Universality of Modern Neurons

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The previous approach of simplifying the biological model of neurons as a simple mathematical model is not made more complicated because of two reasons.

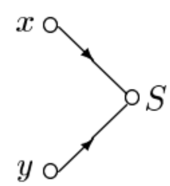
1. Solving for arbitrary control parameters and non-trivial system sizes is already impossible in spite of its apparent simplicity
2. Networks of the type proposed already are found to be universal information processing systems, such that they can perform any computation that can be performed by conventional digital computers, with appropriate selection of the synapses and thresholds.

This can be proven by making sure that the basic logical operations like AND, OR and NOT can be performed by the model neurons. These are explained in the truth tables below. For each of those gates, model neuron of the type has to be defined

$$w_1 x + w_2 y - \theta > 0 : \quad S = 1$$
$$w_1 x + w_2 y - \theta < 0 : \quad S = 0$$

AND:

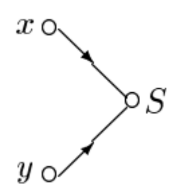
x	y	$x \wedge y$	$x+y-\frac{3}{2}$	S
0	0	0	$-3/2$	0
0	1	0	$-1/2$	0
1	0	0	$-1/2$	0
1	1	1	$1/2$	1



$$w_1 = w_2 = 1$$
$$\theta = \frac{3}{2}$$

OR:

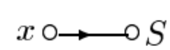
x	y	$x \vee y$	$x+y-\frac{1}{2}$	S
0	0	0	$-1/2$	0
0	1	1	$1/2$	1
1	0	1	$1/2$	1
1	1	1	$3/2$	1



$$w_1 = w_2 = 1$$
$$\theta = \frac{1}{2}$$

NOT:

x	$\neg x$	$-x+\frac{1}{2}$	S
0	1	$1/2$	1
1	0	$-1/2$	0



$$w_1 = -1$$
$$\theta = -\frac{1}{2}$$

If we employ model neurons with potentially large number of input channels, every operation involving binary numbers can be performed with a feed-forward network of at most two layers.
Every operation $\{0,1\}^N$ --> $\{0,1\}^K$ can be reduced into specific number of sub-operations M, each performing a separation of the input signals **x** (vector) - given by N binary numbers into two classes: (described by a truth table with 2^N rows

$$M : \{0,1\}^N \to \{0,1\}$$

M can be built as a neural realization of a network exercise, whose aim is simply to check whether an $\mathbf{x} \in \{0,1\}^N$ is in the set for which M(x) = 1. This set is denoted by Ω, with $L \leq 2N$ elements which can be labeled as follows $\Omega = \{y_1, \ldots, y_L\}$. In order to find if the input signals $\mathbf{y}_l$ belongs to the set of Ω Grandmother neurons G_l can be used.

$$\begin{aligned} w_1 x_1 + \ldots + w_N x_N > \theta : &\quad G_\ell = 1 \\ w_1 x_1 + \ldots + w_N x_N < \theta : &\quad G_\ell = 0 \end{aligned}$$

where

$$w_\ell = 2(2y_\ell - 1) \text{ and } \theta = 2(y_1 + \ldots + y_N) - 1$$

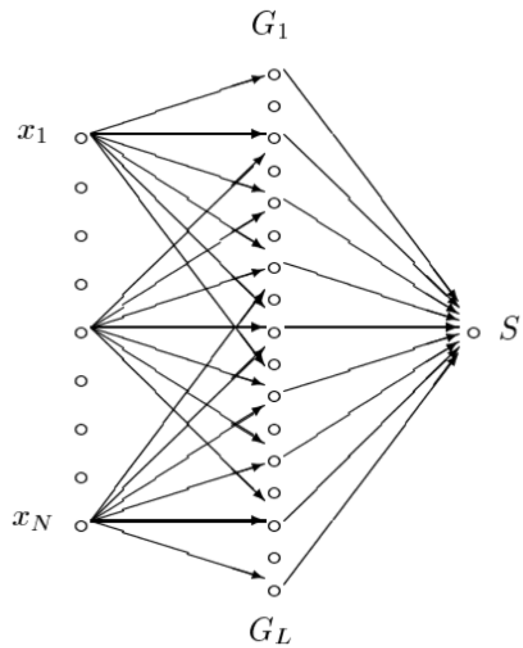
Hence the following relationship holds

$$\begin{aligned} \boldsymbol{x} = \boldsymbol{y}_\ell : &\quad G_\ell = 1 \\ \boldsymbol{x} \neq \boldsymbol{y}_\ell : &\quad G_\ell = 0 \end{aligned}$$

Then the output of the grandmother neurons is fed into a model neuron S, to determine whether or not one of the grandmothers neuron is active:

$$\begin{aligned} y_1 + \ldots + y_L > 1/2 : &\quad S = 1 \\ y_1 + \ldots + y_L < 1/2 : &\quad S = 0 \end{aligned}$$

The following diagram shows one such feed-forward neural network.



For any input **x**, the number of active neurons G_l in the first layer is either 0 (leading to the final output S = 0) or 1 (leading to the final output S = 1). In the first case the input vector **x** is apparently not in the set , in the second case it apparently is. This shows that the network thus constructed performs the separation M.