Artificial Intelligence
——Perceptron Learning Algorithm

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课前问答

- k-NN模型的优缺点有哪些？用k-NN开展分类或回归任务的结果是否稳定？不稳定的话，主要的影响因素有哪些?
- 在基于k-NN的分类任务中，当k取值为训练集的样本个数时，结果是否稳定?
- 朴素贝叶斯分类模型的假设是什么?
- 在实验设计中，验证集的作用是什么?

线性回归

- Least-squares solutions

$$n^{-1}\sum_{i=1}^{n}(y_i - w_0 - w_1 x_i) = 0$$

$$n^{-1}\sum_{i=1}^{n}x_i(y_i - w_0 - w_1 x_i) = 0$$

$$Q(w_0, w_1) = \min_{w_0, w_1}\sum_{i=1}^{n}(y_i - w_0 - w_1 x_i)^2$$

$$\partial Q(w_0, w_1) / \partial w_0 = 0 \qquad \partial Q(w_0, w_1) / \partial w_1 = 0$$

$$-2\sum_{i=1}^{n}(y_i - w_0 - w_1 x_i) = 0 \qquad -2\sum_{i=1}^{n}x_i(y_i - w_0 - w_1 x_i) = 0$$

线性回归

- Least-squares solutions

$$w_0 = \bar{y} - w_1 \bar{x}$$

$$w_1 = \frac{\sum_{i=1}^{n} x_i (y_i - \bar{y})}{\sum_{i=1}^{n} x_i (x_i - \bar{x})}$$

$$= \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Perceptron Learning Algorithm

- Dealing with all attributes jointly which are continuous variables
- Discrete random variables can be changed to continuous variables

Perceptron Learning Algorithm

- Dealing with all attributes jointly which are continuous variables
- Discrete random variables can be changed to continuous variables
- For $\mathbf{x}=(x_1,x_2,\ldots,x_d)$ with d features, compute a weighted 'score' and
  predict +1(good) if $\sum_{k=1}^{d} w_k x_k > threshold$
  predict -1(bad) if $\sum_{k=1}^{d} w_k x_k < threshold$
- $\mathbf{y}$={+1(good), -1(bad)}

$$h(\mathbf{x}) = sign\left(\left(\sum_{k=1}^{d} w_k x_k\right) - threshold\right)$$

Perceptron Learning Algorithm

$$h(\mathbf{x}) = sign\left(\left(\sum_{k=1}^{d} w_k x_k\right) - threshold\right)$$

$$= sign\left(\left(\sum_{k=1}^{d} w_k x_k\right) + \underbrace{(-threshold)}_{w_0} \cdot \underbrace{(+1)}_{x_0}\right)$$

$$= sign\left(\sum_{j=0}^{d} w_j x_j\right)$$

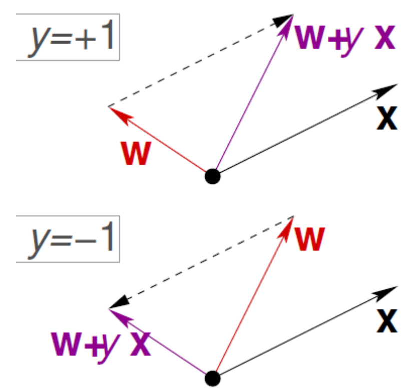
$$= sign\left(\tilde{\mathbf{W}}^{\mathrm{T}} \tilde{\mathbf{X}}\right)$$

Perceptron Learning Algorithm

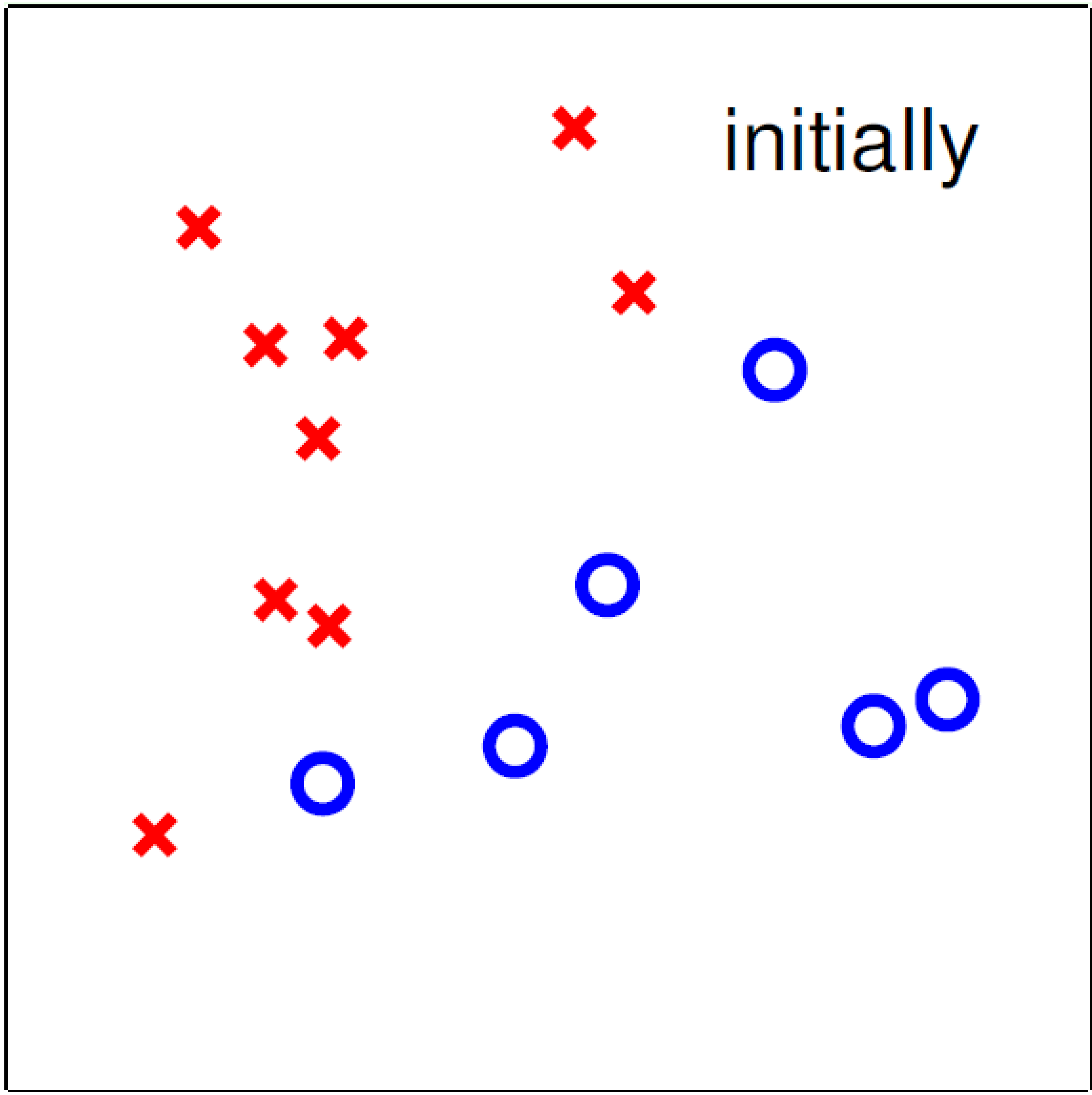
- **Difficult:** the set of $h(\mathbf{x})$ is of infinite size
- **Idea:** start from some initial weight vector $\mathbf{w}_{(0)}$, and "correct" its mistakes on D
- For t = 0, 1,…
  - find a mistake of $\mathbf{w}_{(\mathrm{t})}$ called $(\mathbf{x}_{i(t)}, y_{i(t)})$

    $$sign\left(\tilde{\mathbf{w}}_{(t)}^{\mathrm{T}}\tilde{\mathbf{x}}_{i(t)}\right) \neq y_{i(t)}$$
  - (try to) correct the mistake by

    $$\tilde{\mathbf{w}}_{(t+1)} \leftarrow \tilde{\mathbf{w}}_{(t)} + y_{i(t)}\tilde{\mathbf{x}}_{i(t)}$$
  - until no more mistakes
- Return last $\mathbf{W}$ (called $\mathbf{W}_{\mathrm{PLA}}$)



Perceptron Learning Algorithm



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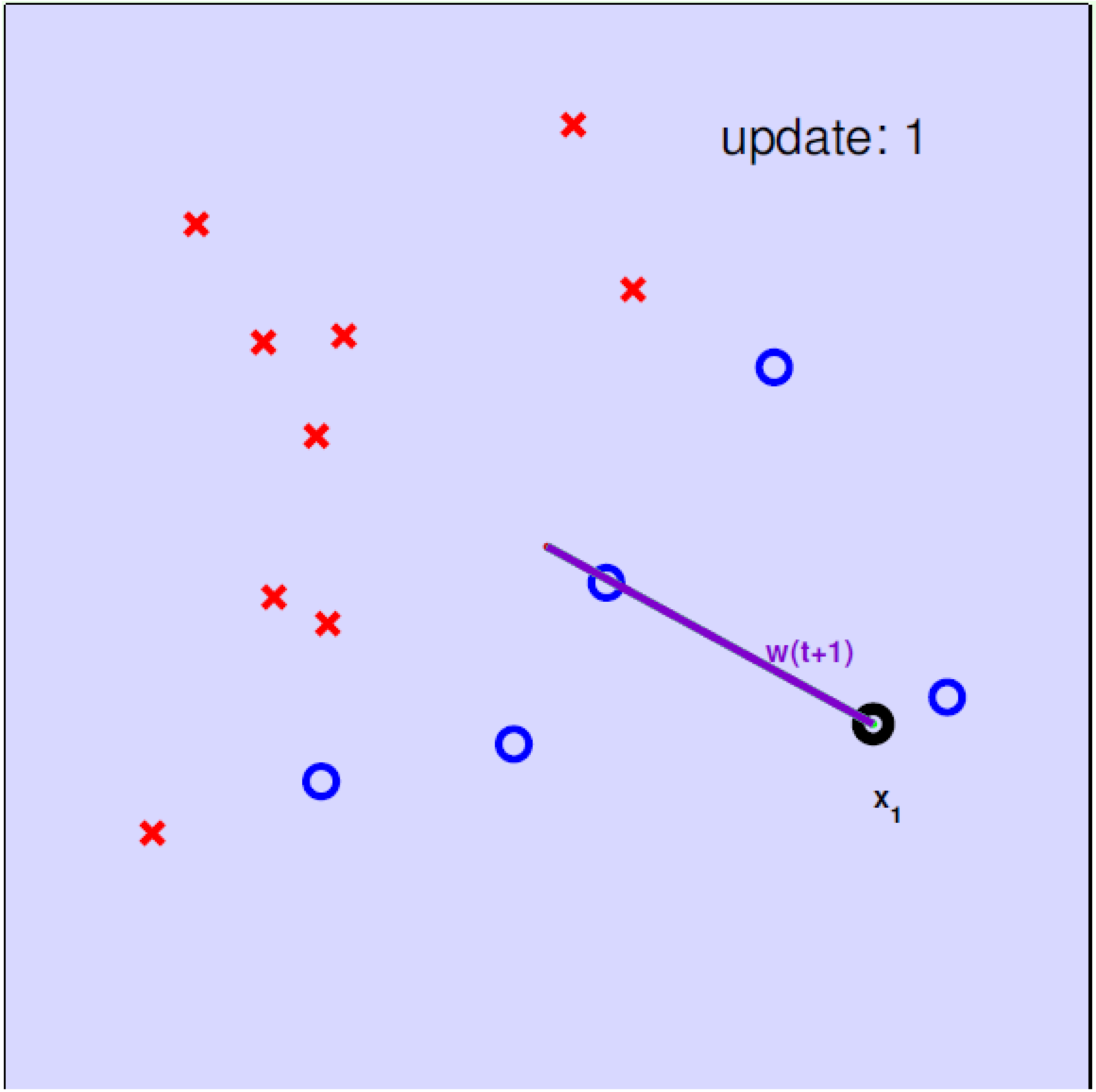

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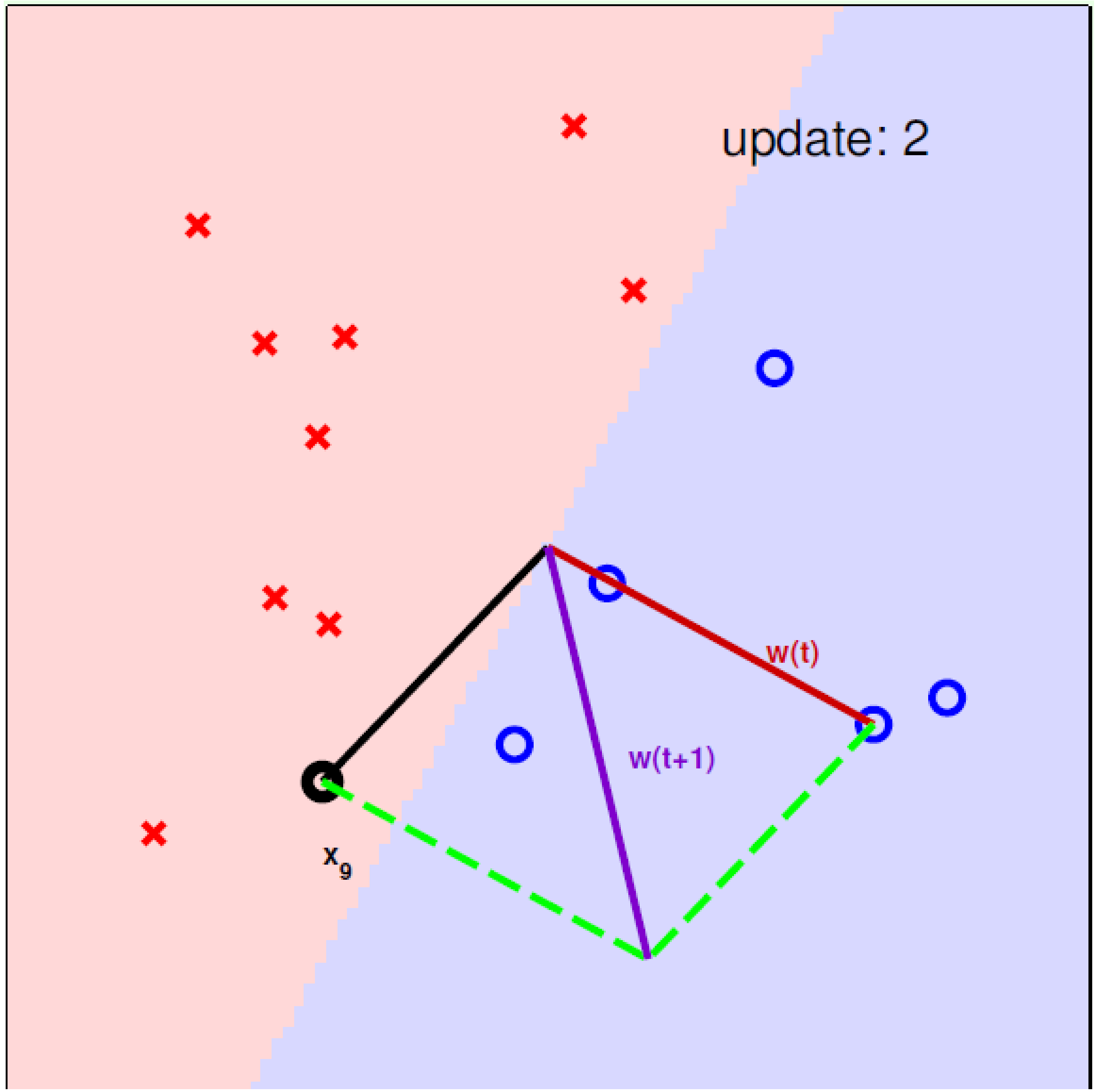

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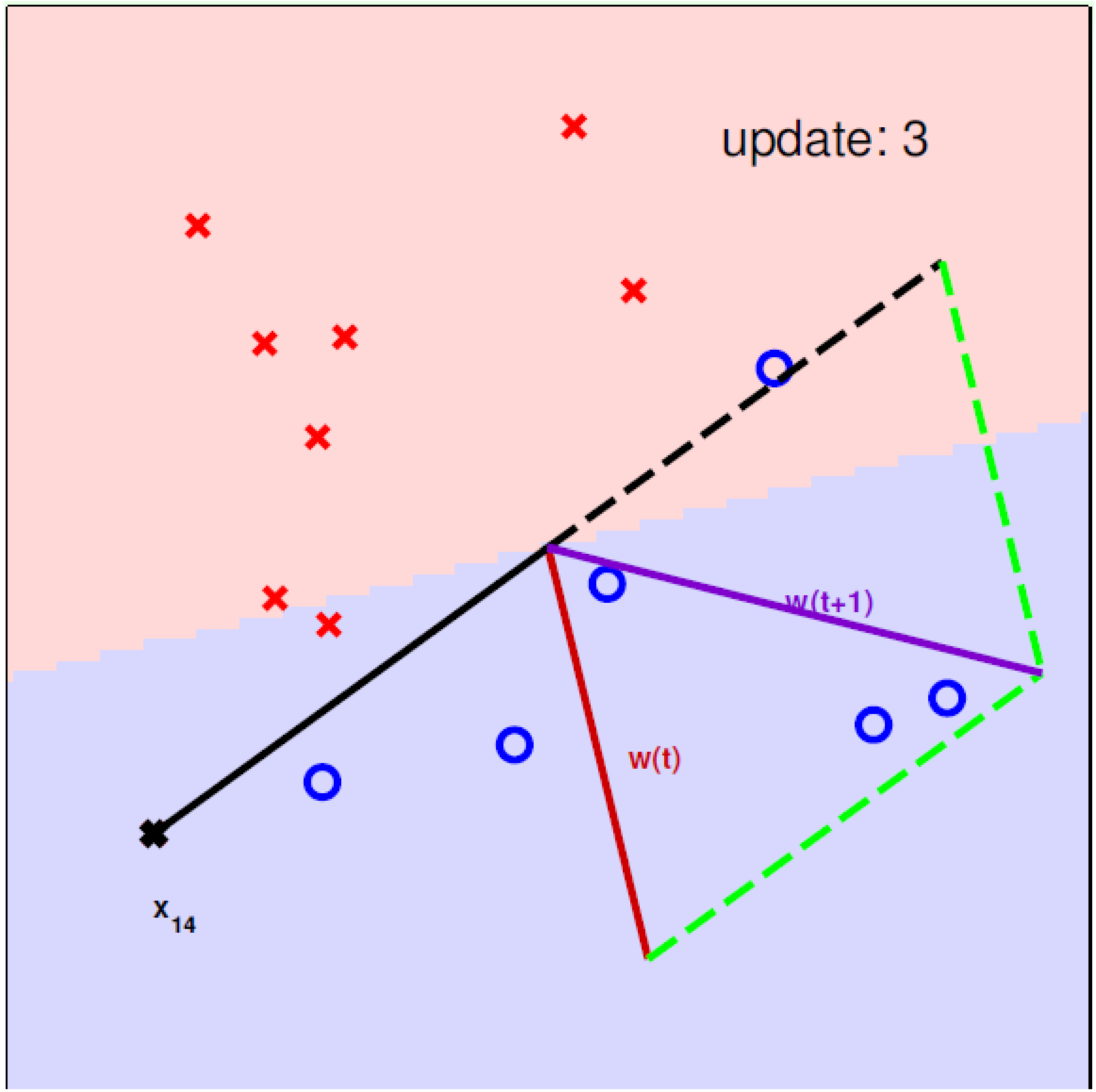

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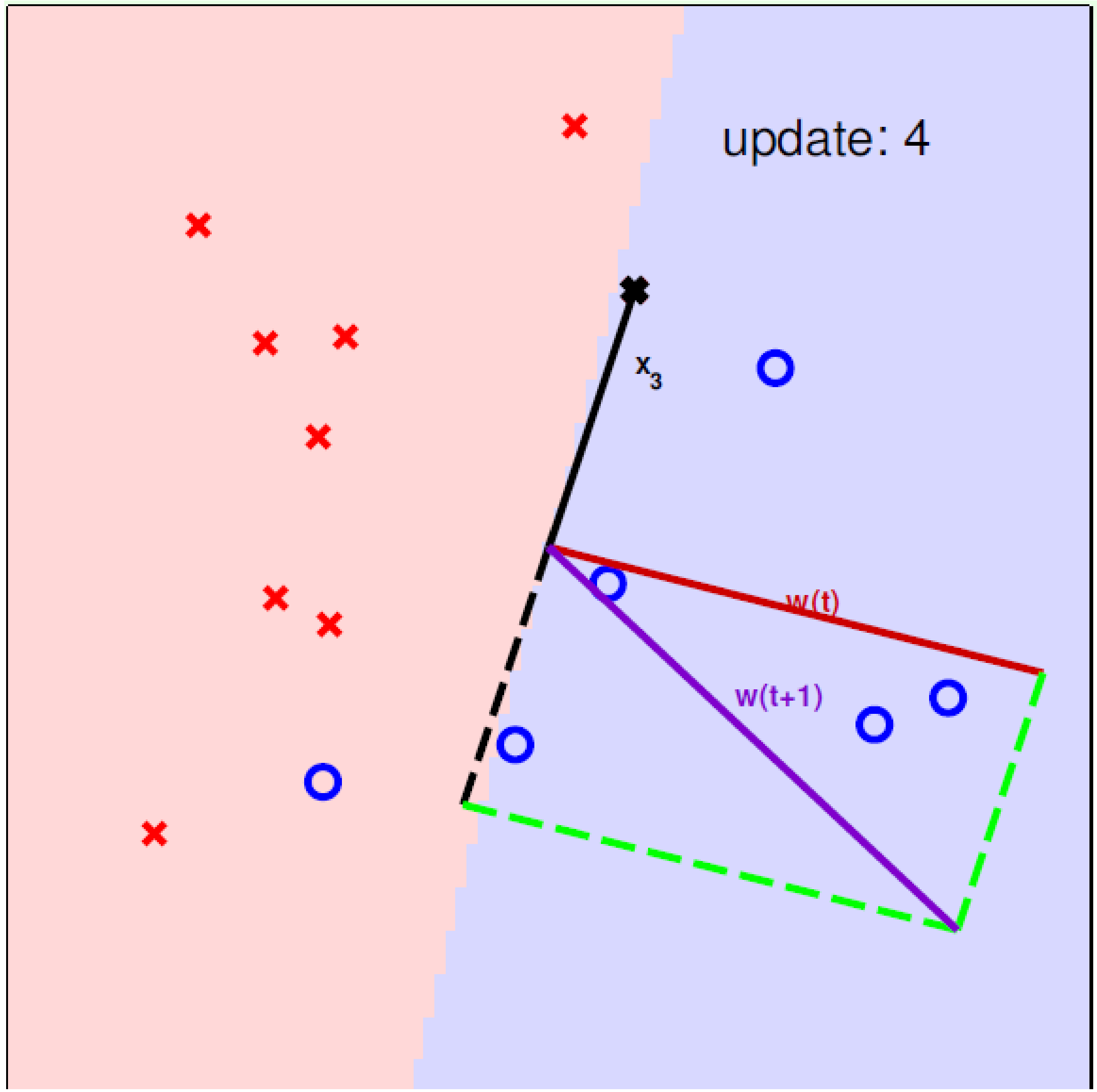

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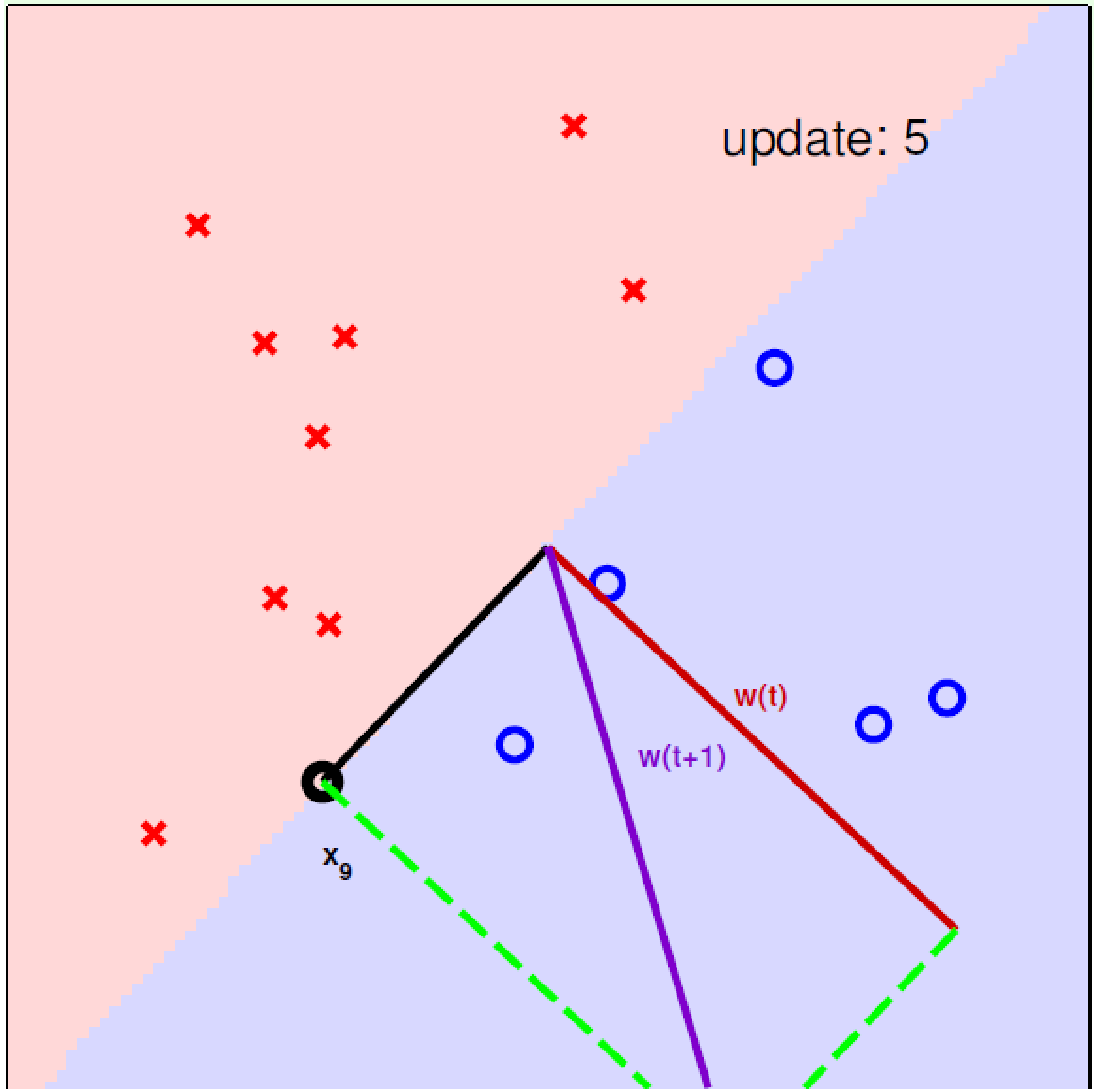

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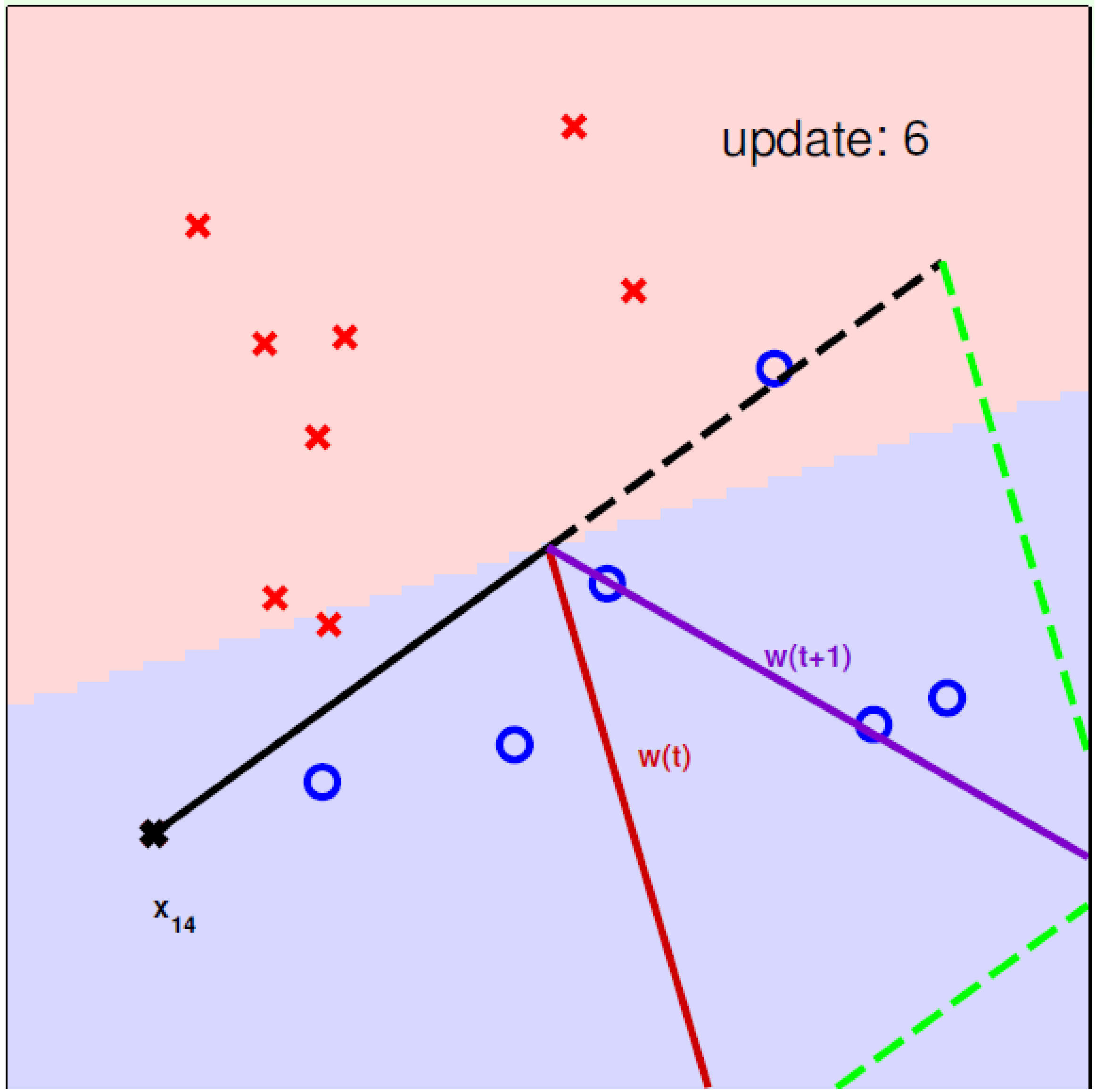

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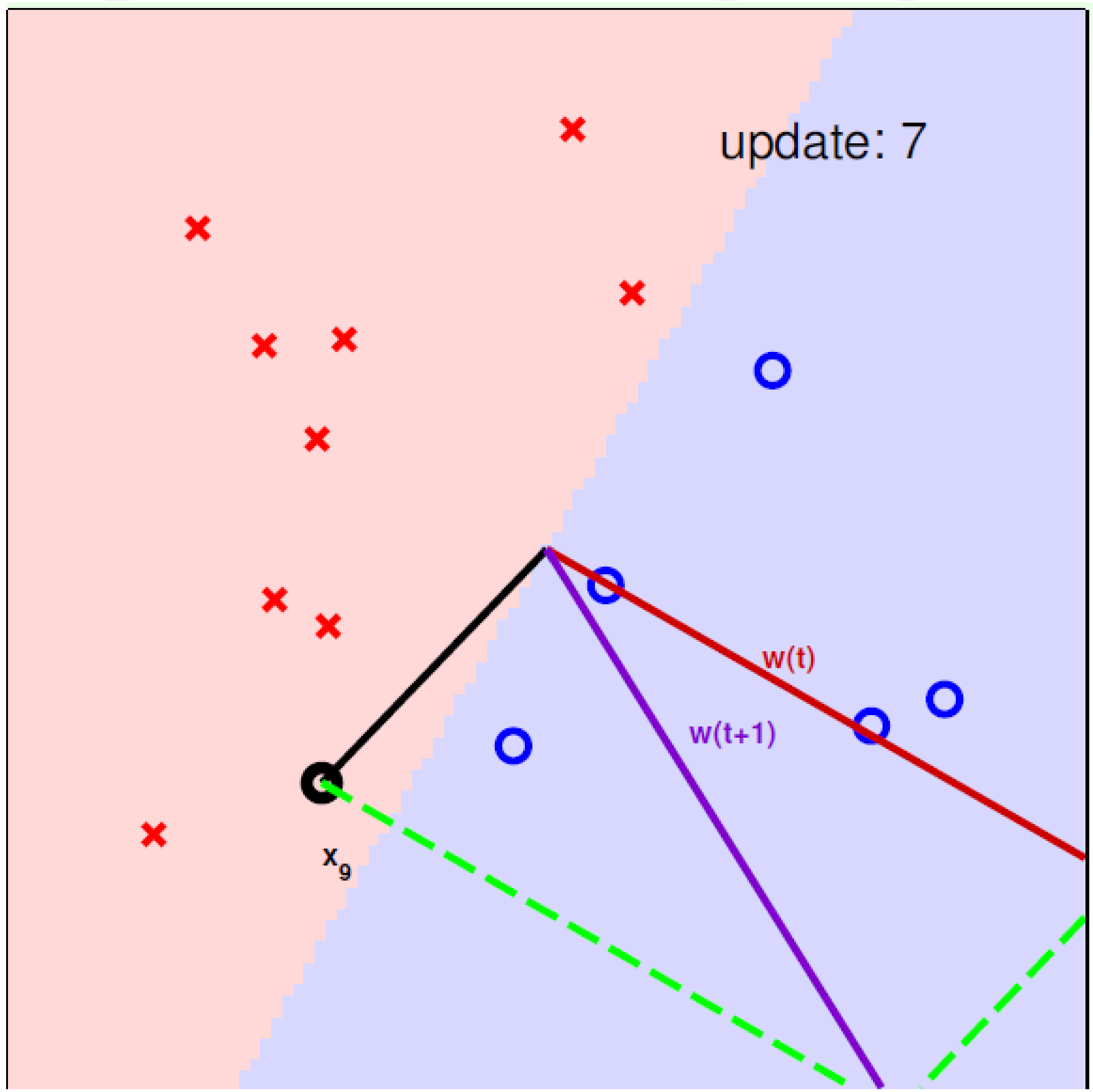

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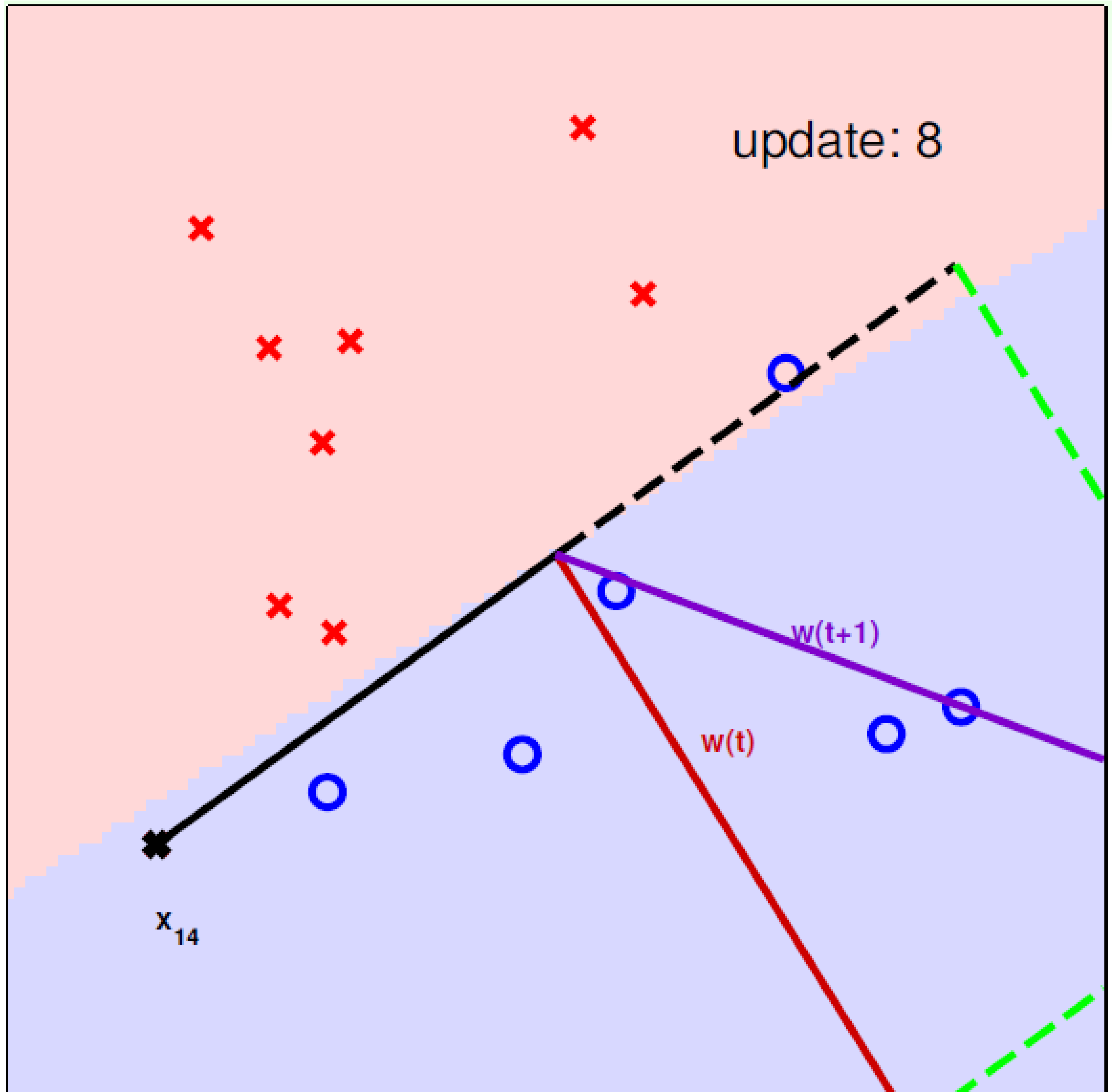

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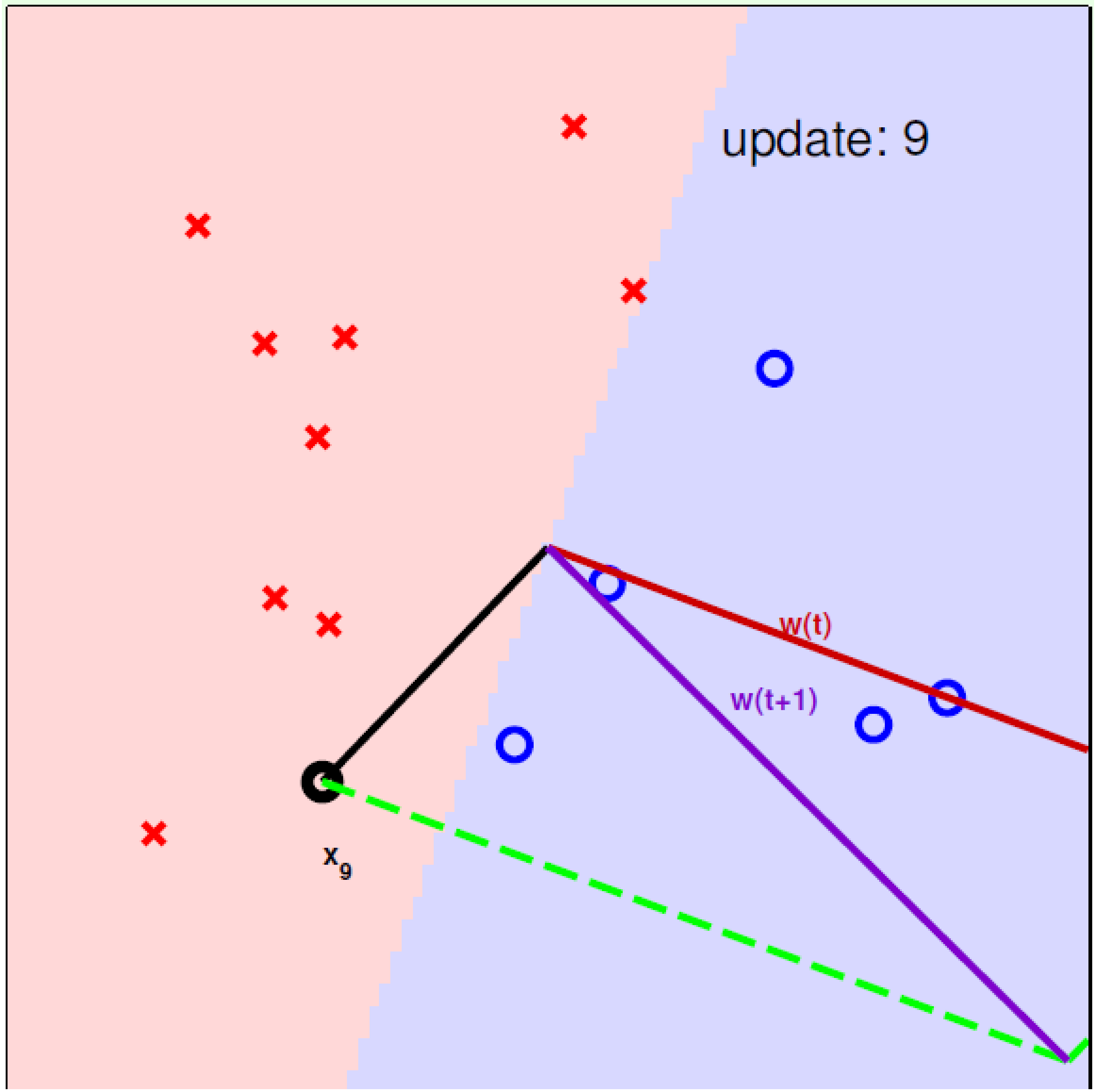

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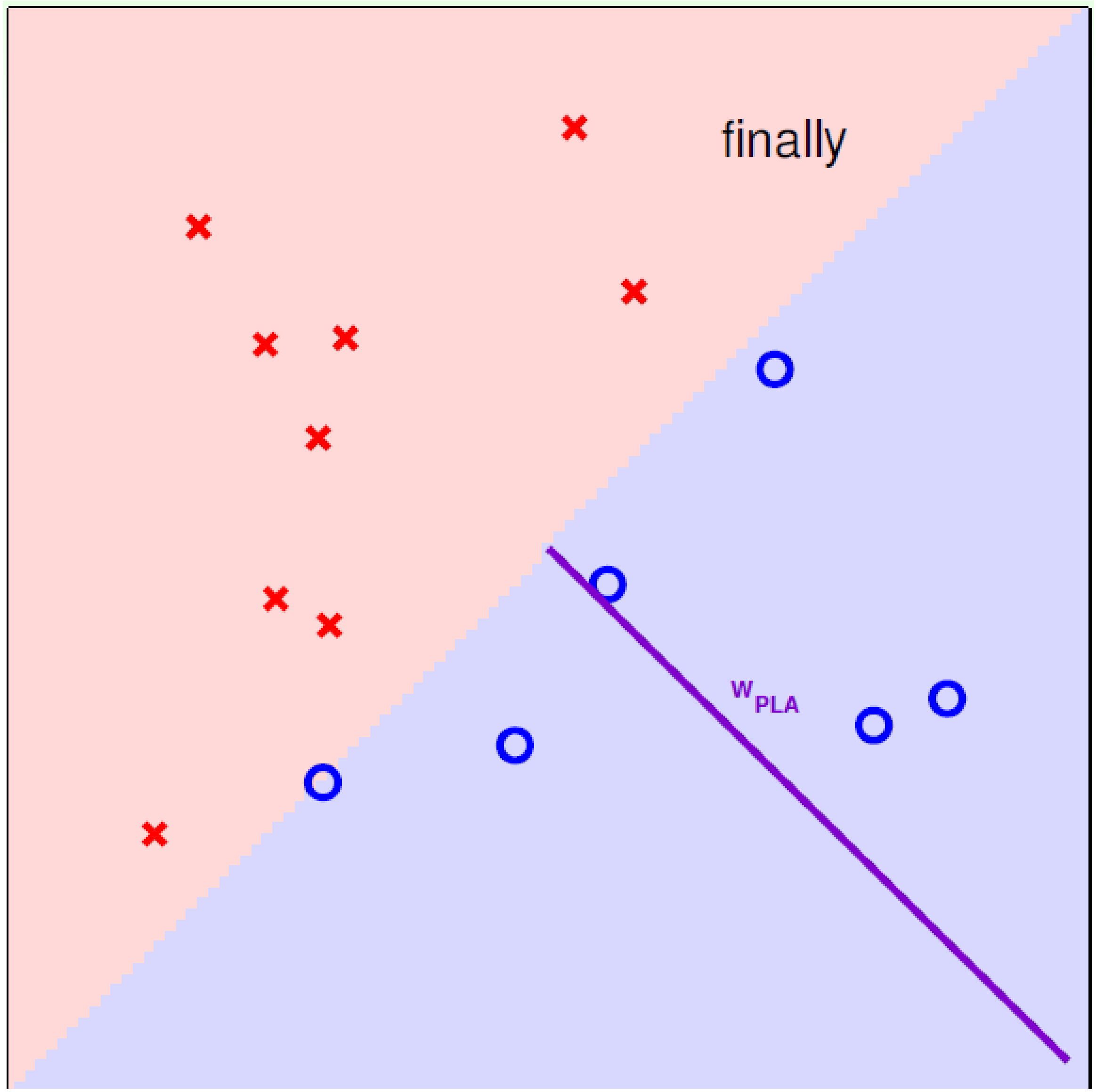

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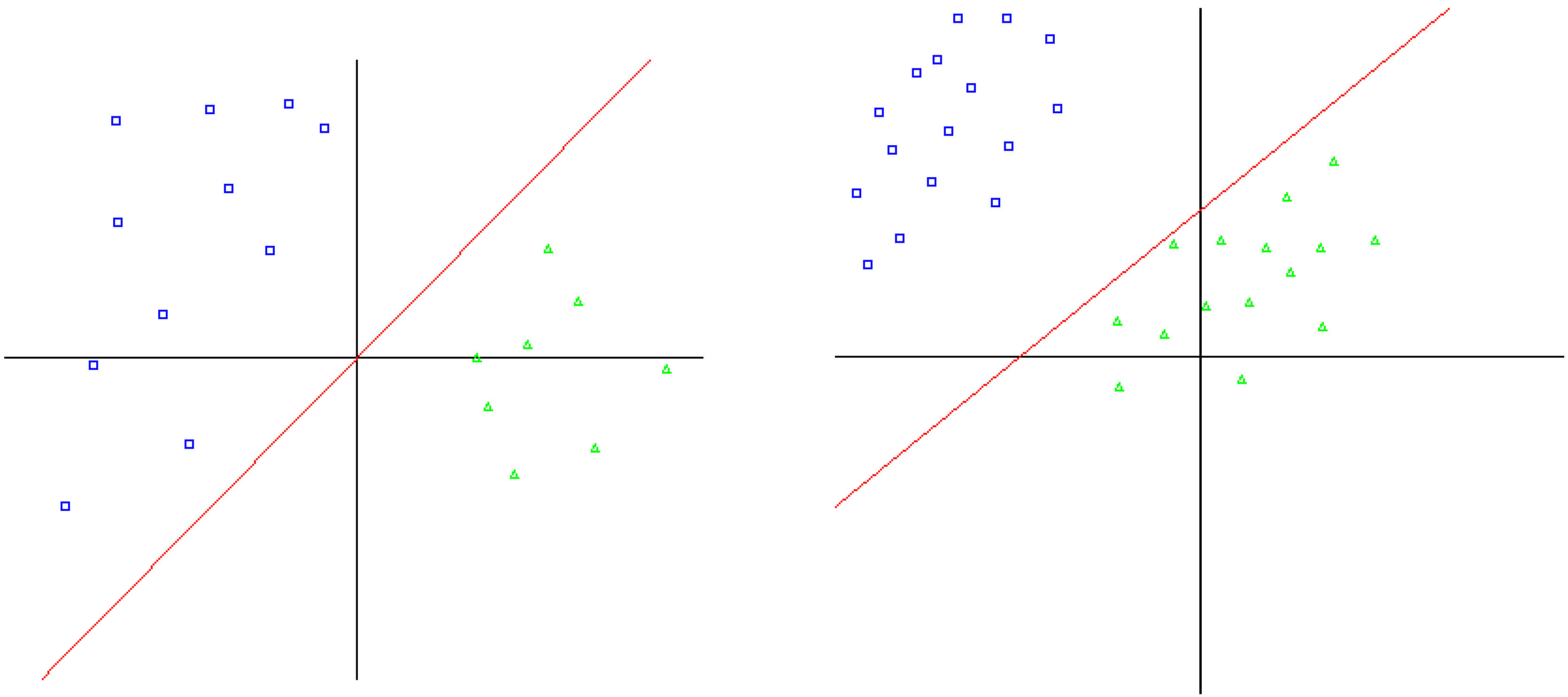

Perceptron Learning Algorithm

- Only if there exists an hyperplane that correctly classifies the data, the Perceptron procedure is guaranteed to converge; furthermore, the algorithm may give different results depending on the order in which the elements are processed, indeed several different solutions exist.

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