Model answer of
10th exam of discrete Math

1)

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	28	29	~~30~~
31	~~32~~	~~33~~	~~34~~	35	~~36~~	37	~~38~~	~~39~~	~~40~~

8

10/12

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37

2) a) $6rs \rightarrow$ yes, $= 2(\underbrace{3rs}_{k}) = \underline{2k}$

b) yes $\rightarrow 2(3r+4s^2) + 2 + 3 + 1$

$2(3r + 4s^2 + 3) + 1 \rightarrow 2k+1$

c) $r^2 + 2rs + s^2 = (r+s)^2 \Rightarrow$ Composite

~~3)~~

3) let m & n are odd integers

$\therefore m = 2k+1 \quad , n = 2l+1$ 6)

$\therefore mn = (2k+1)(2l+1)$

$= 4kl + 2k + 2l + 1$

$= 2(\underbrace{2kl + k + l}_{\text{integer}}) + 1 \Rightarrow$ odd

4) Proof by existance:

let $k = \underbrace{22r + 18s}_{\text{integer}}$ 5)

$\therefore 2k = 2(22r + 18s)$

by distribution law

$2k = 44r + 36s$

5) $n = 1$

$(n+1)^3 = 2^3 = 8 \geq 3^1$ #

$n = 2$

$(n+1)^3 = 3^3 = 27 \geq 3^2 = 9$ #

3

$n = 3$

$(n+1)^3 = 4^3 = 64 \geq 3^4 = 81$ ✗

$n = 4$

$(n+1)^4 = 5^3 = 125 \geq 3^4 = 81$ ✗

6) $3(2x + 3y) = \frac{1}{4}$

$\therefore 2x + 3y = \frac{10}{3}$ not integer

$\therefore x + y = 3\frac{2}{5}$

[illegible] $= \frac{1}{3}$

$\therefore \frac{2}{3}(x + 3y) = 3\frac{2}{5}$

$y = 3\frac{2}{5}$

$\therefore x, y$ must be pos. integer

6)

7- $n = 3p$, $3p+1$, $3p-1$

Case 1

$n=3p \rightarrow n^3 = 27p^3 = 3*9p^3$

Case 2

$3p+1 \rightarrow n^3 = (3p+1)^2(3p+1)$

$= 27p^3 + 27p^2 + 9p + 1$

$= 9\underbrace{(3p^3 + 3p^2 + p)}_{\text{integer}} + 1$

Case 3

$n = 3p-1 \rightarrow n^3 = (3p-1)^3$

$= 27p^3 + 27p^2 + 9p - 1$

$= 9\underbrace{(3p^3 + 3p^2 + p)}_{\text{integer}} - 1$

8)

$n=1$	$n^3 = 1$	$9*0+1$
$n=2$	$n^3 = 8$	$9*1-1$
$n=3$	$n^3 = 27$	$3*9$
$n=4$	$n^3 = 64$	$7*9+1$
$n=5$	$n^3 = 125$	$14*9-1$
$n=6$	$n^3 = 216$	$24*9$
$n=7$	$n^3 = 343$	$38*9+1$
$n=8$	$n^3 = 512$	$57*9-1$
$n=9$	$n^3 = 729$	$81*9$
$n=10$	$n^3 = 1000$	$111*9+1$

5)

8) For Contradiction, there exist a & b

$\rightarrow 21a + 3b = 1$

$3(\underbrace{7a + 10b}) = 1$

must be $1/3$ so a or b must be rational

Contradiction

5)

9) $a_k = \frac{k}{10-k}$

$k=1$ $a_1 = \frac{1}{9}$

$k=2$ $a_2 = \frac{2}{8} = 1/4$

$k=3$ $a_3 = \frac{3}{7}$

$k=4$ $a_4 = \frac{4}{6} = \frac{2}{3}$

4)

10) a) Basic property

$I(n)$: $i = n$ and product $= n \cdot x$

before 1st iteration $I(0) = i = 0$, Product $= 0 \cdot x =$

2)

6)

b) inductive Property

befor iteration

$i \neq m$ (G Passed)

$Product_{old} = k \cdot x$, $I_{old} = k$ 4)

after iteration

$Product_{new} = Product_{old} + x = k \cdot x + x = x(k+1)$

$i_{new} = i_{old} + 1 = k+1$

c) eventual falsity of Guard

guard Condition $G: i \neq m$.

after m iteration 2)

$i = m$ ∴ G false

d) Correctness of the post Condition 2)

after last iteration 3)

G false $i = m$

$I(N)$ true $i = N$, $Product = N \cdot x$

∴ $m = N$ $Product = m\,x$