

# Solution of 1D heat conduction with constant and variable source using FEM

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# Problem definition and analytical solution

- ▶ 1D HC with  $T = 0$  at boundaries and heat source (constant and variable)
- ▶ Analytical solution obtained using the method of manufactured solutions

$$T(x) = x - x^2 \quad (Q = 2)$$

$$T(x) = x^2 - x^3 \quad (Q = 2 - 6x)$$

- ▶ Governing equation

$$K \frac{d^2 T}{dx^2} + Q = 0$$

# Finite element fomulation

- ▶ Galerkin method with linear shape function elements were used

$$\int_0^L \phi(x) \left( K \frac{d^2 T}{dx^2} + Q \right) dx = 0$$

- ▶ Shape functions

$$N_1 = 1 - \frac{x}{h}$$

$$N_2 = \frac{x}{h}$$

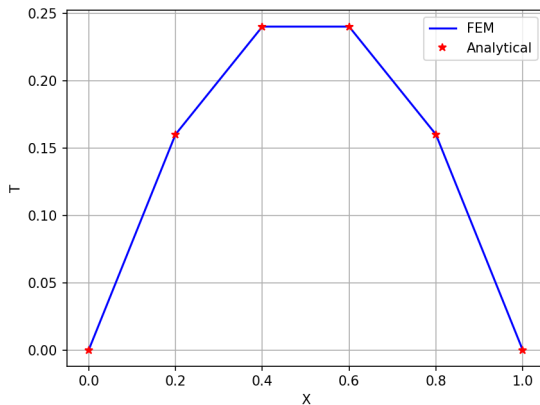
h - spatial step size

- ▶ Matrix equation

$$\frac{K}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \frac{Qh}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

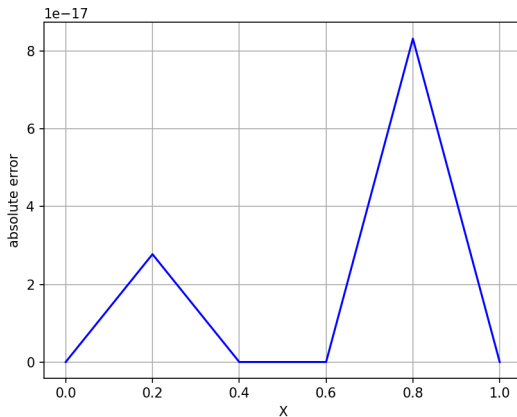
# Results - with 5 elements - constant Q

Computed result



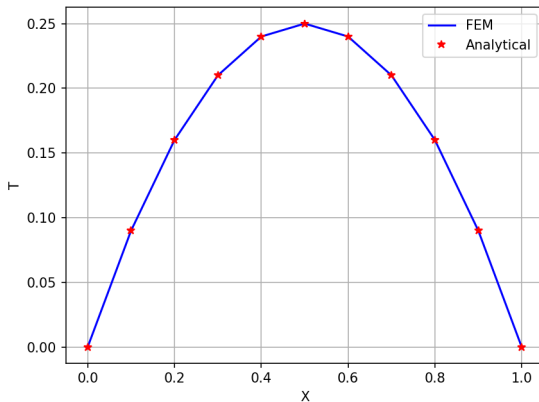
# Results - with 5 elements - constant Q

Computed error



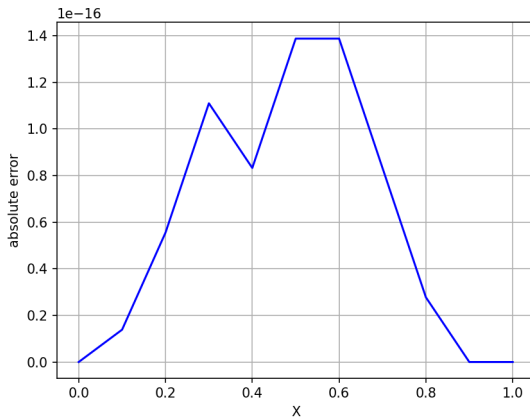
# Results - with 10 elements - constant Q

Computed result



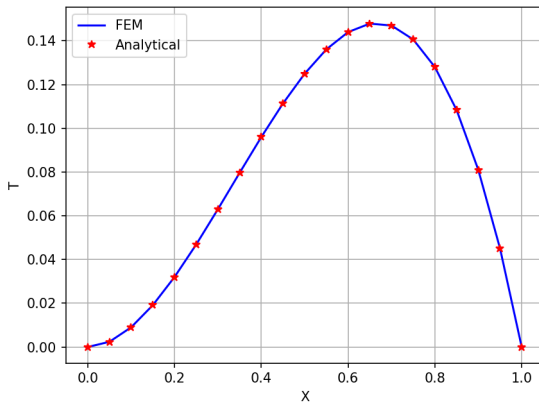
# Results - with 10 elements - constant Q

Computed error



# Results - with 20 elements - variable Q

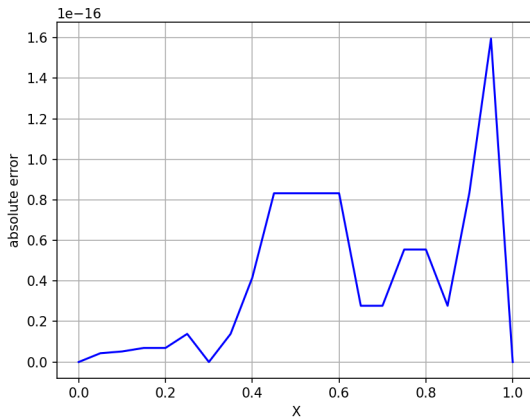
Computed result





# Results - with 20 elements - variable Q

Computed error



# Reference

Pepper, Darrell W., and Juan C. Heinrich. The finite element method: basic concepts and applications. Taylor & Francis, 2005.