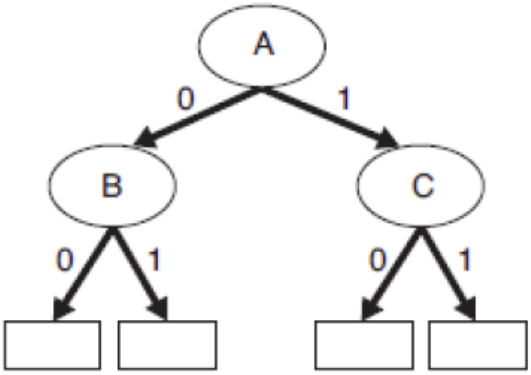
1. Consider the decision tree shown in the following figure:



Training:

Instance	A	B	C	Class
1	0	0	0	+
2	0	0	1	+
3	0	1	0	+
4	0	1	1	–
5	1	0	0	+
6	1	0	0	+
7	1	1	0	–
8	1	0	1	+
9	1	1	0	–
10	1	1	0	–

a) Estimate the training error of the tree.

Instances associated with the left child node of B:

1(+), 2(+)

Instances associated with the right child node of B:

3(+), 4(-)

Instances associated with the left child node of C:

5(+), 6(+), 7(-), 9(-), 10(-)

Instances associated with the right child node of C:

8(+)

Accordingly, the estimated training error is:

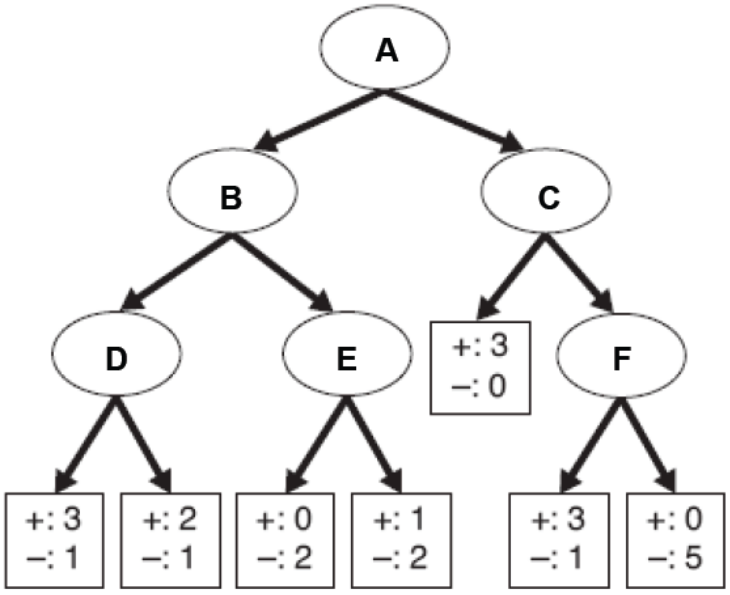
$$\frac{0+1+2+0}{10} = \frac{3}{10}$$

b) Estimate the generalization error by using a penalty term of 0.5 for each leaf node.

Using a penalty term of 0.5 for each leaf node, the estimated generalization error becomes

$$\frac{0+1+2+0+0.5\times 4}{10}=\frac{5}{10}$$

2. We consider the following decision tree example in the lecture notes. Suppose a penalty term of 1.5 is assigned to each leaf node.



a) Estimate the generalization error if the sub-tree associated with node F is pruned and replaced with a leaf node.

If the sub-tree associated with node F is pruned and replaced with a leaf node, the estimated generalization error is

$$\frac{6+1.5\times 6}{24}=\frac{15}{24}$$

b) Estimate the generalization error if the sub-trees associated with nodes D and E are pruned and replaced with leaf nodes.

If the sub-tree associated with nodes D and E are pruned and replaced with leaf nodes, the estimated generalization error is

$$\frac{4+1.5\times 5}{24}=\frac{11.5}{24}$$

c) Estimate the generalization error if the above operations are performed together.

If the above operations are performed together, the estimated generalization error is

$$\frac{6+1.5\times 4}{24}=\frac{12}{24}$$