Given the following three-dimensional points and their actual labels:

$$\mathbf{x}_A = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad , \quad y_A = -1$$

$$\mathbf{x}_{B} = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} \quad , \quad y_{B} = +1$$

$$\mathbf{x}_C = \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \quad , \quad \mathbf{y}_C = +1$$

If we initial the vector of weights for each dimension (including w_0) as

$$\tilde{\mathbf{w}} = \begin{pmatrix} -3 \\ 2 \\ 2 \\ 0 \end{pmatrix}$$
. What's the vector of weights using PLA until convergence?

Answer:

Based on the initial weights, we can predict the label of each point as

$$\begin{split} \hat{y}_A &= sign\Big(\tilde{\mathbf{w}}^T \tilde{\mathbf{x}}_A\Big) \\ &= sign\Bigg(\begin{bmatrix} -3 \\ 2 \\ 2 \\ 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \\ &\text{, where } \hat{y}_A \text{ means the predicted label of } A. \\ &= sign\big(-3 \times 1 + 2 \times 1 + 2 \times 1 + 0 \times 0 \big) \\ &= +1 \end{split}$$

Since the actual label of A is $y_A = -1$, so the label of A is predicted incorrectly by the initial weights, and PLA uses the input vector of A and its actual label to adjust the vector of weights, as follows:

$$\tilde{\mathbf{w}}_{new} = \tilde{\mathbf{w}}_{old} + y_A \tilde{\mathbf{x}}_A$$

$$= \begin{pmatrix} -3 \\ 2 \\ 2 \\ 0 \end{pmatrix} + (-1) \times \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}.$$

$$= \begin{pmatrix} -4 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

Based on the new weights, we can predict the label of each point as

$$\begin{split} \hat{y}_A &= sign\left(-4\times1+1\times1+1\times1+0\times0\right) = -1,\\ \hat{y}_B &= sign\left(-4\times1+1\times3+1\times3+0\times1\right) = +1,\\ \hat{y}_C &= sign\left(-4\times1+1\times4+1\times3-0\times1\right) = +1. \end{split}$$

The vector of weights stops updating since the labels of all points are predicted correctly.