```
import numpy as np

# Step 1: Generate a 3x4 random matrix
matrix_3x4 = np.random.randint(1, 21, size=(3, 4))  # Adjust range as needed
A = matrix_3x4[:, :3]  # Coefficient matrix (first 3 columns)
b = matrix_3x4[:, 3]   # Right-hand side vector (last column)

print("Generated 3x4 Matrix:")
print(matrix_3x4)
print("\nCoefficient Matrix (A):")
print(A)
print("\nRight-hand Side Vector (b):")
print(b)
```

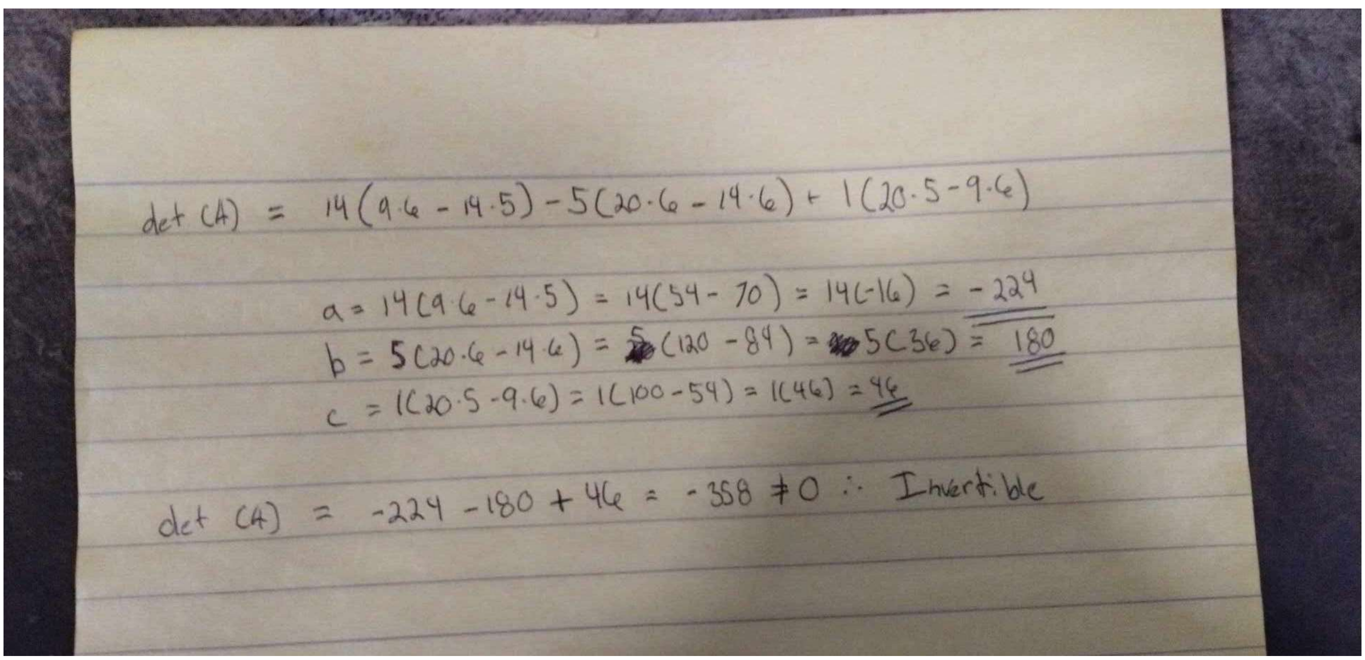
```
Generated 3x4 Matrix:
[[14  5  1  5]
 [20  9 14  1]
 [ 6  5  6 15]]

Coefficient Matrix (A):
[[14  5  1]
 [20  9 14]
 [ 6  5  6]]

Right-hand Side Vector (b):
[ 5  1 15]
```

Matrix Inverse Method

- Determinant should == 0

-
  - det(A) = a (ei − fh) − b (di − fg) + c (dh − eg)

Determinant is -358



Double-click (or enter) to edit

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- Inverse the Matrix
-
  - Cofactor & Transpose the Matrix

Find cofactor matrix

$c_{11} = 9\cdot6 - 14\cdot5 = 54 - 70 = \underline{\underline{-16}}$ $\quad c_{12} = 20\cdot6 - 14\cdot6 = 120 - 84 = \underline{\underline{36}}$

$c_{13} = 20\cdot5 - 9\cdot6 = 100 - 54 = \underline{\underline{46}}$

$c_{21} = 5\cdot6 - 1\cdot5 = 30 - 5 = \underline{\underline{25}}$ $\quad c_{22} = 14\cdot6 - 1\cdot6 = 84 - 6 = \underline{\underline{78}}$

$c_{23} = 14\cdot5 - 5\cdot6 = 70 - 30 = \underline{\underline{40}}$

$c_{31} = 5\cdot14 - 1\cdot9 = 70 - 9 = \underline{\underline{61}}$ $\quad c_{32} = 14\cdot14 - 1\cdot20 = 196 - 20 = \underline{\underline{176}}$

$c_{33} = 14\cdot9 - 5\cdot20 = 126 - 100 = \underline{\underline{26}}$

$c_{33} = 14\cdot9 - 5\cdot20 = 126 - 100 = \underline{\underline{26}}$

$$c = \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \rightarrow \begin{bmatrix} -16 & -(36) & 46 \\ -(25) & 78 & -(40) \\ 61 & -(176) & 26 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} -16 & -25 & 61 \\ -36 & 78 & -176 \\ 46 & -40 & 26 \end{bmatrix} \xleftarrow{\text{Transpose}} \begin{bmatrix} -16 & -36 & 46 \\ -25 & 78 & -40 \\ 61 & -176 & 26 \end{bmatrix}$$

- Computing inverse is c = c / determinant

$$A^{-1} = \begin{bmatrix} 0.0447 & 0.0698 & -0.1704 \\ 0.1006 & -0.2179 & 0.4916 \\ -0.1285 & 0.1117 & -0.0226 \end{bmatrix}$$

- Multiplying Inverse by Matrix B

$$x_1 = 0.0447 \cdot 5 + 0.0698 \cdot 1 - 0.1704 \cdot 15$$
$$= 0.2235 + 0.0698 - 2.556 = -2.2627$$

$$x_2 = 0.1006 \cdot 5 - 0.2179 \cdot 1 + 0.4916 \cdot 15$$
$$= 0.503 - 0.2179 + 7.374 = 7.6591$$

$$x_3 = -0.1285 \cdot 5 + 0.1117 \cdot 1 - 0.0726 \cdot 15$$
$$= -0.6425 + 0.1117 - 1.089 = -1.6198 \to -1.620$$

$$x_1 = -2.2627 \quad x_2 = 7.6591 \quad x_3 = -1.62$$

Answer:

x1 = -2.2627
x2 = 7.6591
x3 = -1.620

Gaussian Elimination Method

- Partial pivoting

$$\cancel{\begin{bmatrix} 14 & 5 & 1 \\ 20 & 9 & 14 \\ 6 & 5 & 6 \end{bmatrix}} \rightarrow \cancel{\begin{bmatrix} 20 & 9 & 14 \end{bmatrix}}$$

Partial Pivoting

$$\begin{bmatrix} 14 & 5 & 1 \\ 20 & 9 & 14 \\ 6 & 5 & 6 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ 15 \end{bmatrix} \rightarrow \begin{bmatrix} 20 & 9 & 14 \\ 14 & 5 & 1 \\ 6 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 15 \end{bmatrix}$$

Forward Elimination - Part 1

Pivot = 20

Row 2 multiplier : $\frac{14}{20} = \frac{7}{10} = \underline{0.7}$

Row 3 multiplier : $\frac{6}{20} = \frac{3}{10} = \underline{0.3}$

~~[14 5 1 5] - [0.7(20) 0.7(9) 0.7(14) 0.7(1)]~~

= [14 5 1 5] - [20(0.7) 9(0.7) 14(0.7) 1(0.7)]

= 14 6.3 9.8 0.7

= [0 -1.3 -8.8 4.3]

= [6 5 6 15] - [20(0.3) 9(0.3) 14(0.3) 1(0.3)]

= 6 2.7 4.2 0.3

= [0 2.3 1.8 14.7]

6 2.7 4.2 0.3

4.7]

$$\begin{bmatrix} 20 & 9 & 14 \\ 0 & -1.3 & -8.8 \\ 0 & 2.3 & 1.8 \end{bmatrix} \begin{bmatrix} 1 \\ 4.3 \\ 14.7 \end{bmatrix}$$

Partial Pivoting (Again)

~~Pivot: -1.3~~

~~Row 3 multiplier: $\frac{2.3}{-1.3}$~~

Partial Pivoting (Again)

$$\begin{bmatrix} 20 & 9 & 14 \\ 0 & -1.3 & -8.8 \\ 0 & 2.3 & 1.8 \end{bmatrix} \begin{bmatrix} 1 \\ 4.3 \\ 14.7 \end{bmatrix} \rightarrow \begin{bmatrix} 20 & 9 & 14 \\ 0 & 2.3 & 1.8 \\ 0 & -1.3 & -8.8 \end{bmatrix} \begin{bmatrix} 1 \\ 14.7 \\ 4.3 \end{bmatrix}$$

Forward Elimination - Part 2

Forward elimination

Pivot: 2.3

Row 3 multiplier: $\frac{-1.3}{2.3} = -0.5652$

Back Substitution

Solve for x3:

$-7.777x_3 = 12.6087 \rightarrow x_3 = \frac{12.6087}{-7.777} = -1.621$

Solve for x2:

$2.3x_2 + 1.8x_3 = 14.7 \rightarrow 2.3x_2 + 1.8(-1.621) = 14.7$

$2.3x_2 - 2.918 = 14.7 \rightarrow 2.3x_2 = 14.7 + 2.918$

$2.3x_2 = 17.618 \rightarrow x_2 = \frac{17.618}{2.3} = 7.660$

Solve for x1:

$20x_1 + 9x_2 + 14x_3 = 1 \rightarrow 20x_1 + 9(7.66) + 14(-1.621)$

$20x_1 + 68.94 - 22.694 = 1 \rightarrow 20x_1 = 1 - 68.94 + 22.694$

$20x_1 = -45.246 \rightarrow x_1 = \frac{-45.246}{20} = -2.2623$

Answer:

x1 = -2.2623 x2 = 7.660 x3 = -1.621

Verifying Answers

```
# Solve using Matrix Inverse Method
try:
    A_inv = np.linalg.inv(A)  # Compute A^-1
    x_manual = np.dot(A_inv, b)  # x = A^-1 * b
    print("\nSolution using Matrix Inverse Method:")
    print(x_manual)
```