AE721 Boundary layer Theory

(2022-23 Even Semester)

Assignment – 4

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1. Numerically solve the compressible Couette flow where the top wall is at rest ($y = h, u = 0, T = T_h$) and the bottom wall ($y = 0, u = V, T = T_0$) is moving at velocity V using shooting method for (i) Both walls at constant wall temperature (T_h), and (ii) Top wall is adiabatic and lower wall at temperature T_0. Use non-dimensional equations for solution. Assume calorically perfect gas and $\mathrm{Pr} = 0.72$.

   Deliverables:

   1. Velocity profile (y/h vs u/V) and temperature profile (y/h vs T/T_0) for values of $A = 0, 5, 10, 20$ ($A = \mathrm{Pr\ Ec},\ \mathrm{Ec} = V^2/(c_p T)$) and compare your results with analytical solution. Use the following formula for μ.

   $$\mu(T) \approx \mu_{ref}\left(\frac{T}{T_{ref}}\right)$$

   Analytical solution:

   1. Equal wall temperature

   $$\frac{y}{h} = 1 - \frac{u}{V}\left(\frac{1 + \frac{1}{4}\mathrm{Pr\ Ec_h}\ \left(\frac{u}{V}\right) - \frac{1}{6}\mathrm{Pr\ Ec_h}\left(\frac{u}{V}\right)^2}{1 + \frac{1}{12}\mathrm{Pr\ Ec_h}}\right)$$

   $$\frac{T}{T_h} = 1 + \frac{1}{2}\mathrm{Pr\ Ec_h}\frac{u}{V}\left(1 - \frac{u}{V}\right)$$

   $$\mathrm{Ec_h} = \frac{V^2}{c_p T_h}$$

   2. Adiabatic wall

   $$\frac{y}{h} = 1 - \frac{u}{V}\left(\frac{1 + \frac{1}{2}\mathrm{Pr\ Ec_0}\left(1 - \frac{u}{2V}\right)}{1 + \frac{1}{4}\mathrm{Pr\ Ec_0}}\right)$$

   $$\frac{T}{T_0} = 1 + \frac{1}{2}\mathrm{PrEc_0}\left(1 - \frac{u^2}{V^2}\right)$$

   $$\mathrm{Ec_0} = \frac{V^2}{c_p T_0}$$