AE721 Boundary layer Theory

(2022-2023, Even Semester)

Assignment – 3

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1. Derive the Falkner-Skan equation (given below) from the boundary layer equations by assuming $u_e = Ax^m$, where s is streamwise direction

$$f''' + \frac{m+1}{2} f f'' + m[1 - (f')^2] = 0$$

2. Solve the Falkner-Skan equation numerically with appropriate boundary conditions for $m = 2.0, 1.0, 0.6, 0.3, 0, -0.05, -0.09043, -0.08$.

$$f''' + \frac{m+1}{2} f f'' + m[1 - (f')^2] = 0$$

Deliverables:

(a) For each m, include a table for the final iteration values of η, f, f', f''

(b) Comparison plot of streamwise velocity profiles $\left(\frac{u}{u_e} \text{ vs } \eta\right)$ and locate the inflectional point in the profile, if any.

(c) Comparison plot of normal velocity profiles $\left(\frac{v\sqrt{Re_x}}{u_e} \text{ vs } \eta\right)$

(d) Comparison plot of shear stress profiles $\left(\frac{\tau\sqrt{Re_x}}{\rho_e u_e^2} \text{ vs } \eta\right)$

(f) Provide a table with $\frac{\delta^*}{\delta_{FS}}$, $\frac{\theta}{\delta_{FS}}$, H, $\sqrt{Re_x}\,\frac{c_f}{2}$, $\lambda, \mathcal{T}$, F_θ for the given values of m, where $\delta_{FS} = \sqrt{\frac{\nu\, x}{u_e}}$. The definition of H, $\lambda, \mathcal{T}$, $\mathcal{F}_\theta$ are given below

- $H = \frac{\delta^*}{\theta}$
- $\mathcal{T} = \sqrt{Re_\theta}\,\frac{c_f}{2}$
- $\lambda = \frac{\theta^2}{\nu}\frac{du_e}{dx}$
- $\mathcal{F}_\theta = 2[\mathcal{T} - (H+2)\lambda]$

(g) Plot λ vs H, λ vs $\mathcal{T}$, and λ vs $\mathcal{F}_\theta$ for different m.

3. Calculate the value of $\frac{\delta^*}{\delta_{FS}}$, $\frac{\theta}{\delta_{FS}}$, H, $\sqrt{Re_x}\,\frac{c_f}{2}$, for the given values of m (refer Problem 2) using analytical Thwaits method and compare with the numerical solutions of Falkner-Skan equation.

$$\widetilde{H}(\lambda) = 2.61 - 4.1\,\lambda + 14\,\lambda^3 + \frac{0.56\,\lambda^2}{(\lambda + 0.18)^2}$$

$$\widetilde{T}(\lambda) = 0.220 + 1.52\,\lambda - 5\,\lambda^3 - \frac{0.072\,\lambda^2}{(\lambda + 0.18)^2}$$