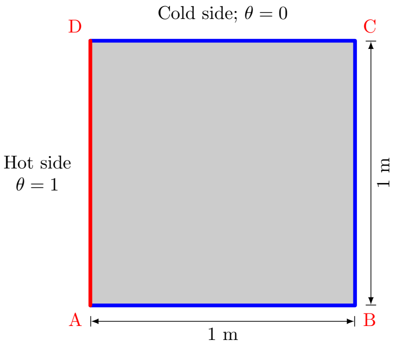
2D Heat Conduction Equation solution using PINNs

November 6, 2023

Problem Definition

2D square plate of side 1 m with one hot side and other 3 cold sides



Present work is to compute the temperature field within plate domain using PINNs with different variations in methodology.

Governing equation and Analytical solution

the governing two-dimensional temperature distribution becomes

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0$$

with the boundary conditions

$$\theta(0, y) = 1$$

$$\theta(x, 0) = 0$$

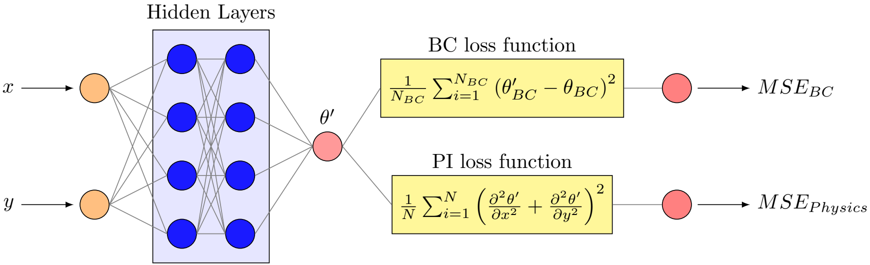
$$\theta(L, y) = 0$$

$$\theta(x, H) = 0$$

Use of the separation of variables method gives the solution for θ as

$$\theta = \frac{4}{\pi}\sum_{n=0}^{\infty} \frac{\sinh\left[(2n+1)\pi(L-x)/H\right]}{\sinh\left[(2n+1)\pi L/H\right]} \frac{\sin\left[(2n+1)\pi y/H\right]}{2n+1}$$

Network Schematic



key points

- pure physics-based training was done on the network with 20NX8L size, with 40 boundary points and 100 internal points.
- the model was trained with 40,000 epochs and took about 3 minutes. Then the model is used to predict the solution on a 1 million grid.
- The grid-extended solution was compared with analytical solution, the numerical solution was attempted for the same grid using FDM on FORTRAN with Gauss-seidel method.

Run-time comparison: PINN vs FDM

On 1 million grid,

- PINN took 172.42 seconds (2.87 minutes), including training time
- FDM took 1635.79 seconds (27.63 minutes)

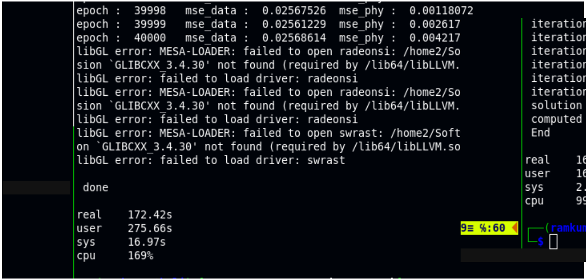


Figure: PINN runtime

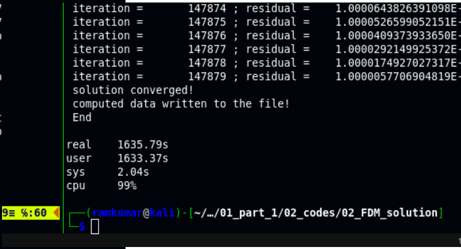
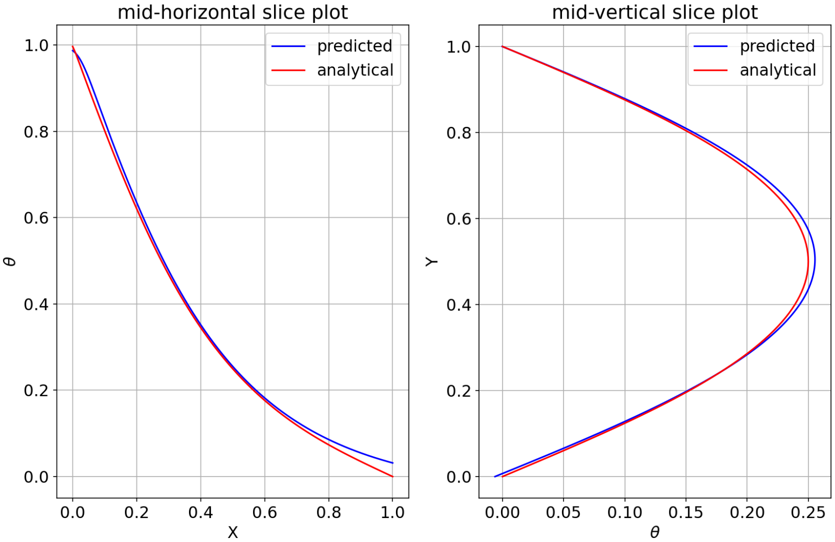
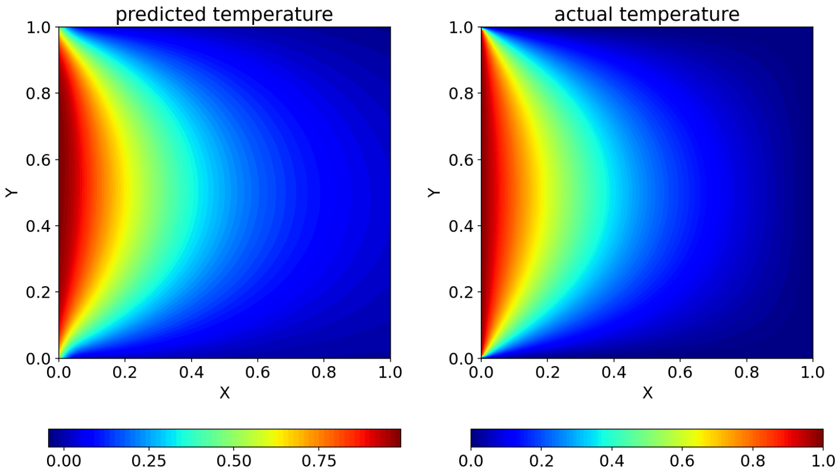


Figure: PINN runtime

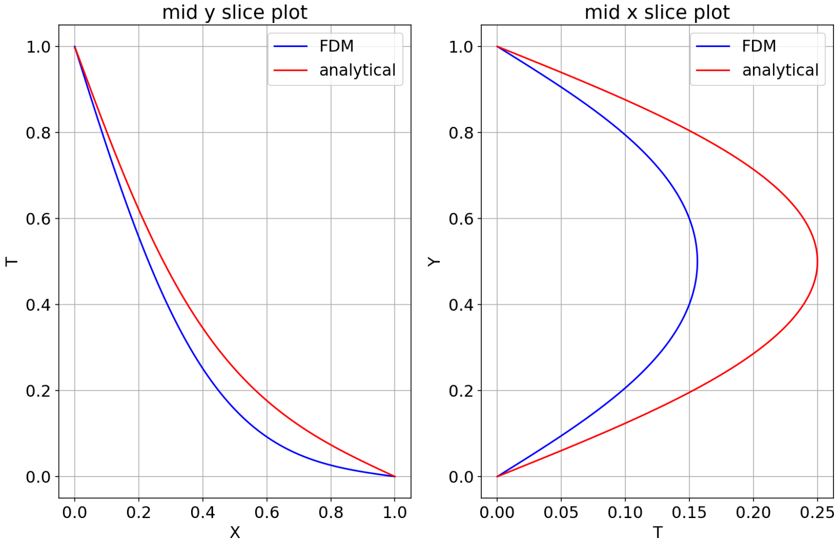
Solution Comparison: PINN vs Analytical : 1 Mil grid



Solution Comparison: PINN vs Analytical : 1 Mil grid



Solution Comparison: FDM vs Analytical : 1 Mil grid



Conclusion

1. FDM solution did not match with analytical solution despite its convergence.
2. PINN's grid-extended solution matches with analytical solution.
3. PINN learns the function quite well by solving the problem with just 140 data points in the domain.
4. The time taken by PINN is 10 times less than FDM for the same grid solution, including training time.
5. PINN solution can be used as initial condition for steady state problems on large grids to save computational time.