Compositional Reasoning for WMMs

COV889 Course Presentation

Ramneet Singh
Mrunmayi Bhalerao

IIT Delhi

May 2023

1. Introduction
2. Abstract Language
3. Basic Proof System
4. Multicopy Atomic Memory Models
5. Non-Multicopy Atomic Memory Models

Section 1

Introduction

Rely/Guarantee Reasoning

- Verification of concurrent programs with shared resources is challenging due to **combinatorial explosion**
- **Abstraction to the rescue!**
- Everything the environment can do: $\mathcal{R}$
- Everything you can do: $\mathcal{G}$

$$\mathcal{R}, \mathcal{G} \vdash P\{\ c\ \}Q$$

- *Compositional!*

Extension to Weak Memory Models

- Judgements using earlier techniques are valid under **sequentially consistent semantics**
  - Can be directly used for data-race free code executing on weak memory models
  - But, lots of code has data races! `seqlock`, `ConcurrentLinkedQueue` in `java.util.concurrent` ...
- How do we extend them to weak memory models?
- What If: We could find a condition under which sequentially consistent rely-guarantee reasoning can be *soundly* preserved

$$(\vdash P\{\ c\ \}Q) \wedge ?? \implies \vdash P\{\ c_{WM}\ \}Q$$

- Benefits:
  - Reuse existing verification techniques
  - Deal with the complexity of weak memory separately as a *side-condition*

What are WMMs anyway?

- Relaxing the memory consistency guarantees provided by hardware enables optimisations
  - Store forwarding (will see later)
  - Write buffers
- (Part 1) **Multicopy Atomic**: One thread's stores become observable to all other threads at the same time.
  - x86-TSO, ARMv8, RISC-V
- (Part 2) **Non-Multicopy Atomic**: Each component has its own *view* of the global memory.
  - Older ARM versions, POWER, C11
- Challenge: Two types of interference now – **Inter-Thread** + **Intra-Thread** (due to reordering)
- How will we deal with this? ...

Teaser

- We want a compositional approach through thread-local reasoning.
- Exploit the reordering semantics of Colvin and Smith: **multicopy atomic memory models can be captured in terms of instruction reordering.**
  - Combinatorial explosion? (n reorderable instructions in a thread $\Longrightarrow$ $n!$ behaviours)
  - Introduce *reordering interference freedom* between $\left(\frac{n(n-1)}{2}\right)$ pairs of instructions (Stay tuned. . .)
- In non-multicopy atomic WMMs, there is **no global shared state**(!!)
  - Judgement for each thread is applicable to *its view* (depends on *propagation* of writes by hardware)
  - How do we know it holds in other threads' views?
  - Represent the semantics using reordering **between** different threads
  - No longer compositional? Hardest part of the talk – *global reordering interference freedom*: use the rely abstraction to represent reorderings between threads

Section 2

Abstract Language

Syntax

- Individual (atomic) instructions α
- Commands (or programs)

$$c \ := \ \epsilon \mid \alpha \mid c_1 ; c_2 \mid c_1 \ \sqcap \ c_2 \mid c^* \mid c_1 \parallel c_2$$

- Iteration, choice are non-deterministic
- Empty program ϵ represents termination

Semantics: Commands

- Each atomic instruction α has a relation $\mathrm{beh}(\alpha)$ (over pre- and post-states) specifying its behaviour
- Program execution is defined by a small-step semantics over commands
- Iteration, non-deterministic choice are dealt with at a higher level (see next slide)

$$\frac{}{\alpha \mapsto_{\alpha} \epsilon} \qquad \frac{c_1 \mapsto_{\alpha} c_1'}{c_1 \,;\, c_2 \mapsto_{\alpha} c_1' \,;\, c_2}$$

$$\frac{c_1 \mapsto_{\alpha} c_1'}{c_1 \;||\; c_2 \mapsto_{\alpha} c_1' \;||\; c_2} \qquad \frac{c_2 \mapsto_{\alpha} c_2'}{c_1 \;||\; c_2 \mapsto_{\alpha} c_1 \;||\; c_2'}$$

Semantics: Configurations

- *Configuration* (c, σ) of a program
  - Command c to be executed
  - State σ (map from variables to values)
- *Action Step*: Performed by component, changes state

$$(c, \sigma) \xrightarrow{as} (c', \sigma') \iff \exists \alpha . c \mapsto_\alpha c' \wedge (\sigma, \sigma') \in \mathrm{beh}(\alpha)$$

- *Silent Step*: Performed by component, doesn't change state

$$(c_1 \sqcap c_2, \sigma) \rightsquigarrow (c_1, \sigma) \quad (c_1 \sqcap c_2, \sigma) \rightsquigarrow (c_2, \sigma)$$

$$(c^*, \sigma) \rightsquigarrow (\epsilon, \sigma) \quad (c^*, \sigma) \rightsquigarrow (c; c^*, \sigma)$$

- *Program Step*: Action Step or Silent Step
- *Environment Step*: Performed by environment, changes state. $(c, \sigma) \xrightarrow{es} (c, \sigma')$.

Section 3

Basic Proof System

Definitions

- Associate a verification condition $\mathrm{vc}(\alpha)$ with each instruction α: Provides finer-grained control (just set to $\top$ if not needed)
- Hoare triple

$$P\{\ \alpha\ \}Q \stackrel{\text{def}}{=} P \subseteq \mathrm{vc}(\alpha) \cap \{\sigma \mid \forall\, \sigma',\ (\sigma, \sigma') \in \mathrm{beh}(\alpha) \Longrightarrow \sigma' \in Q\}$$

- A rely-guarantee pair $(\mathcal{R}, \mathcal{G})$ is well-formed if
  - $\mathcal{R}$ is reflexive and transitive
  - $\mathcal{G}$ is reflexive
- Stability of predicate P under rely condition $\mathcal{R}$

$$\mathrm{stable}_{\mathcal{R}}(P) \stackrel{\text{def}}{=} P \subseteq \{\sigma \in P \mid \forall\, \sigma',\ (\sigma, \sigma') \in \mathcal{R} \Longrightarrow \sigma' \in P\}$$

- Instruction α satisfies guarantee condition $\mathcal{G}$

$$\mathrm{sat}(\alpha, \mathcal{G}) \stackrel{\text{def}}{=} \{\sigma \mid \forall\, \sigma',\ (\sigma, \sigma') \in \mathrm{beh}(\alpha) \Longrightarrow (\sigma, \sigma') \in \mathcal{G}\}$$

- Now introduce rely/guarantee judgements at three levels

Instruction Level ($\vdash_a$)

$$\mathcal{R}, \mathcal{G} \vdash_a P\{\ \alpha\ \}Q \overset{\mathsf{def}}{=} \mathrm{stable}_{\mathcal{R}}(P) \wedge \mathrm{stable}_{\mathcal{R}}(Q) \wedge \mathrm{vc}(\alpha) \subseteq \mathrm{sat}(\alpha, \mathcal{G}) \wedge P\{\ \alpha\ \}Q$$

- Interplay between environmental interference and pre-,post-conditions handled through stability

Component Level ($\vdash_c$)

$$\text{Atom}\ \frac{\mathcal{R},\mathcal{G} \vdash_a P\{\ \alpha\ \}Q}{\mathcal{R},\mathcal{G} \vdash_c P\{\ \alpha\ \}Q}$$

$$\text{Seq}\ \frac{\mathcal{R},\mathcal{G} \vdash_c P\{\ c_1\ \}M \quad \mathcal{R},\mathcal{G} \vdash_c M\{\ c_2\ \}Q}{\mathcal{R},\mathcal{G} \vdash_c P\{\ c_1; c_2\ \}Q}$$

$$\text{Choice}\ \frac{\mathcal{R},\mathcal{G} \vdash_c P\{\ c_1\ \}Q \quad \mathcal{R},\mathcal{G} \vdash_c P\{\ c_2\ \}Q}{\mathcal{R},\mathcal{G} \vdash_c P\{\ c_1\ \sqcap\ c_2\ \}Q}$$

$$\text{Iteration}\ \frac{\mathcal{R},\mathcal{G} \vdash_c P\{\ c\ \}P \quad \mathrm{stable}_{\mathcal{R}}(P)}{\mathcal{R},\mathcal{G} \vdash_c P\{\ c^*\ \}P}$$

$$\text{Conseq}\ \frac{\mathcal{R},\mathcal{G} \vdash_c P\{\ c\ \}Q \quad P' \subseteq P \quad \mathcal{R}' \subseteq \mathcal{R} \quad Q \subseteq Q' \quad \mathcal{G} \subseteq \mathcal{G}'}{\mathcal{R}',\mathcal{G}' \vdash_c P'\{\ c\ \}Q'}$$

Global Level ($\vdash$)

- Global satisfiability needs component satisfiability + **interference check**

$$\text{Comp}\,\frac{\mathcal{R},\mathcal{G} \vdash_c P\{\ c\ \}Q \quad \mathrm{rif}(\mathcal{R},\mathcal{G},c)}{\mathcal{R},\mathcal{G} \vdash P\{\ c\ \}Q}$$

- Usual parallel rule

$$\text{Par}\,\frac{\mathcal{R}_1,\mathcal{G}_1 \vdash_c P_1\{\ c_1\ \}Q_1 \quad \mathcal{R}_2,\mathcal{G}_2 \vdash_c P_2\{\ c_2\ \}Q_2 \quad \mathcal{G}_2 \subseteq \mathcal{R}_1 \quad \mathcal{G}_1 \subseteq \mathcal{R}_2}{\mathcal{R}_1 \cap \mathcal{R}_2, \mathcal{G}_1 \cup \mathcal{G}_2 \vdash P_1 \wedge P_2\{\ c_1 \parallel c_2\ \}Q_1 \wedge Q_2}$$

Section 4

Multicopy Atomic Memory Models

Reordering Semantics: Basics

- Multicopy atomic memory models can be characterised using a *reordering* relation $\hookleftarrow$ over pairs of instructions in a component
- $\hookleftarrow$ is syntactically derivable based on the specific memory model. E.g., in ARMv8
  - Two instructions which don't access (read or write) a common variable can be reordered
  - Various types of memory barriers prevent reordering
- *Forwarding* is another complication
  - $\beta = \texttt{x := 3}; \alpha = \texttt{y := x}$. Can forward the value 3 to *y*, losing dependence between α, β.
  - $\texttt{x := 3 ; y := x} \Longrightarrow \texttt{y := 3 ; x := 3}$
  - Denote α with the value written in an earlier instruction forwarded to it as $\alpha_{<\beta>}$.
- Forwarding may continue arbitrarily and can span multiple instructions

Reordering Semantics: Formal

- $\alpha_{<c>}$: cumulative forwarding effects of the instructions in command c on α
- Ternary relation $\gamma < c < \alpha$: Reordering of instruction α prior to command c, with cumulative forwarding effects producing γ.
- Definition by induction

$$\alpha_{<\beta>} < \beta < \alpha \stackrel{\text{def}}{=} \beta \hookleftarrow \alpha_{<\beta>}$$

$$\alpha_{<c_1;c_2>} < c_1 ; c_2 < \alpha \stackrel{\text{def}}{=} \alpha_{<c_1;c_2>} < c_1 < \alpha_{<c_2>} \wedge \alpha_{<c_2>} < c_2 < \alpha$$

- Example: $\alpha = (\texttt{y := x}), \beta = (\texttt{x := 3}), \gamma = (\texttt{z := 5})$.
  $\alpha_{<\beta>} = (\texttt{y := 3})$,$\alpha_{<\gamma;\beta>} = (\texttt{y := 3})$.

  $\texttt{y := 3} < \texttt{x := 3} < \texttt{y := x}$ and $\texttt{y := 3} < \texttt{z := 5 ; x := 3} < \texttt{y := x}$

- Can execute an instruction which occurs later in the program if reordering and forwarding can bring it (in its new form γ) to the beginning

$$\text{Reorder}\,\frac{c_2 \mapsto_\alpha \quad \gamma < c < \alpha}{c_1 ; c_2 \mapsto_\gamma c_1 ; c_2'}$$

Reordering Interference Freedom

- Insight: Any valid reordering will **preserve thread-local semantics**, thus may only invalidate reasoning when **observed by the environment**.
    - Abstraction to the rescue again! Observed by environment $\implies \mathcal{G}$ violated, or $\mathcal{R}$ not strong enough
- Three Levels: Instructions, Commands, Program

RIF: Instructions

- Two instructions are *reordering interference free*: Reasoning over them in their original order is sufficient to include reordered behaviour.

$$\mathrm{rif}_a(\mathcal{R}, \mathcal{G}, \beta, \alpha) \stackrel{\text{def}}{=} \forall P, Q, M.\ \mathcal{R}, \mathcal{G} \vdash_a P\{\ \beta\ \}M \wedge \mathcal{R}, \mathcal{G} \vdash_a M\{\ \alpha\ \}Q \\ \implies \exists M'.\ \mathcal{R}, \mathcal{G} \vdash_a P\{\ \alpha_{<\beta>}\ \}M' \wedge \mathcal{R}, \mathcal{G} \vdash_a M'\{\ \beta\ \}Q$$

RIF: Commands

- Command c is *reordering interference free* from α under $\mathcal{R}, \mathcal{G}$ if the reordering of α over each instruction of c is reordering interference free, *including those variants produced by forwarding*.

$$\mathrm{rif}_c(\mathcal{R}, \mathcal{G}, \beta, \alpha) \stackrel{\mathsf{def}}{=} \mathrm{rif}_a(\mathcal{R}, \mathcal{G}, \beta, \alpha)$$

$$\mathrm{rif}_c(\mathcal{R}, \mathcal{G}, c_1; c_2, \alpha) \stackrel{\mathsf{def}}{=} \mathrm{rif}_c(\mathcal{R}, \mathcal{G}, c_1, \alpha_{<c_2>}) \wedge \mathrm{rif}_c(\mathcal{R}, \mathcal{G}, c_2, \alpha)$$

RIF: Programs

- Program c is *reordering interference free* if and only if **all possible reorderings** of its instructions over the respective prefixes are reordering interference free.

$$\mathrm{rif}(\mathcal{R}, \mathcal{G}, c) \stackrel{\text{def}}{=} \forall \alpha, r, c'.\ c \mapsto_{\alpha_{<r>}} c' \Longrightarrow \mathrm{rif}_c(\mathcal{R}, \mathcal{G}, r, \alpha) \wedge \mathrm{rif}(\mathcal{R}, \mathcal{G}, c')$$

- Observe: Checking $\mathrm{rif}(\mathcal{R}, \mathcal{G}, c)$ amounts to
  - Checking $\mathrm{rif}_a(\mathcal{R}, \mathcal{G}, \beta, \alpha)$ for all pairs of instructions β, α that can reorder in c
  - Including those pairs for which α is a new instruction generated through forwarding

Gameplan

1. Compute all pairs of reorderable instructions (β, α).
2. Demonstrate reordering interference freedom for as many of these pairs as possible (using $\mathrm{rif}_a(\mathcal{R}, \mathcal{G}, \beta, \alpha)$).
3. If rif_a cannot be shown for some pairs
   - introduce memory barriers to prevent their reordering or
   - modify the verification problem such that their reordering can be considered benign
4. Verify the component in isolation, using standard rely/guarantee reasoning with an assumed sequentially consistent memory model.

For a thread with n reorderable instructions,

$n!$ Possible Behaviours $\longrightarrow n(n-1)/2\ \mathrm{rif}_a$ checks

Thanks for staying tuned :)

Section 5

Non-Multicopy Atomic Memory Models

Non-Multicopy Atomic Memory Models

- There is **no shared state** that all components agree on throughout execution, invalidating a core assumption of standard rely/guarantee reasoning.

Write History Semantics: Representation

- Each component is associated with a unique identifier.
- Shared memory state is represented as a list of variable writes $< w_1, w_2, w_3, \cdots >$, with metadata to indicate which components have performed and observed particular writes.
- The order of events in this write history provides an *overall order* to the system's events, with those later in the list being the most recent.
- Each $w_i = (x \mapsto v)^{wr}_{rds}$ where
  - x is a variable
  - v is a value
  - $\mathrm{writer}((x \mapsto v)^{wr}_{rds}) = wr$ is the writer component's identifier
  - $\mathrm{readers}((x \mapsto v)^{wr}_{rds}) = rds$ is the set of component identifiers that have observed the write
  - $\mathrm{var}((x \mapsto v)^{wr}_{rds}) = x$

Write History Semantics: Manipulation

- Divide instructions into two types: *global* and *local*. Global instructions α are:
  - Store $(x := v)_i$, Load $[x = v]_i$, Memory barrier fence$_i$, Skip instruction (corresponding to some internal step)
- Behaviour of these instructions is formalised as (for skip it's just *id*):

$$\mathrm{beh}((x := v)_i) = \{ (h \circ h', h \circ (x \mapsto v)^i_{\{i\}} \circ h') \mid \forall w \in h'.\ \mathrm{writer}(w) \neq i \wedge (\mathrm{var}(w) = x \Longrightarrow i \notin \mathrm{readers}(w))$$

$$\mathrm{beh}([x = v]_i) = \{ (h \circ (x \mapsto v)^j_r \circ h', h \circ (x \mapsto v)^j_r \circ h') \mid \forall w \in h'.\ (\mathrm{var}(w) = x \Longrightarrow i \notin \mathrm{readers}(w))$$

$$\mathrm{beh}(\mathsf{fence}_i) = \{ (h, h) \mid \forall w \in h.\ (i \in \mathrm{readers}(w) \Longrightarrow \forall y.\ y \in \mathrm{readers}(w$$

- Propagations of writes are modelled as environment effects and can take place at any point during the execution.

$$\mathrm{prp} = \{ (h \circ (x \mapsto v)^j_r \circ h', h \circ (x \mapsto v)^j_{r \cup \{i\}} \circ h') \mid i \notin r \wedge \forall w \in h.\ (\mathrm{var}(w) = x \Longrightarrow i \in \mathrm{readers}(w)) \}$$

More Notation

- New constructor in the language: $\mathrm{comp}(i, m, c)$ indicating a component with
  - identifier i
  - local state m
  - command c
- Assume a local behaviour relation lbeh such that $(m, \alpha', m') \in \mathrm{lbeh}(\alpha)$ if executing α
  - changes the local state from m to m'
  - corresponds to the global instruction α'

$$\mathrm{comp}(i, m, c) \mapsto_{\alpha'_i} \mathrm{comp}(i, m', c') \iff c \mapsto_{\alpha} c' \wedge (m, \alpha', m') \in \mathrm{lbeh}(\alpha)$$

- Go from local semantics/reasoning to global semantics/reasoning using comp and lbeh.
  - Constraint: systems are constructed as the parallel composition of a series of comp commands.
  - Trivial support for local state (e.g., hardware registers).

Meaning of Judgement

- If there is no global state, what does $\mathcal{R}, \mathcal{G} \vdash P\{\ c\ \}Q$ (for a component i with command c) mean?
- For a set of components I, write history h, for all variables x, $\mathrm{view}_I(h, x) = v$ iff

$$h = h' \circ (x \mapsto v)^w_r \circ h'' \wedge I \subseteq r \wedge \forall w_i \in h''.\ \mathrm{var}(w_i) = x \implies I \not\subseteq \mathrm{readers}(w_i)$$

- For all executions of c
  - If
    - the execution operates on a write history h such that $\mathrm{view}_i(h) \in P$
    - all propagations to i modify view_i in accordance with $\mathcal{R}$
  - Then i will
    - modify view_i in accordance with $\mathcal{G}$
    - given termination, end with a write history h such that $\mathrm{view}_i(h) \in Q$
- This state mapping allows for rely/guarantee judgements **over individual components** to be trivially lifted from a standard memory model to their respective views of a write history.

Parallel Composition: Taming the Beast

- Parallel composition is complicated: **Need to relate differing components views**.
- *If* the execution of an instruction α by some component i satisfies its guarantee specification $\mathcal{G}_i$ in state h,

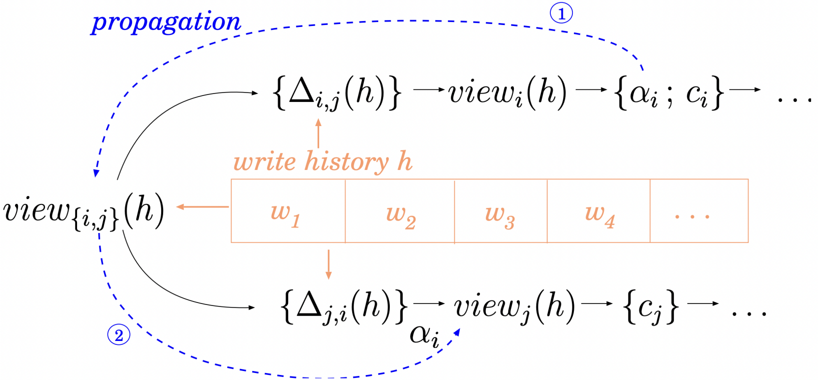
$$\mathrm{view}_i(h) \in \mathrm{sat}(\alpha, \mathcal{G}_i)$$

- *Then* the effects of propagating α's writes to some other component j will satisfy its rely specification $\mathcal{R}_j$ in its view,

$$\mathrm{view}_j(h) \in \mathrm{sat}(\alpha, \mathcal{R}_j)$$

- Insight: It is possible to relate the views of two components by only considering the **difference in their observed writes**, i.e., the writes one component has observed but the other has not.

Travelling Between Components



- Aim to demonstrate rely/guarantee compatibility when propagating an instruction α from component i to component j
  - Given: component i executes α such that $\mathrm{view}_i(h) \in \mathrm{sat}(\alpha, \mathcal{G}_i)$.
  - Step 1: Show that α can be executed in the shared view, i.e., $\mathrm{view}_{\{i,j\}}(h) \in \mathrm{sat}(\alpha, \mathcal{G}_i)$.
  - Step 2: Show that α can be executed in component j's view, i.e., $\mathrm{view}_j(h) \in \mathrm{sat}(\alpha, \mathcal{G}_i)$.

Travelling to Shared View Through Non i, j Writes

- Prove step 1 by induction on length of $\Delta_{i,j}(h)$.
  - Base case trivial $(\mathrm{view}_{\{i,j\}}(h) = \mathrm{view}_i(h))$.
  - Induction step:
    - The write cannot be from j since j hasn't observed it.
    - If the write is from i, then the reordering of α before it has been covered in multicopy reordering interference freedom.
    - If the write is from some $k \neq i, j$, then we do what follows.
- Have some relation $\mathcal{E}$ intended to capture the possible writes i may have observed ahead of j

$$\mathrm{rif}_{nmca}(\mathcal{E}, \alpha, \mathcal{G}_i) = wp(\mathcal{E}, \mathrm{sat}(\alpha, \mathcal{G}_i)) \subseteq \mathrm{sat}(\alpha, \mathcal{G}_i)$$

- Proving this for $\mathcal{E} = \mathcal{R}_i \cap \mathcal{R}_j \cap id_\alpha$ is sufficient.

Endgame

- Define a compatibility relation by universally quantifying over all writes

$$\begin{array}{l}\mathrm{compat}(\mathcal{G}_i, \mathcal{R}_i, \mathcal{R}_j) \stackrel{\mathsf{def}}{=} \forall x, v. \\ wp(\mathcal{R}_i \cap \mathcal{R}_j \cap id_x, \mathrm{sat}(\mathtt{x\ :=\ v}, \mathcal{G}_i)) \subseteq \mathrm{sat}(\mathtt{x\ :=\ v}, \mathcal{R}_j)\end{array}$$

- Modify the rules for parallel composition (note that we need separate relies and guarantees for each component because demonstrating compat requires pairwise checking)

$$\text{Comp'}\ \frac{\mathcal{R}, \mathcal{G} \vdash_c P\{\ c\ \}Q \quad \mathrm{rif}(\mathcal{R}, \mathcal{G}, c)}{[i \mapsto \mathcal{R}], [i \mapsto \mathcal{G}] \vdash P\{\ \mathrm{comp}(i, m, c)\ \}Q}$$

$$\text{Par'}\ \frac{\begin{array}{c}\mathcal{R}_1, \mathcal{G}_1 \vdash P_1\{\ c_1\ \}Q_1 \quad \mathcal{R}_2, \mathcal{G}_2 \vdash P_2\{\ c_2\ \}Q_2 \quad \mathrm{disjoint}(\mathcal{R}_1, \mathcal{R}_2) \\ \forall i \in \mathrm{dom}(\mathcal{R}_1). \forall j \in \mathrm{dom}(\mathcal{R}_2).\ \mathrm{compat}(\mathcal{G}_1(i), \mathcal{R}_1(i), \mathcal{R}_2(j)) \\ \forall i \in \mathrm{dom}(\mathcal{R}_2). \forall j \in \mathrm{dom}(\mathcal{R}_1).\ \mathrm{compat}(\mathcal{G}_2(i), \mathcal{R}_2(i), \mathcal{R}_1(j))\end{array}}{\mathcal{R}_1 \uplus \mathcal{R}_2, \mathcal{G}_1 \uplus \mathcal{G}_2 \vdash P_1 \wedge P_2\{\ c_1 \mid\mid c_2\ \}Q_1 \wedge Q_2}$$