1. $$\frac{dE(\theta)}{d\theta} = \frac{d}{d\theta} \frac{1}{m} \sum_{i=1}^{m} (\theta \Delta R_i^{id} - \Delta R_i)^2$$

$$= \frac{1}{m} \sum_{i=1}^{m} 2 \Delta R_i^{id} (\theta \Delta R_i^{id} - \Delta R_i)$$

$$= \underline{\frac{2}{m} \sum_{i=1}^{m} (\theta \Delta R_i^{id2} - \Delta R_i \Delta R_i^{id})}$$

$$0 \stackrel{!}{=} \frac{2}{m} \sum_{i=1}^{m} (\theta \Delta R_i^{id2} - \Delta R_i \Delta R_i^{id})$$

$$0 = \sum_{i=1}^{m} (\theta \Delta R_i^{id2} - \Delta R_i \Delta R_i^{id})$$

$$\sum_{i=1}^{m} (\theta \Delta R_i^{id2}) = \sum_{i=1}^{m} (\Delta R_i \Delta R_i^{id})$$

$$\underline{\underline{\theta = \frac{\sum_{i=1}^{m} (\Delta R_i \Delta R_i^{id})}{\sum_{i=1}^{m} (\Delta R_i^{id2})}}}$$

2. $\frac{d\,E(\theta_0,\theta_1)}{d\theta_0} = \frac{d}{d\theta_0}\,\frac{1}{m}\sum_{i=1}^{m}(\theta_0+\theta_1 sR_i^{id}-sR_i)^2$

$$= \frac{1}{m}\sum_{i=1}^{m} 2(\theta_0+\theta_1 sR_i^{id}-sR_i)$$

$$= \frac{2}{m}\sum_{i=1}^{m}(\theta_0+\theta_1 sR_i^{id}-sR_i)$$

$$= \frac{2}{m}\sum_{i=1}^{m}(\theta_1 sR_i^{id}-sR_i)+2\theta_0$$

$$\frac{d\,E(\theta_0,\theta_1)}{d\theta_1} = \frac{d}{d\theta_1}\,\frac{1}{m}\sum_{i=1}^{m}(\theta_0+\theta_1 sR_i^{id}-sR_i)^2$$

$$= \frac{1}{m}\sum_{i=1}^{m} 2sR_i^{id}(\theta_0+\theta_1 sR_i^{id}-sR_i)$$

$$= \frac{2}{m}\sum_{i=1}^{m}(\theta_0 sR_i^{id}+\theta_1 R_i^{id2}-sR_i\,sR_i^{id})$$

$$0 \overset{!}{=} \frac{2}{m}\sum_{i=1}^{m}(\theta_1 sR_i^{id}-sR_i)+2\theta_0$$

$$0 = \frac{1}{m}\sum_{i=1}^{m}(\theta_1 sR_i^{id}-sR_i)+\theta_0$$

$$\theta_0 = -\frac{1}{m}\sum_{i=1}^{m}(\theta_1 sR_i^{id}-sR_i) \qquad \text{I}$$

$$0 = \sum_{i=1}^{m}(\theta_1 sR_i^{id}-sR_i)+m\theta_0$$

$$\sum_{i=1}^{m}(\theta_1 sR_i^{id}) = \sum_{i=1}^{m}(sR_i)-m\theta_0$$

$$\theta_1 = \frac{\sum_{i=1}^{m}(sR_i)-m\theta_0}{\sum_{i=1}^{m}(sR_i^{id})} \qquad \text{II}$$

$$0 \doteq \frac{2}{m} \sum_{i=1}^{m} \left( \Theta_0 \Delta R_i^{id} + \Theta_1 \Delta R_i^{id^2} - \Delta R_i \Delta R_i^{id} \right)$$

$$0 = \sum_{i=1}^{m} \left( \Theta_0 \Delta R_i^{id} + \Theta_1 \Delta R_i^{id^2} - \Delta R_i \Delta R_i^{id} \right)$$

$$\sum_{i=1}^{m} \left( \Theta_0 \Delta R_i^{id} \right) = \sum_{i=1}^{m} \left( \Delta R_i \Delta R_i^{id} - \Theta_1 \Delta R_i^{id^2} \right)$$

$$\Theta_0 = \frac{\sum_{i=1}^{m} \left( \Delta R_i \Delta R_i^{id} - \Theta_1 \Delta R_i^{id^2} \right)}{\sum_{i=1}^{m} \left( \Delta R_i^{id} \right)} \qquad \text{II}$$

$$\sum_{i=1}^{m} \left( \Theta_1 \Delta R_i^{id^2} \right) = \sum_{i=1}^{m} \left( \Delta R_i \Delta R_i^{id} - \Theta_0 \Delta R_i^{id} \right)$$

$$\Theta_1 = \frac{\sum_{i=1}^{m} \left( \Delta R_i \Delta R_i^{id} - \Theta_0 \Delta R_i^{id} \right)}{\sum_{i=1}^{m} \left( \Delta R_i^{id^2} \right)} \qquad \text{IV}$$

$$\text{II} + \text{IV}: \quad \frac{\sum_{i=1}^{m} \left( \Delta R_i \right) - m \Theta_0}{\sum_{i=1}^{m} \left( \Delta R_i^{id} \right)} = \frac{\sum_{i=1}^{m} \left( \Delta R_i \Delta R_i^{id} - \Theta_0 \Delta R_i^{id} \right)}{\sum_{i=1}^{m} \left( \Delta R_i^{id^2} \right)}$$

$$\Theta_0 \frac{-m}{\sum_{i=1}^{m} \left( \Delta R_i^{id} \right)} + \frac{\sum_{i=1}^{m} \left( \Delta R_i \right)}{\sum_{i=1}^{m} \left( \Delta R_i^{id} \right)} = \Theta_0 \frac{-\sum_{i=1}^{m} \left( \Delta R_i^{id} \right)}{\sum_{i=1}^{m} \left( \Delta R_i^{id^2} \right)} + \frac{\sum_{i=1}^{m} \left( \Delta R_i \Delta R_i^{id} \right)}{\sum_{i=1}^{m} \left( \Delta R_i^{id^2} \right)}$$

$$\Theta_0 \left( \frac{\sum_{i=1}^{m} \left( \Delta R_i^{id} \right)}{\sum_{i=1}^{m} \left( \Delta R_i^{id^2} \right)} - \frac{m}{\sum_{i=1}^{m} \left( \Delta R_i^{id} \right)} \right) = \frac{\sum_{i=1}^{m} \left( \Delta R_i \Delta R_i^{id} \right)}{\sum_{i=1}^{m} \left( \Delta R_i^{id^2} \right)} - \frac{\sum_{i=1}^{m} \left( \Delta R_i \right)}{\sum_{i=1}^{m} \left( \Delta R_i^{id} \right)}$$

$$\Theta_0 = \left( \frac{\sum_{i=1}^{m} \left( \Delta R_i \Delta R_i^{id} \right)}{\sum_{i=1}^{m} \left( \Delta R_i^{id^2} \right)} - \frac{\sum_{i=1}^{m} \left( \Delta R_i \right)}{\sum_{i=1}^{m} \left( \Delta R_i^{id} \right)} \right) \left( \frac{\sum_{i=1}^{m} \left( \Delta R_i^{id} \right)}{\sum_{i=1}^{m} \left( \Delta R_i^{id^2} \right)} - \frac{m}{\sum_{i=1}^{m} \left( \Delta R_i^{id} \right)} \right)^{-1}$$