

Task 1

$$\begin{aligned} 1. \quad \frac{dE(\theta)}{d\theta} &= \frac{d}{d\theta} \frac{1}{m} \sum_{i=1}^m (\theta \Delta R_i^{id} - \Delta R_i)^2 \\ &= \frac{1}{m} \sum_{i=1}^m 2 \Delta R_i^{id} (\theta \Delta R_i^{id} - \Delta R_i) \\ &= \frac{2}{m} \sum_{i=1}^m (\theta \Delta R_i^{id^2} - \Delta R_i \Delta R_i^{id}) \end{aligned}$$

$$0 \stackrel{!}{=} \frac{2}{m} \sum_{i=1}^m (\theta \Delta R_i^{id^2} - \Delta R_i \Delta R_i^{id})$$

$$0 = \sum_{i=1}^m (\theta \Delta R_i^{id^2} - \Delta R_i \Delta R_i^{id})$$

$$\sum_{i=1}^m (\theta \Delta R_i^{id^2}) = \sum_{i=1}^m (\Delta R_i \Delta R_i^{id})$$

$$\theta = \frac{\sum_{i=1}^m (\Delta R_i \Delta R_i^{id})}{\sum_{i=1}^m (\Delta R_i^{id^2})}$$

$$\begin{aligned}
 2. \quad \frac{dE(\theta_0, \theta_1)}{d\theta_0} &= \frac{d}{d\theta_0} \frac{1}{m} \sum_{i=1}^m (\theta_0 + \theta_1 \Delta R_i^{\text{id}} - \Delta R_i)^2 \\
 &= \frac{1}{m} \sum_{i=1}^m 2(\theta_0 + \theta_1 \Delta R_i^{\text{id}} - \Delta R_i) \\
 &= \frac{2}{m} \sum_{i=1}^m (\theta_0 + \theta_1 \Delta R_i^{\text{id}} - \Delta R_i) \\
 &= \frac{2}{m} \sum_{i=1}^m (\theta_1 \Delta R_i^{\text{id}} - \Delta R_i) + 2\theta_0
 \end{aligned}$$

$$\begin{aligned}
 \frac{dE(\theta_0, \theta_1)}{d\theta_1} &= \frac{d}{d\theta_1} \frac{1}{m} \sum_{i=1}^m (\theta_0 + \theta_1 \Delta R_i^{\text{id}} - \Delta R_i)^2 \\
 &= \frac{1}{m} \sum_{i=1}^m 2\Delta R_i^{\text{id}} (\theta_0 + \theta_1 \Delta R_i^{\text{id}} - \Delta R_i) \\
 &= \frac{2}{m} \sum_{i=1}^m (\theta_0 \Delta R_i^{\text{id}} + \theta_1 \Delta R_i^{\text{id}2} - \Delta R_i \Delta R_i^{\text{id}})
 \end{aligned}$$

$$0 \stackrel{!}{=} \frac{2}{m} \sum_{i=1}^m (\theta_1 \Delta R_i^{\text{id}} - \Delta R_i) + 2\theta_0$$

$$0 = \frac{1}{m} \sum_{i=1}^m (\theta_1 \Delta R_i^{\text{id}} - \Delta R_i) + \theta_0$$

$$\rightarrow \theta_0 = -\frac{1}{m} \sum_{i=1}^m (\theta_1 \Delta R_i^{\text{id}} - \Delta R_i) \quad \text{I}$$

$$\rightarrow 0 = \sum_{i=1}^m (\theta_1 \Delta R_i^{\text{id}} - \Delta R_i) + m\theta_0$$

$$\sum_{i=1}^m (\theta_1 \Delta R_i^{\text{id}}) = \sum_{i=1}^m (\Delta R_i) - m\theta_0$$

$$\underline{\underline{\theta_1 = \frac{\sum_{i=1}^m (\Delta R_i) - m\theta_0}{\sum_{i=1}^m (\Delta R_i^{\text{id}})}}} \quad \text{II}$$

$$0 \doteq \frac{2}{m} \sum_{i=1}^m (\theta_0 \Delta R_i^{id} + \theta_1 \Delta R_i^{id^2} - \Delta R_i \Delta R_i^{id})$$

$$0 = \sum_{i=1}^m (\theta_0 \Delta R_i^{id} + \theta_1 \Delta R_i^{id^2} - \Delta R_i \Delta R_i^{id})$$

$$\sum_{i=1}^m (\theta_0 \Delta R_i^{id}) = \sum_{i=1}^m (\Delta R_i \Delta R_i^{id} - \theta_1 \Delta R_i^{id^2})$$

$$\theta_0 = \frac{\sum_{i=1}^m (\Delta R_i \Delta R_i^{id} - \theta_1 \Delta R_i^{id^2})}{\sum_{i=1}^m (\Delta R_i^{id})} \quad \text{III}$$

$$\sum_{i=1}^m (\theta_1 \Delta R_i^{id^2}) = \sum_{i=1}^m (\Delta R_i \Delta R_i^{id} - \theta_0 \Delta R_i^{id})$$

$$\theta_1 = \frac{\sum_{i=1}^m (\Delta R_i \Delta R_i^{id} - \theta_0 \Delta R_i^{id})}{\sum_{i=1}^m (\Delta R_i^{id^2})} \quad \text{IV}$$

$$\text{II+IV: } \frac{\sum_{i=1}^m (\Delta R_i) - m \theta_0}{\sum_{i=1}^m (\Delta R_i^{id})} = \frac{\sum_{i=1}^m (\Delta R_i \Delta R_i^{id} - \theta_0 \Delta R_i^{id})}{\sum_{i=1}^m (\Delta R_i^{id^2})}$$

$$\theta_0 \frac{-m}{\sum_{i=1}^m (\Delta R_i^{id})} + \frac{\sum_{i=1}^m (\Delta R_i)}{\sum_{i=1}^m (\Delta R_i^{id})} = \theta_0 \frac{-\sum_{i=1}^m (\Delta R_i^{id})}{\sum_{i=1}^m (\Delta R_i^{id^2})} + \frac{\sum_{i=1}^m (\Delta R_i \Delta R_i^{id})}{\sum_{i=1}^m (\Delta R_i^{id^2})}$$

$$\theta_0 \left(\frac{\sum_{i=1}^m (\Delta R_i^{id})}{\sum_{i=1}^m (\Delta R_i^{id^2})} - \frac{m}{\sum_{i=1}^m (\Delta R_i^{id})} \right) = \frac{\sum_{i=1}^m (\Delta R_i \Delta R_i^{id})}{\sum_{i=1}^m (\Delta R_i^{id^2})} - \frac{\sum_{i=1}^m (\Delta R_i)}{\sum_{i=1}^m (\Delta R_i^{id})}$$

$$\theta_0 = \left(\frac{\sum_{i=1}^m (\Delta R_i \Delta R_i^{id})}{\sum_{i=1}^m (\Delta R_i^{id^2})} - \frac{\sum_{i=1}^m (\Delta R_i)}{\sum_{i=1}^m (\Delta R_i^{id})} \right) \left(\frac{\sum_{i=1}^m (\Delta R_i^{id})}{\sum_{i=1}^m (\Delta R_i^{id^2})} - \frac{m}{\sum_{i=1}^m (\Delta R_i^{id})} \right)^{-1}$$