Question 1 :

(a) She is a mathematics major.

⇒ not statement . because it may be true or false .

(b) $128 = 2^6$ .

⇒ statement (false) . indicated by false .

(c) $x = 2^6$

⇒ not statement . because it may be true or false .

Question 2 :

(a) Stocks are increasing [but] interest rates are steady .

⇒ $(S \wedge i)$ .

(b) [Neither] are stocks increasing [nor] interest rates steady .

$(\sim S \wedge \sim i)$ .

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(a) John is healthy and wealthy but not wise.

$(h \wedge w \wedge \sim s)$

(b) John is not wealthy but he is healthy and wise.

$(\sim w \wedge h \wedge s)$

(c) John is neither healthy, wealthy, nor wise.

$(\sim h \wedge \sim w \wedge \sim s) \equiv \sim (h \vee w \vee s)$

3

(a) DataFlag is off, error equals 0 and Sum is less than 1000.

$(p \wedge q \wedge \sim r)$

(b) DataFlag is off but error is not equal to 0.

$(p \wedge \sim q)$

(c) DataFlag is off, however error is not 0 or Sum is greater than or equal to 1000.

$(p \wedge \sim q \vee \sim r)$

(d) DataFlag is on and Error equals 0 but Sum is greater than or equal to 1000.

$(\sim p \wedge q \wedge \sim r)$

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Question 8:

write truth tables

① $\sim p \wedge q$

| p | q | $\sim p$ | $\sim p \wedge q$ |
|---|---|---|---|
| T | T | F | F |
| T | F | F | F |
| F | T | T | T |
| F | F | T | F |

② $(p \wedge q) \vee \sim (p \vee q)$

| p | q | $(p \wedge q)$ | $(p \vee q)$ | $\sim (p \vee q)$ | $(p \wedge q) \vee \sim (p \vee q)$ |
|---|---|---|---|---|---|
| T | T | T | T | F | T |
| T | F | F | T | F | F |
| F | T | F | T | F | F |
| F | F | F | F | T | T |

③ $p \wedge (q \wedge r)$

| p | q | r | $(q \wedge r)$ | $p \wedge (q \wedge r)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | F | F |
| T | F | T | F | F |
| T | F | F | F | F |
| F | T | T | T | F |
| F | T | F | F | F |
| F | F | T | F | F |
| F | F | F | F | F |

(4) $\neg p \wedge (p \vee q)$

| $p$ | $q$ | $r$ | $p \vee q$ | $\neg p$ | $\neg p \wedge (p \vee q)$ | $(\neg p \wedge (p \vee q)) \vee r$ |
|---|---|---|---|---|---|---|
| T | T | T | T | F | F | T |
| T | T | F | T | F | F | F |
| T | F | T | T | F | F | T |
| T | F | F | F | F | F | F |
| F | T | T | T | T | T | T |
| F | T | F | T | T | T | T |
| F | F | T | F | T | F | T |
| F | F | F | F | T | F | F |

Question # 1 :-

(1) $p \vee (p \wedge q) \equiv p$ absorption law

| $p$ | $q$ | $p \wedge q$ | $p \vee (p \wedge q)$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | T |
| F | T | F | F |
| F | F | F | F |

The same ∴ So it is equivalent .

(a) $\sim(p \vee q)$ and $\sim p \wedge \sim q$

$\sim(p \vee q)$ Demorgan's law

$= \sim p \wedge \sim q$

| p | q | $(p \vee q)$ | $\sim(p \vee q)$ | $\sim p$ | $\sim q$ | $\sim p \wedge \sim q$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | T | F | F | T | F |
| F | T | T | F | T | F | F |
| F | F | F | T | T | T | T |

the same ∴ So it is equivalent.

(b) $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$ :

$(p \vee q) \wedge r$

$r \wedge (p \vee q)$ Commutative law

$(r \wedge p) \vee (r \wedge q)$ distributive law

$r \wedge (p \vee r) \wedge q$ associative law

$r \wedge q$ absorption law

$q \wedge r$ //

not equivalent

$(p \vee q) \vee (p \wedge r)$

$(p \wedge r) \vee (p \vee q)$ Commutative law

$(p \wedge r \vee p) \vee (p \wedge r \vee q)$ distributive law

$p \wedge (p \vee r) \vee (p \wedge r \vee q)$ Commutative law

$(p \vee (p \wedge r)) \vee q$ absorption law

$p \vee q$ " "

" " " $(p \vee q)$ //

| p | q | r | (p∨q) | (p∧r) | (p∨q)∨(p∧r) | (p∨q)∧r |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | F | T | F | T | F |
| T | F | T | T | T | T | T |
| T | F | F | T | F | T | F |
| F | T | T | T | F | T | T |
| F | T | F | T | F | T | F |
| F | F | T | F | F | F | F |
| F | F | F | F | F | F | F |

not the same So it isn't equivalent

4) $((r \vee p) \wedge ((\sim r \vee (p \wedge q)) \wedge (r \vee q))$ and $p \wedge q$.

$r \vee (p \wedge q) \wedge (\sim r \vee (p \wedge q))$ distributive law.

$(p \wedge q) \vee r \wedge (\sim r \vee (p \wedge q))$ Commutative law.

$(p \wedge q) \vee (r \wedge \sim r)$ distributive law.

$(p \wedge q) \vee c$ negation law.

$p \wedge q$ identity law.

$\therefore$ equivalent

| p | q | r | (r∨p) | (p∧q) | ¬r | ¬r∨(p∧q) | (r∨q) | (¬r∨(p∧q))∧(r∨q) | (r∨p)∧((¬r∨(p∧q))∧(r∨q)) |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | F | T | T | T | T |
| T | T | F | T | T | T | T | T | T | T |
| T | F | T | T | F | F | F | T | F | F |
| T | F | F | T | F | T | T | F | F | F |
| F | T | T | T | F | F | F | T | F | F |
| F | T | F | F | F | T | T | T | T | F |
| F | F | T | T | F | F | F | T | F | F |
| F | F | F | F | F | T | T | F | F | F |

| (p∧q) | (r∨p)∧((¬r∨(p∧q))∧(r∨q)) |
|---|---|
| T | T |
| T | T |
| F | F |
| F | F |
| F | F |
| F | F |
| F | F |
| F | F |

The Same So it is equivalent.

Question 5:

Demorgan's law

① Hala isn't a math major or Hala's sister isn't a Computer science major.

② Sama dosen't Swim on thursdays or John dosen't play tennis on Saturdays.

(B) the Connector isn't lamp and the machine isn't unplugged (CC)

- the Connector isn't lamp and the machine is plugged.

Question 6, Tautologies and, Contradictions

① $(p \wedge q) \vee (\neg p \vee (p \wedge \neg q))$

| p | q | $(p \wedge q)$ | $\neg p$ | $\neg q$ | $(p \wedge \neg q)$ | $\neg p \vee (p \wedge \neg q)$ | $(p \wedge q) \vee (\neg p \vee (p \wedge \neg q))$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F | T |
| T | F | F | F | T | T | T | T |
| F | T | F | T | F | F | T | T |
| F | F | F | T | T | F | T | T |

∴ Tautology

② $(p \wedge \neg q) \wedge (\neg p \vee q)$

| p | q | $\neg p$ | $\neg q$ | $(p \wedge \neg q)$ | $(\neg p \vee q)$ | $(p \wedge \neg q) \wedge (\neg p \vee q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | F | T | F |
| T | F | F | T | T | F | F |
| F | T | T | F | F | T | F |
| F | F | T | T | F | T | F |

∴ Contradiction

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③ $(\sim P \vee q) \vee (P \wedge \sim q)$

| P | q | ~P | ~q | $(\sim P \vee q)$ | $(P \wedge \sim q)$ | $(\sim P \vee q) \vee (P \wedge \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | T |
| T | F | F | T | F | T | T |
| F | T | T | F | T | F | T |
| F | F | T | T | T | F | T |

∴ Tautology.

Question 7 :

① $(P \wedge \sim q) \vee P \equiv P$

$(P \wedge \sim q) \vee P$

$\equiv P \vee (P \wedge \sim q)$ Commutative law.

$\equiv P$ Absorption law.

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(2) $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge \sim q)$ Demorgan's law.

$\equiv \sim p \wedge (q \vee \sim q)$ Distributive law.

$\equiv \sim p \wedge t$ negation law.

$\equiv \sim p$ universal bound laws.

(3) $\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv p$

$\equiv \sim(\sim p \wedge (q \vee \sim q)) \vee (p \wedge q)$ distributive law

$\equiv \sim(\sim p \wedge t) \vee (p \wedge q)$ negation law.

$\equiv (p \vee \sim) \vee (p \wedge q)$ demorgan law.

$\equiv p \vee (p \wedge q)$ identity law.

$\equiv \boxed{p}$ absorption law.

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# sheet 2

## Question 1.

truth table

① $\sim p \vee q \rightarrow \sim q$

| p | q | $\sim p$ | $\sim p \vee q$ | $\sim q$ | $\sim p \vee q \rightarrow \sim q$ |
|---|---|---|---|---|---|
| T | T | F | T | F | F |
| T | F | F | F | T | T |
| F | T | T | T | F | F |
| F | F | T | T | T | T |

② $p \vee (\sim p \wedge q) \rightarrow q$

| p | q | $\sim p$ | $(\sim p \wedge q)$ | $p \vee (\sim p \wedge q)$ | $p \vee (\sim p \wedge q) \rightarrow q$ |
|---|---|---|---|---|---|
| T | T | F | F | T | T |
| T | F | F | F | T | F |
| F | T | T | T | T | T |
| F | F | T | F | F | F |
| | F | | | T | |

③ $p \wedge \sim r \leftrightarrow q \vee r$

| p | q | r | $\sim r$ | $(p \wedge \sim r)$ | $q \vee r$ | $(p \wedge \sim r) \leftrightarrow q \vee r$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | T | F |
| T | T | F | T | T | T | T |
| T | F | T | F | F | T | F |
| T | F | F | T | T | F | F |
| F | T | T | F | F | T | F |
| F | T | F | T | F | T | F |
| F | F | T | F | F | T | F |
| F | F | F | T | F | F | T |

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④ $(p \to r) \leftrightarrow (q \to r)$

| p | q | r | $(p \to r)$ | $q \to r$ | $(p \to r) \leftrightarrow (q \to r)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | T | F | F | F | T |
| T | F | T | T | T | T |
| T | F | F | F | T | F |
| F | T | T | T | T | T |
| F | T | F | T | F | F |
| F | F | T | T | T | T |
| F | F | F | T | T | T |

Question 2 :

use truth table to verify that

① $p \to q \equiv \sim p \vee q$

| p | q | $p \to q$ | $\sim p$ | $\sim p \vee q$ | |
|---|---|---|---|---|---|
| T | T | T | F | T | |
| T | F | F | F | F | ∴ equivalent |
| F | T | T | T | T | |
| F | F | T | T | T | |

The Same

② $\sim (p \to q) \equiv p \wedge \sim q$

| p | q | $(p \to q)$ | $\sim (p \to q)$ | $\sim q$ | $(p \wedge \sim q)$ | |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | The Same |
| T | F | F | T | T | T | is equivalent |
| F | T | T | F | F | F | |
| F | F | T | F | T | F | |

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④ if Tom isn't Ann's father, then Jim isn't her uncle or Sue isn't her aunt.

Question 4: Contrapositives $\frac{p \to q}{\sim q \to \sim p}$

① if p isn't a rectangle, then p isn't a square.

② if n isn't odd and n isn't 2, then n isn't Prime.

③ if x isn't Positive and x isn't 0, then x isn't nonnegativ

④ if Jim isn't her uncle or Sue isn't her aunt, then Tom isn't Ann's father.



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Question 5 :

Converse $p \to q$ / $q \to p$

① if P is a rectangle, then P is a square

② if n is odd or n is 2, then n is prime.

③ if x is positive or x is 0, then x is nonnegative.

④ if Jim is her uncle and Sue is her aunt, then Tom is Ann's father.

Inverse $p \to q$ / $\sim p \to \sim q$

① if P isn't a square, then P isn't a rectangle.

② if n isn't prime, then n isn't odd and n isn't 2.

③ if x isn't nonnegative, then x isn't positive and x isn't 0.

④ if Tom isn't Ann's father, then Jim isn't her uncle or Sue isn't her aunt.

Question 6 : truth tables

① $p \to q$

$\therefore q \to p$

$\therefore p \vee q$

| p | q | ① $p \to q$ | ② $q \to p$ | Conclusion $(p \vee q)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | T | T | F |

$\therefore$ not valid

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② $p$
$p \to q$
$\sim q \vee r$
$\therefore r$

| p | q | r | $p \to q$ | $\sim q$ | $\sim q \vee r$ |
|---|---|---|---|---|---|
| T | T | T | T | F | T |
| T | T | F | F | F | F |
| T | F | T | F | T | T |
| T | F | F | F | T | T |
| F | T | T | T | F | T |
| F | T | F | T | F | F |
| F | F | T | T | T | T |
| F | F | F | T | T | T |

$\therefore$ valid.

③ $p \vee q$
$p \to \sim q$
$p \to r$
$\therefore r$

| p | q | r | $p \vee q$ | $\sim q$ | $p \to \sim q$ | $p \to r$ |
|---|---|---|---|---|---|---|
| T | T | T | T | F | F | T |
| T | T | F | T | F | F | F |
| T | F | T | T | T | T | T |
| T | F | F | T | T | T | F |
| F | T | T | T | F | T | T |
| F | T | F | F | T | T | T |
| F | F | T | F | T | T | T |
| F | F | F | F | T | T | T |

not valid.

④ $p \to q$
$p \to r$
$\therefore p \to q \wedge r$

| p | q | r | $(q \wedge r)$ | $p \to q$ | $p \to r$ | $p \to q \wedge r$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | F | F | T | F | F |
| T | F | T | F | F | T | T |
| T | F | F | F | F | F | F |
| F | T | T | T | T | T | T |
| F | T | F | F | T | T | T |
| F | F | T | F | T | T | T |
| F | F | F | F | T | T | T |

$\therefore$ valid.

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Question 7 :- Prove modus tollens

| $p \to q$ | p | q | $\sim q$ | $\sim p$ | $p \to q$ |
|---|---|---|---|---|---|
| $\sim q$ | T | T | F | F | T |
| $\sim p$ | T | F | T | F | F |
| | F | T | F | T | T |
| | F | F | (T) | (T) | (T) |

∴ Valid. as premisses true and conclusion true.

Question 8 :- use truth table

① $p \to q$

$\therefore q$

$\therefore p$

| (p) | q | $p \to q$ |
|---|---|---|
| (T) | (T) | (T) |
| T | F | F |
| (F) | (T) | (T) |
| F | F | T |

∴ not valid

② $p \to q$

$\sim p$

$\therefore \sim q$

| p | q | $p \to q$ | $\sim p$ | $\sim q$ |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | F | F | T |
| F | T | (T) | (T) | (F) |
| F | F | T | T | T |

∴ not valid.