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D

6. Suppose that when the polynomial $p(x)$ is divided by $x - 5$, the quotient is $3x^4 - 5x^2 + 2x - 5$ with a remainder of 4. Then, we conclude that

A. $x - 4$ *is a factor of* $p(x)$ *and* 4 *is a zeros of* $p(x)$.

B. $x + 5$ *is not a factor of* $p(x)$ *and* -5 *is not a zero of* $p(x)$.

C. $x - 5$ *is a factor of* $p(x)$ *and* -5 *is a zero of* $p(x)$. ×

D. $x - 5$ *is not a factor of* $p(x)$ *and* 5 *is not a zero of* $p(x)$. ✓

C

7. Which type of asymptote will never intersect the graph of a rational function?

A. Horizontal ×

B. Oblique ×

C. Vertical

D. Slant ×

A

8. Let C be a circle of radius 4 and centered at $(2, 2)$. Then, which one of the following is true about the point $P(5, 0)$?

$r = \sqrt{9 + 4} = \sqrt{13}$

A. P is inside the circle C.

B. P is outside of the circle C. ×

C. P is on the circle C. ×

D. All ×

B

9. Which of the following pair of lines are perpendicular?

A. $3x - 6y + 1 = 0$ and $x - 2y = 3$ × $\frac{6y}{6} = \frac{3x+1}{6} \Rightarrow \frac{2y}{2} = \frac{x-3}{2}$

B. $2x - y + 1 = 0$ and $2x + 4y = 3$ $\frac{4y}{4} = \frac{-2x+3}{4}$

C. $y = 3x + 2$ and $y + 3x = 2$ ×

D. $\frac{x}{3} + \frac{y}{2} = 1$ and $4x + 6y - 12 = 0$ × $2x + 3y = 6 \Rightarrow \frac{3y}{3} = \frac{-2x+6}{3}$ | $6y = \frac{-4x+12}{6}$

A

10. Among the following which one does not represent equation of a circle?

A. $x^2 + y^2 + 2x - 2y + 3 = 0$

B. $x^2 + y^2 + 2x - 6y + 7 = 0$

C. $x^2 + y^2 - 8x + 12y - 12 = 0$

D. $x^2 + y^2 - 4x - 6y + 3 = 0$

A) $x^2 + 2x + 1 + y^2 - 2y + 1 = -3 + 2$

$(x+1)^2 + (y-1)^2 = -1$

B) $x^2 + 2x + 1 + y^2 - 6y + 9 = -7 + 10$

C) $x^2 - 8x + 16 + y^2 + 12y + 36 = 12 + 52$

D) $x^2 - 4x + 4 + y^2 - 6y + 9 = -3 + 13$

$p(x) = (x-5)(3x^4 - 5x^2 + 2x - 5) + 4$

$= 3x^5 - 5x^3 + 2x^2 - 5x - 15x^4 + 25x^2 - 10x + 25 + 4$

$p(x) = 3x^5 - 15x^4 - 5x^3$

Section:________ Total Earned Points: ____ ID:________

Part I: Write "True" if the statement is correct and "False" if incorrect **(5 points)**

True 1. Let $f: A \to B$ be a relation, then the domain of f is always equal to A.

True 2. A function is one to one if and only if no horizontal line intersects its graph more than once.

False 3. If $f(x) = \frac{x}{x-1}$, then the domain of $(f o f)(x)$ is the set of all real numbers.

$\frac{\frac{x}{x-1}}{\frac{x}{x-1} - \frac{1}{1}} \Rightarrow \frac{\frac{x}{x-1}}{\frac{x-x+1}{x-1}} \Rightarrow \frac{x}{1} = x$

True 4. Two lines with positive slopes are non-perpendicular lines.

True 5. A circle is a set of points in a plane which are equidistant from a fixed point in the plane.

Part II: Choose the best answer from the given alternatives **(10 points)**

$z + w = 3 + 2i$

E 1. Let z and w be complex numbers. Then, which one of the following is true.

A. $Re(z + w) = Re(z) + Re(w)$

B. $Im(z + w) = Im(z) + Im(w)$

C. $Re(zw) = Re(z)Re(w) - Im(z)Im(w)$

D. $Im(zw) = Re(z)Im(w) + Re(w)Im(z)$

E. All

$a + bi = z$

$x + yi = w$

$ax = ax -$

$|2 + 2i|$

$\sqrt{2^2 + 2^2} = \sqrt{8}$

None 2. The value of $|i^{50} + i^{52} + i^{54} + i^{56}|$ is $= |(i^2)^{25} + (i^2)^{26} + (i^2)^{27} + (i^2)^{28}| = |-1 + 1 - 1 + 1| = 0$

A. 1

B. -1

C. 2

D. i

$i^{-25} \cdot i^{-25}$

$(i^2)^{12} \cdot i \cdot$

$i \cdot i = -1$

$z = 2 + i$

$w = 1 + i$

$Re(3 + 2i) = 2 + 1$

$= 2 + 2i + i - 1$

$zw \quad 1 + 3i$

C 3. The range of a relation $R = \{(x, x^2): \underline{x \text{ is a prime number less than } 13}\}$ is

A. $\{2, 3, 5, 7\}$ ×

B. $\{2, 3, 5, 7, 11\}$ ×

C. $\{4, 9, 25, 49, 121\}$

D. $\{1, 4, 9, 25, 49, 121\}$

$R = \{(2, 4), (3, 9), (5, 25), (7, 49), (11, \quad)\}$

D 4. If the ordered pairs $(a + 2, 4)$ and $(5, 2a + b)$ are equal, then (a, b) is

A. $(2, -2)$

B. $(5, 1)$

C. $(3, 10)$

D. $(3, -2)$

$a + 2 = 5 \quad 2 - 1 = 1$

$a = 3$

$2a + b = 4$

$6 + b = 4$

$b = -2$

A 5. Let f and g be two functions given by

$f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 7)\}$ and $g = \{(0, 2), (1, 0), (2, 4), (-4, 2), (7, 0)\}$.

Then, the range of $f o g$ is

A. $\{0, 1, 2\}$ B. $\{-4, 1, 0, 2, 7\}$ C. $\{1, 2, 3, 4, 5\}$ D. $\{0, 2, 3, 4, 5\}$

$fog = f(g(x))$

$= (0, 1)(1, 1)(2, 0)$

$f(g(x))$

$f(g(x)) = (2, 0)(0, 1)(4, 2)(2, 0)(0, 1)$

$f(2) = 0 \quad f(0) = 1 \quad f(4) = 2 \quad f(2) = 0 \quad f(0) = 1$

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Part III: Short Answer (Write the most simplified answer on the space provided) **(25 points)**

1. Given a complex number $z = \frac{3i}{-1-i}$, find $= \boxed{\frac{-3}{2} - \frac{3i}{2}}$ (5 Points)

(a) conjugate of z

Answer: $-\frac{3}{2} + \frac{3}{2}i$

(b) modulus of z

Answer: $\frac{3\sqrt{2}}{2}$

(c) multiplicative inverse of z

Answer: $-\frac{1}{3} + \frac{1}{3}i$

(d) principal argument of z

Answer: $-\frac{3\pi}{4}$

(e) polar form of z. $\boxed{\cos\theta = \cos(-\theta)}$ since $\boxed{\cos\left(-\frac{3\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)}$

Answer: $z = \frac{3\sqrt{2}}{2}\left(\cos\left(+\frac{3\pi}{4}\right) - i\sin\left(\frac{3\pi}{4}\right)\right)$

2. If $7x - 2iy + 3ix + 2y = 4 - i$, then find x and y. (2 Points)

Answer: $x = \frac{3}{10}$ and $y = \frac{19}{20}$

3. Let $A = \{1, 2, 3, 4\}$. Let R be a relation on A defined by $R = \{(a, b) : a, b \in A, a \text{ is a multiple of } b\}$. Then, find (4 Points)

(a) the relation R

Answer: $\{(1,1),(2,1),(3,1),(4,1),(2,2),(4,2),(3,3),(4,4)\}$

(b) domain of R

Answer: D_o of $R = \{1, 2, 3, 4\}$

(c) range of R

Answer: R_a of $R = \{1, 2, 3, 4\}$

(d) $R^{-1} = \{(1,1),(1,2),(1,3),(1,4),(2,2),(2,4),(3,3),(4,4)\}$

Answer: $\{(1,1),(1,2),(1,3),(1,4),(2,2),(2,4),(3,3),(4,4)\}$

4. Find the domain of $f(x) = \frac{x+5}{4-\sqrt{16-x^2}}$. (2 Points)

Answer: $[-4,0) \cup (0,4]$

5. Find the inverse function of $f(x) = \frac{4x-1}{2x+3}$ (2 Points)

Answer: $f(x) = \frac{3x+1}{4-2x}$, $x \neq 2$

6. The polynomial equation $2x^3 - 5x^2 + cx - 5 = 0$, $c \in \mathcal{R}$ has a root $1 - 2i$, then find (3 Points)

(a) the other two roots of the equation

Answer: $\{1+2i, 5\}$

(b) the value of c.

$y = -x + 1$

Answer: $c = -24$

7. Find the exact value of $\tanh(2\ln 5)$. (2 Points)

Answer: $4\ln 5$

8. If $x + cy = 1$ represents equation of a line, then determine the value of c for which the following conditions hold true. (3 Points)

(a) The line has slope $m = 4$.

Answer: $c = -1/4$

(b) The line passes through $(-2, 1)$.

Answer: $c = 3$

(c) The line is vertical line.

Answer: $c = 0$

9. Given two points $P(-3, 3)$ and $Q(7, 8)$. Then, find the coordinate of the points R on the line segment PQ such that $|PR| : |RQ| = 2 : 3$. (2 Points)

Answer: $R(1, 5)$

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C. $\{4,9,25,49,121\}$

D. $\{1,4,9,25,49,121\}$

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$6+b=4$

$b=-2$

A. $(2, -2)$

B. $(5, 1)$

C. $(3, 10)$

D. $(3, -2)$

$f o g = f(g(x))$

$= (0,1)(1,\;)(2,0)$

$f(g(x))$

$f(g(x)) = (2,0)(0,1)(4,2)(2,0)(0,1)$

A 5. Let f and g be two functions given by

$f = \{(0,1), (2,0), (3,-4), (4,2), (5,7)\}$ and $g = \{(0,2), (1,0), (2,4), (-4,2), (7,0)\}$.

Then, the range of $f o g$ is

A. $\{0,1,2\}$ B. $\{-4,1,0,2,7\}$ C. $\{1,2,3,4,5\}$ D. $\{0,2,3,4,5\}$

$f(2)=0$ $f(0)=1$ $f(4)=2$ $f(2)=0$ $f(0)=1$

2. Consider the rational function $f(x) = \frac{x-2}{x^2-4}$. Then, find **(6 points)**

(a) domain of f

(b) intercepts (if any)

(c) asymptotes (if any)

(d) sketch the graph of f.

solution

$$f(x) = \frac{x-2}{(x+2)(x-2)}$$

(a) Domain $= \mathbb{R} \setminus \{-2, 2\}$

$\Rightarrow$ but, the graph has a hole at $x = 2$, so we can find that point of hole after simplification

(*) Let us simplify:

$$f(x) = \frac{\cancel{x-2}}{(x+2)\cancel{(x-2)}} = \boxed{\frac{1}{x+2}}$$

(*) point of hole $= \left(2, \frac{1}{2+2}\right) = \left(2, \frac{1}{4}\right)$

(b) Intercepts

(1) x-intercept $(y=0)$

$$\frac{0}{1} = \frac{1}{x+2}$$

$1 = 0$ (False)

we have no x-inter

$\Rightarrow$ Therefore, we have no x-intercept

(2) y-intercept $(x=0)$

$$y = \frac{1}{0+2} = \frac{1}{2}$$

$\Rightarrow$ We have y-intercept at $\left(0, \frac{1}{2}\right)$

(c) Asymptotes

(1) Vertical Asymptote (VA)

$d(x) = 0$

$x + 2 = 0$

$\boxed{x = -2} \leftarrow$ VA

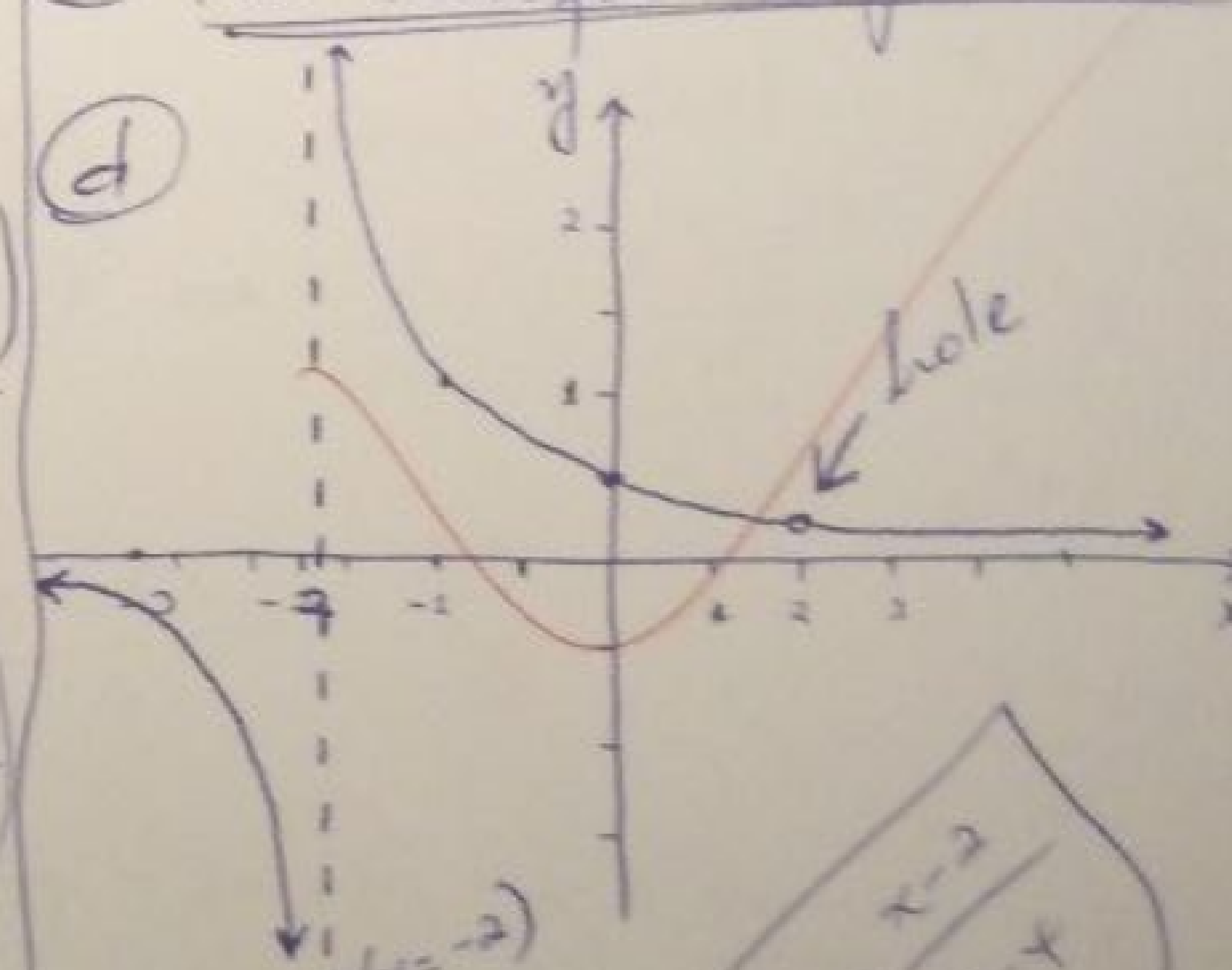
(2) HA (Horizontal Asymptote)

$\hookrightarrow$ since the degree of $d(x)$ is greater than the degree of $n(x)$, then

$\boxed{y = 0 \text{ (x-axis)}} \leftarrow$ HA

(3) No Oblique Asymtote $\leftarrow$

(d)



$f(x) = \frac{x-2}{x^2-4}$

Additional pt

$f(-3) = \frac{1}{-3+2} = -1$

$f(-20) = \frac{1}{-20+2} = \frac{-1}{18} =$

Part IV: Work Out (Show all the necessary steps clearly and neatly) **(10 points)**

1. Find cube roots of the complex number $z = -8i$. **(4 points)**

$\Rightarrow$ To find cube root of z,

$$C_k = r^{\frac{1}{n}} e^{i\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)}$$

$$C_k = 8^{\frac{1}{3}} e^{i\left(\frac{-\frac{\pi}{2}}{3} + \frac{2k\pi}{3}\right)}$$

$$\boxed{C_k = 2e^{i\left(-\frac{\pi}{6} + \frac{2k\pi}{3}\right)}}$$

* $r = 8$, $\theta = \tan^{-1}\left(\frac{-8}{0}\right) = -\frac{\pi}{2}$

* $\theta = -\frac{\pi}{2}$

* $n = 3$, then $k = 0, 1, 2$

* $C_0 = 2e^{i\left(-\frac{\pi}{6}\right)} = 2\left(\cos\left(\frac{\pi}{6}\right) - i\sin\left(\pi/6\right)\right)$ but $\Rightarrow$ $\cos(-\theta) = \cos\theta$

$= 2\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)$

* $\boxed{C_0 = \sqrt{3} - i} \leftarrow$

* $C_1 = 2e^{i\left(-\frac{\pi}{6} + \frac{2\pi}{3}\right)} = 2e^{i\left(\frac{\pi}{2}\right)} = 2\left(\cos(\pi/2) + i\sin(\pi/2)\right)$

$= 2(0 + 1i)$

* $\boxed{C_1 = 2i} \leftarrow$

* $C_2 = 2e^{i\left(-\frac{\pi}{6} + \frac{4\pi}{3}\right)} = 2e^{i\left(\frac{7\pi}{6}\right)} = 2\left(\cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right)\right)$

$= 2\left(-\frac{\sqrt{3}}{2} - \frac{i}{2}\right)$

* $\boxed{C_2 = -\sqrt{3} - i} \leftarrow$