Chapter 3

Thursday, January 3, 2019 1:15 PM

Finite Markov Decision Processes (Finite MDP):
The Finite MDP problem involves both evaluative feedback and also an associative aspect (choosing different actions in different situations). They are a classical formalization of sequential decision making, where actions influence not only the immediate rewards but also subsequent situations, or states and through those future rewards also. So they involve delayed reward and the need to tradeoff between immediate and delayed reward.
In the bandit problem only the value of each action ($q_*(a)$) is estimated but in MDPs, the value $q_*(s,a)$ of each action is each state 's' has to be estimated or the value $v_*(s)$ of each state given optimal actions selection have to be estimated. These state dependent quantities are essential to accurately assign credit for long-term consequences to individual action selections.

3.1. The Agent-Environment Interface
MDPs are a straight forward way to frame the problem of learning from interaction to achieve a goal.

Agent:
The learner and decision maker.

Environment:
The thing the agent interacts with, it comprises everything outside the agent.

Interactions:
The agent and environment interacts continually. The agent selects actions and the environment responds to those actions and present new situations to the agent. The environment also provides the rewards, which are nothing but the numerical values that the agent has to maximize over time through its choice of actions.

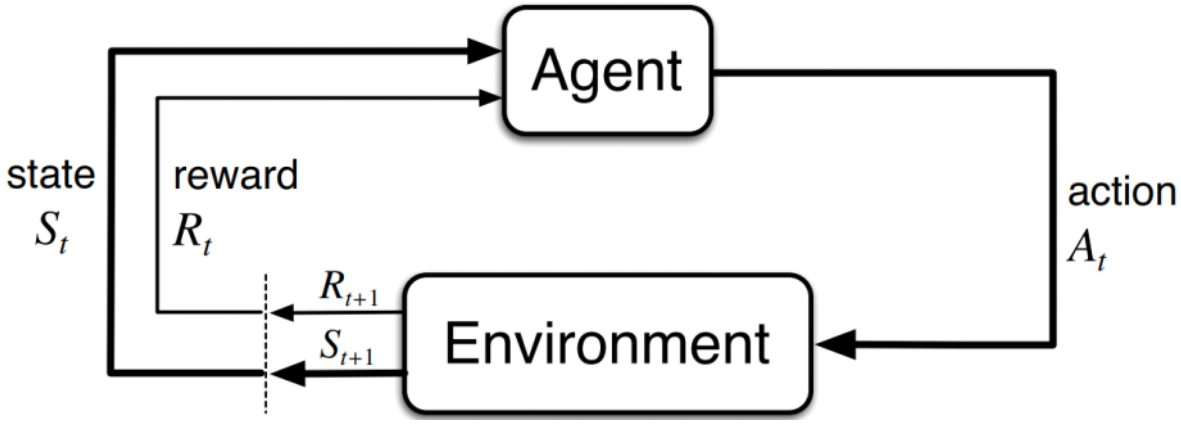


Figure Description: The agent-environment interaction in a Markov Decision Process

The agent and environment interact at each of a sequence of discrete time steps, t = 0, 1, 2, 3, ... At each time step t, the agent receives some representation of the environment's state, $S_t \in S$, and on that basis selects an action, $A_t \in A(s)$. One time step later, in part as a consequence of its action, the agent receives a numerical reward, $R_{t+1} \in R$, and finds itself in a new state, S_{t+1}.
The MDP and agent together thereby give rise to a sequence or *trajectory* that begins like this:

$$S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \ldots$$

In finite MDP, the sets of states, actions and rewards (S, A, and R) have a finite number of elements. The random variables R_t and S_t have well defined discrete probability distributions which depend only on the previous state and the action. For particular values of the random variables, s' ∈ S and r ∈ R, there is a probability of those values occurring at time t, given particular values of the preceding state and action:

$$p(s', r \,|\, s, a) \doteq \Pr\{S_t = s', R_t = r \mid S_{t-1} = s, A_{t-1} = a\}$$

This function 'p' defines the dynamics of MDP. 'p' specifies a probability distribution for each choice of s and a, that is, that

$$\sum_{s' \in \mathcal{S}} \sum_{r \in \mathcal{R}} p(s', r \,|\, s, a) = 1, \text{ for all } s \in \mathcal{S}, a \in \mathcal{A}(s)$$

The probabilities given by 'p' completely characterize the environment's dynamics. So the probability of each possible value for S_t and R_t depend only on the immediate preceding state and action, S_{t-1} and A_{t-1} not on the earlier states and actions.

This is a restriction to the states and this restriction implies that state must include information about all aspects of the past agent-environment interaction that make a difference for the future. It is called as the "Markov Property".

The four argument dynamics function 'p' can be used to compute anything about the environment.
The state transition probabilities can be found out using the following relation:

$$p(s' \,|\, s, a) \doteq \Pr\{S_t = s' \mid S_{t-1} = s, A_{t-1} = a\} = \sum_{r \in \mathcal{R}} p(s', r \,|\, s, a)$$

which becomes a three argument function.

$$p : \mathcal{S} \times \mathcal{S} \times \mathcal{A} \to [0, 1])$$

The expected rewards can also be computed for the state action pairs using the following equation and this is a two argument function.

$$r(s, a) \doteq \mathbb{E}[R_t \mid S_{t-1} = s, A_{t-1} = a] = \sum_{r \in \mathcal{R}} r \sum_{s' \in \mathcal{S}} p(s', r \,|\, s, a)$$

The two - argument function:

$$r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$$

The expected rewards for the state-action-next-state triples as a three-argument function.

$$r(s, a, s') \doteq \mathbb{E}[R_t \mid S_{t-1} = s, A_{t-1} = a, S_t = s'] = \sum_{r \in \mathcal{R}} r \frac{p(s', r \,|\, s, a)}{p(s' \,|\, s, a)}$$

The three argument function:

$$r : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to \mathbb{R}$$

The MDP framework is abstract and flexible. They can be applied to different problems in different ways.

Example Considerations:

Example 1: (Time steps)

The time steps need not to refer to the fixed intervals of real time. They can refer to arbitrary successive stages of decision making and acting.

Example 2: (Actions)

The actions can be low-level controls, such as voltages applied to motors of robot arm or high-level decisions such as whether or not to have lunch or to go to graduate school.

Example 3: (States)

States can be completely determined by low-level sensations, such as direct sensor readings, or they can be more high-level and abstract, such as symbolic description of objects in a room. Some of what makes up a state can be based on memory of past sensations or even be entirely mental or subjective.

Broader Considerations:

An agent can be in the state of not being sure where an object is or of having just been surprised in some clearly defined sense.
Similarly some actions might be totally mental or computational. Some actions might control what an agent chooses to think about or where it focuses its attention.

In general, actions can be any decisions that have to be learned how to make, and the states can be anything to know which might be useful in making them.

Boundary between the agent and the environment:

Boundary between the agent and the environment is not necessarily to be same as the physical boundary of a robot's or animal's body.
For example, the motors and mechanical linkages of a robot and its sensing hardware should be considered as part of the environment rather than parts of the agent. If the MDP is applied to a human or animal muscles, skeleton, and sensory organs should be considered part of the environment.
Rewards may be calculated inside the physical bodies of natural and artificial learning systems, but should be considered external to the agent.
So the general rule is, anything that cannot be changes arbitrarily by the agent is considered as outside of it and belongs to the part of its environment.

It is not necessary to consider everything in the environment is unknown to the agent. The agent often knows quite a bit about how the rewards are computed as a function of its actions and the states in which they are taken. But the reward computation is considered as external to the agent, because it defines the task facing the agent and beyond the agent's ability to change arbitrarily.

For example, in a complicated robot, many different agents may be operating at once, each with its own boundary. One agent may take high-level decisions which form part of the states faced by a lower-agent that implements the high-level decisions.

Agent - Environment boundary is selected once particular states, actions and rewards were selected and also specific decision making task of interest is identified.

The MDP framework suggests that any problem of goal-directed learning can be reduced to three signals passing back and forth between an agent and its environment:

1. One signal to represent the choices made by the agent (actions)
2. One signal to represent the basis on which choices are made (state)
3. One signal to define the agent's goal (reward)

The particular states and actions vary greatly from task to task and how they are represented can

strongly affect the performance. Such representational choices are more art than science.

3.2. Goals and Rewards:

In RL, purpose / goal of agent is denoted by a special signal called reward, passed by environment to the agent.

At each timestep, the reward is a simple number $R_t \in \mathbb{R}$.

The agent's goal is to maximize the total amount of reward. So the agent has to maximize not only the immediate reward but also the cumulative reward in the long run.

So the "reward hypothesis" can be stated as follows:

> *That all of what we mean by goals and purposes can be well thought of as the maximization of the expected value of the cumulative sum of a received scalar signal (called reward).*

The use of reward signal to formalize the idea of a goal is one of the most distinctive features of RL.

Example cases of reward assignment:

Walking robot:

To make a robot to learn to walk, reward will be provided on each time proportional to the robot's forward motion.

Maze solving agent:

The reward is often -1 or every time step that passes prior to escape, which encourages the agent to escape as quickly as possible.

Empty Soda Can Collecting Agent:

Reward of zero for most of the time and +1 for each can collected. Give negative rewards when the robot bumps into things or when somebody yells at it.

Checkers / Chess Playing Agent:

The natural rewards are +1 for winning, -1 for losing, and 0 for drawing and for all nonterminal positions.

So if we want the agent to do something, the rewards have to be provided in such a way that in maximizing them the agent will also achieve the goals. Reward signal is not the place to impart to the agent prior knowledge about how to achieve what we want to do.

This can be visualized using the following example:

A chess playing agent should be rewarded only for actually winning, not for achieving sub goals such as taking its opponent's pieces or gaining control of the center of the board. If achieving these sorts of sub-goals were rewarded, then the agent might find a way to achieve them without achieving the real goal. For example, it might find a way to take the opponent's pieces even at the cost of losing the game. *The reward signal is your way of communicating to the robot what you want it to achieve, not how you want it achieved*

3.3. Returns and Episodes:

The agent has to maximize the *expected return.* The return can be denoted as G_t and it is defined as some function of the reward sequence. The return is the sum of rewards.

$$G_t \doteq R_{t+1} + R_{t+2} + R_{t+3} + \cdots + R_T$$

where T is the final time step. This approach is suitable for the applications in which there is a final step. In these applications the agent-environment interaction breaks naturally into subsequences, called episodes. Some of the examples of episode are plays of a game, trips through a maze, or any sort of repeated interaction.

Each episode ends in a special state called the terminal state and then reset to a standard starting state or to a sample from standard distribution of starting states. Each new episode begins independently of

how the previous episode ended. Episodes can be considered to end in the same terminal state with different rewards for different outcomes. Tasks with episode of this kind are called as *episodic tasks*. It is needed to distinguish the set of all nonterminal states (S) from the set of all states plus the terminal state (S^+). The time of termination (T) is a random variable that normally varies from episode to episode.

In many-cases the agent-environment interaction does not break into identifiable episodes, but goes on continually without limit. These are called as *continuing tasks*. It is the natural way to formulate an on-going process-control task or an application to a robot with a long life span. The return formulation for the continuing tasks is challenging because the final time step would be $T = \infty$, and the return which the agent tries to maximize will also be infinite.

Concept of Discounting:

The agent tries to select actions so that the sum of the discounted rewards it receives over the future is maximized. So, the agent chooses A_t to maximize the expected discounted return:

$$G_t \doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

where $0 \leq \gamma \leq 1$ is called as the discount rate.

The *discount rate* determines the present value of future rewards. Because the reward received k time steps into the future is worth only γ^{k-1} times what it would be worth if it was received immediately.

If $\gamma < 1$, the infinite sum in the above equation has a finite value as long as the reward sequence $\{R_k\}$ is bounded.

If $\gamma = 0$, then the agent is concerned only with maximizing the immediate rewards, this agent is "myopic" (short-sighted). It's objective is only to learn how to choose A_t so as to maximize only R_{t+1}. Acting to maximize the immediate reward can reduce access to future rewards so that the return is reduced. As γ approaches 1, the return objective takes future rewards into account more strongly and hence the agent becomes far sighted.

Return at each time steps are related to each other in a way which is important for the theory and algorithms of reinforcement learning.

$$\begin{aligned} G_t &\doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \gamma^3 R_{t+4} + \cdots \\ &= R_{t+1} + \gamma\left(R_{t+2} + \gamma R_{t+3} + \gamma^2 R_{t+4} + \cdots\right) \\ &= R_{t+1} + \gamma G_{t+1} \end{aligned}$$

The above relationship works for all time steps $t < T$, even if the termination occurs at t+1, if $G_T = 0$.
This formula makes it easy to compute the returns (a form of incremental computation).

Although the return is a sum of an infinite number of terms, it is still finite if the reward is nonzero and constant if $\gamma < 1$.
For example if the reward is a constant +1, then the return can be considered as below:

$$G_t = \sum_{k=0}^{\infty} \gamma^k = \frac{1}{1-\gamma}$$

3.4 Unified Notation for Episodic and Continuing Tasks

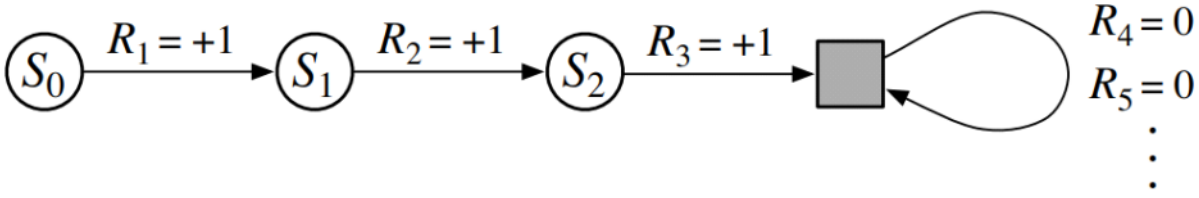
It is essential to develop common notations to discuss about both the episodic and the continuing tasks.

Episodic tasks require some additional notation. It is needed to consider a series of episodes, each of which contains finite sequence of time steps. So the state representation has to be given in the form of $S_{t,i}$ which represents the state at time 't' and episode 'i'. This has to be followed for the other terms like $A_{t,i}$, $R_{t,i}$, $\Pi_{t,i}$, T_i. During discussion of episodic tasks it is not needed to distinguish between different episodes.

Either a particular single episode is considered or something that is true for all episodes is stated.

So there is a need to represent the "return" in both the episodic and continuing tasks.
In episodic tasks the return is a sum over a finite number of terms and in continuing tasks it is a sum over infinite number of terms. These two can be unified by considering the episode termination to be the entering of a special absorbing state that transitions only to itself and that generates only rewards of zero.

The following example states the same consideration discussed above.



The solid square represents the end of an episode (which is special absorbing state).
So starting from S_0, the reward sequence is +1,+1,+1,0,0,… So when these are added it same as that of the first T reward (here T is 3) or over the full infinite sequence.
It is also true in case of discounting. So the return can be defined in general using the convention of omitting the episode numbers when they are not needed and including the possibility that γ = 1 if the sums remain defined (all the episodes terminate). So the return can be written as follows:

$$G_t \doteq \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$$

Including the possibility that T = ∞ or γ = 1, but not the both.

3.5. Policies and Value Functions:

Value Functions:
RL algorithms involve estimating the value functions. These are the functions of states (or of state-action pairs) that estimate how good it is for the agent to be in a given state (or how good is to perform a given action in a given state).
So the notion "how good" is defined as the future rewards that can be expected or the expected return. Value functions are also defined with respect to particular ways of acting, called policies.

Policies:
A policy is a mapping from states to probabilities of selecting each possible action.
Π is an ordinary function; the "|" in the middle of Π (a|s) merely reminds that it defines a probability distribution over A ∈ A(s) for each S ∈ S.

The value function of a state 's' under a policy 'Π' denoted as V_Π(S), is the expected return when starting

in 's' and following 'Π' thereafter.

For MDPs $V_\Pi(S)$ can be denoted as follows.

$$v_\pi(s) \doteq \mathbb{E}_\pi[G_t \mid S_t = s] = \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s\right], \text{ for all } s \in \mathcal{S},$$

The function $V_\Pi(S)$ is called as the state-value function for policy Π.

The value of taking action 'a' in state 's' under policy 'Π', $q_\Pi(s,a)$ can be denoted using the following equation.

$$q_\pi(s,a) \doteq \mathbb{E}_\pi[G_t \mid S_t = s, A_t = a] = \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s, A_t = a\right]$$

$q_\Pi(s,a)$ is called as the action-value function for policy Π.

Both the state value function and action value function can be estimated from the experience.

If an agent follows policy Π and maintains an average, for each state encountered, of the actual returns that have followed that state, then the average will converge to the state's value, $v_\Pi(s)$, as the number of times that state is encountered approaches infinity.
Same way if separate averages are kept for each action taken in each state, then these averages will similarly converge to the action values, $q_\Pi(s,a)$.

The estimation methods of this type are called as Monte-Carlo methods. They involve averaging over any random samples of actual returns.

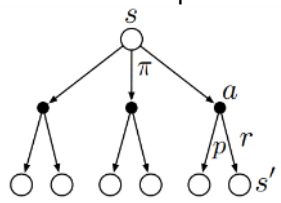
A fundamental property of value functions used throughout reinforcement learning and dynamic programming is that they satisfy recursive relationships.

$$\begin{aligned} v_\pi(s) &\doteq \mathbb{E}_\pi[G_t \mid S_t = s] \\ &= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1} \mid S_t = s] \\ &= \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r \,|\, s, a)\Big[r + \gamma \mathbb{E}_\pi[G_{t+1} | S_{t+1} = s']\Big] \\ &= \sum_a \pi(a|s) \sum_{s', r} p(s', r \,|\, s, a)\Big[r + \gamma v_\pi(s')\Big], \quad \text{for all } s \in \mathcal{S}, \end{aligned}$$

where it is implicit that the actions, a, are taken from the set A(s), that the next states, s_0, are taken from the set S (or from S+ in the case of an episodic problem), and that the rewards, r, are taken from the set R.

The above equation is called the Bellman Equation for v_Π. This equation describes the relationship between the value of a state and the values of its successor states.

The following diagram actually represents that relationship.



Backup diagram for v_π

Description about the previous example:
Each open circle represents a state and each solid circle represents a state-action pair. Starting from the root node 's' which is a state at the top, the agent can take any of some set of actions (three) based on the policy Π.
The environment could then respond with one of several next states, s' (two), along with a reward 'r', depending on the dynamics given by the function 'p'.
The Bellman equation averages over all the possibilities , weighting each by its probability of occurring. So, the value of the start state must equal the (discounted) value of the expected next state, plus the reward expected along the way.

The backup diagrams form the basis of the backup operations which are the heart of RL.

These backup operations transfer the value information back to a state(or a state action pair) from its successor state (or successor state action pairs).

3.6. Optimal Policies and Optimal Value Functions:
Solving a RL problem means, roughly finding a policy that achieves a lot of reward over the long run.

Value functions define a partial ordering over policies. A policy (Π) is defined to be better than or equal to a policy(Π') if it expected return is greater than or equal to that of Π' for all states.

So for $\Pi \geq \Pi'$, if and only if $v_\Pi(s) \geq v_{\Pi'}(s)$ for all $s \in S$.

Optimal Policy:
There is always at least one policy that is better than or equal to all other policies. This is called as "optimal policy". There may be more than one optimal policy and the optimal policies can be denoted by Π_*.

Optimal State-Value Function:
All of the optimal policies share the same state-value function, which is called as the "optimal state-value function".
This can be denoted using the term v_* and can be denoted as follows:

$$v_*(s) \doteq \max_\pi v_\pi(s)$$
for all $s \in \mathcal{S}$

Optimal Action-Value Function:
Optimal policies also share the same "optimal action-value function", which can be denoted by q_*. It can be

$$q_*(s,a) \doteq \max_\pi q_\pi(s,a).$$
for all $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$

The optimal action-value function can also be written in terms of optimal state-value function as follows.

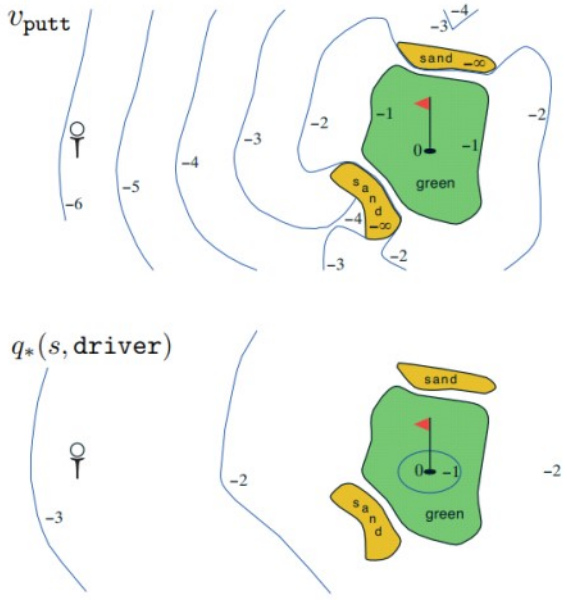
$$q_*(s,a) = \mathbb{E}[R_{t+1} + \gamma v_*(S_{t+1}) \mid S_t = s, A_t = a]$$

Example : Golf

Playing Golf can be formulated as a RL problem by counting a penalty of -1 for each stroke until the ball is hit into the hole. Here *state is the location of the ball*. The *value of state is the negative of the number of strokes to the hole from that location*. *Actions are how we aim and swing at the ball and which club is selected*. In order to simplify the problem, *how we aim and swing are considered given* and just only the choice of club is considered. Here the *club choice is either a putter or a driver*.

The first diagram shows the possible state value function, $v_{putt}(s)$, for the policy that always uses putter. The terminal state in-the-hole has a value of 0. It is assumed that from anywhere on the green a putt can be made, so these states have value -1. But off the green it is not possible to reach the hole by putting , so the value is greater. So if from one state we can reach to the green by putting, then this state has value one less than the green's value, that is -2.

It is assumed that we can putt very precisely and deterministically but with a limited range. Because of this a sharp contour line labeled with -2 is obtained, so all the region between that line and green require exactly two strokes to complete the hole. This is followed for all the contour lines in the figure. In the sand zone, putting can't get us out of it, so they have a value of -∞.



The lower part of the figure shows the contours of a possible optimal action-value function $q_*(s, \text{driver})$. These are the values of each state if first stroke is played with driver and afterward select either the driver or putter, whichever is better. The driver can be used to hit the ball farther away but with less accuracy.

Using driver the hole can be reached in one shot only when the ball is very close denoted by the contour -1 in the diagram below. When there are two strokes allowed then the hole can be reached from further away, shown by contour -2. In this case it is not required to drive all the way within the small -1 contour, but only to anywhere on the green, then from there putter can be used.

The optimal action-value function gives the values after committing to a particular first action, in this case the choice of driver is considered, but after that best actions will be chosen. The -3 contour is still farther out, which is having the starting tee. So the best sequence of actions is two drives and one putt, sinking the ball in three strokes.

It is required for the optimal value function v_* to satisfy the self-consistency condition provided by the Bellman Equation for state values.

Since it is optimal value function, v_*'s consistency condition can be written in a special form w/o reference to any specific policy.

This equation is called as the ***Bellman Optimality Equation*** or ***Bellman Equation for v∗***.

The derivation for that equation is shown below.

$$
\begin{aligned}
v_*(s) &= \max_{a \in \mathcal{A}(s)} q_{\pi_*}(s,a) \\
&= \max_a \mathbb{E}_{\pi_*}[G_t \mid S_t = s, A_t = a] \\
&= \max_a \mathbb{E}_{\pi_*}[R_{t+1} + \gamma G_{t+1} \mid S_t = s, A_t = a] \\
&= \max_a \mathbb{E}[R_{t+1} + \gamma v_*(S_{t+1}) \mid S_t = s, A_t = a] \\
&= \max_a \sum_{s',r} p(s', r \mid s, a)\big[r + \gamma v_*(s')\big].
\end{aligned}
$$

The Bellman optimality equation for q∗ is

$$
\begin{aligned}
q_*(s,a) &= \mathbb{E}\Big[R_{t+1} + \gamma \max_{a'} q_*(S_{t+1}, a') \;\Big|\; S_t = s, A_t = a\Big] \\
&= \sum_{s',r} p(s', r \mid s, a)\Big[r + \gamma \max_{a'} q_*(s', a')\Big].
\end{aligned}
$$

The backup diagrams below show graphically the spans of future states and actions considered in the Bellman Optimality Equation for v∗ and q∗.

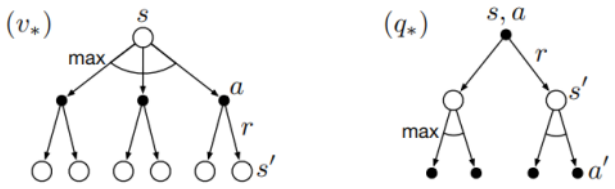


Figure 3.4: Backup diagrams for v_* and q_*

For finite MDP , the Bellman optimality equation has a unique solution.

If we have v∗ then it is easy to determine an optimal policy. For each state 's' there will be one or more actions at which the maximum is obtained in the Bellman optimality equation. So any policy that assigns nonzero probability only to these actions is called as "Optimal Policy".

So *any policy that is greedy with respect to the optimal evaluation function v∗ is an optimal policy*.

The term 'greedy' says that any search or decision procedure that selects alternatives based only on local or immediate considerations, without considering the possibility that such a selection may prevent future access to even better alternatives.

But the beauty about v∗ is that it has already taken into account the reward consequences of all the possible future behaviors. So by means of v* the optimal expected long-term reward is turned into a quantity that is locally and immediately available for each state. So, the one-step-ahead search yields the long-term optimal actions.

So this search can be even made quickly with q∗. So with q∗ the agent doesn't have to do a one-step-ahead search: for any state 's', it can find any action that maximizes q∗(s,a). Since the action-value function caches the result of all one-step-ahead searches. So, at the cost of representing a function state-action pairs, instead of just of states, the optimal action-value function allows optimal actions to be selected without having to know anything about possible successor states and their values, that is ,

without having to know about the environment's dynamics.

Explicit Solution for Bellman Optimality Equation:
Explicitly solving the Bellman Optimality equation provide one way to find an optimal policy and thus solves the RL problem. It is similar to exhaustive search, looking ahead at all possibilities, computing their probabilities of occurrence and their desirabilities in terms of expected rewards.

This solutions follow these three assumptions:
1. Dynamics of the environment is known accurately
2. Availability of enough computational resources to complete the computation of the solution
3. The Markov property

In some tasks, this solution finding method is not possible, because the task violates various combinations of these assumptions.

Example of such task is the game of backgammon. First and third assumption can be applicable, but due to presence of 10^{20} states, it would take 1000 years to solve the Bellman Equation for v_* or q_*.

So RL has to settle for approximate solutions.

Approximate Solution for the Bellman Optimality Equation:
For example *heuristic search methods (such as A*),* can be used to expand the right hand side of the bellman equation several times to some depth, forming a "tree" of possibilities and then using a heuristic evaluation function to approximate v_*, at the "leaf" nodes.

"Dynamic Programming" can be also related to Bellman Equation.

3.7. Optimality and Approximation:
- An agent that learns an optimal policy has done very well, but in practice this happens rarely.
- Optimal policies can be generated only with extreme computational cost.
- In ideal situations, the agents can only approximate to varying degrees.
- A critical constraint is the computational power available to the agent, to perform computation per time step.
- Another constraint is the requirement of large amount of memory for the approximations of value functions, policies and models.
- In small tasks, these approximations can be handled via tables with one entry for each state (or state - action pair)
- These are called "tabular cases" and methods to solve them are called as "tabular methods".
- In many practical problems, there are more states that could be possible to represent as table
- The function approximation has to be used in these cases

Some unique opportunities occur when we settle for achieving useful approximations.

For example:
In approximating optimal behavior, there may be many states that the agent faces with low probability, so that selecting sub-optimal actions for them has little impact on the amount of reward the agent receives.

Tesauro's backgammon player play with exceptional skill though it might take bad decisions on board configs that never occur in games against experts. So, it is possible that TD-Gammon makes bad decisions for a large fraction of game's state set.

The online nature for RL makes it possible to approximate optimal policies in ways that put more effort

into learning to make good decisions for frequently encountered states, at the expense of less effort for infrequently encountered states.

3.8. Summary

1. The reinforcement learning agent and its environment interact over a sequence of discrete time steps.
2. The specification of their interface defines a particular task: the actions are the choices made by the agent; the states are the basis for making the choices; and the rewards are the basis for evaluating the choices.
3. Everything inside the agent is completely known and controllable by the agent; everything outside is incompletely controllable but may or may not be completely known.
4. A policy is a stochastic rule by which the agent selects actions as a function of states.
5. The agent's objective is to maximize the amount of reward it receives over time
6. A finite MDP is an MDP with finite state, action, and (as we formulate it here) reward sets.
7. The return is the function of future rewards that the agent seeks to maximize (in expected value).
8. The *undiscounted formulation* is appropriate *for episodic tasks*, in which the agent–environment interaction *breaks naturally into episodes*; the *discounted formulation* is appropriate *for continuing tasks*, in which the interaction *does not naturally break into episodes* but continues without limit.
9. *A policy's value functions* assign to each state, or state–action pair, the expected return from that state, or state–action pair, given that the agent uses the policy.
10. The *optimal value functions* assign to each state, or state–action pair, the largest expected return achievable by any policy.
11. A policy whose value functions are optimal is an optimal policy
12. *Any policy that is greedy with respect to the optimal value functions must be an optimal policy.*
13. Even if the agent has a complete and accurate environment model, the agent is typically *unable to perform enough computation per time step to fully use it.*
14. The *memory availablity is also an important constraint*. Memory may be required to build up accurate approximations of value functions, policies, and models. *In most cases of practical interest there are far more states than could possibly be entries in a table, and approximations must be made.*