Neural Networks as Associative Memories

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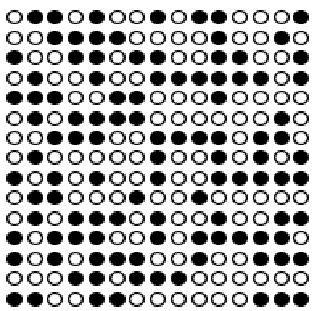
By using the neuron models discussed before we can represent a global network state by specifying for each neuron the value of its associated binary variable. As a picture with black circles denoting the active neurons and white circles denoting neurons at rest. If the neural thresholds are chosen such that a disconnected neuron would be precisely critical, we can choose the neuron states by using {-1, 1 instead of {0,1} with the following rules.

$$w_{i1}S_1(t) + \ldots + w_{iN}S_N(t) > 0 : \quad S_i(t+1) = 1$$
$$w_{i1}S_1(t) + \ldots + w_{iN}S_N(t) < 0 : \quad S_i(t+1) = -1$$

and can be represented in a picture as follows:



$$\bullet : \quad S_i = 1 \quad \text{(neuron } i \text{ firing)}$$
$$\circ : \quad S_i = -1 \quad \text{(neuron } i \text{ at rest)}$$

$$input_i > 0 : \quad S_i \to 1$$
$$input_i < 0 : \quad S_i \to -1$$

$$input_i = w_{i1}S_1 + \ldots + w_{iN}S_N$$

If the neural network is set to operate as a memory, for storing and retrieving patterns (such as pictures, words, sounds, etc.,), it is assumed the information (physically) is stored in the synapses and the pattern retrieval corresponds to dynamical process of neuron states. This leads to represent patterns to be stored as global network states, i.e., each pattern corresponds, to a specific set of binary numbers {S_1, …, S_N} or equivalently considered as a specific way of coloring circles.
In computers such pattern information is retrieved by specifying the label of each pattern, which codes for the address of its physical memory location.

Following are the principles behind the neural way of storing and retrieving information, by working out the details for a very simple model example.
Biologically realistic learning rules for synapses are required to meet the constraint that the way a given synapse w_{ij} is modified can depend only on the information locally available: the electro-chemical state properties of the neuron i and j. Simplest rule: increase w_{ij} if the neurons i and j are in the same state, decrease otherwise.
With the allowed states of neurons being {-1, 1}, this can be written as

$$S_i = S_j : \quad w_{ij} \uparrow \qquad\qquad w_{ij} \to w_{ij} + S_i S_j$$
$$S_i \neq S_j : \quad w_{ij} \downarrow$$

If that rule is applied to just one pattern

$$\{\xi_1, \ldots, \xi_N\}$$

Each component

$$\xi_i \in \{-1, 1\}$$

represents specific state of a single neuron. The following recipe can be obtained for the synapses.

$$w_{ij} = \xi_i \xi_j$$

Thus

$$input_i = w_{i1}S_1 + \ldots + w_{iN}S_N = \xi_i\left[\xi_1 S_1 + \ldots + \xi_N S_N\right]$$

So the dynamical rules for the neurons become

$$\xi_i\left[\xi_1 S_1(t) + \ldots + \xi_N S_N(t)\right] > 0 : \quad S_i(t+1) = 1$$
$$\xi_i\left[\xi_1 S_1(t) + \ldots + \xi_N S_N(t)\right] < 0 : \quad S_i(t+1) = -1$$

Note also that $\xi_i S_i(t) = 1$ if $\xi_i = S_i(t)$, and that $\xi_i S_i(t) = -1$ if $\xi_i \neq S_i(t)$. Therefore, if at time t more than half of the neurons are in the state $S_i(t) = \xi_i$ then $\xi_1 S_1(t) + \ldots + \xi_N S_N(t) > 0$. It subsequently follows from (4) that

$$\text{for all } i: \quad \text{sign}(input_i) = \xi_i \quad \text{so} \quad S_i(t+1) = \xi_i$$

If the dynamics is of the parallel type then this convergence is completed in a single iteration step. For sequential dynamics, in which neurons change state one after the another, the convergence is a gradual process.