
- How is component analysis applied to solve non-linear collision problems?
- Can it be proven, beyond a reasonable doubt, that Kathy was moving at the speed limit just prior to the collision described in the accident scenario in the Big Picture?

Module 2: Lesson 3 Assignments

Your teacher-marked Module 2: Lesson 3 Assignment requires you to submit a response to the following:

- Assignment—A 1, A 2, A 3, and A 4

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

You will also complete your Module 2 Project that you started in the previous lessons and saved to your course folder. The finished project will be submitted to your teacher for marks once you have completed Module 2.

Explore

Watch and Listen

What is the difference between a linear and non-linear interaction? To view an example of a non-linear collision, watch the "Non-Linear Collision Animation" on the Physics 30 Multimedia DVD.

Is Momentum Conserved in a Non-linear Collision?

In the previous lesson you learned about momentum and the law of conservation of momentum in the context of one-dimensional collisions: for any isolated system, the total momentum does not change. In a collision, momentum is conserved. The total momentum before the collision is equal to the total momentum after the collision.

Is the same true of two-dimensional collisions?

Watch and Listen

If you don't feel very comfortable with vector analysis, you can choose to open and try this animation titled "Component Vector Analysis" on the Physics 30 Multimedia DVD.

Module 2: Lesson 3 Assignment

Remember to submit the answer to A1 to your teacher as part of your Module 2: Lesson 3 Assignment.

Watch and Listen

LAB 1. A simulation can be used to determine if momentum is conserved in two-dimensional collisions. Use the "Collision 2D simulator" on the Physics 30 Multimedia DVD to help you answer the following questions.

Once the simulation is open, follow these steps:

- Toggle "Show CM" and "Show CM Frame" to the off position (☐ Show CM ☐ Show CM Frame).
- Leave the "Show Trails" box checked.
- Move the "Impact Parameter" slider to a value other than zero.
- Press "Play" and observe the collision.
- Press "Pause" before either of the objects leaves the viewing area.
- Press the "Data" button to display the data describing the collision.

Perform one two-dimensional collision; then complete the following table. To generate a new collision, press the "New" button ( ) and move the Impact Parameter slider to any value other than zero. Press the "Data" button ( ) to view the collision information that is required to complete the following table.

Collision 1 (sample data)							
Object	Mass (kg)	$\vec{v}_i$ (m/s)	$\vec{v}_f$ (m/s)	$\Delta\vec{v}$ (m/s)	$\vec{p}_i$ (kg·m/s)	$\vec{p}_f$ (kg·m/s)	$\Delta\vec{p}$ (kg·m/s)
blue							
green							

a. How would you calculate the total momentum before and after a two-dimensional collision?

b. Would the same analysis that you used for one-dimensional situations work here?

c. According to your observations, is momentum conserved in a two-dimensional collision?

According to the data you collected for LAB 1 and using the typical analysis performed for a one-dimensional collision, you should have found that momentum in a two-dimensional collision is *not conserved.* However, this is contrary to the law of conservation of momentum, which means a new type of analysis must be used for two-dimensional collisions.

This new type of analysis is based on the following principles, which apply to two-dimensional interactions:

- Momentum in the x direction is conserved.
- Momentum in the y direction is conserved.

Try This

TR 1. With the following data from the simulation, carry out a calculation of the total momentum before and after the collision using the analysis method that was just introduced to you.

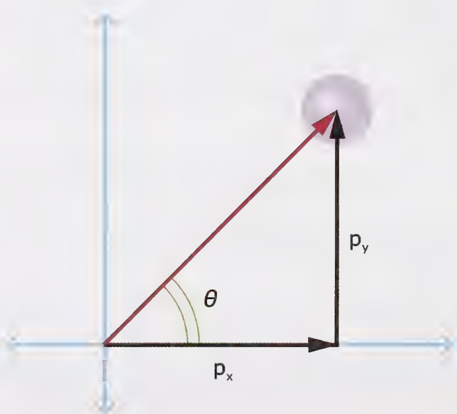
Object	Mass (kg)	$\vec{v}_i$		$\vec{v}_f$	
		Magnitude (m/s)	Direction (degrees)	Magnitude (m/s)	Direction (degrees)
blue ball	5.00	8.00	0	2.80	69.51
green ball	5.00	0.0	0	7.50	−20.49

a. What is the initial total momentum in the x direction?
b. What is the initial total momentum in the y direction?
c. What is the final total momentum in the x direction?
d. What is the final total momentum in the y direction?

On the next page is an example of how to describe vectors (such as momentum) using components and how to apply the law of conservation of momentum correctly to a two-dimensional collision. You may wish to look at the vector review in Module 1 if you're feeling a bit rusty on vector analysis.

Describing Momentum Using Vector Components

Any two-dimensional vector, such as momentum, can be resolved into components. Generally, we resolve vectors into horizontal (x) and vertical (y) components. The diagram to the right shows a vector, $\vec{p}$, resolved into its x and y components. Using this diagram and some basic trigonometric identities (sin, cos, tan), answer the Self-Check questions below.



Self-Check

SC 1. What is the expression that gives p_x as a function of $\vec{p}$ and θ?

SC 2. What is the expression that gives p_y as a function of $\vec{p}$ and θ?

SC 3. Write a general expression for $\vec{p}$, if both p_x and p_y are known.

SC 4. If both p^x and p_y are known, what is the general expression for θ?

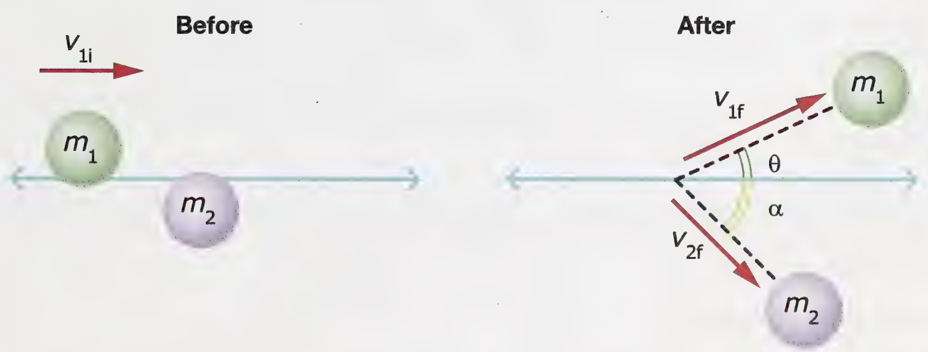
Check your work with the answer in the Appendix.

Analyzing Collisions Using Component Analysis

See this demonstration of adding vectors for a 2-D collision. Applying the law of conservation of momentum in two-dimensional collisions is more complex than that of a one-dimensional collision. Essentially, the law of conservation of momentum is applied twice—once in the *x* and once in the *y* directions. Following is an example of how to apply the law of conservation of momentum correctly to solve two-dimensional collision problems.

Example Problem

An 8.0-kg mass collides elastically with a 5.0-kg mass that is at rest. Initially, the 8.0-kg mass was travelling to the right at 4.5 m/s. After the collision, it is moving with a speed of 3.65 m/s and at an angle of 27° to its original direction.



What is the final velocity for the 5.0-kg mass?

Example Solution

Given

mass 1 $m_1 = 8.0$ kg

initial velocity of mass 1 $\vec{v}_{1i} = 4.5 \text{ m/s}[0°]$

$\vec{v}_{1xi} = +4.5 \text{ m/s}$

$\vec{v}_{1yi} = 0$

final velocity of mass 1 $\vec{v}_{1f} = 3.65 \text{ m/s}[27°]$

$$\begin{aligned}\vec{v}_{1xf} &= \vec{v}_{1f}\cos[27°]\\ \vec{v}_{1yf} &= \vec{v}_{1f}\sin[27°]\end{aligned}$$

mass 2 $m_2 = 5.0 \text{ kg}$

initial velocity of mass 2 $\vec{v}_{2i} = 0$

$$\begin{aligned}\vec{v}_{2xi} &= 0\\ \vec{v}_{2yi} &= 0\end{aligned}$$

Required

The final velocity of mass 2, $\vec{v}_{2f}$.

Analysis and Solution

Initially, mass 1 moves to the right, and mass 2 is at rest. Thus, the total momentum is in the *x* direction (to the right). After the collision, mass 1 travels at an angle of 27° to its original direction of motion; it now has momentum in the *y* direction as well as the *x* direction. For momentum to be conserved, mass 2 must have momentum in both the *x* and *y* directions.

Define motion to the right as positive, which follows the usual convention. As well, measure angles counterclockwise from the horizontal.

Use the law of conservation of momentum to solve this question. Since momentum is conserved in both the *x* and *y* directions, we can set up two sets of equations—one each to solve for v_{2fx} and v_{2fy}.

Conservation of momentum in the *x* direction:

$$\begin{aligned}p_{1xi} + p_{2xi} &= p_{1xf} + p_{2xf}\\ m_1 v_{1xi} + m_2 v_{2xi} &= m_1 v_{1xf} + m_2 v_{2xf}\\ m_1 v_{1xi} + m_2 \times 0 &= m_1 v_{1xf} + m_2 v_{2xf}\\ m_1 v_{1xi} &= m_1 v_{1xf} + m_2 v_{2xf}\\ m_2 v_{2xf} &= m_1 v_{1xi} - m_1 v_{1xf}\\ v_{2xf} &= \frac{m_1 v_{1xi} - m_1 v_{1xf}}{m_2}\\ v_{2xf} &= \frac{m_1 v_{1xi} - m_1 v_{1f}\cos\theta}{m_2}\end{aligned}$$

Conservation of momentum in the y direction:

$$\begin{aligned}
p_{1yi} + p_{2yi} &= p_{1yf} + p_{2yf} \\
m_1 v_{1yi} + m_2 v_{2yi} &= m_1 v_{1yf} + m_2 v_{2yf} \\
m_1 \times 0 + m_2 \times 0 &= m_1 v_{1yf} + m_2 v_{2yf} \\
0 &= m_1 v_{1yf} + m_2 v_{2yf} \\
m_2 v_{2yf} &= -m_1 v_{1yf} \\
v_{2yf} &= \frac{-m_1 v_{1fy}}{m_2} \\
v_{2yf} &= -\frac{m_1 v_{1f} \sin\theta}{m_2}
\end{aligned}$$

The final velocity for mass 2 in the x direction:

$$\begin{aligned}
v_{2xf} &= \frac{m_1 v_{1xi} - m_1 v_{1f} \cos\theta}{m_2} \\
v_{2xf} &= \frac{(8.0\text{ kg})(4.5\text{ m/s}) - (8.0\text{ kg})(3.65\text{ m/s})\cos(27°)}{5.0\text{ kg}} \\
v_{2xf} &= 1.996\,521\,899\text{ m/s}
\end{aligned}$$

The final velocity for mass 2 in the y direction:

$$\begin{aligned}
v_{2yf} &= -\frac{m_1 v_{1f} \sin\theta}{m_2} \\
v_{2yf} &= -\frac{(8.0\text{ kg})(3.65\text{ m/s})\sin(27°)}{5.0\text{ kg}} \\
v_{2yf} &= -2.651\,304\,508\text{ m/s}
\end{aligned}$$

Once you know the final velocity in the x and y directions, you can find the final velocity and direction of motion for mass 2. Remember, do not use the rounded off values in your calculations—use rounded numbers for the final paraphrase.

Final velocity for mass 2:

$$\begin{aligned}
v_{2f} &= \sqrt{(v_{2xf})^2 + (v_{2yf})^2} \\
v_{2f} &= \sqrt{(1.996\,521\,899\text{ m/s})^2 + (-2.651\,304\,508\text{ m/s})^2} \\
v_{2f} &= 3.318962984\text{ m/s} \\
v_{2f} &= 3.3\text{ m/s, correct to 2 significant digits}
\end{aligned}$$

Direction of motion:

$$\tan\alpha = \frac{v_{2yf}}{v_{2xf}} \Rightarrow \alpha = \tan^{-1}\left(\frac{v_{2yf}}{v_{2xf}}\right)$$
$$\alpha = \tan^{-1}\left(\frac{-2.651\,304\,508\text{ m/s}}{1.996\,521\,899\text{ m/s}}\right)$$
$$\alpha = -53.01901707^\circ$$
$$\alpha = -53^\circ\text{, correct to 2 significant digits}$$

This answer seems to make sense—mass 2 has to move to the right and down in order to conserve momentum. Furthermore, this answer can be verified with the simulation. Check it yourself!

Paraphrase

The final velocity of mass 2 is 3.3 m/s [−53°] or 3.3 m/s [307°].

Try This

TR 2.A 9.0-kg ball is travelling to the right at 8.0 m/s until it impacts a 3.0-kg stationary ball. After the collision, the 9.0-kg ball is travelling at 5.61 m/s at an angle of 7.8º above the horizontal. Calculate the velocity of the 3.0-kg ball after the impact. (**Note:** To view and verify this collision using the simulation, open the "Options" ( ) and set e = 0.44. Then set the blue mass to 9.0 kg and the green mass to 3.0 kg.)

Read

Read "Collisions in Two Dimensions" on pages 487 to example 9.14 on page 495 of your physics textbook.

Module 2: Lesson 3 Assignment

Remember to submit the answers to A 1 and A 2 to your teacher as part of your Module 2: Lesson 3 Assignment.

A1. A car with a mass of 1400 kg is westbound at 50 km/h. It collides at an intersection with a northbound truck having a mass of 2000 kg and travelling at 40 km/h. What is the initial common velocity of the car and truck immediately after the collision if they have an inelastic collision?

A2. A police car with a mass of 1460 kg is headed west at 60 km/h when it has an inelastic collision with a southbound 2000-kg ambulance. The wreckage ended up travelling at 45 km/h at 236°. What was the initial speed of the ambulance?

Read

Read "Elastic and Inelastic Collision in Two Dimensions" on pages 495 to 498 of your physics textbook.

Try This

TR 3. Answer questions 1 to 4 in "9.4 Check and Reflect" on page 499 of your physics textbook.

Module 2: Lesson 3 Assignment

Remember to submit the answers to A 3 and A 4 to your teacher as part of your Module 2: Lesson 3 Assignment.

A3. A billiard cue ball with a mass of 0.60 kg and an eight ball with a mass of 0.55 kg are rolled toward each other. The cue ball has a velocity of 3.0 m/s at 360°, and the eight ball has a velocity of 2.0 m/s at 90°. After the collision, the cue ball moves off at a velocity of 2.0 m/s at 40°.

a. What is the velocity of the eight ball?
b. Prove whether or not the collision is elastic.

A4. A proton with an initial velocity of 2800 m/s [90°] is struck by a neutron heading at 320° with a speed of 2900 m/s. The neutron bounces off of the proton with a velocity of 1864 m/s [270°].

a. What is the final velocity of the proton?
b. Prove whether or not the collision is elastic.

Reflect and Connect

Watch and Listen

The Alberta Traffic Safety Plan, found on the Physics 30 Multimedia DVD, reports the following statistics:

- A traffic collision occurs every five minutes in Alberta.
- On average, one person is killed and 65 injured in collisions every day.
- Collisions kill six times as many people as homicide, eight times as many as AIDS, and 100 times as many as meningitis.
- The costs associated with collisions in Alberta are $12 million per day, or more than $4 billion per year.


Maps courtesy of ERCB/AGS

The statistics indicate that collisions are common and have significant costs both to property and people.


Maps courtesy of ERCB/AGS

An intersection is essentially a planned point of conflict. Two roads, travelling in different directions, meet with drivers travelling in a variety of directions. The greater the number of points of conflict, the greater the number of collisions.

To understand the scope of this problem, open an online map site that allows a satellite view. Then find your house. Use the satellite view, and zoom in as far as you can with a clear satellite image. How many intersections are in your neighbourhood? Now find the intersection of 75th street NW and 98th Avenue NW in Edmonton by entering the address in the search area. Zoom out and look at more of the city. How many intersections do you think there are? Now try the intersection of 17th Avenue SW and 14th Street SW in Calgary. In cities such as Edmonton and Calgary, there are many intersections.

Reflect on the Big Picture

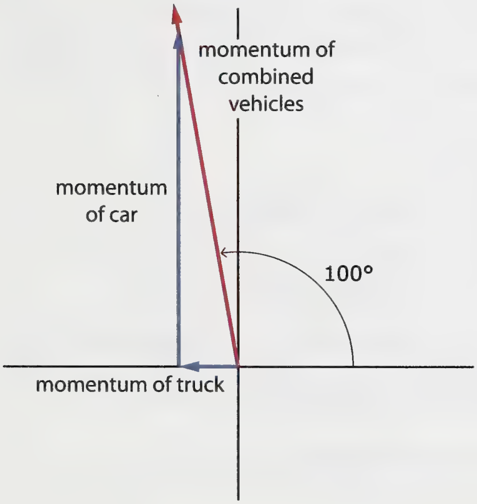
© Katherine Welles/shutterstock

Recall the collision of Kathy and Bill in the Big Picture section. Based on what you learned in this lesson, continue to analyze this collision.

Complete your Module 2 Project. Add the following questions and answers to your Crash Analysis stored in your Physics 30 course folder.

In previous lessons you determined the magnitude of the momentum of the combined vehicles just after impact and the kinetic energy before and after the impact. The following vector diagram illustrates the direction of the momentum vector based on evidence from the scene.

The two momentum vectors before the collisions are represented in blue. The momentum after the collision is in red. Adding the two momentum vectors before the collision equals the momentum after the collision, according to the law of conservation of momentum.



While the photo suggests a much more complex collision than the given diagram, assume that the diagram shows exactly what happened.

- Using this vector diagram and your calculated value for the momentum of the combined vehicles, verify the police calculations and determine the momentum and velocity of Bill's truck just before the collision. (Analytic)

- Using this vector diagram and your calculated value for the momentum of the combined vehicles, verify the police calculations and determine the momentum and velocity of Kathy's car just before the collision. (Analytic)

- Does your analysis prove, beyond a reasonable doubt, that Kathy was, in fact, travelling at the speed limit of 50 km/h just prior to the accident? What assumptions did you make in your solution to determine this? (Holistic)

Before submitting your answers for your Module 2 Project double check your answers with the following rubrics:

The Scoring Guide for Analytic Questions and the Scoring Guide for Holistic Questions can be found in the Appendix.

Answering these questions completes your Module 2 Project. Submit your Crash Analysis to your teacher for marks.

Module 2: Lesson 3 Assignment

Remember to submit the following to your teacher:

- Module 2: Lesson 3 Assignment
- Module 2 Project

Lesson Summary

At the start of this lesson, you were asked the following essential questions:

- Is momentum conserved in a non-linear collision?

- How is component analysis applied to solve non-linear collision problems?

- Can it be proven, beyond a reasonable doubt, that Kathy was moving at the speed limit just prior to the collision described in the accident scenario in the Big Picture?

You looked at collisions that occur in two dimensions. You verified that the total momentum of a system is conserved during a collision. Specifically, the total momentum of the *x* direction and the *y* direction is also conserved during a collision. The key principles are as follows:

- Momentum is a vector and can be resolved into components.
- During a collision, the total momentum of the components is conserved:

$\sum \vec{p}_{\text{initial x}} = \sum \vec{p}_{\text{final x}}$ and $\sum \vec{p}_{\text{initial y}} = \sum \vec{p}_{\text{final y}}$

Using this conservation principle, it is possible to analyze two-dimensional collisions, such as those of vehicles that meet in an intersection.

Module 2—The Conservation of Momentum in Isolated Systems

Module Summary

In Lesson 1 you investigated the momentum in an isolated system during a collision. You verified one of the fundamental laws that govern physical processes, the law of conservation of momentum. In summary, the law states: momentum is "mass in motion" or a measure of how much motion an object has. Expressed as an equation, it is as follows:

$$\vec{p} = m\vec{v}$$

In a collision between two objects, the change in momentum of one object is equal and opposite to the change in momentum of the other object. Expressed as an equation, it is as follows:

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

The law of conservation of momentum states that the sum of the momentum in a system before a collision equals the sum of the momentum in the same system after a collision. Expressed as an equation, it is as follows:

$$\sum\vec{p}_{\text{initial}} = \sum\vec{p}_{\text{final}}$$

You learned that we can analyze collisions to determine initial velocities or predict final velocities by applying the conservation of momentum.

In Lesson 2 you investigated the total kinetic energy of a system before and after a collision and found that kinetic energy is not always conserved. This is a criterion that distinguishes between two types of collisions. Furthermore, the amount of lost kinetic energy can be used to produce a spectrum of "elasticity," including the following three generalizations for collisions:

- *Perfectly elastic collisions* result in the total kinetic energy of the system being conserved. Perfectly elastic collisions generally occur only at the subatomic level.
- *Inelastic collisions* involve some loss of kinetic energy, generally as sound or thermal energy. This is a broad range, and most collisions fall within this class.
- *Perfectly inelastic collisions* are ones in which the colliding objects stick together upon impact. There is the greatest loss of kinetic energy in this type of collision.

All linear and non-linear collisions are described by one of these three general classifications, including the one analyzed for the module assessment in the form of a crash reconstruction analysis.

In Lesson 3 you studied collisions that occur in two dimensions. You verified that the total momentum of an isolated system is conserved during a collision. Specifically, the total momentum in the x direction and in the y direction is also conserved during a collision.

Therefore, $\sum \vec{p}_{\text{initial x}} = \sum \vec{p}_{\text{final x}}$ and $\sum \vec{p}_{\text{initial y}} = \sum \vec{p}_{\text{final y}}$.

Using this conservation principle, it is possible to analyze two-dimensional collisions, such as those of vehicles that meet in an intersection.

Module Assessment

Module 2 Project

In each lesson you have been working on a crash analysis of the collision between Kathy and Bill. The last part of the analysis was completed at the end of Lesson 3. Submit the entire analysis including answers to all of the questions in the Reflect on the Big Picture sections of Lessons 1, 2, and 3 of Module 2.

Module 2—Glossary

Module Glossary

momentum: the product of the mass and velocity of an object

Appendix

Scoring Guide for Analytic Questions

Physics Principles: Conservation of Momentum

Score	Description
4	Both relevant physics principles are stated and both are clearly related to the response. Physics principles for questions involving linear vector addition require explicit communication of the vector nature; e.g., a situational diagram or a free-body diagram (FBD) for forces and a vector addition diagram.
3	Both relevant physics principles are stated, but only one is clearly related to the response.
2	Both relevant physics principles are stated but neither is clearly related to the response. **or** One relevant physics principle is stated and is clearly related to the response.
1	One relevant physics principle is stated.
0	No relevant physics principle is stated.

Substitutions

Score	Description
1	All substitutions are shown. Significant digits are not required in intermediate steps. A response with at most one implicit unit conversion may receive this score. An incomplete or incorrect response may receive this score if all the values substituted are appropriate; i.e., length measurements into length variables, energy measurements into energy variables.
0	Too many substitutions are missing. **or** The response contains one invalid substitution; i.e., electric field strength for energy or speed for electric potential difference.

Formulas

Score	Description
3	All relevant formulas required for the complete solution are present and have been written as they appear on the equations sheet or in the information given with the question.
2	Most relevant formulas are stated. **or** Derived formulas are used as starting points.
1	One relevant formula from the formula sheet is stated.
0	No relevant formula is stated.

Final Answer

Score	Description
2	The value of the answer to the complete problem is stated and calculated consistently with the solution presented. The final answer is stated with the appropriate number of significant digits and with appropriate units. A response in which an inappropriate substitution has been made may receive this score if the incorrect units are consistently carried forward.
1	The value of the final answer is stated, calculated consistently with the solution presented. Units or significant digits are incorrect. **or** The response is incomplete, but an intermediate value is stated and calculated consistently with the solution presented with appropriate units (significant digits not required).
0	The answer stated is unrelated to the solution shown. **or** No answer is given.

Scoring Guide for Analytic Questions

Physics Principles: conservation of energy and conservation of momentum

Score	Description
4	Both relevant physics principles are stated and both are clearly related to the response. Physics principles for questions involving linear vector addition require explicit communication of the vector nature; e.g., a situational diagram or a free-body diagram (FBD) for forces and a vector addition diagram.
3	Both relevant physics principles are stated, but only one is clearly related to the response.
2	Both relevant physics principles are stated but neither is clearly related to the response. **or** One relevant physics principle is stated and is clearly related to the response.
1	One relevant physics principle is stated.
0	No relevant physics principle is stated.

Substitutions

Score	Description
1	All substitutions are shown. Significant digits are not required in intermediate steps. A response with at most one implicit unit conversion may receive this score. An incomplete or incorrect response may receive this score if all the values substituted are appropriate; i.e., length measurements into length variables, energy measurements into energy variables.
0	Too many substitutions are missing. **or** The response contains one invalid substitution; i.e., electric field strength for energy or speed for electric potential difference.

Formulas

Score	Description
3	All relevant formulas required for the complete solution are present and have been written as they appear on the equations sheet or in the information given with the question.
2	Most relevant formulas are stated. **or** Derived formulas are used as starting points.
1	One relevant formula from the formula sheet is stated.
0	No relevant formula is stated.

Final Answer

Score	Description
2	The value of the answer to the complete problem is stated and calculated consistently with the solution presented. The final answer is stated with the appropriate number of significant digits and with appropriate units. A response in which an inappropriate substitution has been made may receive this score if the incorrect units are consistently carried forward.
1	The value of the final answer is stated, calculated consistently with the solution presented. Units or significant digits are incorrect. **or** The response is incomplete, but an intermediate value is stated and calculated consistently with the solution presented with appropriate units (significant digits not required).
0	The answer stated is unrelated to the solution shown. **or** No answer is given.

Holistic Scoring Guide

Major Concepts: conservation of momentum, conservation of energy

Score	Description
5	The nature of a response that will receive a score of 5 has the following characteristics: • The response addresses, with appropriate knowledge, all the major concepts in the question (all bullets must be attempted). • The student applies major physics principles in the response (appropriate physics principles are stated). • The relationships between ideas contained in the response are explicit* (physics principles are clearly linked to the application). • The reader has no difficulty following the strategy or solution presented by the student. • Statements made in the response are supported explicitly.* **Note:** The response may contain minor errors or have minor omissions.
4	The nature of a response that will receive a score of 4 has the following characteristics: • The response addresses, with appropriate knowledge, all the major concepts in the question (all bullets must be attempted). • The student applies major physics principles in the response (appropriate physics principles are stated). • The relationships between the ideas contained in the response are implied**(physics principles are stated but not properly linked to the application). • The reader has some difficulty following the strategy or solution presented by the student. • Statements made in the response are supported implicitly.** **Note:** The response is mostly complete and mostly correct, although it may contain errors or have omissions, and contains some application of physics principles.
3	The nature of a response that will receive a score of 3 has the following characteristics: • The response addresses, with some appropriate knowledge, all the major concepts in the question (all bullets must be attempted). • The student does not apply major physics principles in the response (all appropriate physics principles are not stated). • There are no relationships between the ideas contained in the response (physics principles are stated but not applied). • The reader may have difficulty following the strategy or solution presented by the student.

2	The nature of a response that will receive a score of 2 has the following characteristic: • The response addresses, with some appropriate knowledge, two of the major concepts in the question (only two bullets are attempted).
1	The nature of a response that will receive a score of 1 has the following characteristic: • The response addresses, with some appropriate knowledge, one of the major concepts in the question (only one bullet is attempted).
0	• The student provides a solution that is invalid for the question.

*Explicit means the response is clearly stated; the marker does not have to interpret.
**Implicit (implied) means the response is not clearly stated; the marker must interpret.

For example:

Explicit: An electron has a negative charge while a proton has a positive charge.

The answer is clear with no possible misinterpretation.

Implicit: An electron has a negative charge while a proton does not.

The answer is not clear because the marker does not know if a proton is neutral or positively charged. There is more than one possible way to interpret the answer.

Holistic Scoring Guide

Major Concepts: Momentum

Score	Description
5	The nature of a response that will receive a score of 5 has the following characteristics: • The response addresses, with appropriate knowledge, all the major concepts in the question (all bullets must be attempted). • The student applies major physics principles in the response (appropriate physics principles are stated). • The relationships between ideas contained in the response are explicit* (physics principles are clearly linked to the application). • The reader has no difficulty following the strategy or solution presented by the student. • Statements made in the response are supported explicitly.* **Note:** The response may contain minor errors or have minor omissions.

4	The nature of a response that will receive a score of 4 has the following characteristics: • The response addresses, with appropriate knowledge, all the major concepts in the question (all bullets must be attempted). • The student applies major physics principles in the response (appropriate physics principles are stated). • The relationships between the ideas contained in the response are implied**(physics principles are stated but not properly linked to the application). • The reader has some difficulty following the strategy or solution presented by the student. • Statements made in the response are supported implicitly.** **Note:** The response is mostly complete and mostly correct, although it may contain errors or have omissions, and contains some application of physics principles.
3	The nature of a response that will receive a score of 3 has the following characteristics: • The response addresses, with some appropriate knowledge, all the major concepts in the question (all bullets must be attempted). • The student does not apply major physics principles in the response (all appropriate physics principles are not stated). • There are no relationships between the ideas contained in the response (physics principles are stated but not applied). • The reader may have difficulty following the strategy or solution presented by the student.
2	The nature of a response that will receive a score of 2 has the following characteristic: • The response addresses, with some appropriate knowledge, two of the major concepts in the question (only two bullets are attempted).
1	The nature of a response that will receive a score of 1 has the following characteristic: • The response addresses, with some appropriate knowledge, one of the major concepts in the question (only one bullet is attempted).
0	• The student provides a solution that is invalid for the question.

*Explicit means the response is clearly stated; the marker does not have to interpret.

**Implicit (implied) means the response is not clearly stated; the marker must interpret.

For example:

Explicit: An electron has a negative charge while a proton has a positive charge.

The answer is clear with no possible misinterpretation.

Implicit: An electron has a negative charge while a proton does not.

The answer is not clear because the marker does not know if a proton is neutral or positively charged. There is more than one possible way to interpret the answer.

Discussion Scoring Guide

Principles involved: Conservation of momentum, isolated systems				
Criteria	**Level 1 (Below Standard)**	**Level 2 (Approaching Standard)**	**Level 3 (Standard)**	**Level 4 (Above Standard)**
Knowledge				
Demonstrates understanding of the situation, physics principles and technology, and their connections.	Demonstrates a vague and sometimes incorrect understanding of the physics principles involved. Obvious irrelevant or missing information.	Demonstrates a basic understanding of the physics principles involved. May exhibit minor mistakes or vague information or application to the situation.	Demonstrates a good understanding of the physics principles involved and applies them properly to the given situation. All necessary information is given.	Demonstrates a superior understanding of the physics principles involved and their application to the situation. All applications are considered in detail.
Reflection				
The post shows reflection on one's own and other students' work. Contributes to the group discussion.	Does not make an effort to participate. Seems indifferent to discussion.	Occasionally makes meaningful reflections on the group's efforts or discussions. Marginal effort is shown to become involved with the group or discussion.	Frequently makes meaningful reflections on the group's efforts and presents relevant viewpoints for consideration by the group. Interacts freely with group members.	Regularly attempts to motivate the group discussion and delve deeper into concepts. Interacts freely and encourages all group members.

Content and presentation of discussion summary				
The information is logically arranged in a clear and concise manner.	The information is poorly organized with many concepts implied. Irrelevant or rambling sentences make reading difficult.	The information is somewhat organized with implied concepts. Excessive words or awkward sentences are used, which hinder reading.	The information is well-organized and logically arranged. All concepts are explicitly explained. There are a few awkward but understandable sentences.	The information is well-organized and very easy to understand. Well-worded sentences make reading pleasurable.

Self-Check Answers

Lesson 1

SC 1.

Given

mass 1 $m_1 = 5.0$ kg
mass 2 $m_2 = 7.0$ kg

mass 1 initial velocity $\vec{v}_{1i} = +10.0$ m/s

mass 2 initial velocity $\vec{v}_{2i} = 0$ m/s

mass 1 final velocity $\vec{v}_{1f} = +1.17$ m/s

Required

The final velocity of the 7.0-kg object $\left(\vec{v}_{2f}\right)$.

Analysis and Solution

Choose the two objects as an isolated system. Since the 7.0-kg object is stationary, its initial momentum is 0.

$$\begin{aligned}
\vec{p}_{\text{sys}_\text{i}} &= \vec{p}_{\text{sys}_\text{f}} \\
\vec{p}_{1\text{i}} + \vec{p}_{2\text{i}} &= \vec{p}_{1\text{f}} + \vec{p}_{2\text{f}} \\
m_1\vec{v}_{1\text{i}} + 0 &= m_1\vec{v}_{1\text{f}} + m_2\vec{v}_{2\text{f}} \\
m_2\vec{v}_{2\text{f}} &= m_1\vec{v}_{1\text{i}} - m_1\vec{v}_{1\text{f}} \\
\vec{v}_{2\text{f}} &= \frac{m_1\vec{v}_{1\text{i}} - m_1\vec{v}_{1\text{f}}}{m_2} \\
&= \frac{(5.0\text{ kg})(+10.0\text{ m/s}) - (5.0\text{ kg})(+1.17\text{ m/s})}{7.0\text{ kg}} \\
\vec{v}_{2\text{f}} &= 6.3\text{ m/s}
\end{aligned}$$

Paraphrase

The final velocity of the 7.0-kg object is 6.3 m/s.

SC 2. This is not possible. The initial momentum of the system is to the right; therefore, the final momentum must also be to the right.

Lesson 2

SC 1.

a. $$\begin{aligned}
\vec{p}_1 &= m\vec{v}_1 \\
&= (50\text{ kg})(+0.75\text{ m/s}) \\
&= +38\text{ kg}\bullet\text{m/s}
\end{aligned}$$

$$\begin{aligned}
\vec{p}_2 &= m\vec{v}_2 \\
&= (50\text{ kg})(-0.75\text{ m/s}) \\
&= -38\text{ kg}\bullet\text{m/s}
\end{aligned}$$

Each car has a different momentum. They are identical in magnitude but opposite in direction.

b. $$\begin{aligned}
\sum\vec{p} &= \vec{p}_1 + \vec{p}_2 \\
&= (+38\text{ kg}\bullet\text{m/s}) + (-38\text{ kg}\bullet\text{m/s}) \\
&= 0\text{ kg}\bullet\text{m/s}
\end{aligned}$$

The total momentum of the system is zero.

c. $$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(50\ \text{kg})(0.75\ \text{m/s})^2 \\ &= 14\ \text{J} \end{aligned}$$

Both cars have the same kinetic energy because it is a scalar quantity.

d. $$\begin{aligned} E_k &= E_{k1} + E_{k2} \\ &= (14\ \text{J}) + (14\ \text{J}) \\ &= 28\ \text{J} \end{aligned}$$

The total kinetic energy of the system is 28 J.

SC 2.

a. before: **spherical** during: **squished** after: **spherical**

b. The kinetic energy of the ball is nearly conserved, with some kinetic energy being used as work done to momentarily change the shape of the ball.

SC 3.

a. before: **blob** during: **flat** after: **stays flat**

b. The kinetic energy of the playdough is not conserved. Most of the kinetic energy is used in order to change the shape of the playdough permanently.

SC 4.

Given

$$\begin{aligned} m_b &= 2.59\ \text{g} \\ &= 0.00259\ \text{kg} \end{aligned} \qquad m_p = 1.00\ \text{kg} \qquad \begin{aligned} \Delta h &= 5.20\ \text{cm} \\ &= 0.0520\ \text{m} \end{aligned}$$

Required

The velocity of the bullet just before impact, $\vec{v}_{b_i}$.

Analysis and Solution

Choose the system of bullet and pendulum as an isolated system. Since the pendulum is stationary before the bullet hits, its initial velocity is zero; thus, its initial momentum $\vec{p}_{\mathrm{p_i}} = 0$. After the collision, the bullet and pendulum move together as a unit. The kinetic energy of the pendulum-bullet system is converted into gravitational potential energy:

$E_k = E_p$

Apply the law of conservation of energy to find the speed of the pendulum-bullet system just after impact:

$$
\begin{aligned}
E_{\mathrm{k}} &= E_{\mathrm{p}} \\
\frac{1}{2}\left(m_{\mathrm{b}} + m_{\mathrm{p}}\right)\left(v_{\mathrm{f}}\right)^2 &= \left(m_{\mathrm{b}} + m_{\mathrm{p}}\right) g\left(\Delta h\right) \\
\left(v_{\mathrm{f}}\right)^2 &= 2g\left(\Delta h\right) \\
v_{\mathrm{f}} &= \sqrt{2g\left(\Delta h\right)} \\
&= \sqrt{2\left(9.81\ \mathrm{m/s^2}\right)\left(0.0520\ \mathrm{m}\right)} \\
&= 1.010069305\ \mathrm{m/s} \\
v_{\mathrm{f}} &= 1.010069305\ \mathrm{m/s}
\end{aligned}
$$

Apply the law of conservation of momentum to find the initial velocity of the bullet:

$$
\begin{aligned}
\vec{p}_{\mathrm{sys_i}} &= \vec{p}_{\mathrm{sys_f}} \\
\vec{p}_{\mathrm{b_i}} + \vec{p}_{\mathrm{p_i}} &= \vec{p}_{\mathrm{sys_f}} \\
m_{\mathrm{b}}\vec{v}_{\mathrm{b_i}} + 0 &= \left(m_{\mathrm{b}} + m_{\mathrm{p}}\right)\vec{v}_{\mathrm{f}} \\
\vec{v}_{\mathrm{b_i}} &= \left(\frac{m_{\mathrm{b}} + m_{\mathrm{p}}}{m_{\mathrm{b}}}\right)\left(\vec{v}_{\mathrm{f}}\right) \\
&= \frac{0.00259\ \mathrm{kg} + 1.00\ \mathrm{kg}}{0.00259\ \mathrm{kg}}\left(+1.010069305\ \mathrm{m/s}\right) \\
&= \frac{1.00259}{0.00259}\left(+1.010069305\ \mathrm{m/s}\right) \\
&= +391\ \mathrm{m/s} \\
\vec{v}_{\mathrm{b_i}} &= 391\ \mathrm{m/s}\ \ [\text{forward}]
\end{aligned}
$$

Paraphrase

The initial velocity of the bullet immediately before impact was 391 m/s [forward].

SC 5.

Given

mass of dart $m_d = 0.012$ kg

mass of block-glider $m_b = 0.200$ kg

final velocity of system $\vec{v}_f = 0.78$ m/s [right]

initial velocity of dart $\vec{v}_i = 14$ m/s [right]

Required

The amount of kinetic energy lost immediately after the interaction.

Analysis and Solution

the initial energy of the system comes from the kinetic energy of the dart:

$$\begin{aligned} E_{k_i} &= \frac{1}{2}m_d\left(v_d\right)^2 \\ &= \frac{1}{2}(0.012\ \text{kg})(14\ \text{m/s})^2 \\ &= 1.176\ \text{J} \\ E_{k_i} &= 1.2\ \text{J} \end{aligned}$$

The final kinetic energy comes from the dart-block-glider system.

$$\begin{aligned} E_{k_f} &= \frac{1}{2}\left(m_d + m_g\right)\left(v_f\right)^2 \\ &= \frac{1}{2}(0.212\ \text{kg})(0.78\ \text{m/s})^2 \\ &= 0.0644904\ \text{J} \\ E_{k_f} &= 0.064\ \text{J} \end{aligned}$$

The difference between the initial and final kinetic energies is the kinetic energy lost.

$$\begin{aligned} E_{k_{\text{lost}}} &= E_{k_i} - E_{k_f} \\ &= 1.176\ \text{J} - 0.0644904\ \text{J} \\ E_{k_{\text{lost}}} &= 1.1\ \text{J} \end{aligned}$$

Paraphrase

There was 1.1 J lost immediately after the interaction of the dart and the block-glider system.

SC 6. Inelastic collisions involve a significant reduction in the kinetic energy of the objects after the collision since very little *bouncing* or reversal of direction occurs. This means that the overall change in velocity is less, so the change in momentum is less. Therefore, less impulse is required to cause that change in momentum. In addition, the time interval for the inelastic collision is increased because *crumpling* takes time. As was mentioned in the previous module, if the time interval for an interaction lengthens, then the force required reduces to achieve the same impulse.

Lesson 3

SC 1. $\rho_{\mathrm{x}} = \vec{\rho}(\cos\theta)$

SC 2. $\rho_{\mathrm{y}} = \vec{\rho}(\sin\theta)$

SC 3. $\vec{\rho} = \sqrt{{\rho_{\mathrm{x}}}^2 + {\rho_{\mathrm{y}}}^2}$

SC 4. $\theta = \tan^{-1}\left(\dfrac{\rho_{\mathrm{y}}}{\rho_{\mathrm{x}}}\right)$

Unit A Conclusion

In Module 1 you explored the concepts of momentum and impulse as they relate to the function and design requirements of vehicle safety devices. In this context, you discovered that momentum is the product of the mass and velocity of an object and that it is a vector quantity. The direction of the momentum vector is the same as the direction of the velocity vector.

You then applied Newton's second law to explain that the external, non-zero, net force acting on an object will be equal to the rate of change of the momentum of that object. This has particular relevance to situations involving large changes in momentum, such as those of vehicle collisions that occur at a high rate of speed. This relationship helps makes sense of the phrase "speed kills." The faster an object is going, the greater the momentum that object has—and the greater the potential for more serious collisions.

From this, you learned that impulse is the change in momentum, $\Delta\vec{p}$, and is the product of net force and time. Understanding impulse helps you understand how we can attempt to reduce injury and death in vehicle collisions where the occupants experience a large change in momentum. The force associated with a given impulse can be reduced by applying it over a greater time period, a principle related to the design of vehicle safety devices such as air bags and crumple zones.

In Module 2 you explored collisions in an isolated system and gained an understanding of the law of conservation of momentum, which states that the sum of the momentum in a system before a collision equals the sum of the momentum in the same system after a collision. Expressed as an equation, it is as follows:

$$\sum \vec{p}_{\text{initial}} = \sum \vec{p}_{\text{final}}$$

You also learned that there are two different types of collisions and that we can analyze elastic and inelastic collisions to determine initial velocities or predict final velocities or mass of the objects by applying the conservation of momentum principle. You first analyzed this in linear collisions and then applied that knowledge to analyze non-linear collisions in two dimensions, such as two cars meeting in an intersection. You discovered that momentum is still conserved in a two-dimensional collision. Specifically, the total momentum in the *x* direction and in the *y* direction is also conserved during a collision.

Therefore, $\sum \vec{p}_{\text{initial x}} = \sum \vec{p}_{\text{final x}}$ and $\sum \vec{p}_{\text{initial y}} = \sum \vec{p}_{\text{final y}}$

Lastly, you investigated the total kinetic energy of a system before and after a collision and discovered that kinetic energy is not always conserved. Furthermore, the amount of lost kinetic energy can be used to produce a spectrum of elasticity:

- *Perfectly elastic collisions* result in the total kinetic energy of the system being conserved. Perfectly elastic collisions generally occur only at the subatomic level.

- *Inelastic collisions* involve some loss of kinetic energy, generally as sound or thermal energy. This is a broad range, and most collisions fall within this class.

- *Perfectly inelastic collisions* are ones in which the colliding objects stick together upon impact. There is the greatest loss of kinetic energy in this type of collision.

Throughout this unit, you have learned how to explain how momentum is conserved when objects interact in an isolated system.

Unit A Assessment

There is no unit-level assessment. In each module you completed a significant number of assessment activities. Some activities were assessed by you (e.g., Self-Check) or your classmates (e.g., Discuss) and others were assessed by your teacher (e.g., Lesson Assignment, and Module Project).