

Given the following 4 points with 2 attributes:

A: (2, 2), B: (2, 3), C: (3, 5), D: (4, 3).

The distance function is Euclidean distance.

Perform agglomerative hierarchical clustering using the single link (or MIN) approach and the complete link (or MAX) approach, respectively. Show the order in which the points are merged.

$$d(A, B) = 1, \quad d(A, C) = \sqrt{10}, \quad d(A, D) = \sqrt{5},$$

$$d(B, C) = \sqrt{5}, \quad d(B, D) = 2, \quad d(C, D) = \sqrt{5}. \text{ Thus,}$$

(1) For the single link (or MIN) approach:

A and B are merged firstly. We denote the cluster containing A and B by C1.

Then, for C, D, C1:

$$d(C, C1) = \sqrt{5}, \quad d(D, C1) = 2, \quad d(C, D) = \sqrt{5}.$$

Thus, D and C1 (A and B) are then merged, which is denoted by C2.

Finally, C and C2 (A, B and D) are merged.

(2) For the complete link (or MAX) approach:

A and B are merged firstly. We denote the cluster containing A and B by C1.

Then, for C, D, C1:

$$d(C, C1) = \sqrt{10}, \quad d(D, C1) = \sqrt{5}, \quad d(C, D) = \sqrt{5}.$$

Thus, we have the following two orders:

(a) D and C1 (A and B) are then merged, which is denoted by C2.

Finally, C and C2 (A, B and D) are merged.

(b) C and D are then merged, which is denoted by C2.

Finally, C1 (A and B) and C2 (C and D) are merged.