Question 1:

(a)

for $m = 25$ and $n = 10$

* m is a factor of n^2 (100) but it isn't a factor of 10

(b) $m = 4$, $n = 6$

(c) for $m = 5$ and $n = 10$ m is a factor of n^2 (100) and it is also a factor of 10 .

(d) $m = 2$, $n = 4$.

Question 2:

(a) Predicate $\frac{6}{d}$ is an integer , domain Z

Domain : $Z \{ x \in Z \mid 6x \in Z \}$

$= \{ -6, -3, -2, -1, 1, 2, 3, 6 \}$

(b) Predicate $\frac{6}{d}$ is an integer , domain $Z+$.

Domain : $N \{ x \in N \mid 6x \in Z \}$

$= \{ 1, 2, 3, 6 \}$.

© Predicate : $1 \le x^2 \le 4$, domain : R

   Domain = R $x \in R \mid 1 \le x^2 \le 4$

   = $[-2, -1] \cup [1, 2]$

ⓓ predicate : $1 \le x^2 \le 4$, domain : Z

   Domain : Z $x \in Z \mid 1 \le x^2 \le 4$

   = $\{-2, -1, 1, 2\}$

Question 3 :

ⓐ $a > 0$ and $b > 0 \Rightarrow ab > 0$

   The statement is true . Since a and b are
   Positive .

ⓑ $a < 0$ and $b < 0 \Rightarrow ab < 0$

   the statement is false .

[ex]:

   let $a = -5$ (< 0) and $b = -6$ (< 0)
   Then $ab = (-5)(-6) = 30 > 0$
   30 isn't less than 0 .

© $ab = 0 \Rightarrow a = 0$ or $b = 0$

   The statement is true .

(d) $a < b$ and $c < d \Rightarrow ac < bd$

The statement is false

Counterexample,

Let $a = -2$, $b = 2$, $c = -3$ and $d = 3$

then $a * c < b * d = (-2)(-3) < (2)(3) = 6 \not< 6$

Questions 4

(a) $\forall$ Positive integers m and n , $m \cdot n \geq m + n$

Counterexample,

Let $m = 2$ and $n = 1$ $\therefore$ $2 * 1 \geq 2 + 1$

$$2 \not\geq 3$$

(b) $\forall$ Real numbers x and y , $\sqrt{x + y} = \sqrt{x} + \sqrt{y}$

Counterexample,

let $x = 16$ and $y = 9$.

$\therefore$ $\sqrt{x + y} = \sqrt{16 + 9} = \sqrt{25} = \pm 5$.

and $\sqrt{x} = \sqrt{16} = \pm 4$

and $\sqrt{y} = \sqrt{9} = \pm 3$ $\therefore$ $\sqrt{16 + 9} \neq \sqrt{16} + \sqrt{9}$.

© $\forall x \in R, \, x > 1/x$

Let $x = -2$ ∴ $-2 \not> \dfrac{-1}{2}$

④ $\nexists \, a \in Z \, (a-1)/a$ isn't an integer

Counterexample:

Let $a = 1$ ∴ $\dfrac{a-1}{a} = \dfrac{1-1}{1} = \dfrac{0}{1} = 0$

Question 5:

ⓐ $\forall$ fish x, x has gills.

negation:

$\exists$ a fish x such that x doesn't have gills.

$\exists$ fish x, x has no gills.

ⓑ $\forall$ Computer c, c has a cpu.

$\exists$ Computer c, c has no cpu.

ⓒ $\exists$ a movie m such that m is over 6 hours long.

$\forall$ movies m, m is less than or equal to 6 hours long.

④

(d) $\exists$ a band b such that b has won at least 10 grammy awards.

* $\forall$ bands b, b has won fewer than 10 grammy awards.

Question 5:

(a) All dogs are Carnivorous.
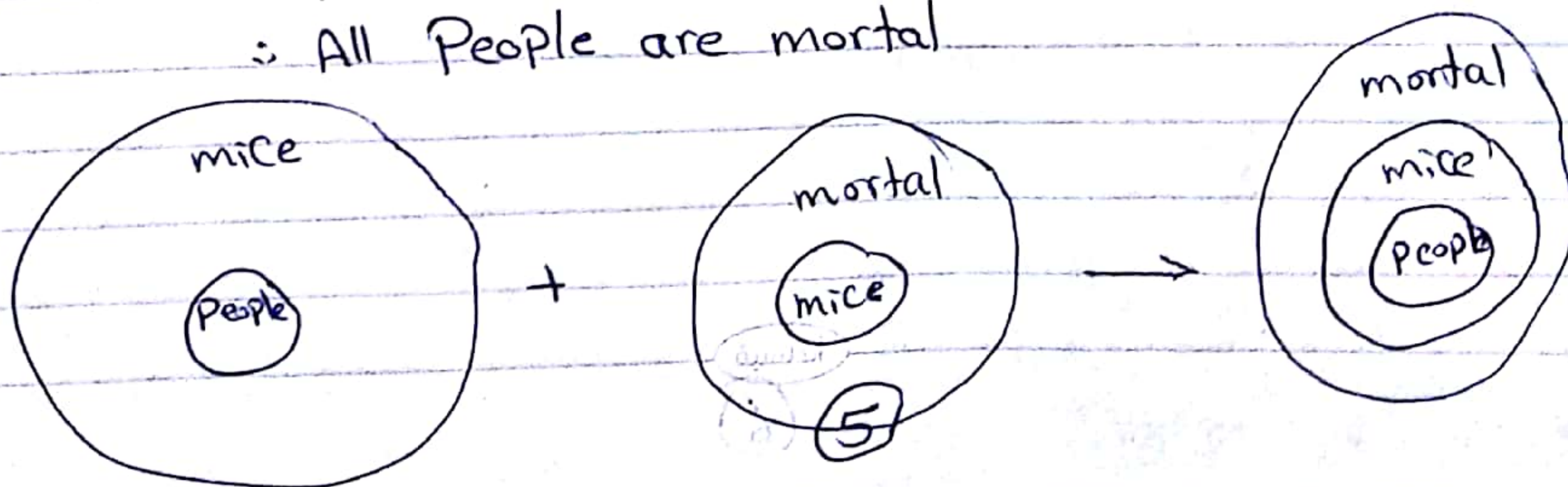Felix is not a dog.
$\therefore$ Felix is not Carnivorous.

invalid $\implies$ the premises are true and the conclusion is false.

(b) All People are mice   Valid.
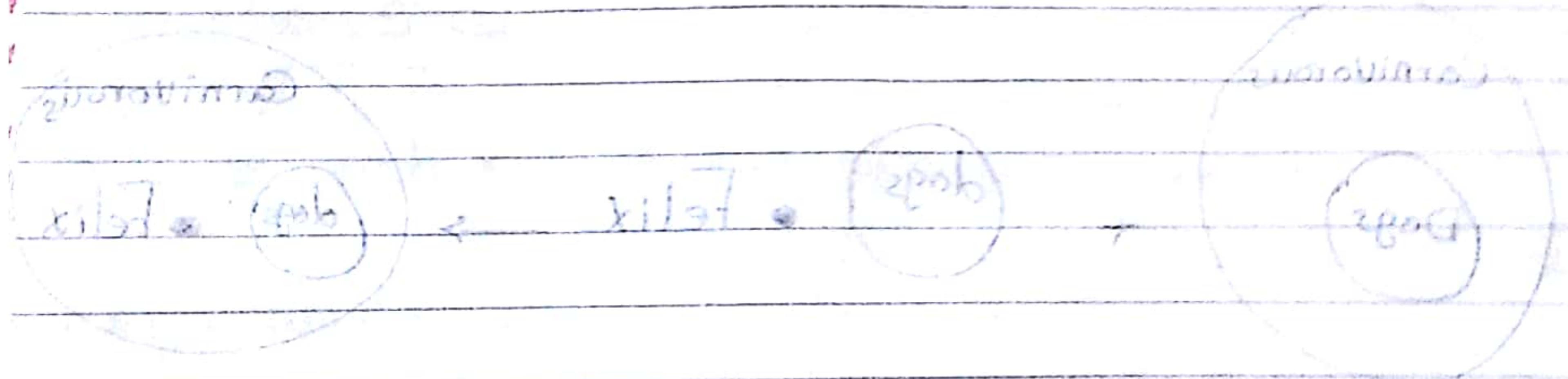All mice are mortal
$\therefore$ All People are mortal

(c)

All honest People Pay their taxes.
Darth Isn't honest.
∴ Darth does not Pay his taxes.

this is invalid