Given Tables on Final Exam

Z Transform Table

f (t)	F (Z)
c	$\dfrac{cZ}{Z-1}$
$\delta(t)$	1
$u(t)$	$\dfrac{Z}{Z-1}$
e^{at}	$\dfrac{Z}{Z-e^{aT}}$
t	$\dfrac{TZ}{(Z-1)^2}$
a^{t}	$\dfrac{Z}{Z-a^{T}}$
$\sin(\omega t)$	$\dfrac{Z\sin(\omega T)}{Z^2-2Z\cos(\omega T)+1}$
$\cos(\omega t)$	$\dfrac{Z^2-Z\cos(\omega T)}{Z^2-2Z\cos(\omega T)+1}$
$\sinh(\omega t)$	$\dfrac{Z\sinh(\omega T)}{Z^2-2Z\cosh(\omega T)+1}$
$\cosh(\omega t)$	$\dfrac{Z^2-Z\sinh(\omega T)}{Z^2-2Z\cosh(\omega T)+1}$

Property	$Z\{f(t)\} \longleftrightarrow F(Z)$
Linearity	$Z\{f(t)+g(t)\} \longleftrightarrow Z\{f(t)\}+Z\{g(t)\}$
Scaling	$Z\{a^t f(t)\} \longleftrightarrow F(Z/a^T)$
Complex translation	$Z\{e^{-at} f(t)\} \longleftrightarrow F(Ze^{aT})$
Multiplication by t	$Z\{t\, f(t)\} \longleftrightarrow -ZT\dfrac{d}{dZ}F(Z)$
Shift to left	$Z\{f(K+n)T\} \longleftrightarrow Z^n F(Z)-(Z^n f(0)+Z^{n-1}f(T)+\dots\dots+Zf(n-1)T)$
Shift to Right	$Z\{f(K-n)T\,u(K-n)T\} \longleftrightarrow Z^{-n}F(Z)$

Legendre Polynomials

$$P_n(x)=\sum_{k=0}^{N}(-1)^k\frac{(2n-2k)!}{2^n k!(n-2k)!(n-k)!}x^{n-2k}$$

"Recurrence Relations"

I) $(n+1)P_{n+1}(x)-(2n+1)xP_n(x)+nP_{n-1}(x)=0$

II) $nP_n(x)=xP'_n(x)-P'_{n-1}(x)$

III) $(2n+1)P_n(x)=P'_{n+1}(x)-P'_{n-1}(x)$

IV) $(n+1)P_n(x)=P'_{n+1}(x)-xP'_n(x)$

V) $(1-x^2)P'_n(x)=n(P_{n-1}(x)-xP_n(x))$

1) $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$

2) $\frac{d}{dx}[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$

3) $J'_n(x) = J_{n-1}(x) - \frac{n}{x} J_n(x)$

4) $J'_n(x) = \frac{n}{x} J_n(x) - J_{n+1}(x)$

5) $J'_n(x) = \frac{1}{2}\{J_{n-1}(x) - J_{n+1}(x)\}$

6) $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$

Fourier Transform Table

$f(t)$	$F(\omega)$
$Rect(t)$	$2sinc(\omega)$
$e^{-\lvert t\rvert}$	$\frac{2}{1+\omega^2}$
e^{-t^2}	$\sqrt{\pi}e^{-\frac{\omega^2}{4}}$
$\delta(t)$	1
C	$2\pi C\delta(\omega)$
$e^{-at}u(t)$	$\frac{1}{a+i\omega}$
$e^{at}u(-t)$	$\frac{1}{a-i\omega}$
$sgn(t)$	$\frac{2}{i\omega}$
$Tri(t)$	$sinc^2(\frac{\omega}{2})$
$u(t)$	$\pi\delta(\omega) + \frac{1}{i\omega}$

Fourier Transform proberties

Proberty	$f(t)$	$F(\omega)$
Linearity	$A f_1(t) + B f_2(t)$	$A F_1(\omega) + B F_2(\omega)$
Scaling	$f(at)$	$\frac{1}{\lvert a\rvert} F(\frac{\omega}{a})$
Time Shift	$f(t \pm a)$	$F(\omega)e^{\pm ia\omega}$
Frequency Shift	$f(t)e^{\pm iat}$	$F(\omega \mp a)$
Symmetry (Duality)	$F(t)$	$2\pi f(-\omega)$
Time Differentiation	$\frac{d}{dt} f(t)$	$i\omega F(\omega)$
Frequency Diffentiation	$-it f(t)$	$\frac{d}{d\omega} F(\omega)$
Integration in time domain	$\int_{-\infty}^{t} f(x)dx$	$\frac{F(\omega)}{i\omega}$
Time Convolution	$f_1(t) * f_2(t)$	$F_1(\omega) F_2(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} \lvert f(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert F(\omega)\rvert^2 d\omega$	