Sheet (5)
Sequences and Mathematical Induction

Question 1:

Write the first four terms of the sequences defined by the following formulas:

1. $a_k = \dfrac{k}{10-k}$, for all integers $k \geq 1$.
2. $b_j = 1 + 2^j$, for all integers $j \geq 0$.
3. $c_i = \dfrac{(-1)^i}{3}$, for all integers $i \geq 0$.
4. $d_m = 1 - \left(\dfrac{1}{10}\right)^m$, for all integers $m \geq 1$.

Question 2:

Find explicit formulas for the following sequences with the initial terms given:

1. 0, 1, -2, 3, -4, 5.
2. $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}, \frac{6}{8}$.
3. $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \frac{6}{49}$.

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4. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, -\frac{6}{7}.$

Question 3:

Let $a_0 = 2$, $a_1 = 3$, $a_2 = -2$, $a_3 = 1$, $a_4 = 0$, $a_5 = -1$ and $a_6 = -2$. Compute each of the summations and products below.

1. $\sum_{i=0}^{0} a_i$.
2. $\sum_{j=1}^{3} a_j$.
3. $\prod_{k=0}^{6} a_k$.
4. $\prod_{k=2}^{2} a_k$.

Question 4:

Write each of the following summation or product notations.

1. $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2$.
2. $(1^3 - 1)(2^3 - 1)(3^3 - 1)(4^3 - 1)$.
3. $n(n-1)(n-2)\ldots\ldots\ldots\ldots 1$.
4. $n + \frac{n-1}{2!} + \frac{n-2}{3!} + \frac{n-3}{4!} + \ldots\ldots\ldots + \frac{1}{n!}$

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Question 5:
Transform each of the following by making the change of variable $j = i - 1$.

1. $\sum_{i=1}^{n+1} \frac{(i-1)^2}{i}$.
2. $\sum_{i=3}^{n+1} \frac{i}{i+n-1}$.

Question 6:

a) Prove that $n! + 2$ is divisible by 2, for all integers $n \geq 2$.
b) Prove that $n! + k$ is divisible by k, for all integers $n \geq 2$ and $k = 1, 2, 3, \ldots, n$.

Question 7:
Using theorem 4.2.2 to solve the following:

$$2 + 4 + 6 + \ldots + 2n = n^2 + n.$$

For all integers $n \geq 1$.

Question 8:
Without using theorem 4.2.2, use mathematical induction to prove that:

$$1 + 5 + 9 + \ldots + (4n - 3) = n(2n - 1).$$

For all integers $n \geq 1$.

Question 9:
Without using theorem 4.2.3, use mathematical induction to prove that: $1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$.

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