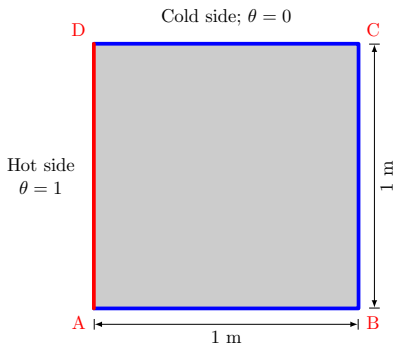


# 2D Heat Conduction Equation solution using PINNs

November 6, 2023

# Problem Definition

2D square plate of side 1 m with one hot side and other 3 cold sides



Present work is to compute the temperature field within plate domain using PINNs with different variations in methodology.

## Governing equation and Analytical solution

the governing two-dimensional temperature distribution becomes

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$$

with the boundary conditions

$$\theta(0, y) = 1$$

$$\theta(x, 0) = 0$$

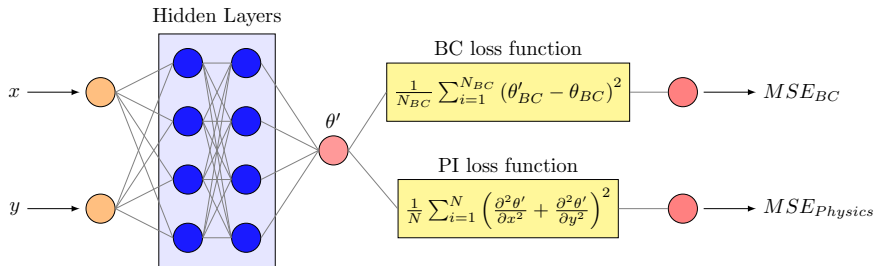
$$\theta(L, y) = 0$$

$$\theta(x, H) = 0$$

Use of the separation of variables method gives the solution for  $\theta$  as

$$\theta = \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{\sinh [(2n+1)\pi(L-x)/H]}{\sinh [(2n+1)\pi L/H]} \frac{\sin [(2n+1)\pi y/H]}{2n+1}$$

# Network Schematic



## key points

- ▶ pure physics-based training was done on the network with 20NX8L size, with 40 boundary points and 100 internal points.
- ▶ the model was trained with 40,000 epochs and took about 3 minutes. Then the model is used to predict the solution on a 1 million grid.
- ▶ The grid-extended solution was compared with analytical solution, the numerical solution was attempted for the same grid using FDM on FORTRAN with Gauss-seidel method.

# Run-time comparison: PINN vs FDM

On 1 million grid,

- ▶ PINN took 172.42 seconds (2.87 minutes), including training time
- ▶ FDM took 1635.79 seconds (27.63 minutes)

```
epoch : 39998 mse_data : 0.02567526 mse_phy : 0.00118072
epoch : 39999 mse_data : 0.02561229 mse_phy : 0.002617
epoch : 40000 mse_data : 0.02568614 mse_phy : 0.004217
libGL error: MESA-LOADER: failed to open radeonsi: /home2/Soft
sion 'GLIBCXX 3.4.30' not found (required by /lib64/libLLVM.
libGL error: failed to load driver: radeonsi
libGL error: MESA-LOADER: failed to open radeonsi: /home2/Soft
sion 'GLIBCXX 3.4.30' not found (required by /lib64/libLLVM.
libGL error: failed to load driver: radeonsi
libGL error: MESA-LOADER: failed to open swrast: /home2/Soft
on 'GLIBCXX 3.4.30' not found (required by /lib64/libLLVM.so
libGL error: failed to load driver: swrast

done

real    172.42s
user    275.66s
sys     16.97s
cpu     169%
```

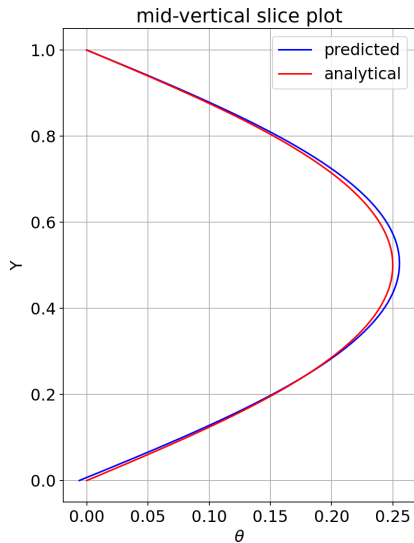
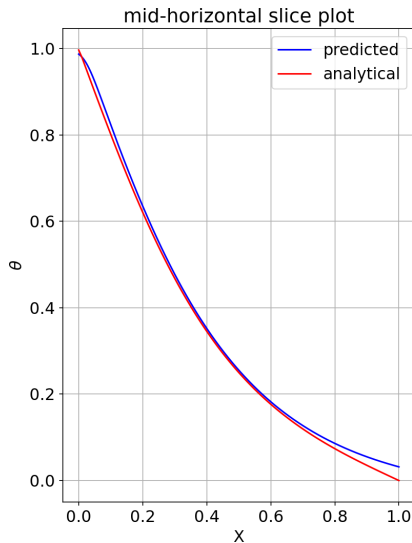
Figure: PINN runtime

```
iteration = 147874 ; residual = 1.0000643826391098E
iteration = 147875 ; residual = 1.0000526599052151E
iteration = 147876 ; residual = 1.0000409373933650E
iteration = 147877 ; residual = 1.0000292149925372E
iteration = 147878 ; residual = 1.0000174927027317E
iteration = 147879 ; residual = 1.0000057706904819E
solution converged!
computed data written to the file!
End

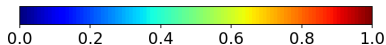
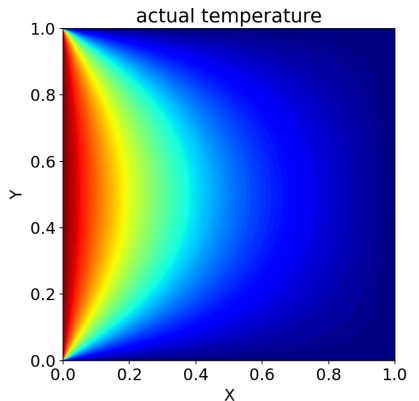
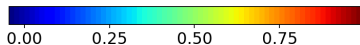
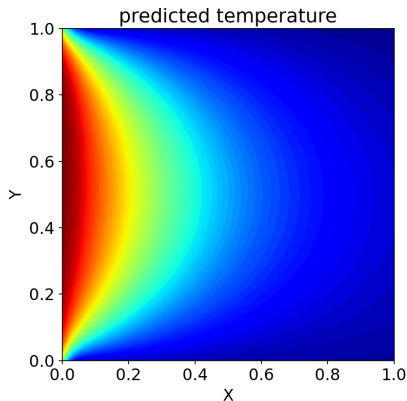
real    1635.79s
user    1633.37s
sys      2.04s
cpu      99%
```

Figure: PINN runtime

# Solution Comparison: PINN vs Analytical : 1 Mil grid

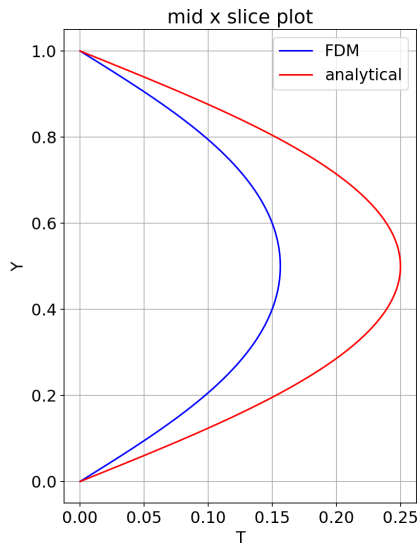
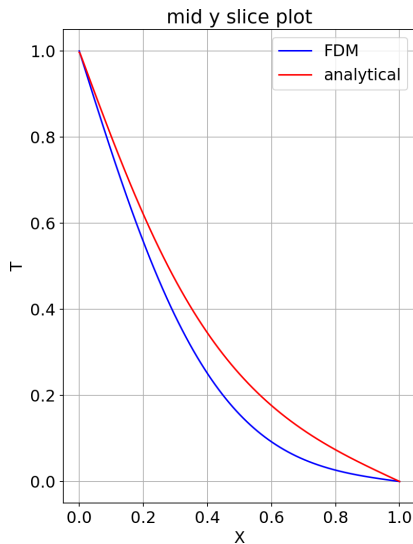


# Solution Comparison: PINN vs Analytical : 1 Mil grid





# Solution Comparison: FDM vs Analytical : 1 Mil grid



# Conclusion

1. FDM solution did not match with analytical solution despite its convergence.
2. PINN's grid-extended solution matches with analytical solution.
3. PINN learns the function quite well by solving the problem with just 140 data points in the domain.
4. The time taken by PINN is 10 times less than FDM for the same grid solution, including training time.
5. PINN solution can be used as initial condition for steady state problems on large grids to save computational time.