HARAMAYA UNIVERSITY

College of Natural and Computational Sciences

Department of Mathematics

Final Exam: Mathematics for Natural Sciences (Math 1011)

Date: June 26, 2023 Time allowed: 3hrs Total Marks: 50%

Name [redacted] ID: [redacted]

Section: [redacted] : Instructor Name: Mohammed Sani

General Instructions:

- Write your Name, ID and section on space provided!
- Make sure that the exam contains **8 pages** including the cover page!
- Put the answer only in **space provided**. If you need extra room to answer the questions, use the back of the pages, do not take the exam **apart**!
- Your hand writing should be **clear** and **legible!**
- Mobile Phone or any related electronic materials are **NOT ALLOWED** to be with you during the examination!
- Any types of mishaps and cheating during examination will **invalidate** your result!
- All other **EXAM RULES AND REGULATIONS** of the University are also binding!

Excellent!

FOR INSTRUCTORS' USE ONLY:

Part no & weight	Part I (8%)	Part II (12%)	Part III (10%)	Part IV (20%)	Total (50%)
Mark earned	6	11	10.5	20	47.5

Good luck !!

$2\pi = -\frac{2\pi}{3}$

Math 1011

thematics for NS

$180 - 120 = 60$

$z = \frac{-3+\sqrt{27}i}{2}$

$z = \frac{-3-\sqrt{27}i}{2}$

$z = \frac{-3 \pm \sqrt{9-36}}{2}$

$z-3 \;\overline{)\; z^3+3z+9}$

$z^3 - 27$

$-(z^3 - 3z^2)$

$3z^2 - 27$

Part I: Write "True" if the statement is correct and "False" if it is incorrect. (1pt. each)

TRUE 1. The two lines are coinciding line when they have the same slope and the same y-intercepts.

True 2. The range of relation is always a subset of the codomain.

False 3. A function is a relation in which each element of the range corresponds to exactly one element of the domain.

False 4. A polynomial function of degree 3 has at least one real zero.

False 5. The domain of $csch x$ is the set of real number.

False 6. If θ is an argument of the non-zero complex number z, then $-\theta$ is an argument of $\frac{1}{z}$.

True 7. The domain of a composite function $(f \circ g)(x)$ is a subset of the domain of $g(x)$.

False 8. There is no complex number that is equal to its conjugate.

Part II: Choose the best answer from the given alternatives. (1pt. each)

D 1. Let R be a relation given by $R = \{(a, b) \in \mathbb{N}: a = b - 2,\ b > 6\}$. Which of the following does not belongs to R?

A. $\{(8,10)\}$

B. $\{(11, 13), (16, 18), (15,17), (21, 23)\}$

C. $\{(6, 8), (10,12), (14,16), (22,24)\}$

D. $\{(5, 7), (7, 9), (9,11), (17, 19), (12,13), (47,49)\}$

C 2. What is the principal argument of the complex number $(1 + \sqrt{3}i)^{16}$?

A. $\frac{-2\pi}{3}$ B. $\frac{\pi}{3}$ C. $\frac{2\pi}{3}$ D. $\frac{5\pi}{3}$

D 3. Which of the following function is a one to one correspondence?

A. $f: [1, \infty) \to [1, \infty)$ defined by $f(x) = |x - 1|$

B. $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^x$

C. $f: [1, \infty) \to [0, \infty)$ defined by $f(x) = (x - 1)^2 + 1$

D. $f: [1, \infty) \to [0, \infty)$ defined by $f(x) = (x - 1)^2$

C 4. The value of $(x,\ y)$ that satisfies the complex number $\frac{1}{x+yi} - \frac{1}{1+i} = i$, for $x, y \in \mathbb{R}$?

A. $(1, 1)$ B. $(-1, -1)$ C. $(1, -1)$ D. $(-1, 1)$

A 11. Which one of the following is necessarily true about a p
$6x^3 - 2x^2 + 4x - 8$?

A. $x - 1$ is a factor of $p(x)$

B. $p(x)$ have at least 5 zeros in complex number

C. $p(x)$ have at most 5 turning point

D. -1 is a zero of $p(x)$

D 12. Which one of the following is not true about a function $f(x) = \log_a(x-2)$, where $a > 0$ and $a \neq 1$?

A. f has x – intercept at $x = 3$

B. f has vertical asymptote at $x = 2$

C. f is negative if $0 < a < 1$ and $x \geq 3$

D. f is increasing function if $0 < a < 1$

(1.5pt. each)

Part III: Write the most simplified answer on the provided space.

1. The inverse of a function $f(x) = e^{(3x-1)}$ is

Answer: $y = \frac{\ln x + 1}{3}$ or $f^{-1}(x) = \frac{\ln x + 1}{3}$

2. Let $f(x) = \frac{3x}{x^2-4}$ and $g(x) = \sqrt{2x}$, then find

a) Domain of $(fog)(x)$ **Answer:** $[0,2) \cup (2,\infty)$

b) $(gof)(2)$ **Answer:** $\nexists$

3. The exact value of $sinh(-2ln3)$ is

Answer: $-40/9$

4. If $f(x) = -2\,cosx$ and $g(x) = f\left(\frac{x}{2}\right)$, then find

a) Period of g **Answer:** 4π

b) Amplitude of g **Answer:** 2

5. If $z = \frac{\alpha+4i}{1+\alpha i}$ and z is real number, then all possible value of α is

Answer: $\alpha = \pm 2$ or $\alpha = 2 \equiv \alpha = -2$

C 5. All of the following is true about the line $L: 2y - 4x + 14 = 0$ except?

A. The line $x + 2y + 2 = 0$ is perpendicular to L ✓

B. The line $2x - y + 2 = 0$ is parallel to L ✓

C. The distance from $P(1, -1)$ to L is $2\sqrt{5}$

D. The point $P(6, 5)$ is on L ✓

D ✓ 6. Which of the following is not true about the complex number $z = -\sqrt{3} - i$?

A. $Arg(z) = \frac{-5\pi}{6}$ ✓

B. $z = 2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

C. $\bar{z} = -2\left(-\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

D. $z^3 = 4e^{\frac{-5\pi}{6}}$

D ✓ 7. If z is a complex number such that $|z| = 4$ and $Arg(z) = \frac{2\pi}{3}$, then z is equal to?

A. $2i - 2\sqrt{3}$

B. $2 - 2\sqrt{3}i$

C. $2\sqrt{3} - 2i$

D. $2\sqrt{3}i - 2$

C ✓ 8. Which of the following is true about the rational function $f(x) = \frac{x^2-2x-3}{1-x^2}$?

A. f has vertical asymptote at $x = \pm 1$ ✗

B. f has zero at $x = 3$ and $x = -1$ ✓

C. f has horizontal asymptote $y = -1$ ✓

D. f crosses its horizontal asymptote at $x = 2$

A ✓ 9. Which one of the following is not true about a function $f(x) = 5^{1-x} - 1$?

A. Range of f is $(-\infty, -1]$

B. Domain of f is $(-\infty, \infty)$

C. $x-$intercept of f is 1

D. f has horizontal asymptote at $y = -1$

D 10. Given that equation of a circle $(x + 1)^2 + (y - 1)^2 = 13$. Which of the following equation of a line is tangent to a circle?

A. $2y + 3x + 4 = 0$

B. $2y + 3x + 8 = 0$

C. $2y - 3x + 4 = 0$

D. $2y - 3x + 8 = 0$

[illegible] of a line segment PQ are P(3, 1) and Q(8, 6). If R is a point which [illegible] in the ratio 3 : 2, then find an equation of a circle passes through R and has tangent [illegible] (2, 6).

(5pt.)

$y_1 = 1 \quad r_2 = 2$
$x_1 = 3 \quad x_2 = 8$
$r_1 = 3 \quad y_2 = 6$

(3, 1) —3— R(h, k) —2— (8, 6)

$R(x_0, y_0) = \left[\frac{x_1 r_2 + x_2 r_1}{r_1 + r_2}, \frac{y_1 r_2 + y_2 r_1}{r_1 + r_2}\right]$

$= \left[\frac{3(2) + 8(3)}{5}, \frac{1(2) + 6(3)}{5}\right]$

$R(x_0, y_0) = [6, 4]$ then see at the back Teacher.

$y = 2x + 2$

P(6, 4)

(2, 6)

$m_2 = -\frac{1}{2}$

$\Rightarrow \frac{k-6}{h-4} = -\frac{1}{2}$

$\Rightarrow 2h - 12 = k - 4$

$\Rightarrow k = 2h - 8 \quad \ldots (1)$

$\overline{PC} = \overline{QC}$

$\sqrt{(6-h)^2 + (4-k)^2} = \sqrt{(2-h)^2 + (6-k)^2}$

$36 - 12h + h^2 + 16 - 8k + k^2 = 4 - 4h + h^2 + 36 - 12k + k^2$

$\Rightarrow -12h + 16 - 8k = 4 - 4h - 12k$

$\Rightarrow -8h + 4k = -12$

$4k = -12 + 8h$

$k = -3 + 2h$

$(6-(2h-8))^2 + (4-(2h-8))^2 = (2-(2h-8))^2 + (6-(2h-8))^2$

$\Rightarrow (14-2h)^2 + (12-2h)^2 = (10-2h)^2 + (14-2h)^2$

$\Rightarrow 196 - 56h + 4h^2 + 144 - 48h + 4h^2 = 100 - 40h + 4h^2 + 196 - 56h + 4h^2$

$36 - 12h + h^2 + 144 - 48h + 4h^2 = 4 - 4h + h^2 + 196$

$180 - 60h = 200 - 60h$

$(14-2h)(14-2h)$

$196 - 28h - 28h + 4h^2$

$196 - 56h + 4h^2$

$(10-2h)(10-2h)$

$100 - 40h + 4h^2$

$(14-2h)(14-2h)$

$196 - 56h$

$(12-2h)$

$-24h + 4h^2$

C 5. All of the following is true about the line $L: 2y - 4x + 14 = 0$ except?

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B. The line $2x - y + 2 = 0$ is parallel to L

C. The distance from $P(1, -1)$ to L is $2\sqrt{5}$

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C. $z = -2\left(-\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

D. $z^2 = 4e^{\frac{-5\pi}{6}}$

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B. $2 - 2\sqrt{3}i$

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D. $2\sqrt{3}\,i - 2$

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A. f has vertical asymptote at $x = \pm 1$

B. f has zero at $x = 3$ and $x = -1$

C. f has horizontal asymptote $y = -1$

D. f crosses its horizontal asymptote at $x = 2$

A 9. Which one of the following is not true about a function $f(x) = 5^{1-x} - 1$?

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B. Domain of f is $(-\infty, \infty)$

C. x −intercept of f is 1

D. f has horizontal asymptote at $y = -1$

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B. $2y + 3x + 8 = 0$

C. $2y - 3x + 4 = 0$

D. $2y - 3x + 8 = 0$

Given the rational function $f(x) = \frac{2x^3 - 8x^2 + 6x}{x^2 - 1}$, then

a) Find the domain of f
b) Find the x and y intercept of f
c) Find asymptote of f (if any)
d) Sketch the graph of f

(6)

a) Domain
$(R) \{x^2 - 1 = 0\}$
$\Rightarrow x^2 - 1 = 0$
$\Rightarrow (x-1)(x+1) = 0$
$\Rightarrow x = 1 \quad x = -1$
$\Rightarrow D = \mathbb{R} \setminus \{-1, 1\}$

Soln
$\Rightarrow \frac{2x(x^2 - 4x + 3)}{(x-1)(x+1)}$
$\Rightarrow \frac{2x(x-1)(x-3)}{(x-1)(x+1)}$
$\Rightarrow \frac{2x(x-3)}{x+1}$

we have a hole at $x = 1$
$\frac{2(1)(1-3)}{1+1} = \frac{2(-2)}{2} = -2$
hole $= (1, -2)$

after we Find Domain we simplify the function if it is simplify

(b) x & y - intercept

x-intercept
$y = 0$
$0 = \frac{2x(x-3)}{x+1}$
$2x(x-3) = 0$
$x = 0 \; \& \; x = 3$
$(0,0) \; \& \; (3,0)$

y-intercept
$x = 0$
$y = \frac{2(0)(0-3)}{0+1}$
$y = 0$
$(0,0)$

(c) Asymptote

① VA. $x = -1$

② HA → no horizontal asymptote.

③ oblique asymptote

$y = 2x - 8$

x	-3	-2	-1	0	1	2	3	4
y	-14	-12	-10	-8	-6	-4	-2	0

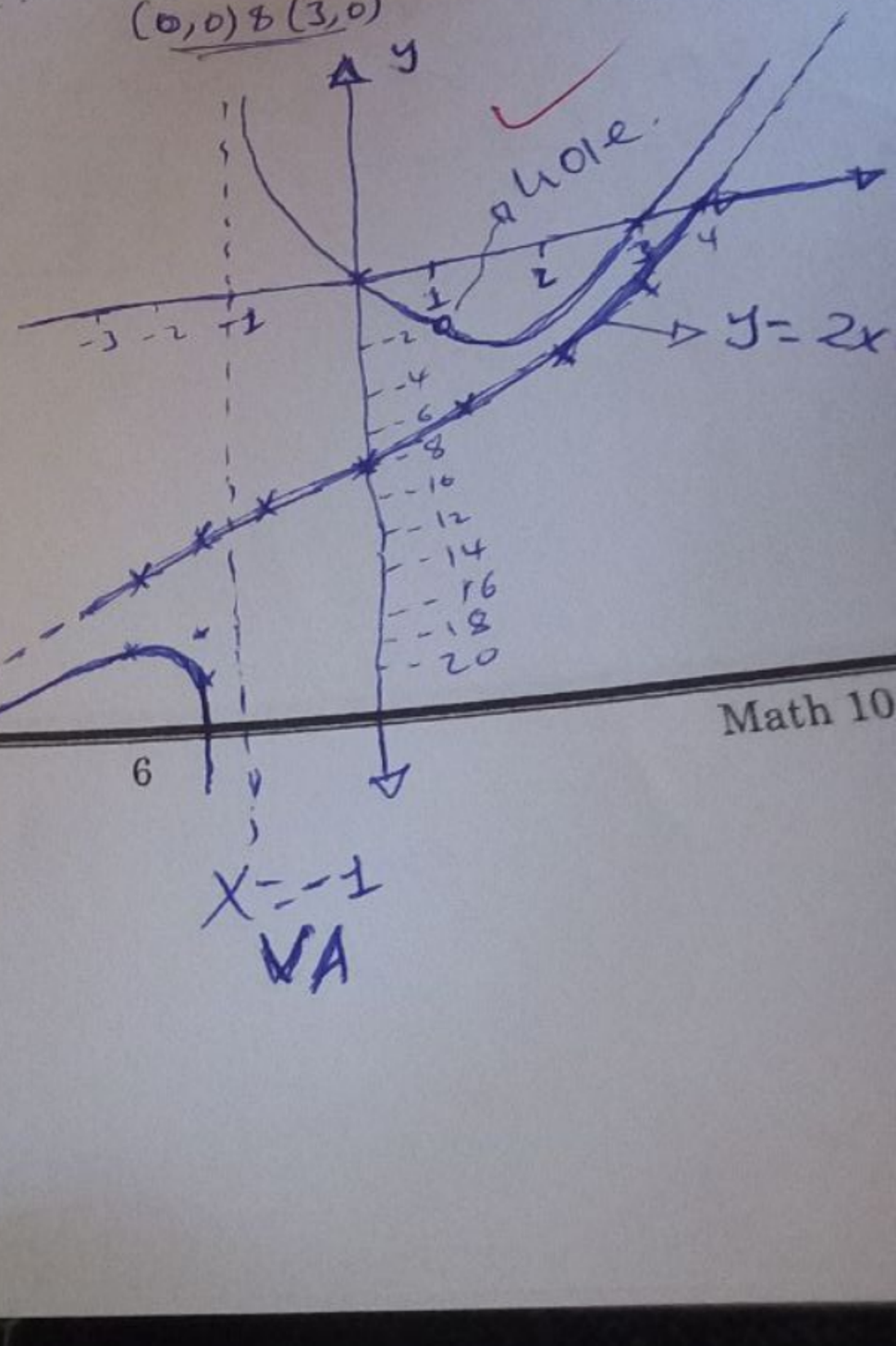
$2x - 8 = \frac{2x^2 - 6x}{x+1}$
$2x^2 - 6x - 8 = 2x^2 - 6x$
$(+6) \quad = 18 \quad -8 = 0$ never touch (cross) oblique asymptote

(-2, -3)

Additional Point

x	-2
y	-20

$\frac{(-4)(-7)}{-3} = -\frac{56}{3}$



11. Which one of the following is necessarily true about a polynomial function $p(x) = 6x^5 - 2x^4 + 4x - 8$?

A. $x - 1$ is a factor of $p(x)$

B. $p(x)$ have at least 5 zeros in complex number

C. $p(x)$ have at most 5 turning point

D. -1 is a zero of $p(x)$

12. Which one of the following is not true about a function $f(x) = \log_a(x-2)$, where $a > 0$ and $a \neq 1$?

A. f has x −intercept at $x = 3$

B. f has vertical asymptote at $x = 2$

C. f is negative if $0 < a < 1$ and $x > 3$

D. f is increasing function if $0 < a < 1$

Part III: Write the most simplified answer on the provided space. (1.5pt. each)

1. The inverse of a function $f(x) = e^{(3x-1)}$ is

Answer: $y = \frac{\ln x + 1}{3}$ or $f^{-1}(x) = \frac{\ln x + 1}{3}$

2. Let $f(x) = \frac{3x}{x^2-4}$ and $g(x) = \sqrt{2x}$, then find

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Answer: $[0, 2) \cup (2, \infty)$

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Answer: $\nexists$

3. The exact value of $sinh(-2ln3)$ is

Answer: $-40/9$

4. If $f(x) = -2\,cosx$ and $g(x) = f\left(\frac{x}{2}\right)$, then find

a) Period of g

Answer: 4π

b) Amplitude of g

Answer: 2

5. If $z = \frac{a+4i}{1+ai}$ and z is real number, then all possible value of a is

Answer: $a = \pm 2$ or $a = 2$ $\not$ $a = -2$

(4) Soln

$R(x_0, y_0) = (6, 4)$

$y = 2x + 2$, $m_1 = 2$, $m_2 = -\frac{1}{2}$

$P(2,6)$, $Q(6,4)$

$\Rightarrow \frac{k-6}{h-2} = -\frac{1}{2}$ b/c perpendicular to line $y = 2x + 2$

$2k - 12 = 2 - h$

$h = -2k + 14$ ---(1)

$y = 2x - 2$

$\Rightarrow \overline{PC} = \overline{QC}$

$\sqrt{(6-h)^2 + (4-k)^2} = \sqrt{(2-h)^2 + (6-k)^2}$

$\Rightarrow k = 3 + 2h = \frac{k+3}{2} = \frac{2h}{2}$

$\Rightarrow \frac{k+3}{2} = h$, but $h = -2k + 14$ in (1)

$\Rightarrow 14 - 2k = \frac{k+3}{2}$

$\Rightarrow 28 - 4k = k + 3$

$\Rightarrow \frac{25}{5} = \frac{5k}{5}$

$k = 5$

$h = 14 - 2k$

$h = 14 - 10$

$h = 4$

$(x-4)^2 + (y-5)^2 = r^2$

$(4-4)^2 + (4-5)^2 = r^2$

$0 + 1 = r^2$

$5 = r^2$

$36 - 12h + h^2 + 16 - 8k + k^2 = 4 - 4h + h^2 + 36 - 12k + k^2$

$\Rightarrow -12h + 4h - 8k + 12k = 4 - 16$

$\Rightarrow -8h + 4k = -12$

$\Rightarrow 4k = -12 + 8h$

$\Rightarrow k = -3 + 2h$ ---

Therefore eq n of circle is.

$(x-4)^2 + (y-5)^2 = 5$