Parameter estimation of 1D ODE with only boundary conditions

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1 Introduction

In this work, a simple 1D ODE given in Equation (1) is solved for its coefficient α with only boundary conditions data. This is done as a trial work for the following.

- trial to combine the physics loss and bc loss, then feed to a single optimizer function. (Previous works include individual models for bc and physics that share same NN and use two identical optimizer functions)
- trial for the System Identification problem in UAV dataset.

$$\frac{dx}{dt} = \alpha cos(t) + t; \quad 0 \leq t \leq \frac{\pi}{4} \tag{1}$$

2 Tensorflow Model

The code is developed using Tensorflow v2 and the following are the key points regarding code development.

- A custom model was built for this work, as the architecture needed is quite complex for the inbuilt model.
- A custom training loop was constructed as the model has to be trained based on both the boundary data condition and equation constraint, and both the data for training are of different size.
- the loss values were combined and fed to a single optimizer.
- the model is made to run in eager execution mode only. The custom training loop is incompatible with graph execution.
- the model contains 3 hidden layers with 10 neurons on each layer. And one input (t) and one output (x). TanH is the activation function. RMSprop optimizer is used.

3 Results

The training data is generated using $\alpha = 0.5$. The model has trained to the expected level within 5000 epochs with learning rate as $1e-4$. Still the model is trainned till 10,000 epochs to get the best accuracy. The results obtained were shown in Figures 1 to 3.

The value of coefficient computed by the model is shown in Table 1

Table 1: predicted coefficient value

predicted value	actual value	error percentage
0.49910807	0.5	0.1783

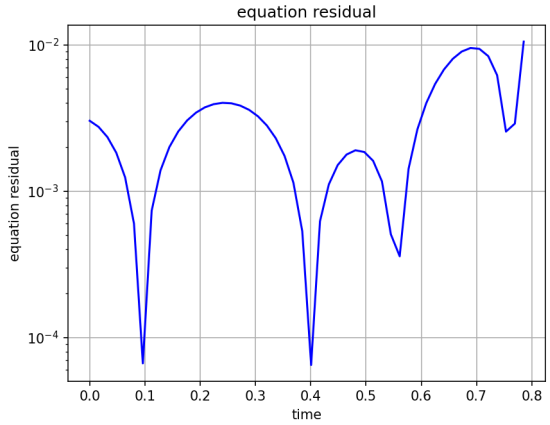

Figure 1: the residual of equation during prediction

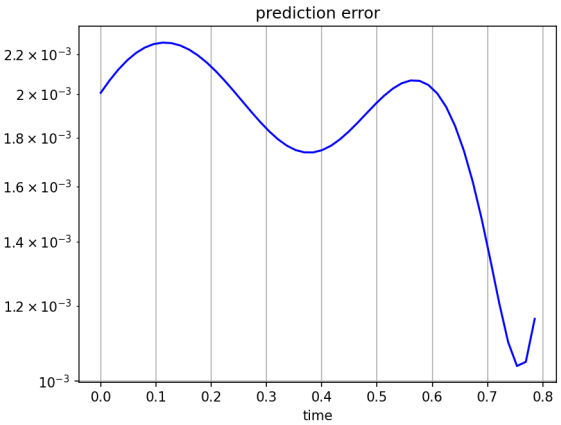

Figure 2: error of predicted vs actual data

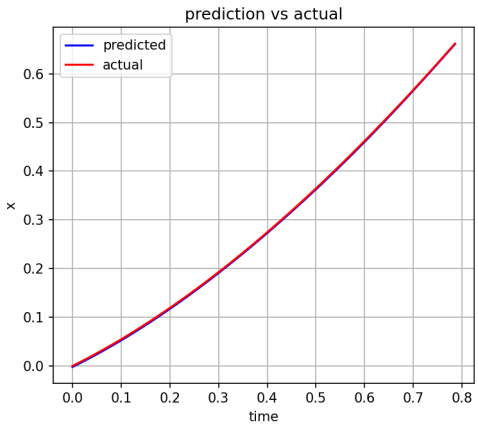

Figure 3: predicted vs actual data comparison

4 conclusion

The present work shows that it is possible to do the parameter estimation with limited dataset and the way to do optimization with combined loss values. Hence, further works can be developed from this.

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