From Biology to Mathematical Models

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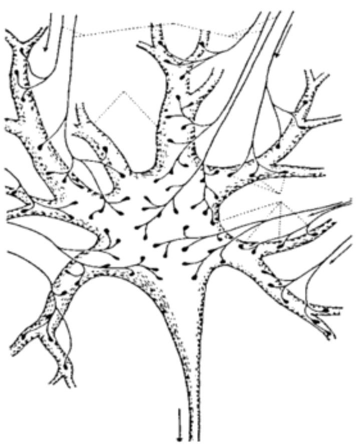
Operation of Biological Neurons:

The cell membrane of a neuron maintains concentration differences between inside and outside the cell, of various ions by a combination of the action of active ion pumps and controllable ion channels. When the neuron is at rest, the channels are closed and due to the activity of the pumps and the resultant concentration differences, the inside of neuron has a net negative electric potential of around -70mV, compared to the fluid outside.

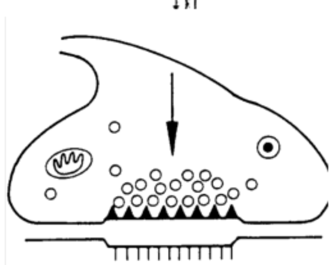
A sufficiently strong local electric excitation, which make the cell potential temporarily less negative, opens specific ion channels, in turn causes a chain reaction of other channels opening and/or closing, results in the generation of an electrical peak of around +40 mV, in a duration about 1 msec, which will propagate along the membrane at a speed of about 5m/s. This is called as action potential. This action potential serves as an electric communication signal, propagating and bifurcating along the output channel of the neuron, which is axon, to the other neurons.

Since the propagation of an action potential along an axon is the result of an active electro/chemical process, the signal will retain shape and strength, even after the bifurcation.



Drawing of a neuron.

The black blobs attached to the cell body and the dendrites (input channels) represent the synapses (adjustable terminals which determine the effect communicating neurons will have on one another's membrane potential and firing state).



Close-up of a typical synapse:

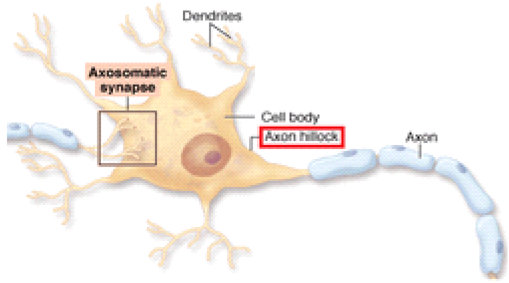
The junction between an output channel (axon) of one neuron and an input channel (dendrite) of another neuron, is called synapse. The arrival of an action potential at a synapse can trigger the release of a chemical (neurotransmitter) into the synaptic-cleft which separates the cell membranes of two neurons.

The neurotransmitter selectively opens the channels. If Na^+ channels are opened it will produce the local increase of potential at the receiving end of the synapse, which will increase the probability of the receiving neuron to start firing itself, hence that synapse is called excitatory. If Cl^- channels are opened it will decrease the local potential at the receiving end of synapse, which decrease the probability of the receiving neuron to start firing itself, hence that synapse is called inhibitory.

There is also a possibility that the arriving potential will not succeed in releasing neurotransmitter, since neurons are not perfect and introduces noise or element of uncertainty.

The case in which the receiving neuron will be triggered to fire itself depends on the cumulative effect of all excitatory and inhibitory signals arriving to the receiving neuron.

The region in the neuron membrane that is most sensitive to be triggered into sending an action potential is the hillock zone near the root of axon. The figure below shows the axon hillock. It also depicts the complete structure of neuron, synapses and dendrites.



If the potential in the axon hillock region (post synaptic potential) exceeds some neuron-specific threshold, then the neuron will fire an action potential. The firing threshold can vary randomly around some average value, which constitutes the second main source of uncertainty.
The synapses and firing thresholds are not fixed and are being updated all the time and this is the key to the adaptive and self-programming properties of neural tissue and also give the ability to store information. The amount of neurotransmitter in a synapse, available for release and the effective contact surface of a synapse are modified when the neural tissue is learning.

Creation of simple model having similar functionality as a Biological neuron:

A simple model of a neuron can be considered as having a single binary variable S denoting the state of firing. S = 1 indicates the neuron fires and S = 0 indicates the neuron is at rest. The state will be decided if the total input the neuron receives is exceeding a threshold (fire) or not (rest). The neuron's firing threshold is denoted by Θ.
The firing state of the networks is denoted by the following illustration

$$\text{firing} = \bullet, \text{ rest } = \circ.$$

The individual signals will add up linearly, weighted by the strengths of the associated synapses. The weights are represented by real variables as w_ℓ whose sign denotes the type of interaction. ($w_\ell > 0$: excitation and $w_\ell < 0$: inhibition) and the absolute value $|w_\ell|$ denotes the magnitude of the interaction.

$$input = w_1 S_1 + \ldots + w_N S_N$$

The neurons are labelled by subscripts $\ell = 1,2,\ldots,N$.
The effect on the input of a dormant neuron when it starts to fire is given by the following formula.

$$S_\ell \to 1: \quad input \to input + w_\ell \quad \begin{cases} w_\ell > 0: & input \uparrow, \quad excitation \\ w_\ell < 0: & input \downarrow, \quad inhibition \end{cases}$$

In order to indicate explicitly at which time t, the various neuron states are observed, the synaptic strength at a junction j --> i (j - sender , i - receiver) by w_{ij} and the threshold of a neuron i by Θ_i . Then this results in the following operation rules.

$$\begin{aligned} w_{i1}S_1(t) + \ldots + w_{iN}S_N(t) > \theta_i: \quad & S_i(t+1) = 1 \\ w_{i1}S_1(t) + \ldots + w_{iN}S_N(t) < \theta_i: \quad & S_i(t+1) = 0 \end{aligned}$$

These can be applied to all neurons at the same time or to one neuron at a time, using sequential dynamics.
Specifying the values of the synapses $\{w_{ij}\}$ and the thresholds Θ_i, as well as the initial network state $\{S_i(0)\}$, the system will evolve in time in a deterministic manner, and the operation of network can be characterized by giving the states $\{S_i(t)\}$ of the N neurons at subsequent times, e.g.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
$t = 0:$	1	1	0	1	0	0	1	0	0
$t = 1:$	1	0	0	1	0	1	1	1	1
$t = 2:$	0	0	1	1	1	0	0	0	1
$t = 3:$	1	0	0	1	1	1	1	0	1
$t = 4:$	0	1	1	1	0	0	1	0	1

Equivalently, neuron states at different times can be represented as a collection of colored circles following the below illustration:

$$\text{firing} = \bullet, \ \text{rest} = \circ.$$

$t=0$	$t=1$	$t=2$	$t=3$	$t=4$
●●○	●○○	○○●	●○○	○●●
●○○	●○●	●●○	●●●	●○○
●○○	●●●	○○●	●○●	●○●

Thus the operation of the neural networks have been reduced to a well-defined manipulation of a set of (binary) numbers, whose rules are extensively simplified versions of biological reality.
Binary numbers represent the states of the information processors and describes the system operation. Details of the operation depend on a set of control parameters (synapses and thresholds) which can be interpreted as representing the program.