

Neural Networks and the XOR Problem

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x_1	x_2	$x_1 \oplus x_2$	$x_1 \wedge x_2$	$x_1 \vee x_2$
0	1	1	0	1
0	0	0	0	0
1	1	0	1	1
1	0	1	0	1

The perceptron is a classification algorithm. It takes a set of inputs and returns a single output. In the case of the XOR problem, the inputs are x_1 and x_2 and the output is $x_1 \oplus x_2$. The perceptron is defined as follows:

Perceptron
<ul style="list-style-type: none">• Input: x_1, x_2• Output: $x_1 \oplus x_2$

The problem is that it's not possible to calculate the XOR function with a single-layer perceptron because XOR is not a linearly separable function. XOR requires a non-linear decision boundary to accurately classify the inputs.

The activation function is the function that determines the output of the perceptron. The sigmoid function is a specific type of activation function commonly used in neural networks. The sigmoid function is a non-linear function. The sigmoid function is defined as follows:

Sigmoid Function
<ul style="list-style-type: none">• Input: z• Output: $\sigma(z) = \frac{1}{1+e^{-z}}$

Foward propagation is the process of calculating the output of the perceptron.

The Perceptron: Forward Propagation

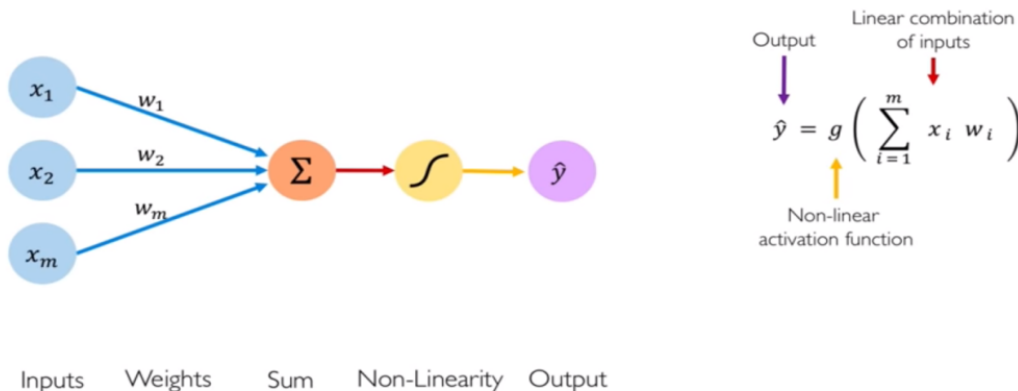


Figure 1: Foward propagation

The decision boundary is the line that separates the two classes.

Terminology

1. The **inputs** are the values that are fed into the perceptron. The inputs are x_1 and x_2 .
2. The **weights** are the values that are multiplied by the inputs. The weights are w_1 and w_2 .
3. The **bias** is a constant value that is added to the weighted sum of the inputs. The bias is b .

References

- [1] Neural Networks and Deep Learning <https://docs.google.com/presentation/d/1e1DGnBZZLHsMNtpDREBT438/edit?usp=sharing>
- [2] How Neural Networks Solve the XOR Problem <https://towardsdatascience.com/how-neural-networks-solve-the-xor-problem-59763136bdd7>
- [3] Perceptions In Neural Networks, What Is It? <https://medium.com/@anton.franzen/perceptions-in-neural-networks-what-is-it-15d982f920e1>