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A simulation analysis of the microstructure of double auction markets*

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Abstract

We introduce an order-driven market model with heterogeneous agents trading via a central order matching mechanism. Traders set bids and asks and post market or limit orders according to exogenously fixed rules. We investigate how different trading strategies may affect the dynamics of price, bid–ask spreads, trading volume and volatility. We also analyse how some features of market design, such as tick size and order lifetime, affect market liquidity. The model is able to reproduce many of the complex phenomena observed in real stock markets.

1. Introduction

Improvements in information technology and organization/deregulation of exchanges have led to a growing interest in the way financial markets are structured. Market microstructure is the burgeoning field of finance research that deals with many of the questions that arise in this area. An excellent survey of the current status of this research area is Madhaven (2000). One important set of questions relates to market design issues, that have taken on an increasing importance for the development of new, fully automated trading systems.

To understand the relative merits of one market design over another in the competition for order flow several questions need to be investigated. Markets compete for order flow on several dimensions but it is commonly accepted that the ability to provide greater liquidity, and therefore cheaper trading, will be a major factor in determining a successful market design.

In quote-driven systems, competing market makers supply liquidity by quoting bid and ask prices and the number of shares at which they are willing to trade. Investors demand liquidity through the submission of market orders. In order-driven markets investors can, but are not obliged to, submit limit orders. Orders are stored in the exchange's book and executed in the sequence they arrive at the market. A transaction occurs when a trader hits the quote on the opposite side of the market. Transactions are executed using time priority at a given price and price priority across prices. Thus an electronic trading mechanism is comparable to a continuous auction system with automatic order matching and anonymous traders interacting via computer screens.

Limit orders trade on the opposite side of the market. If the market is rising, the upward price movements trigger limit orders to sell; if the market is falling, the downward movements trigger limit orders to buy. Limit orders thus provide liquidity and immediacy to the market and therefore are frequently characterized as competing with the market maker, the main difference being that specialists provide liquidity and adjust their quotes while market orders arrive. Limit orders, on the contrary, represent precommitments to provide liquidity to market orders which will arrive in the future. In this respect, as pointed out by Glosten (1994), limit orders are analogous to short option positions with time to maturity equal to the

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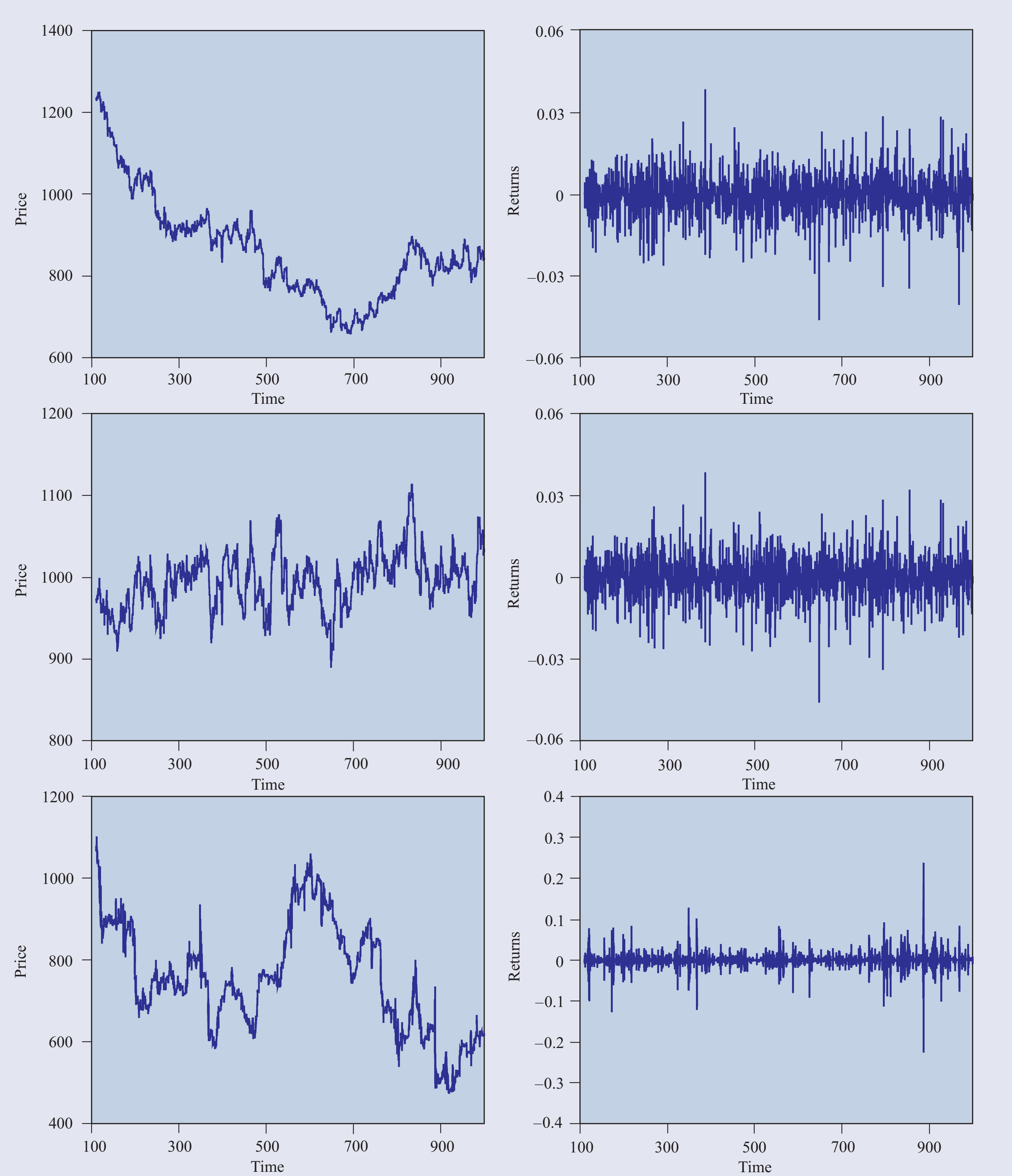

Figure 1. Prices (left) and returns (right) when (top) only noise traders participate in the markets, $\sigma_1 = \sigma_2 = 0$; (middle) fundamentalists and noise traders participate in the markets, $\sigma_1 = 5$, $\sigma_2 = 0$; (bottom) fundamentalists, chartists and noise traders participate in the market, $\sigma_1 = 1$, $\sigma_2 = 1.4$. The other parameters are, for all cases, $n = 3$, $\lambda = 0.5$, $k_{max} = 0.5$.

orders' lifetime. For this reason it is claimed that in order-driven exchanges, the price discovery process, particularly when prices change very quickly, may be delayed by the withdrawal of old quotes while new quotes are submitted.

The pros and cons of hybrid specialist/limit order markets and pure limit order markets have been extensively discussed in the economics literature (Seppi 1997, Martens 1998, Bloomfield and O'Hara 2000, Parlour and Seppi 2001).

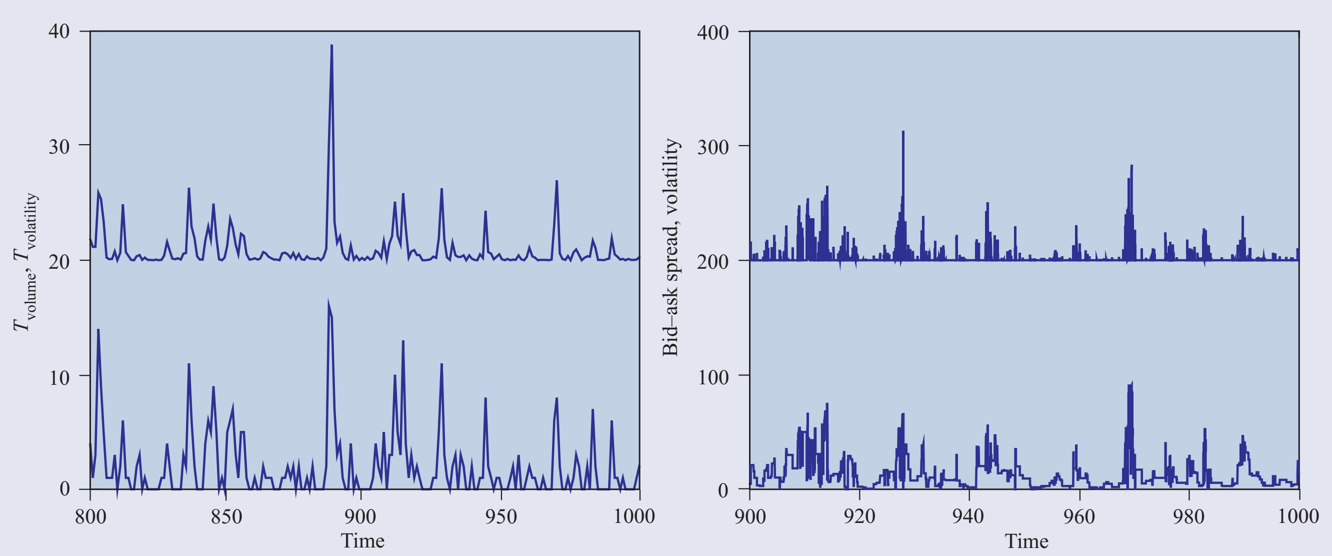

Figure 2. For the same parameters as in figure 1(c) we plot (a) volatility (top) and daily trading volume (bottom); (b) volatility (top) and bid–ask spread (bottom).

Empirical studies, see e.g. Ellul (2001), seem to indicate that different types of markets provide better liquidity on particular trade sizes, with large investors getting better deals in dealer markets and small investors in auction markets. Multiple exchanges, thus, could co-exist in equilibrium with different types of investor preferring different types of trading system. Nonetheless electronic trading employing a public limit order book is continuing to gain a greater share of worldwide security trading and many of the major exchanges in the world rely, at least in part, upon limit orders for the provision of liquidity. Therefore it is important to understand how the placement of limit orders contributes to liquidity and to the price formation mechanism.

Many of the existing theoretical models of market microstructure of necessity considerably simplify the complexity of the trading process in order to remain mathematically tractable. The aim of this paper is to introduce a modelling framework for the analysis of an order-driven market based on numerical simulations. The model is here kept very simple in order to display its essential structure. Nevertheless we are able to employ it to investigate how the price discovery process is affected by factors such as the tick size, market liquidity and the average life of a market order. However, because the model is not shackled by the simplifications imposed by mathematical tractability, it has the ability to be considerably embellished in subsequent research to allow a study of the effect of a number of market protocols, such as rules to halt trading, circuit breakers and trade-by-trade price continuity requirements, to name but a few that are currently of interest.

2. The model of an order-driven market

Our model dispenses with the central market maker role and limit orders are matched automatically. Orders arrive on the market sequentially and at random times and have a finite lifetime τ.

Here we abstract from one of the traditional informational issues considered in the market microstructure literature and assume that agents know the fundamental value p^f of the asset, which we take to be constant. They also know the past history of prices. At any time t the price is given by the price at which a transaction occurs, if any. If no new transaction occurs, a proxy for the price is given by the average of the quoted ask a_t^q (the lowest ask listed in the book) and the quoted bid b_t^q (the highest bid listed in the book): $p_t = (a_t^q + b_t^q)/2$ (we call this value the mid-point). If no bids or asks are listed in the book a proxy of the price is given by the previous traded or quoted price. Bids, asks and prices need to be positive and investors can submit limit orders at any price on a prespecified grid, defined by the tick size Δ.

For simplicity we will assume that each agent can only trade one stock at a time. More realistic extensions could be considered by allowing agents to submit larger order sizes, either chosen randomly or determined through a portfolio optimization scheme which would take into account the risk aversion profile of agents. Nonetheless empirical evidence (Bias *et al* 1995, Ellul 2001) suggests that limit order trading is preferred by small investors while large investors obtain better prices on the floor. In fact generally limit order books do not contain enough large size orders. Hence, quoted spreads for large orders in order-driven markets are high compared to the one quoted in dealer-driven markets. In light of this observation, the restriction on order volumes in our model is not too unrealistic.

The demands of traders are assumed to consist of three components, a fundamentalist component, a chartist component and a noise-induced component. At any time t a trader is chosen, with a given probability λ, to enter the market. The chosen agent, using both the fundamental value and chartist rules, makes an expectation about the spot return, $\hat{r}^i_{t,t+\tau}$, that will prevail in the interval $(t, t+\tau)$ during which

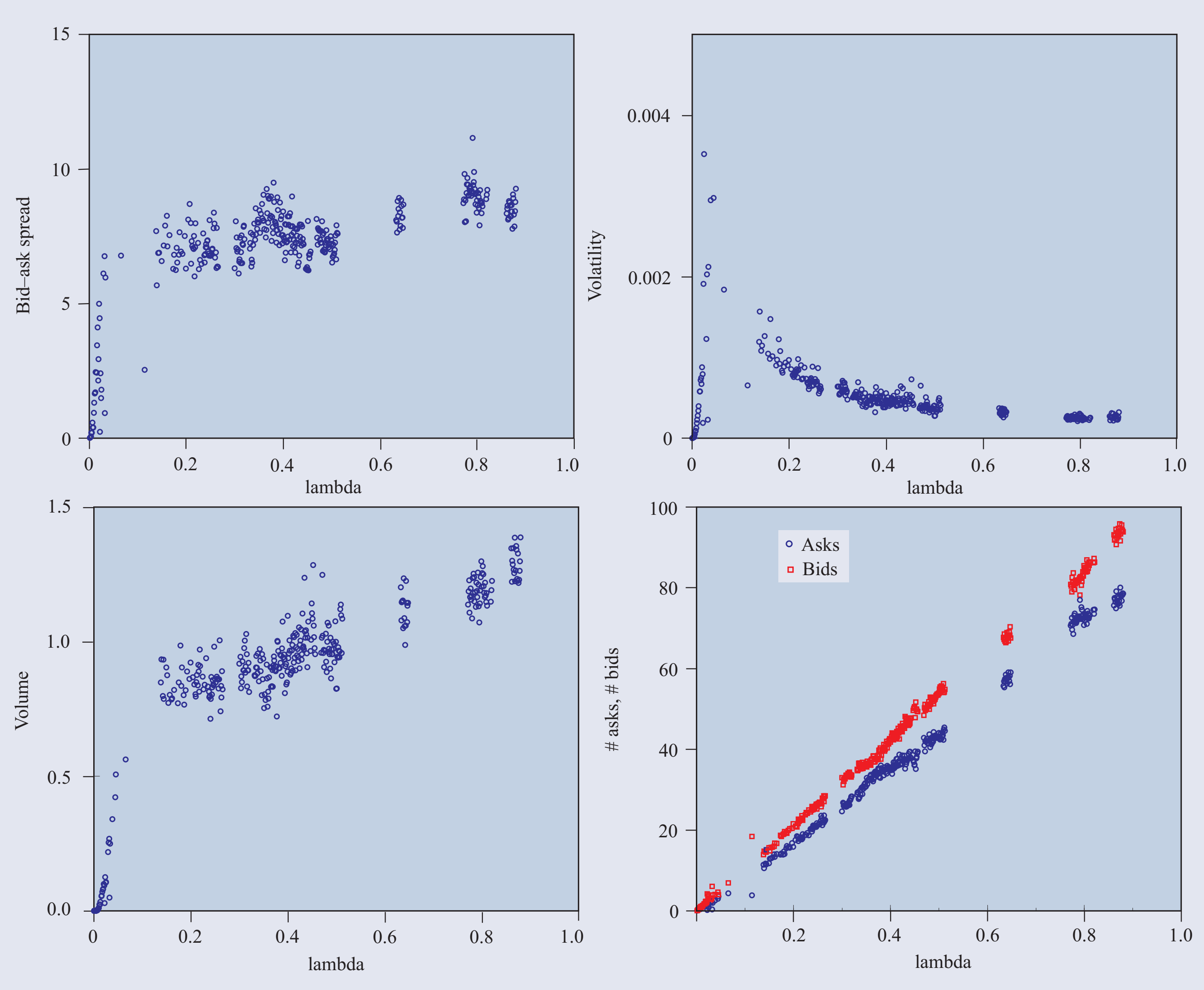

Figure 3. Bid–ask spread (a), volatility (b), trading volume (c) and number of bid and ask orders in the book (d) as a function of λ.

his order will be active

$$\hat{r}^i_{t,t+\tau} = g^i_1 \frac{(p^f - p_t)}{p_t} + g^i_2 \bar{r}_{L_i} + n_i \epsilon_t. \tag{1}$$

The quantities $g^i_1 > 0$ and g^i_2 represent the weights given to the fundamentalist and chartist component respectively. The sign of g^i_2 indicates a trend chasing (>0) or contrarian (<0) chartist strategy. Since the degree of fundamentalism and chartism will be spread across agents we model these parameters as random variables independently chosen for each agent with $g^i_1 \sim |N(0, \sigma_1)|$, $g^i_2 \sim N(0, \sigma_2)$, $n_i \sim N(0, n_0)$ and $\epsilon \sim N(0, 1)$. A trader for whom $g^i_1 = g^i_2 = 0$ is a noise trader. The quantity $\bar{r}^i_L$ is the spot return averaged over the interval L_i and the L_i are uniformly and independently distributed across agents, over the interval $(1, L_{max})$, thus

$$\bar{r}_{L_i} = \frac{1}{L_i} \sum_{j=1}^{L_i} \frac{p_{t-j} - p_{t-j-1}}{p_{t-j-1}}. \tag{2}$$

The future price expected, at time $t+\tau$, by agent i is given by

$$\hat{p}^i_{t+\tau} = p_t e^{\hat{r}^i_{t,t+\tau}\tau}. \tag{3}$$

If the agent expects a price increase (decrease) he decides to buy (sell) one unit of the stock. We assume that the agent is willing to buy (sell) at a price b^i_t (a^i_t) lower (higher) than his expected future price $\hat{p}^i_{t+\tau}$:

$$b^i_t = \hat{p}^i_{t+\tau}(1 - k^i) \tag{4}$$

$$a^i_t = \hat{p}^i_{t+\tau}(1 + k^i) \tag{5}$$

where the k^i are uniformly distributed in the interval $(0, k_{max})$ with $k_{max} \leqslant 1$. If b^i_t (a^i_t) is smaller (larger) than the current quoted ask (bid) the trader submits a limit order at b^i_t (a^i_t) while if b^i_t (a^i_t) is larger (smaller) than or equal to the current quoted ask (bid) the trader submits a market order and trade at a^q_t (b^q_t). At the end of the period τ unmatched orders are removed from the book.

We study our model of the order-driven market by simulations, focusing in particular on how the dynamics of

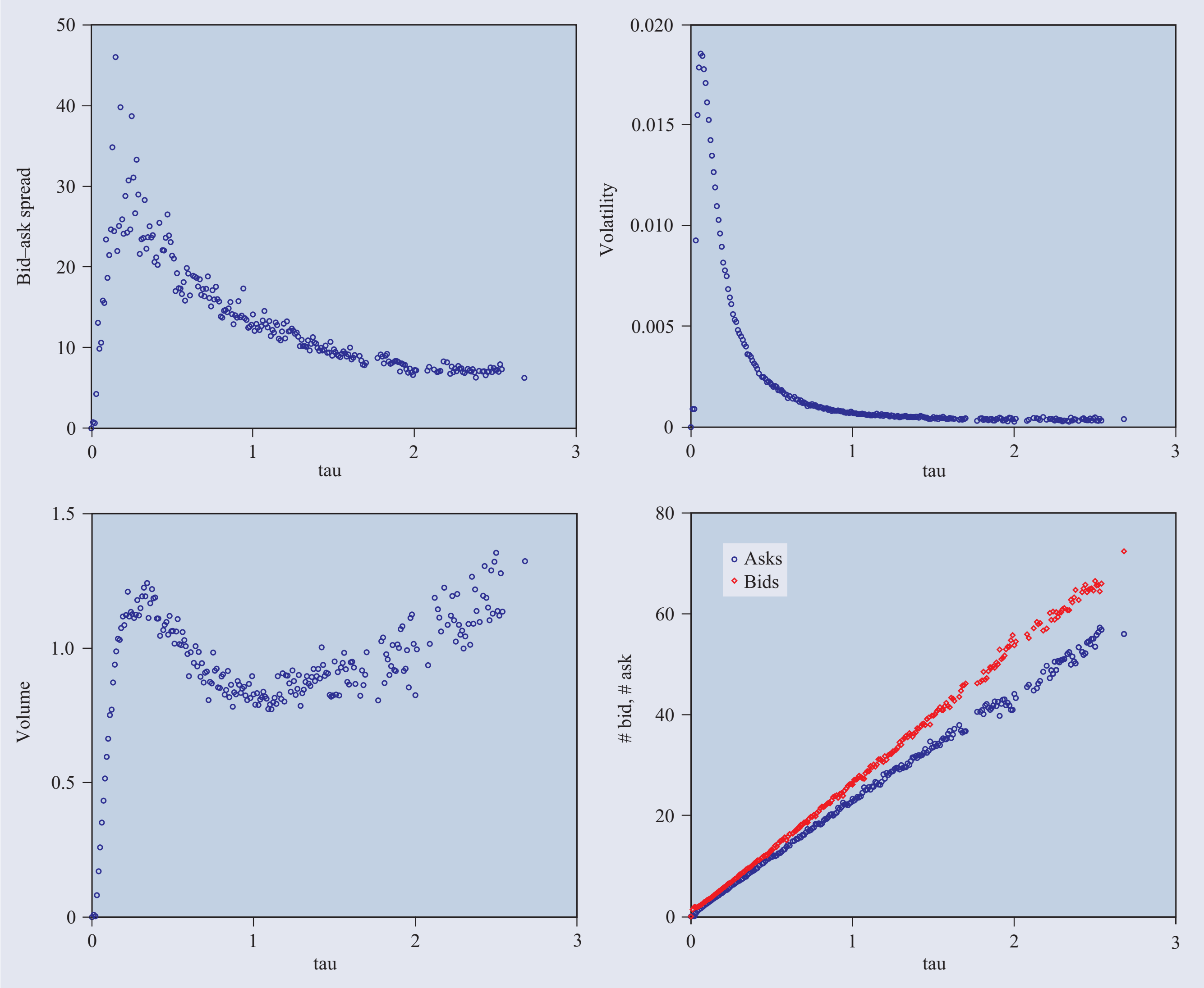

Figure 4. Bid–ask spread (a), volatility (b), trading volume (c) and number of bid and ask orders in the book (d) as a function of τ.

prices and spreads is affected by liquidity (λ), tick size (Δ) and the average life of an order (τ). We assume that the k^i are fixed exogenously. In analogy with the search-bargaining literature (Gale 2000), we would expect that the probability of entering the market λ and the distribution of the k factors, which reflect the liquidity need of agents and their market power, can affect significantly the properties of prices and spreads.

3. Simulations and results

We consider a market of 1000 traders. In the numerical simulations we take as a time unit $T = 100$ time steps and fix $L_{max} = T$. Initially we choose $\Delta = 0.01$ and $\tau = 2T$. The fundamental price is $p_f = 1000$ for all the reported simulations. Parameters which vary across the reported simulations are identified separately. The economic significance of these parameter values will be discussed below.

We start by analysing the impact on the market of the three different trading strategies: noise traders, fundamentalists and chartists. We have tried several different values for the parameters σ_1, σ_2 and n_0 and the results remain qualitatively the same.

In figure 1(a) (top) we plot prices (left) and returns (right) when only noise traders are active in the markets, i.e. $\sigma_1 = \sigma_2 = 0$; in figure 1(b) (middle) fundamentalists are added to the market, i.e. $\sigma_1 = 5$, $\sigma_2 = 0$; in figure 1(c) both fundamentalists and chartists participate in the market, i.e. $\sigma_1 = 1$, $\sigma_2 = 1.4$. The other parameters are, for all cases, $n_0 = 3$, $\lambda = 0.5$, $k_{max} = 0.5$.

In figure 1(b) we chose a value of σ_1 five times greater than that of figure 1(c) to stress the effect of purely fundamentalist strategies in the market. In this case we observe that the volatility of returns remains unrealistically low at around 1% and there is no extreme value beyond about 4%.

These results indicate that all three trader types are necessary in some form to generate realistic looking return dynamics.

Such a result is known from simple theoretical models of financial market dynamics, e.g. Lux and Marchesi (1999), Brock and Hommes (1998) and Chiarella and He (2001).

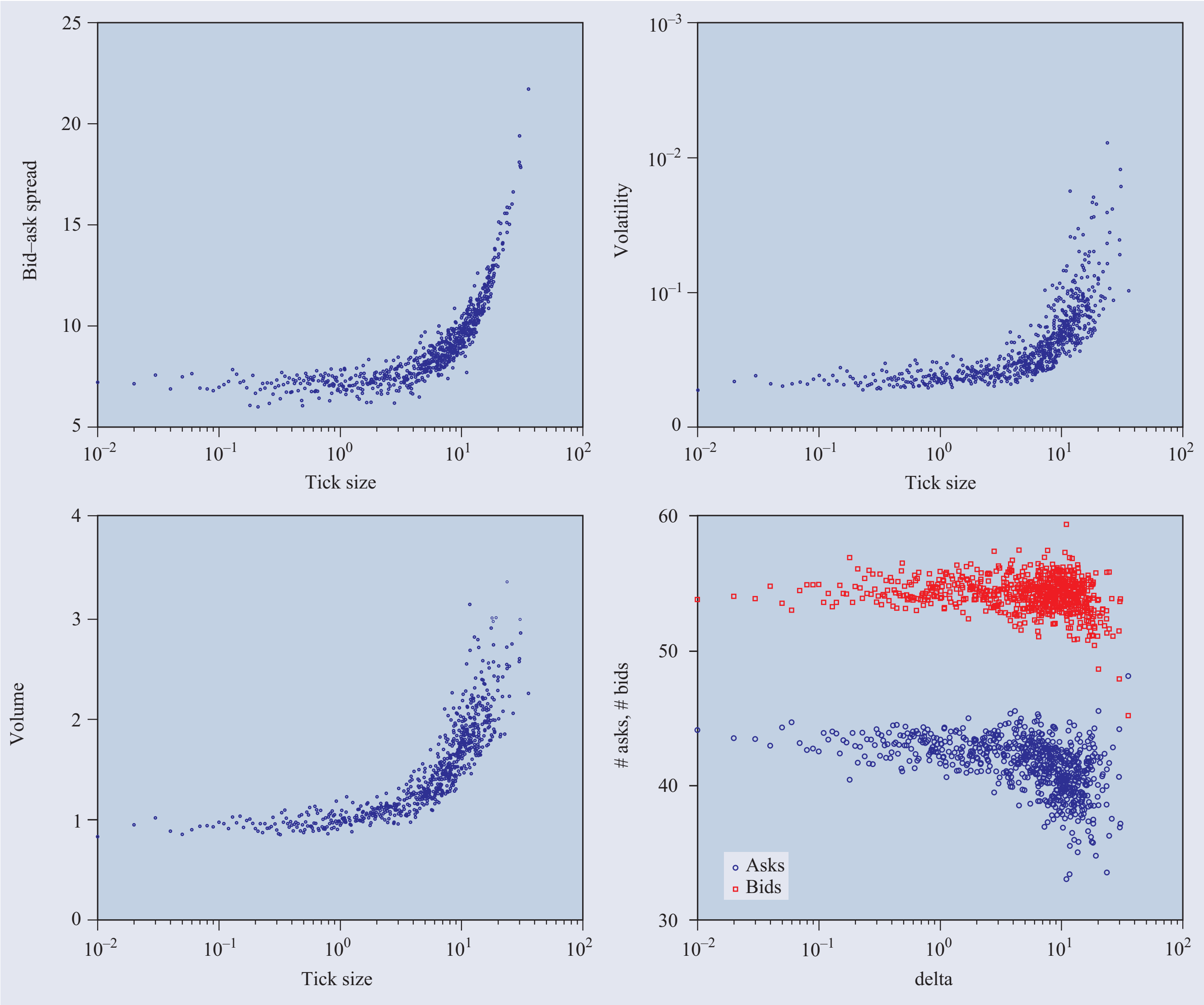

Figure 5. Bid–ask spread (a), volatility (b), trading volume (c) and number of bid and ask orders in the book (d) as a function of Δ.

Our results reinforce these theoretical insights in a much richer (and more realistic) economic environment. The effect of the fundamentalist behaviour is to stabilize the market by reducing the amplitude of price excursions, whereas the chartist behaviour has the opposite effect, very clearly, of generating large price jumps and also clustering in the volatility.

In the following we fix the parameters as in figure 1(c) and analyse the order flow dynamics as discussed at the end of the previous section.

In figure 2(a) we compare volatility and trading volume. Given that at any time step the amount traded can only be zero or one, we calculate the overall trading volume over a period T,

$$V^T = \sum_{t=1}^{T} V(t)$$

and compare it with the average T-volatility,

$$\sigma^T = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{p_t - p_{t-1}}{p_{t-1}}\right|\sqrt{T}.$$

In figure 2(a) we can see that both volatility (top) and daily volume (bottom) are persistent and positively cross-correlated (volatility has been rescaled and shifted to facilitate the comparison). This suggest that chartists' rules favour a higher submission of marketable orders during periods of high volatility.

In figure 2(b) we compare volatility (top) and bid–ask spread (bottom). Volatility has been rescaled and shifted to facilitate the comparison. We observe that also these two quantities are positively cross-correlated. This is a consequence of the increased amount of trading which depletes the book around the mid-point, so widening the spread.

We can interpret the length of T in real time units. For our choice of the parameters we find that the average spot volatility is $\sim 2 \times 10^{-4}$ which gives a value of $\sigma^T \sim 2 \times 10^{-3}$. If we choose an annual volatility of traded assets around 10% we find that 2×10^{-3} is the value of the volatility per hour. So we can identify our time unit T as an hour and the numerical time step t as about half a minute. In these units the life of limit orders is about 2 h and the average trading volume per hour is $V^T \sim 1$. Also notice that the correlations in the volatility extend over several hours.

In figure 3 we plot bid–ask spread (a), volatility (b), trading volume (c) and number of bid and ask orders in the book (d) as a function of λ, i.e. the rate of arrival of limit and market orders in the market. We observe an abrupt change of behaviour at $\lambda_c \sim 0.15$. Below this value (very illiquid markets) the model is very sensitive to changes in λ. Spread, volatility and trading volume increase very rapidly with λ. Increasing λ above λ_c, the spread, while still showing a linear dependence on λ, becomes much less sensitive to changes in this parameter. A similar behaviour is observed in the trading volume. The number of orders stored in the book also increases linearly with λ.

This suggests that, while the market becomes more liquid, more limit orders are stored in the book and more market orders are executed. Nonetheless the rate at which orders are matched close to the mid-point is balanced by the rate at which new limit orders fill the gap between the bid and the ask so that the average bid–ask spread is only little affected by the change in λ. The volatility, which was fast increasing in the illiquid market regime, decreases for $\lambda > \lambda_c$ and appears to saturate to a non-zero value as λ approaches 1. This agrees with the empirical observation that more liquid markets are generally less volatile.

In figure 4 we plot bid–ask spread (a), volatility (b), trading volume (c) and number of bid and ask orders in the book (d) as a function of the orders' lifetime τ.

Longer τ implies that orders stay longer in the book. The first three quantities show a maximum around $\tau_c \sim 0.3$. At values of $\tau < \tau_c$ we observe that prices, volume and volatility decrease to zero when τ decreases. This may be explained by the fact that if the life span of orders is very short then the book is often empty, in which case the bid–ask spread is zero. Furthermore, because of lack of trading the volatility is also very small in this regime.

At large τ volatility and spread decrease and volume increases. To explain this behaviour imagine that the price path shows a positive trend. Chartists will forecast a price increase in the following period and post limit orders (both to sell and to buy) at higher prices than the current level. It is new buy orders and the old sell orders (posted when prices were lower) which accumulate around the mid-point with the effect of reducing the spread and the price volatility. Furthermore, if agents are willing to buy at a high price, it is likely that they will submit market orders to match the old orders to sell (which were posted at a relatively low price). The effect is then that of an increase in the volume of trading.

Finally we investigated the sensitivity to the tick size Δ (figure 5). We find that for values of Δ up to $\Delta \sim 1$ (which in our simulations corresponds to 0.1% of the initial price) all the analysed quantities are independent of this parameter. At $\Delta < 1$ spread and volatility are larger than the tick size and hence independent of it. For larger Δ, spread, volume and volatility increase while the number of orders stored in the book decreases. This is because if prices are allowed only in multiples of Δ the spread and the smallest price change, and hence the volatility, will increase linearly with Δ. On the other hand a larger Δ causes more orders to collapse into the same price tick and overlapping between bids and asks becomes more likely. The overall effect is that trading volume increases and the number of orders stored in the book decreases.

4. Conclusion

We have constructed a model of an order-driven market that captures a number of real world factors, in particular

(i) a spread of trader behaviour, fundamentalist, chartist and noise traders, suggested by empirical studies of a number of markets, and

(ii) parameters representing key determinants of order flow dynamics such as tick size, liquidity and the average lifetime of a market order.

The model has abstracted from important informational issues by assuming that agents have full knowledge of the market fundamental. Our simulations of the model have given us important insights into the determinants of order flow dynamics, in particular tick size, liquidity and average life time of a market order.

The present study has achieved our stated aim of developing an understanding of the essential mechanisms at work in our model of an order-driven market. The model has the advantage of the capability of extensions in a number of directions to tackle issues in market microstructure research from a fresh perspective. Such extensions can complement the studies based on overly simplified theoretical models, empirical studies of one-off events and experimental laboratory studies. The framework presented here offers the prospect of the additional research tool of simulation of theoretical models rich in details of market structure.

Future research with the model presented here could proceed in a number of directions. The assumption that all agents know the fundamental could be relaxed; this would allow a study of the influence of informational issues on order flow dynamics. Whilst much has been written on this issue (see e.g. Madhavan 2000), to our knowledge it has not been studied from the perspective of the framework presented here. More importantly the model could be extended to gauge the effects of market structure and design issues, for instance the formulation of rules governing program trading, the choice of tick size, trade-by-trade price continuity requirements, circuit breakers and rules concerning trading halts. The model needs to be extended to take account of the budget constraints of the heterogeneous investors through consideration of their portfolio optimization decisions. A random order size proportional to the number of stocks owned by the traders has been implemented in Bak *et al* (1997) and Raberto *et al* (2001). For a more detailed approach the frameworks used in Brock and Hommes (1998) or Chiarella and He (2001) can be adapted to this end.

On a broader front the model could be developed further to study important public policy issues such as the imposition of a Tobin type tax to attenuate excessive speculation in certain markets.

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