Language Technologies Institute

Carnegie Mellon University

Algorithms for NLP

CS 11-711 · Fall 2020

Lecture 5: Language modeling

Emma Strubell

Announcements

- Recitation this week will be a P1 Q&A with Han
- P1 bakeoff: No pre-trained word embeddings or BERT. Out-of-the box linguistic annotations (e.g. pos tags), tf-idf, etc. are fine.

Probabilistic language models

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  - Summarization, question answering, …

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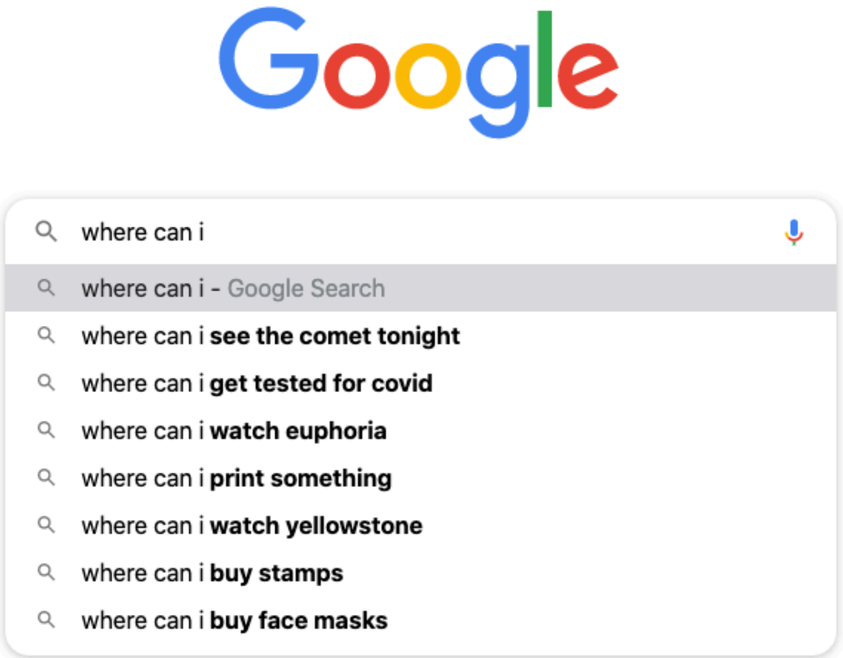

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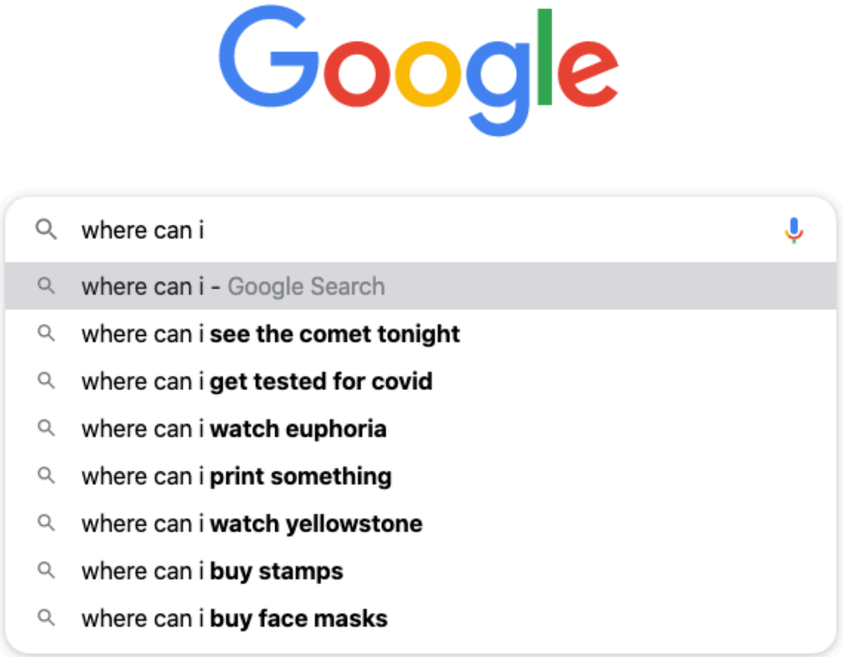

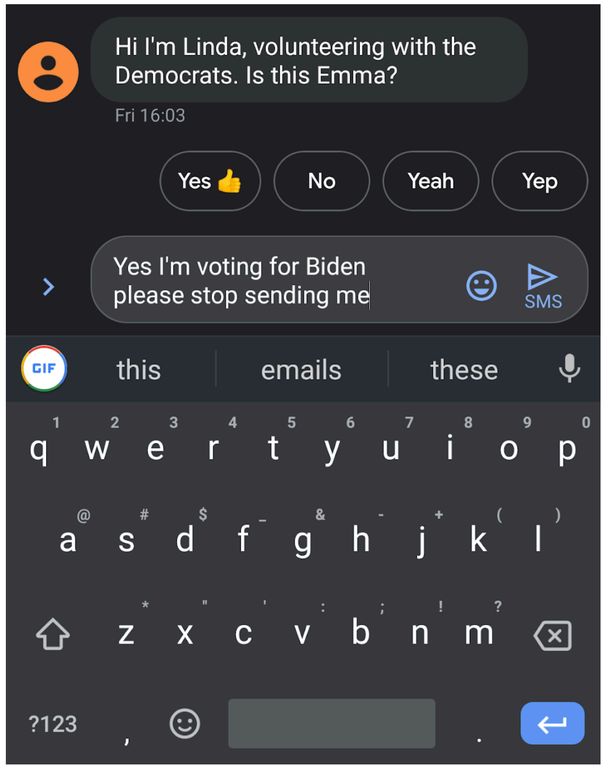

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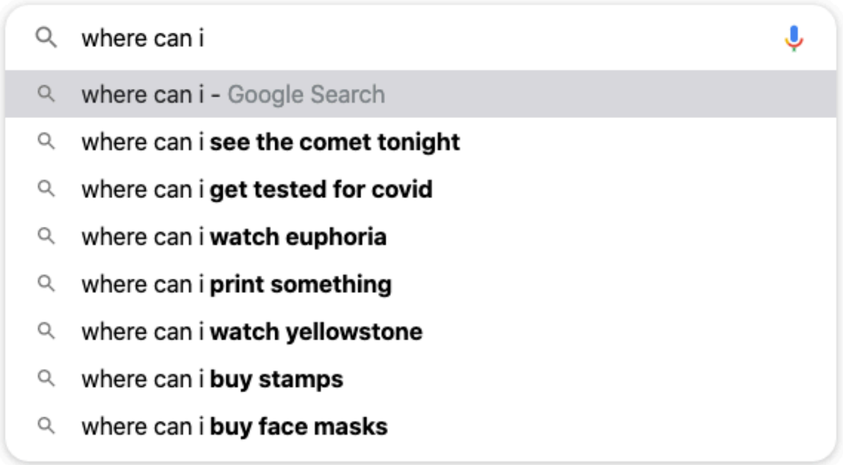

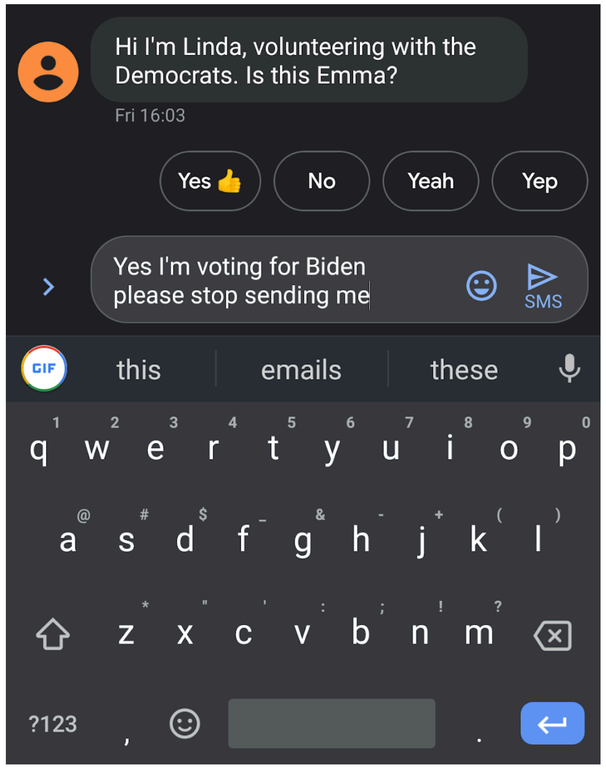

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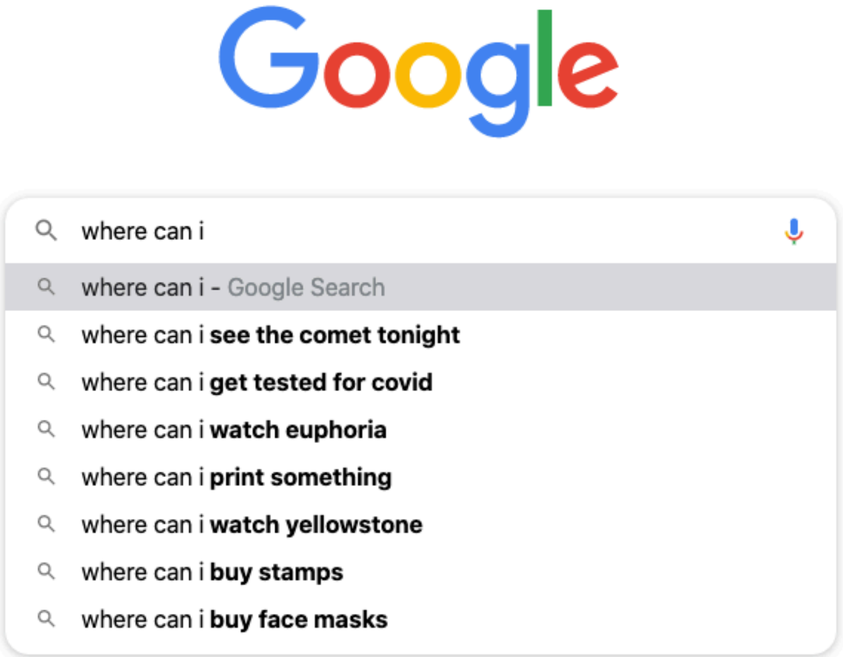

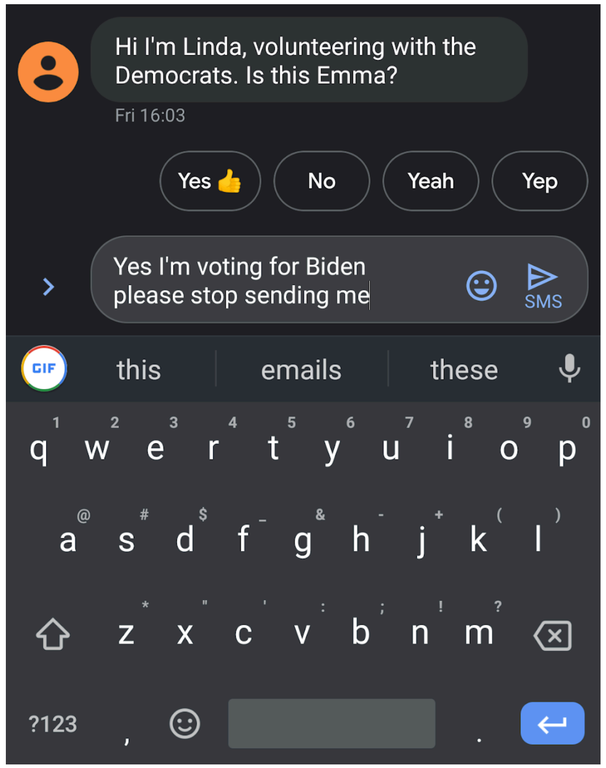

Kannan et al. 2016

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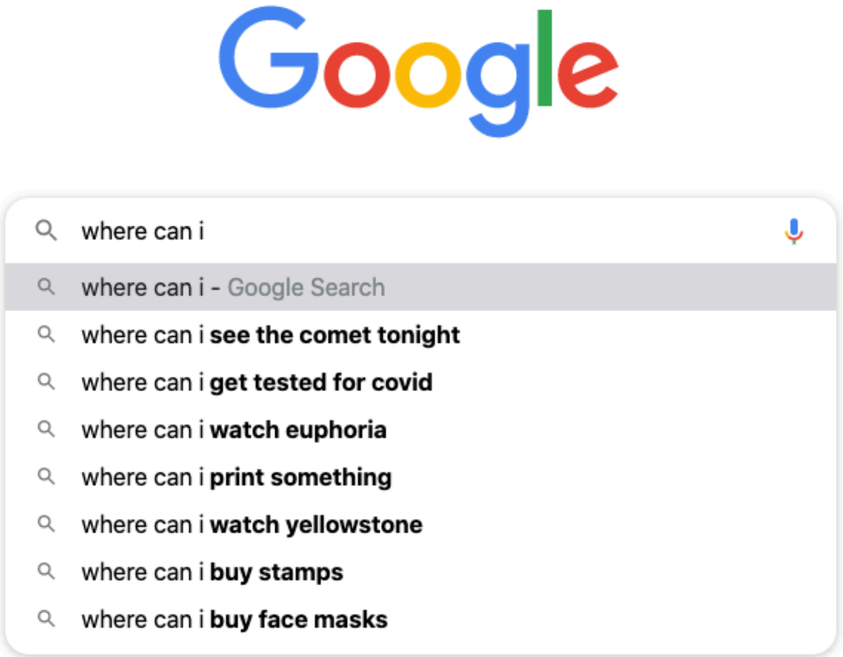

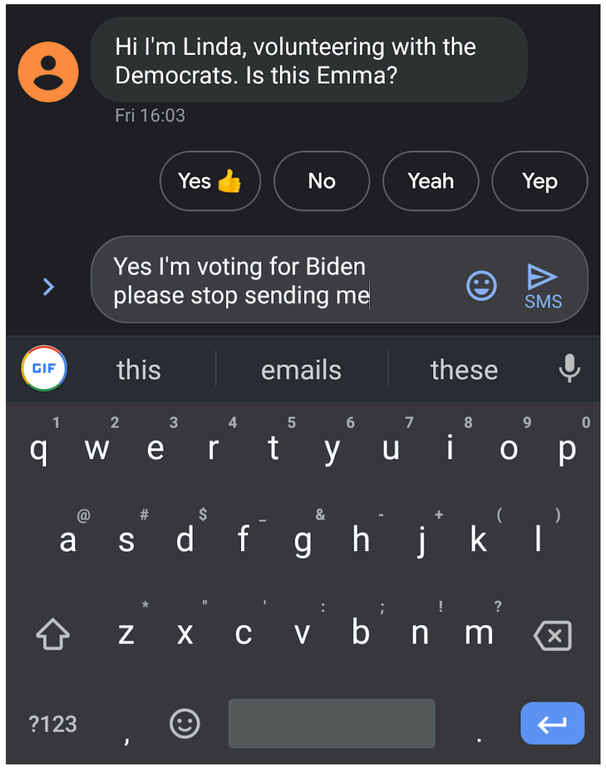

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language model

- and since ~2018, state-of-the-art vector representations of text.

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- A model that computes either of these is called a **language model** (or **LM**).

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- Chain rule in general:

$$P(x_1, x_2, x_3, ..., x_n) = P(x_1)P(x_2 \mid x_1)P(x_3 \mid x_1, x_2)...P(x_n \mid x_1, ..., x_{n-1})$$

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- For example:

$$P(\mathit{its}, \mathit{water}, \mathit{is}, \mathit{so}, \mathit{transparent}, \mathit{that}) =$$
$$P(\mathit{its}) \cdot P(\mathit{water} \mid \mathit{its}) \cdot P(\mathit{is} \mid \mathit{its\ water})$$
$$\cdot P(\mathit{so} \mid \mathit{its\ water\ is}) \cdot P(\mathit{transparent} \mid \mathit{its\ water\ is\ so})$$

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- No! Too many (infinitely) possible sentences.
- Too sparse: we'll never observe enough data to estimate.

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Andrei Markov
1856 – 1922

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- In other words, approximate each component of the product:

$$P(w_i \mid w_{i-1}, \ldots, w_2, w_1) \approx P(w_i \mid w_{i-1}, \ldots, w_{i-k})$$

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- Example sentences generated by a unigram model trained on financial news:

fifth an of futures the an incorporated a a the inflation most dollars quarter in is mass

thrift did eighty said hard ‘m july bullish

that or limited the

Bigram model

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- **Bigram model**: condition on the previous word

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texaco rose one in this issue is pursuing growth in a boiler house said mr. gurria mexico 's motion control proposal without permission from five hundred fifty five yen

outside new car parking lot of the agreement reached

this would be a record november

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1 gram	–To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have –Hill he late speaks; or! a more to leg less first you enter
2 gram	–Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow. –What means, sir. I confess she? then all sorts, he is trim, captain.
3 gram	–Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done. –This shall forbid it should be branded, if renown made it empty.
4 gram	–King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in; –It cannot be but so.

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The **computer**(s) that I just put into the machine room on the fifth floor **is** (are) crashing.

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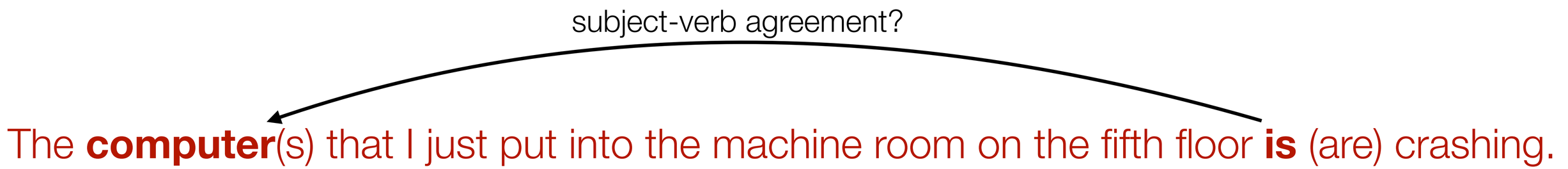

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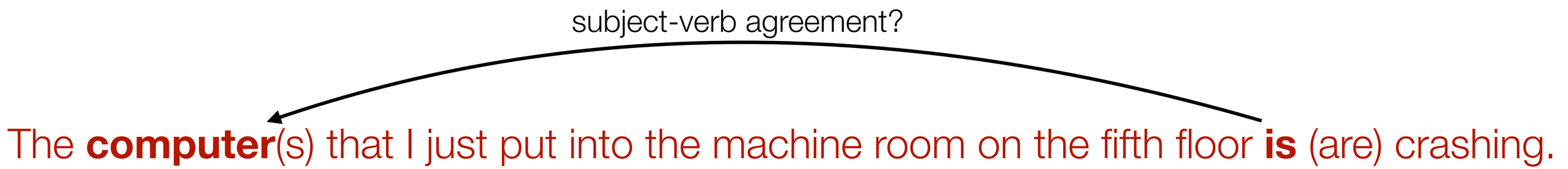

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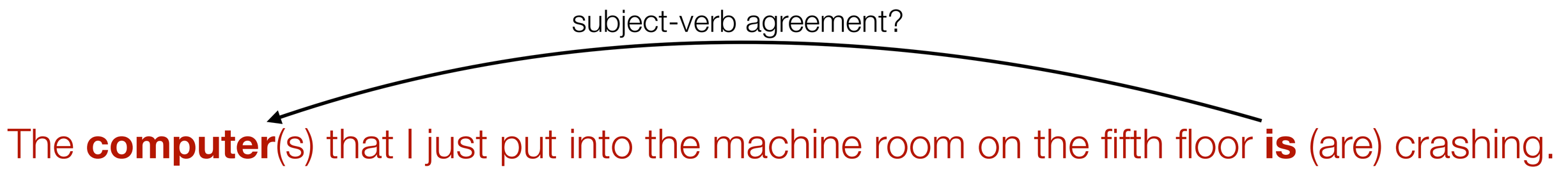

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- Another example of **bias-variance tradeoff**

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A bigger example

Berkeley Restaurant Project sentences

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can you tell me about any good cantonese restaurants close by

mid priced thai food is what i’m looking for

tell me about chez panisse

can you give me a listing of the kinds of food that are available

i’m looking for a good place to eat breakfast

when is caffe venecia open during the day

Raw bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw bigram counts

- Out of 9222 sentences:

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- Result:

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Bigram estimates of sentence probabilities

$$
\begin{aligned}
P(\langle s\rangle\ I\ want\ english\ food\ \langle /s\rangle) = \ & P(I \mid \langle s\rangle) \\
& \times P(want \mid I) \\
& \times P(english \mid want) \\
& \times P(food \mid English) \\
& \times P(\langle /s\rangle \mid food) \\
& = 0.000031
\end{aligned}
$$

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& = 0.000031
\end{aligned}
$$

- In practice:

$$\log P(I \mid \langle s\rangle) + \log P(want \mid I) + \log P(english \mid want) + \ldots$$

What kinds of knowledge?

- P(english | want) = 0.0011
- P(chinese | want) = 0.0065
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people like to talk about themselves

Evaluating language models

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  - **Perplexity**: A bad approximation for future performance.

Perplexity: Intuition

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I always order pizza with cheese and _____

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I saw a _____

mushrooms	0.1
pepperoni	0.1
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…	
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- How confused (*perplexed*) is the model on test data?
- Minimizing perplexity is the same as maximizing probability.

Lower perplexity = better model

https://paperswithcode.com/sota/language-modelling-on-penn-treebank-word

Lower perplexity = better model

- Training 38 million words, test 1.5 million words, financial news (Wall Street Journal)

n-gram order	unigram	bigram	trigram
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RANK	MODEL	TEST PERPLEXITY ↓	VALIDATION PERPLEXITY	PARAMS	EXTRA TRAINING DATA	PAPER	CODE	RESULT	YEAR
1	**GPT-3** (Zero-Shot)	20.5		175000M	✓	Language Models are Few-Shot Learners			2020
2	**BERT-Large-CAS**	31.3	36.1	395M	✗	Language Models with Transformers			2019
3	**GPT-2**	35.76		1542M	✓	Language Models are Unsupervised Multitask Learners			2019
4	**Mogrifier LSTM + dynamic eval**	44.9	44.8	24M	✗	Mogrifier LSTM			2019

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Lower perplexity = better model?

1 gram	–To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have –Hill he late speaks; or! a more to leg less first you enter
2 gram	–Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow. –What means, sir. I confess she? then all sorts, he is trim, captain.
3 gram	–Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, ’tis done. –This shall forbid it should be branded, if renown made it empty.
4 gram	–King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv’d in; –It cannot be but so.

1 gram	Months the my and issue of year foreign new exchange’s september were recession exchange new endorsed a acquire to six executives
2 gram	Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her
3 gram	They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

Zero probability bigrams

Zero probability bigrams

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

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$$P(offer \mid denied\ \ the) = 0$$

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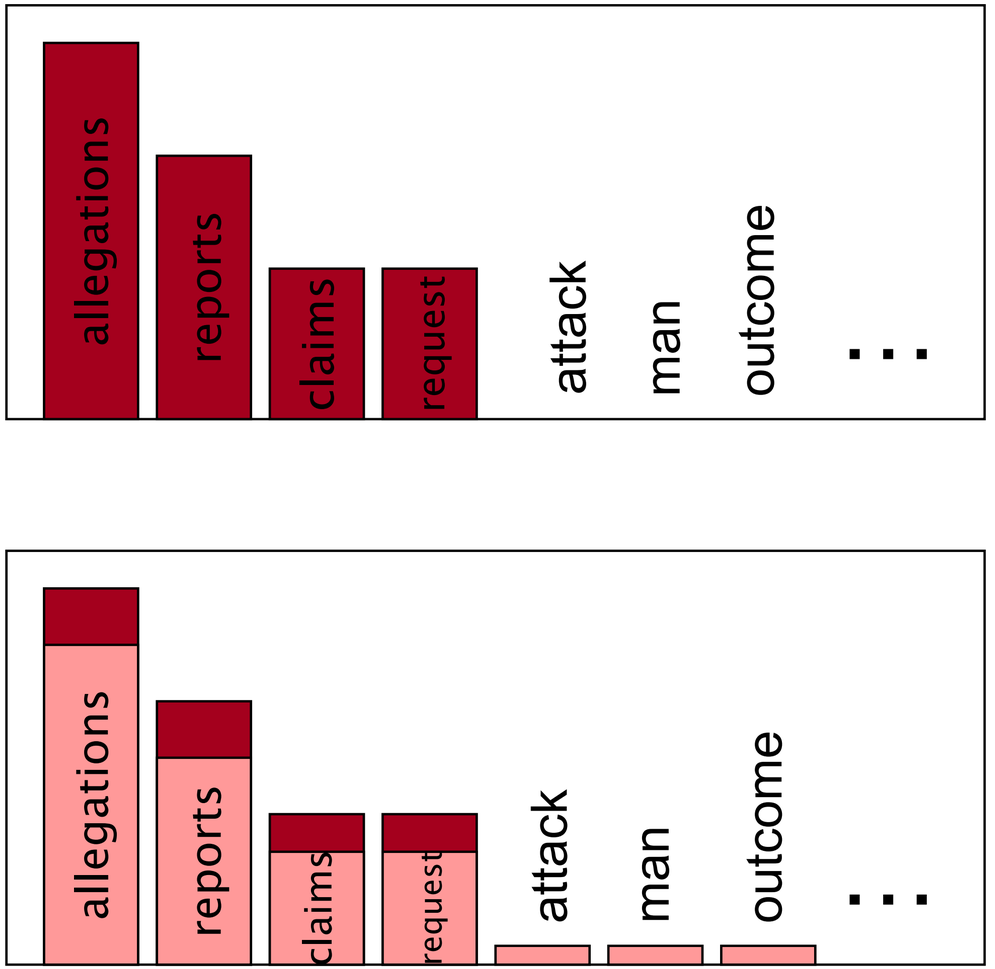
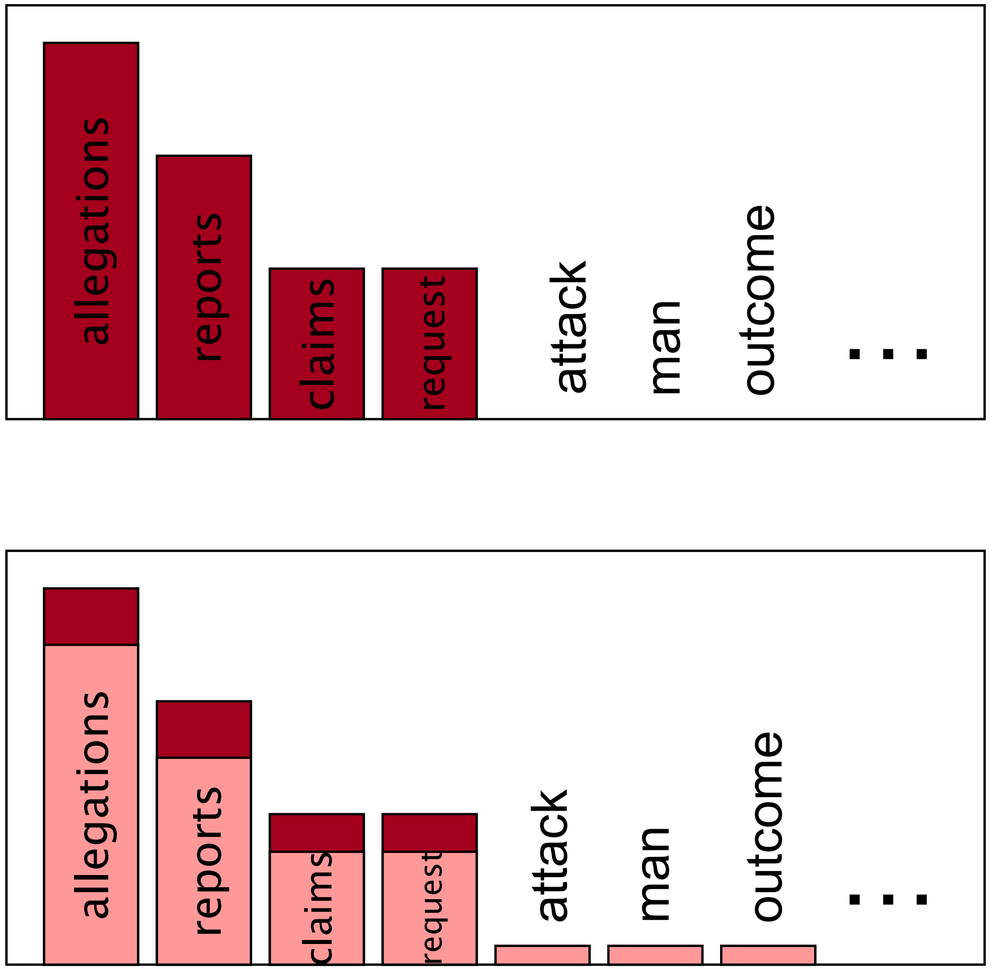


Figure from Dan Klein

Smoothing!

- As with counts for text classification, can use **smoothing** to improve generalization.
- Intuition: steal some probability mass from observed n-grams, distribute the wealth.
- Can use add-one (Laplace) smoothing:

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$



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Laplace smoothing for bigrams?

$$c^*(w_{i-1}, w_i) = \frac{(c(w_{i-1}, w_i) + 1) \cdot c(w_{i-1})}{c(w_{i-1}) + V}$$

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- Compare “reconstituted” counts with original: $c^*(w_{i-1}, w_i) = \dfrac{(c(w_{i-1}, w_i) + 1) \cdot c(w_{i-1})}{c(w_{i-1}) + V}$

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	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
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i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

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discount: $d_c = \dfrac{c^*}{c}$

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- Interpolation works better
- State-of-the-art: Extended interpolated Kneser-Ney smoothing

Linear interpolation

Linear interpolation

- Simple interpolation:

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- Better: λ_i depends on specific context

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skiing = `{^skiing$, ^ski, skii, kiin, iing, ing$}`

Huge, web-scale n-grams

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All Our N-gram are Belong to You

Thursday, August 3, 2006

Posted by Alex Franz and Thorsten Brants, Google Machine Translation Team

We processed 1,024,908,267,229 words of running text and are publishing the counts for all 1,176,470,663 five-word sequences that appear at least 40 times. There are 13,588,391 unique words, after discarding words that appear less than 200 times.

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Number of unigrams:          13,588,391
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⋮

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serve as the incoming 92
serve as the incubator 99
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serve as the indispensible 40
serve as the individual 234
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serve as the industry 607
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Scaling to huge, web-scale n-grams?

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  - Don't use strings: Huffman coding to stuff many words into two bytes.
  - Quantization: store probabilities in 4-8 bits rather than 32 bit float.

Language modeling toolkits

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  http://www.speech.sri.com/projects/srilm/

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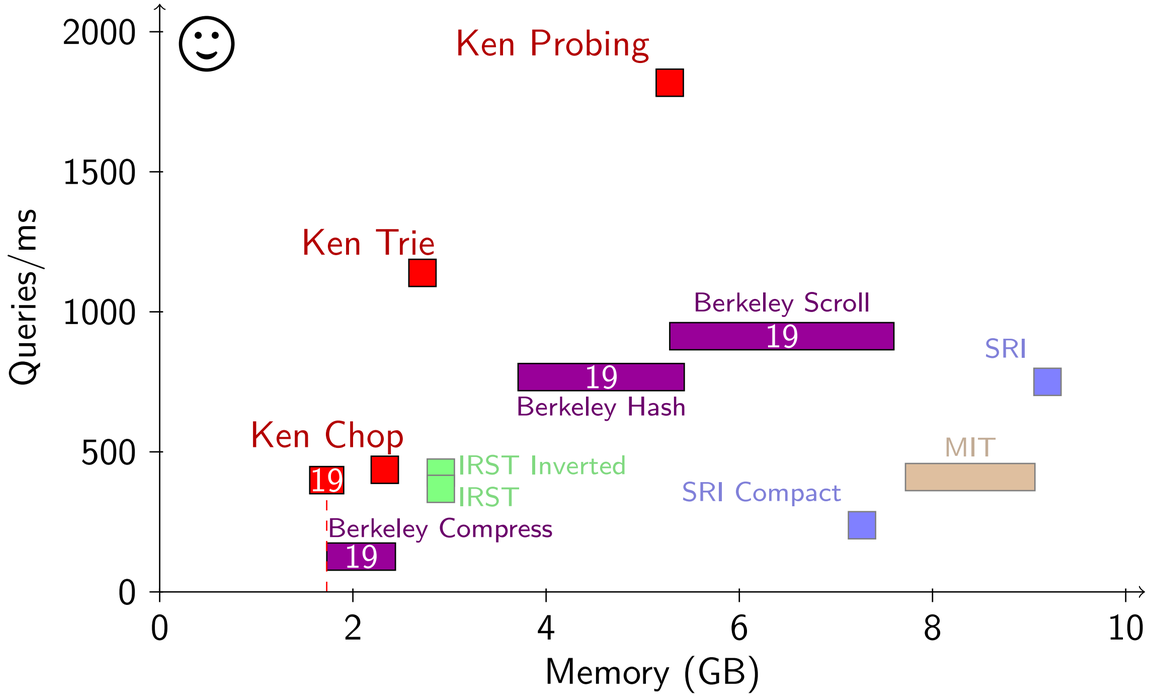


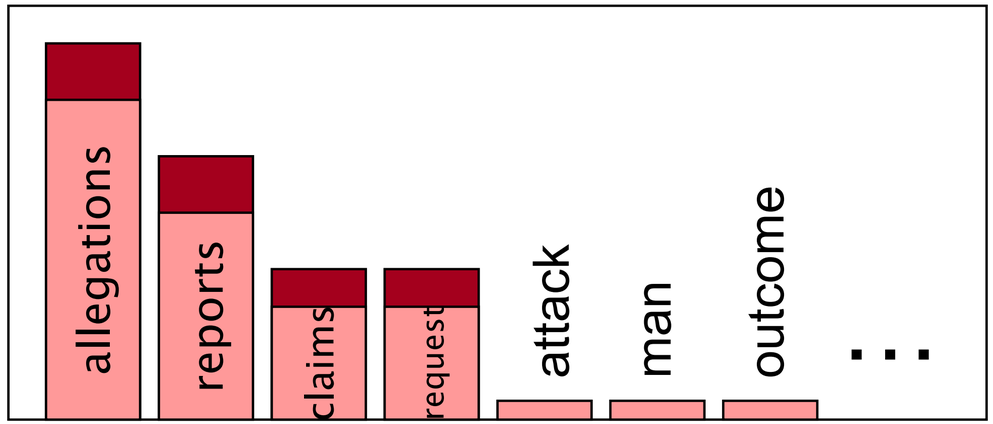
Backoff and interpolation

- Sometimes it helps to use *less* context. Intuition: Condition on less context for contexts you haven't learned much about.
- Two ways to use less context (in e.g. trigram model):
  - **Backoff**: Use trigram counts if you have good evidence; otherwise, bigram; otherwise, unigram.
  - **Interpolation**: Always mix unigram, bigram, trigram statistics.
- Interpolation works better
- State-of-the-art: **Extended interpolated Kneser-Ney smoothing**

Absolute discounting

Absolute discounting

- Removing counts from frequent n-grams, rather than adding to rare



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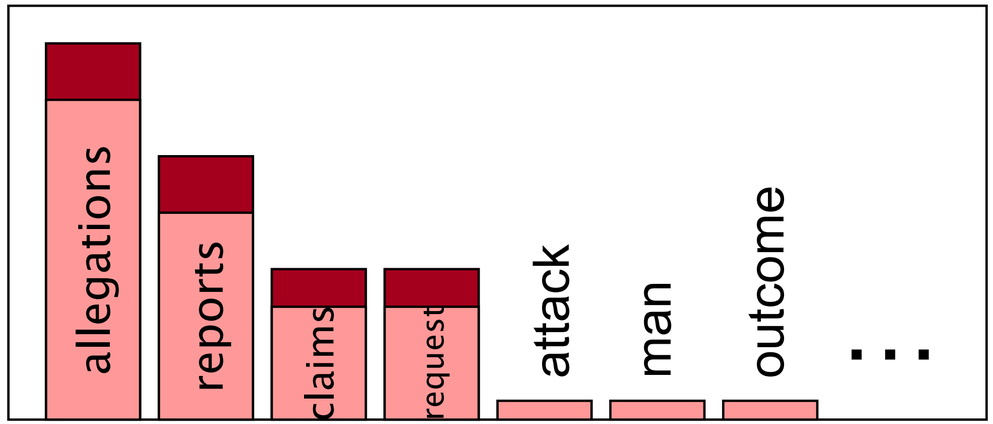

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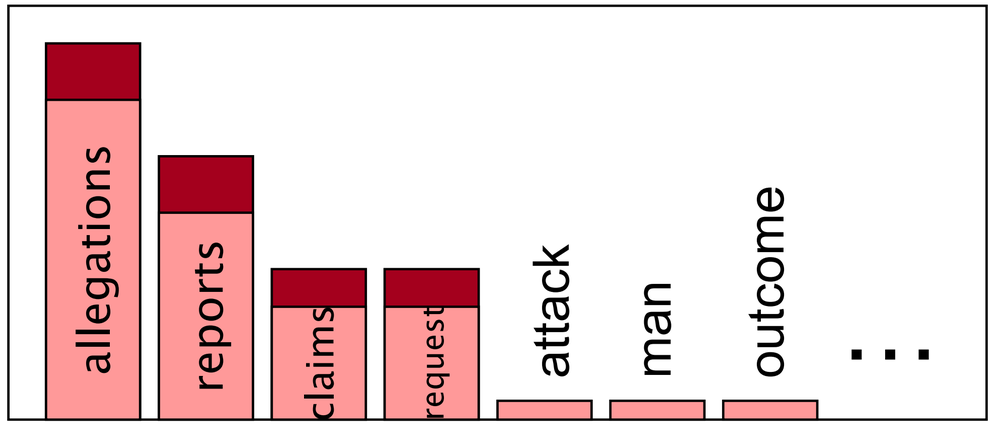

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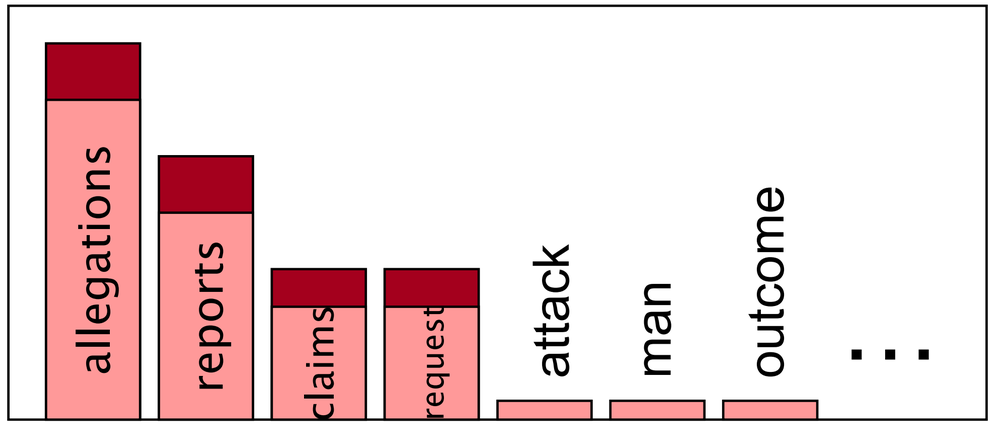

Bigram count in training	0	1	2	3	4	5	6	7	8	9
Bigram count in held-out	0.0000270	0.448	1.25	2.24	3.23	4.21	5.23	6.21	7.21	8.26
Difference	-0.000027	0.552	0.75	0.76	0.77	0.79	0.77	0.79	0.79	0.74

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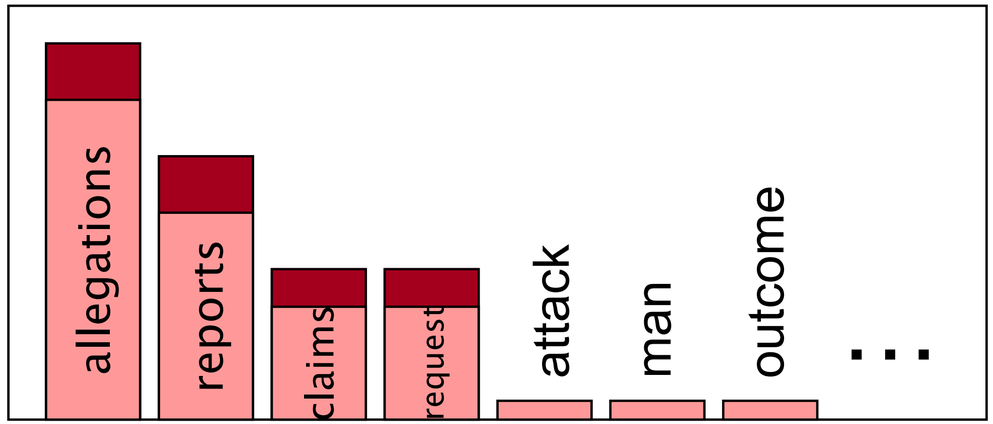

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- But, should we really use the unmodified unigram $P(w_i)$?

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- A frequent word (Kong) occurring mostly in one context (Hong) will have a low continuation probability.

Kneser-Ney Smoothing

$$\lambda(w_{i-1}) = \frac{d}{c(w_{i-1})}|\{w : c(w_{i-1}, w) > 0\}|$$

Kneser-Ney Smoothing

- Interpolated Kneser-Ney smoothing bigram probability:

$$P_{KN}(w_i \mid w_{i-1}) = \frac{\max(c(w_{i-1}, w_i) - d, 0)}{c(w_{i-1})} + \lambda(w_{i-1}) P_{continuation}(w_i)$$

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normalized discount

word types that can follow w_{i-1}
= # word types that we discounted
= # of times we applied normalized discount

Kneser-Ney Smoothing

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- Recursive formulation for n-grams (typically ~5):

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- **Modified Kneser-Ney smoothing**: Uses three different discounts for n-grams with counts 1, 2, and 3+ [Chen and Goodman 1998].

n-gram language model challenges

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- Bias/variance trade-off

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- Generalization

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  - Provide state-of-the-art dense word representations.

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- Solution: **neural language models**. Next class.
  - Better empirical performance.
  - Provide state-of-the-art dense word representations.
  - But: a lot more computation. Slower to train and infer.

Announcements

- Recitation this week will be a P1 Q&A with Han
- P1 bakeoff: No pre-trained word embeddings or BERT. Out-of-the box linguistic annotations (e.g. pos tags), tf-idf, etc. are fine.